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# OPTIMAL COST DESIGN OF REINFORCED CONCRETE T-BEAMS WITH STRAIGHT HAUNCHES

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Abstract. *This mathematical research shows an optimal design for reinforced concrete T-section beams with straight haunches to obtain the minimum cost that considers the cost of concrete and the cost of reinforcing steel, and the constraint functions are the geometric properties (rules of current practice) and the equations provided by the ACI (American Concrete Institute) code. This model is shown in function of any type of vertical load on the beam, negative moments at support A and support B, and maximum positive moment. The paper presents a numerical example for seven different cases, and a comparison is made between the T-section beams and the rectangular section beams for the first four cases. The results show that T-beams are cheaper, lighter and have less volume than rectangular beams.*
**Keywords:** Optimal design, Reinforced concrete T-section beams, Minimum cost, Straight haunches, Rectangular section beams

1. **Introduction.** The optimal design of structures has been the subject of many studies in the field of structural design. The goal of a designer is to develop an optimal solution for structural design under certain considerations. An optimal solution usually involves the most economical structure without impairing to the functional purposes that the structure is supposed to serve.

The optimal design is generally considered to be the one that best meets the project criteria. Typically, there is some kind of objective function that can be calculated from the variables that define a design. The value of the objective function is used to compare feasible designs in order to determine the best or optimal design.

In structural engineering, the objective statement can also be put in the form of an objective function. Some statements of typical objectives and their associated objective functions are the least cost concept (minimize cost), the least weight concept (minimize weight) and the least volume concept (minimize volume) of any structural member or the entire structure.

Studies on the topic of optimal design of reinforced concrete T-section beams are as follows. Subramanyam and Adidam used the mathematical programming techniques to minimize the cost of T-beam for continuous floors slabs and simply supported T-beams

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are designed by the limit state [1]. Ferreira et al. presented a work of optimization of the steel area and the steel localization in a reinforced concrete T-beam under bending for singly or doubly reinforced T-section [2]. Jasim et al. studied the minimum cost for a floor slab reinforced concrete T-beam based on an elastic analysis followed by the ultimate strength design method with the consideration of the serviceability constraints according to the ACI (American Concrete Institute) code [3]. Tiliouine and Fedghouche investigated the structural design optimization of reinforced concrete T-section beams under ultimate design loads based on a minimum cost design criterion and a reduced number of design variables [4]. Hawileh et al. presented a numerical investigation by developing a nonlinear FE (Finite Element) model to study the response of a cantilever reinforced concrete T-beam strengthened in shear with side bonded CFRP (Carbon Fiber Reinforced Polymers) fabric strips and subjected to cyclic loading [5]. Panda et al. realized an experimental investigation on the performance of 2.5 m long RC (Reinforced Concrete) T-beams strengthened in shear using epoxy bonded glass fiber fabric [6]. Panda et al. studied the shear strengthening performance of simply supported Reinforced Concrete (RC) T-beams bonded by GFRP (Glass Fiber Reinforced Polymer) strips in different configuration, orientations and transverse steel reinforcement in different spacing [7]. Fedghouche and Tiliouine estimated the minimization optimal cost designs of reinforced concrete T-beams at ultimate design loads considering the costs of concrete, steel and formwork, and design constraints defined in accordance with Eurocode2 [8]. Bekdas and Nigdeli presented a cost optimization of T-shaped reinforced concrete beams under flexural effect for different compressive strengths of concrete using HS (Harmony Search) algorithm [9]. Tiliouine and Fedghouche developed the optimal cost design for high-strength reinforced concrete T-section beams to obtain the economic designs under ultimate limit state conditions, and the cost objective function, behavior constraints that includes the material nonlinearities of steel and high-strength concrete, conditions of strain compatibility in steel and concrete, geometric design constraints using the generalized reduced gradient optimization algorithm [10,11]. Kale et al. proposed a cost optimization approach of reinforced concrete T-beam girder, and the objective function is to minimize the total cost in the design process of the bridge system considering the cost of materials, labor cost and formwork cost according to IRC-21:2000 (Indian Road Congress) standard specifications [12]. Mittal et al. presented the optimal cost design for reinforced concrete T-section beams using GSA (Gravitational Search Algorithm) considering the flexural and shear effects according to the Indian reinforced cement concrete code (IS 456:2000) [13]. Mansour et al. estimated a minimum cost design for reinforced concrete T-section beams according to the Syrian code using the nonlinear programming method provided by Lingo14.0 software [14]. Correia et al. proposed an optimal cost design as the objective function, the dimensions of the T-section as design variables, and the imposition of technical standards as restrictions, the optimization problem is formulated in a Microsoft Excel spreadsheet, and it is solved by the Analytic Solver Platform optimization tool using evolutionary algorithms [15]. Fedghouche presented a method to optimize the design cost of a doubly reinforced HSC (High Strength Concrete) T-beam, the objective function considers the costs of HSC, steel, and formwork, and the constraint functions are adjusted with the design requirements of the Eurocode 2 (EC-2) [16]. Zhou et al. proposed a shear-strengthening of RC (Reinforced Concrete) continuous T-beams with spliced CFRP (Carbon Fiber Reinforced Polymers) U-strips around bars to upgrade shear performance of RC beams, and particularly of the segments under negative moment within continuous T-section beams [17]. Eldin et al. examined the effect of changing the positions of CFRP (Carbon Fiber Reinforced Polymers) laminates used for the strengthening of the hogging moment zone across the beam flange of two-span-T-section beams by the Finite Element (FE) Package ANSYS to create 3-D theoretical models [18]. Fazili and Malhotra formulated a mathematical relationship between the design parameters and cost elements for simply supported reinforced concrete T-beams, and it is compared between the PSO (Particle Swarm Optimization) method and the GWO (Grey Wolf Optimization) method [19]. Imran et al. studied the optimal design of reinforced concrete ribbed slab (waffle slab) according to the Indian reinforced cement concrete code (IS 456:2000), and the objective function is the cost of the ribbed slab, which is the sum of the steel cost, concrete cost and formwork cost [20]. All these documents are for uniform or prismatic T-sections, but no documents present variable or non-prismatic sections.

Other researchers have developed methods to solve the bending vibration equation for a viscoelastic cantilever beam [21], and the evaluating of the efficiency associated with the optimization model [22].

Some works of rectangular cross-section beams with symmetric straight haunches under uniformly distributed load and/or concentrated load are presented to obtain the fixed-end moments factors, the carry-over factors and the stiffness factors (These papers do not take account of shear deformations) [23,24].

Other works of rectangular cross-section beams with straight or parabolic haunches (symmetrical and/or nonsymmetrical) under uniformly distributed load and/or concentrated load are presented to obtain the fixed-end moments factors, the carry-over factors and the stiffness factors (These papers take account of shear and bending deformations) [25-28].

Other works on the topic of optimization for non-prismatic beams are as the following. Luévanos-Rojas et al. proposed the optimal cost design of reinforced concrete beams for rectangular sections with straight haunches that takes account of the minimum cost (concrete cost and reinforcing steel cost) as objective function and the constraint functions consider the equations presented by the code (ACI 318S-14) [29]. García-Canales et al. investigated the minimum cost design for reinforced concrete rectangular beams with parabolic haunches taking account of the concrete cost and reinforcing steel cost, and the constraint functions consider the equations of the code (ACI 318S-19) [30].

Therefore, the review of the state of the art clearly shows that there is no close relationship to the topic for optimal design or minimum cost for T-section reinforced concrete beams with straight haunches, which is discussed in this document.

This paper shows an optimized cost design for reinforced concrete T-section beams with straight haunches under concept of minimum cost that takes account of the cost of concrete and the cost of reinforcing steel, and the constraint functions consider the geometric properties (rules of current practice) and the equations provided by the ACI code (ACI 318S-19). This model can be applied to any type of T-beam, because it is shown in function of any type of vertical load, the negative moments at support A and support B, and the maximum positive moment. The main justification for the use of haunches is that when a rigid frame analysis is developed, the greatest moments occur at the ends of the beams and tend to decrease in the center, reinforcing steel required by end bending (negative moment) is provided at the top extended along the haunches and reinforcing steel in the center (positive moment) is provided at the bottom along the constant section. The paper presents a numerical example under seven different considerations, and a comparison is made between the T-section beams and the rectangular section beams for the first four cases to observe the differences.

The paper is organized as follows. Section 2 describes the main concepts for the optimal solution. Subsection 2.1 shows the formulation of the optimal model for T-section beam with straight haunches. Subsection 2.2 presents the objective function. Subsection 2.3 describes the constraint functions. Section 3 shows the examples of application of model for T-section beam. Section 4 presents the results. Section 5 shows the conclusions to complete the paper.

## 2. Methodology.

A model is the product of an abstraction of a real system: eliminating the complexities and making pertinent assumptions, a mathematical technique is applied and a symbolic representation of the same is obtained.

A mathematical model consists of at least three basic sets of elements.

1) Decision variables and parameters. The decision variables are unknowns that must be determined from the solution of the model. The parameters represent the known values of the system or that can be controlled.

2) Constraints. Constraints are relationships between decision variables and quantities that give meaning to the solution of the problem and limit them to feasible values.

3) Objective function. The objective function is a mathematical relationship between the decision variables, parameters and a quantity that represents the objective or product of the system.

The optimal solution is obtained when the weight, volume or the cost value is minimal for a set of feasible values of the variables.

The objective and constraint can be expressed as follows:

$$Min\,Z = f(x_1, x_2, x_3, \ldots, x_n) \tag{1}$$

$$\text{Subject to } h_j(x) \leq 0, \; j = 1, \ldots, m \tag{2}$$

$$x_i^k \leq x_i \leq x_i^s, \; i = 1, \ldots, n$$

where $x$ is the vector of the design variables, $f(x_1, x_2, x_3, \ldots, x_n)$ is the objective function to minimize, $h_j(x)$ is the vector of the constraint functions, $x_i^k$ and $x_i^s$ are the lower and upper bounds of typical design variable $x_i$.

Figure 1 represents a T-section beam with straight haunches subjected to any type of vertical load, and a moment at each end, and these moments are obtained from the structural analysis.

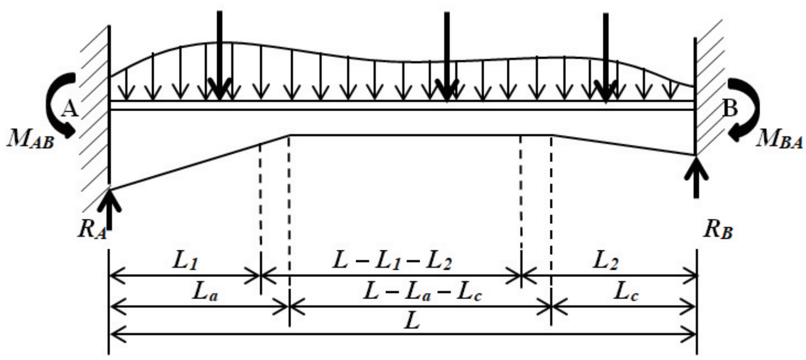


FIGURE 1. T-section beam with straight haunches at the ends under any type of vertical load

The reactions "$R_A$ and $R_B$" in their respective supports are obtained by sum of the moments in one of the supports, and subsequently the other reaction is found by sum of forces. The moment equation "$M_x$" at a distance "$x$" is generated from support A to obtain: The maximum positive moment "$M_{\max}$" is obtained by deriving "$M_x$" with respect to "$x$" ($\mathrm{d}M_x/\mathrm{d}x$), and this equation is made equal to zero to find the position of the maximum moment "$x_{\max}$", and subsequently, "$x_{\max}$" is substituted in the equation "$M_x$" to obtain the maximum positive moment "$M_{\max}$". Distance "$L_1$" that is the inflection point from support A and distance "$L_2$" that is the point of inflection from support B are obtained by equating to zero the equation "$M_x$". "$L_a$" is the horizontal distance of the beam from support A to the constant section (left haunch), and "$L_c$" is the horizontal distance of the beam from support B to the constant section (right haunch).

2.1. **Formulation of the optimal model.** Figure 2 shows the geometry properties of the T-cross section, the strain that is generated, the simplified rectangular stresses block and the separately internal forces acting on the beam.

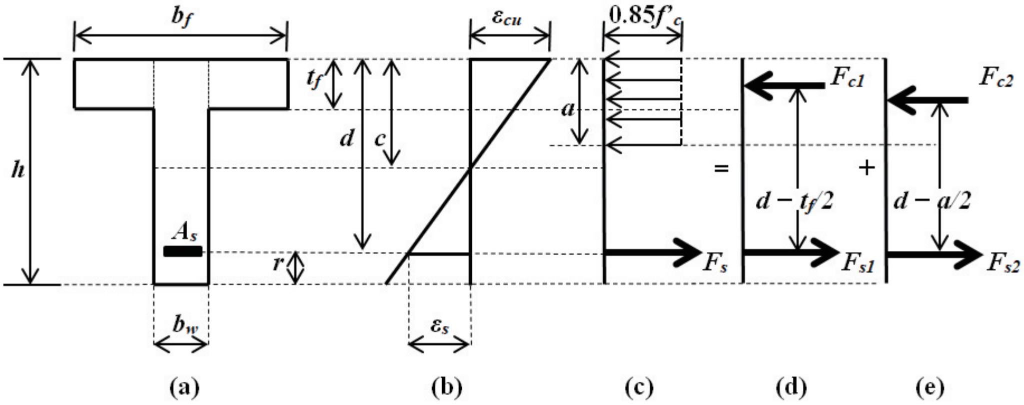


Figure 2. Typical T-cross section beam: (a) Cross section; (b) strain; (c) complete internal forces; (d) forces in the beam due to the flange; (e) forces in the beam due to the web

Forces acting on the T-beam according to Figures 2(d) and 2(e) are

$$F_{c1} = 0.85{f'}_c t_f \left(b_f - b_w\right) \tag{3}$$

$$F_{s1} = A_{sf} f_y \tag{4}$$

$$F_{c2} = 0.85{f'}_c a b_w \tag{5}$$

$$F_{s2} = A_{sw} f_y \tag{6}$$

where $\varepsilon_{cu}$ is concrete unit strain, $\varepsilon_s$ is steel unit strain, $F_{c1}$ is the resultant force of concrete in the beam due to the flange, $F_{s1}$ is the resultant force of steel in the beam due to the flange, $F_{c2}$ is the resultant force of concrete in the beam due to the web, $F_{s2}$ is the resultant force of steel in the beam due to the web, ${f'}_c$ is the specified compressive strength of concrete, $f_y$ is the specified yield strength of steel, $b_w$ is web width, $b_f$ is effective width of compressive flange, $h$ is overall depth or height of beam, $d$ is effective depth, $t_f$ is flange depth, $r$ is the coating, $c$ is the distance from the neutral axis to the furthest fiber in compression, $a = \beta_1 c$, $A_{sf}$ is the steel area due to the flange, $A_{sw}$ is the steel area due to the web, and $A_s$ is the total steel area.

In Equations (3) to (6) above, it is assumed that the neutral axis position is under the beam flange which ensures that the section is behaving as the T-beam shown in Figure 2(a). If the neutral axis position is within the beam flange, the section behaves like a rectangular beam.

Equations for bending moment of T-beams are [31]

$$M_u = \emptyset_f M_n \tag{7}$$

$$M_n = A_{sf} f_y \left(d - \frac{t_f}{2}\right) + A_{sw} f_y \left(d - \frac{a}{2}\right) \tag{8}$$

$$a = \frac{A_{sw} f_y}{0.85 f'_c b_w} \tag{9}$$

$$\rho = \frac{A_s}{b_w d} \tag{10}$$

$$\rho_b = \frac{0.85 f'_c}{f_y} \left[ \frac{t_f (b_f - b_w)}{b_w d} + \frac{600\beta_1}{600 + f_y} \right] \tag{11}$$

$$0.65 \leq \beta_1 = \left( 1.05 - \frac{f'_c}{140} \right) \leq 0.85 \tag{12}$$

$$\rho_{\max} = 0.75 \rho_b \tag{13}$$

$$\rho_{\min} = \begin{cases} \dfrac{0.25\sqrt{f'_c}}{f_y} \\ \dfrac{1.4}{f_y} \end{cases} \tag{14}$$

where $M_n$ is the nominal strength for moment, $M_u$ is the factored moment or ultimate moment at the section considered, $\emptyset_f$ is the strength reduction factor by moment ($\emptyset_f = 0.90$), $\rho_{\min}$ is minimum steel percentage, $\rho_{\max}$ is maximum steel percentage, $\rho_b$ is the balanced relationship between steel and concrete (when both materials fail at the same time), and $\beta_1$ is the factor relating depth of equivalent rectangular compressive stress block to neutral axis depth.

Design variables constraints for geometric properties (rules of current practice) are [31]

$$\frac{L}{30} \leq h \leq \frac{L}{22} \tag{15}$$

$$0.30 \leq \frac{b_w}{d} \leq 0.50 \tag{16}$$

$$\frac{(b_f - b_w)}{2} \leq \frac{L}{10} \tag{17}$$

$$\frac{b_f}{t_f} \leq 8 \tag{18}$$

$$t_f \geq h_{\min} \tag{19}$$

Some specifications of the code mention the following [31]: At least 1/3 of the total tensile reinforcement in the support provided to resist negative moment must have an embedded length beyond the inflection point, not less than $d$, $12d_b$ or $L/16$, whichever is greater.

Figure 3 shows the dimensions and the main reinforcement in general form for the T-beam.

2.2. **Objective function to minimize the cost.** The total cost "$C_t$" is equal to the cost of the flexural reinforcement steel plus the cost of the concrete. These costs include materials and labor. The cost of the T-section beam is

$$C_t = V_c C_c + V_s \gamma_s C_s \tag{20}$$

where $C_c$ is cost of concrete for one cubic meter of ready mix concrete (dollars), $C_s$ is cost of reinforcement steel for one Kilonewtons of steel (dollars), $V_s$ is volume of reinforcing steel (cubic meter), $\gamma_s$ is steel density = 76.94 kN/m$^3$, and $V_c$ is volume of concrete (cubic meter).

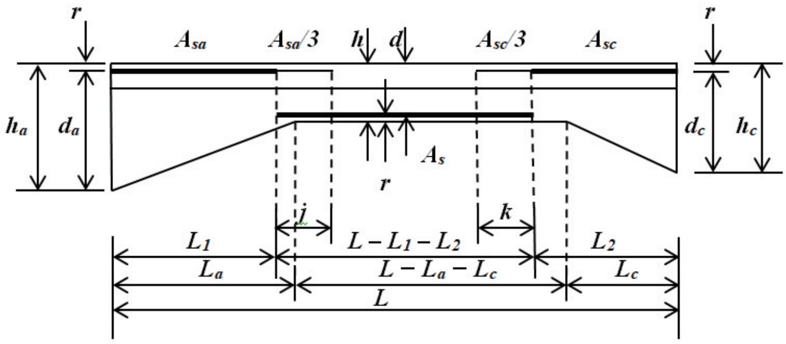


FIGURE 3. Dimensions and the main reinforcement of the T-section beam

The volumes for the T-beam are

$$V_s = A_{swa}\left(L_1 + \frac{j}{3}\right) + A_s(L - L_1 - L_2) + A_{swc}\left(L_2 + \frac{k}{3}\right) \tag{21}$$

$$\begin{aligned} V_c = {} & t_f b_f L + b_w\left[\frac{(h_a - h)L_a}{2} + (h - t_f)L + \frac{(h_c - h)L_c}{2}\right] \\ & - \left[A_{swa}\left(L_1 + \frac{j}{3}\right) + A_s(L - L_1 - L_2) + A_{swc}\left(L_2 + \frac{k}{3}\right)\right] \end{aligned} \tag{22}$$

where $A_{swa}$ is the area of reinforcing steel at the top of the left haunch (support A), $A_s = A_{sf} + A_{sw}$ is the area of reinforcing steel at the bottom of the central part, and $A_{swc}$ is the area of reinforcing steel at the top of the right haunch (support B).

The heights of the T-beam can be expressed as follows: $h_a = d_a + r$, $h = d + r$, $h_c = d_c + r$.

Substituting Equations (21) and (22) into Equation (20) is obtained of

$$\begin{aligned} C_t = {} & \gamma_s C_s\left[A_{swa}\left(L_1 + \frac{j}{3}\right) + (A_{sf} + A_{sw})(L - L_1 - L_2) + A_{swc}\left(L_2 + \frac{k}{3}\right)\right] \\ & + C_c\left\{t_f b_f L + b_w\left[\frac{(h_a - h)L_a}{2} + (h - t_f)L + \frac{(h_c - h)L_c}{2}\right]\right. \\ & \left. - \left[A_{swa}\left(L_1 + \frac{j}{3}\right) + (A_{sf} + A_{sw})(L - L_1 - L_2) + A_{swc}\left(L_2 + \frac{k}{3}\right)\right]\right\} \end{aligned} \tag{23}$$

Now, substituting $h_a = d_a + r$, $h = d + r$, $h_c = d_c + r$, $j = d_a$, $k = d_c$, $\alpha = \gamma_s C_s / C_c \rightarrow \gamma_s C_s = \alpha C_c$ into Equation (23) is obtained of

$$\begin{aligned} C_t = {} & C_c\left\{t_f b_f L + b_w\left[\frac{(d_a - d)L_a}{2} + (d + r - t_f)L + \frac{(d_c - d)L_c}{2}\right]\right. \\ & - \left[A_{swa}\left(L_1 + \frac{d_a}{3}\right) + (A_{sf} + A_{sw})(L - L_1 - L_2)\right. \\ & \left.\left. + A_{swc}\left(L_2 + \frac{d_c}{3}\right)\right](1 - \alpha)\right\} \end{aligned} \tag{24}$$

where $\alpha$ is the relationship between the cost of steel and the cost of concrete.

2.3. **Constraint functions.** The constraint functions to be considered in a general way for the T-beam are

$$\frac{M_{uAB}}{\emptyset_f f_y} = A_{swa}\left(d_a - \frac{a_{swa}}{2}\right) \tag{25}$$

$$0.85{f'}_c a_{swa} b_w = A_{swa} f_y \tag{26}$$

$$\frac{M_{umax}}{\emptyset_f f_y} = A_{sf}\left(d - \frac{t_f}{2}\right) + A_{sw}\left(d - \frac{a_{sw}}{2}\right) \tag{27}$$

$$0.85{f'}_c a_{sw} b_w = A_{sw} f_y \tag{28}$$

$$0.85{f'}_c t_f\left(b_f - b_w\right) = A_{sf} f_y \tag{29}$$

$$\frac{M_{uBA}}{\emptyset_f f_y} = A_{swc}\left(d_c - \frac{a_{swc}}{2}\right) \tag{30}$$

$$0.85{f'}_c a_{swc} b_w = A_{swc} f_y \tag{31}$$

$$\frac{A_{swa}}{b_w d_a} \leq 0.75\left[\frac{0.85\beta_1 {f'}_c}{f_y}\left(\frac{600}{600 + f_y}\right)\right] \tag{32}$$

$$\frac{A_{sw} + A_{sf}}{b_w d} \leq 0.75\left\{\frac{0.85{f'}_c}{f_y}\left[\frac{t_f\left(b_f - b_w\right)}{b_w d} + \frac{600\beta_1}{600 + f_y}\right]\right\} \tag{33}$$

$$\frac{A_{swc}}{b_w d_c} \leq 0.75\left[\frac{0.85\beta_1 {f'}_c}{f_y}\left(\frac{600}{600 + f_y}\right)\right] \tag{34}$$

$$\frac{A_{swa}}{b_w d_a}, \frac{A_{sw} + A_{sf}}{b_w d}, \frac{A_{swc}}{b_w d_c} \geq \begin{cases} \dfrac{0.25\sqrt{{f'}_c}}{f_y} \\ \dfrac{1.4}{f_y} \end{cases} \tag{35}$$

$$d + r \leq \frac{L}{22} \tag{36}$$

$$0.30 \leq \frac{b_w}{d} \tag{37}$$

$$\frac{\left(b_f - b_w\right)}{2} \leq \frac{L}{10} \tag{38}$$

$$b_f \geq b_w \tag{39}$$

$$\frac{b_f}{t_f} \leq 8 \tag{40}$$

$$t_f \geq 0.15 \tag{41}$$

where $M_{uAB}$ is the negative moment at support A, $a_{swa}$ is the depth of the rectangular stress block at support A, $M_{umax}$ is the positive maximum moment in the central part, $a_{sw}$ is the depth of the rectangular stress block in the central part, $M_{uBA}$ is the negative moment at support B, and $a_{swc}$ is the depth of the rectangular stress block at support B.

It is assumed that all variables are non-negative.

This optimal model can be applied to rectangular section beam with straight haunches, setting $b_f = b_w$ in Equation (39), removing Equation (40), and setting $t_f = 0$ in Equation (41).

The volumes of the concrete and reinforced steel are

$$V_t = t_f b_f L + b_w\left[\frac{(d_a - d)L_a}{2} + \left(d + r - t_f\right)L + \frac{(d_c - d)L_c}{2}\right] \tag{42}$$

$$V_c = t_f b_f L + b_w\left[\frac{(d_a - d)L_a}{2} + \left(d + r - t_f\right)L + \frac{(d_c - d)L_c}{2}\right] - \left[A_{swa}\left(L_1 + \frac{d_a}{3}\right) + \left(A_{sf} + A_{sw}\right)\left(L - L_1 - L_2\right) + A_{swc}\left(L_2 + \frac{d_c}{3}\right)\right] \tag{43}$$

$$V_s = A_{swa}\left(L_1 + \frac{d_a}{3}\right) + (A_{sf} + A_{sw})(L - L_1 - L_2) + A_{swc}\left(L_2 + \frac{d_c}{3}\right) \tag{44}$$

The total weight of the T-beam "$W_t$" is obtained as follows:

$$W_t = W_s V_s + W_c V_c \tag{45}$$

where $W_c$ is the weight of the concrete, and $W_s$ is the weight of the reinforcing steel.

## 3. Numerical Experimentation.

Figure 4 shows a T-section reinforced concrete beam for a road bridge with the dead load and the live load to be supported by the beam (loads without factored), and the shear forces and moment diagrams (factored loads or ultimate loads).

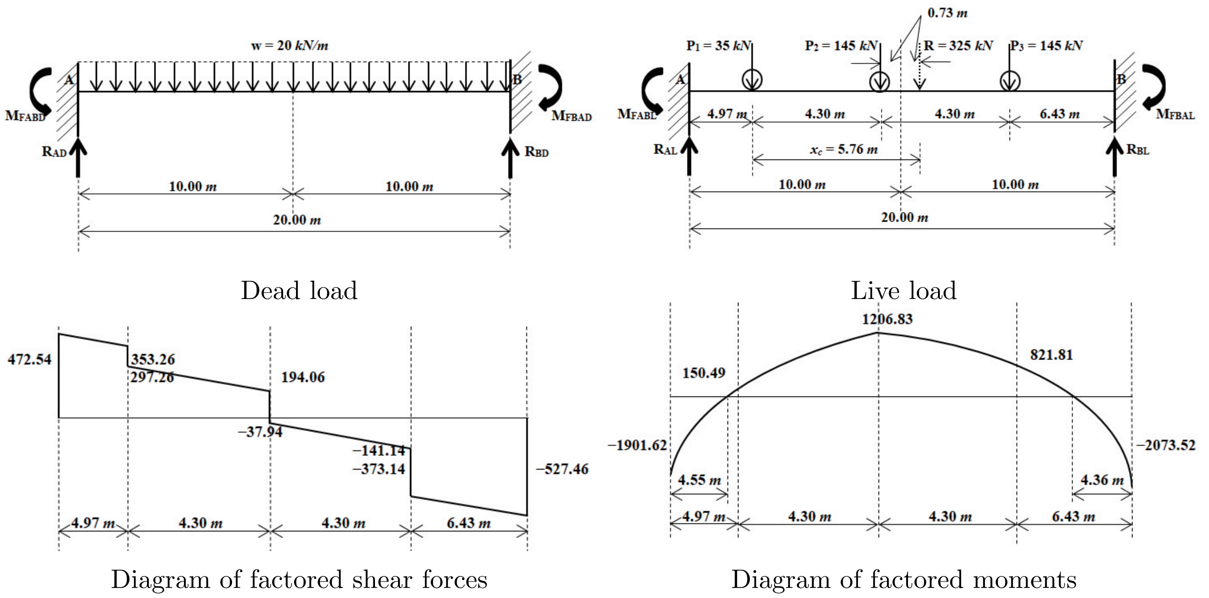


FIGURE 4. T-section reinforced concrete beam for a road bridge

Design of the T-section reinforced concrete beam is shown with the information following: $f'_c = 21$ MPa, $f_y = 420$ MPa (this value is used in Mexico, and it will depend on each country), $r = 8$ cm (by the ACI code), $\alpha = \gamma_s C_s / C_c = (76.94\ \text{kN/m}^3)(124.05\ \text{Dollars/kN}) / (106.04\ \text{Dollars/m}^3) = 90$ (this value depends on the cost of materials in each region or country).

The example is developed for seven different cases: Case 1 considers $L_a = L_1$ and $L_c = L_2$, i.e., the inflection points coincide with the section changes. Case 2 takes account of $d_a = d_c$, $L_a = L_c = 4.36$ m (symmetrical section). Case 3 considers $d_a = d_c$, $L_a = L_c = 4.55$ m (symmetrical section). Case 4 takes account of $d_a = d_c$, $L_a = L_c = L/4 = 5.00$ m (symmetrical section). Case 5 considers $L_a = 3.25$ m and $L_c = 3.00$ m ("$L_a$" is the point where the moment that resists the beam with the same cross section of the central part and the reinforcing steel of the support A, and "$L_c$" is the point where the moment that resists the beam with the same cross section of the central part and the reinforcing steel of the support B). Case 6 is the same as case 5, but the section is symmetric ($d_a = d_c$, $L_a = L_c = 3.25$ m). Case 7 takes into account that the section is constant.

For all cases the following are considered:

1) For positive moments: the compression is located at the top and the tension is located at the bottom of the beam.

2) For negative moments: the compression is located at the bottom and the tension is located at the top of the beam.

Now, substituting this information into Equations (24) to (41) the objective function and the constraint functions are obtained.

These problems assumed that the constant parameters are $M_{uAB}$, $M_{uBA}$, $M_{umax}$, $L_1$, $L_2$, $L_a$, $L_c$, $L$, $f'_c$, $\emptyset_f$, $f_y$, $\alpha$, $r$, and the decision variables are $b_f$, $b_w$, $t_f$, $d_a$, $d$, $d_c$, $a_{swa}$, $a_{sw}$, $a_{swc}$, $A_{swa}$, $A_{sw}$, $A_{sf}$, $A_{swc}$.

The optimal solution is obtained using the MAPLE-15 software. Figure 5 shows the flowchart to obtain the optimal solution of the proposed model.

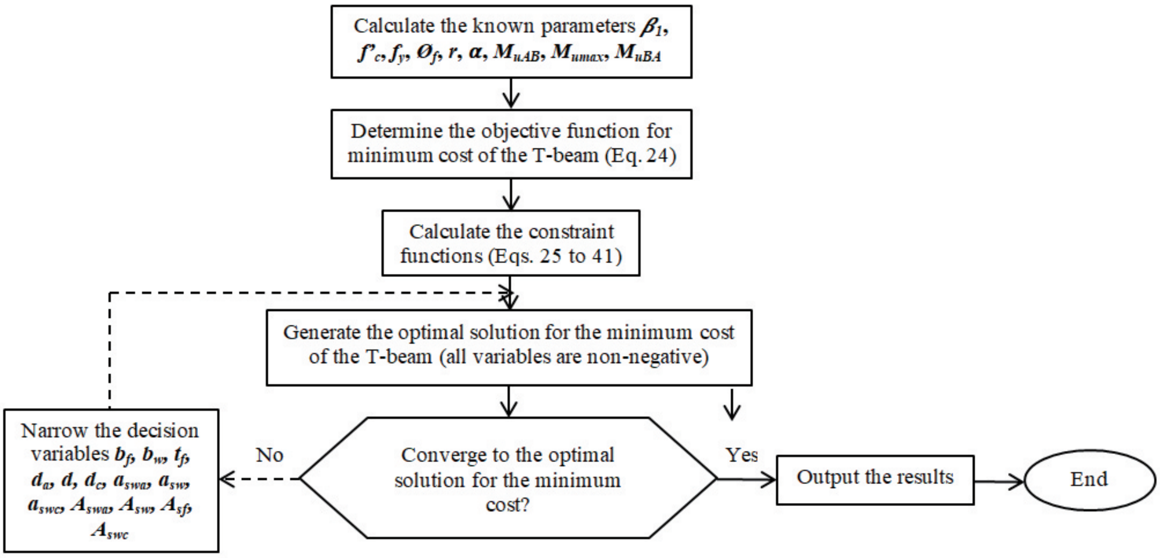


FIGURE 5. Flowchart for the optimal solution of the reinforced concrete T-beam

Table 1 shows the optimal solution for the seven cases.

TABLE 1. Cases 1, 2, 3, 4, 5, 6 and 7

| Concept | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 | Case 6 | Case 7 |
|---|---|---|---|---|---|---|---|
| $L_a$ (m) | 4.55 | 4.36 | 4.55 | 5.00 | 3.25 | 3.25 | 0.00 |
| $L_c$ (m) | 4.36 | 4.36 | 4.55 | 5.00 | 3.00 | 3.25 | 0.00 |
| $b_f$ (cm) | 49.64 | 49.64 | 49.64 | 49.64 | 49.64 | 49.64 | 33.66 |
| $b_w$ (cm) | 24.87 | 24.87 | 24.87 | 24.87 | 24.87 | 24.87 | 33.66 |
| $t_f$ (cm) | 15.00 | 15.00 | 15.00 | 15.00 | 15.00 | 15.00 | 0.00 |
| $d_a$ (cm) | 209.58 | 216.19 | 212.42 | 204.43 | 241.68 | 244.81 | 112.19 |
| $d$ (cm) | 82.91 | 82.91 | 82.91 | 82.91 | 82.91 | 82.91 | 112.19 |
| $d_c$ (cm) | 218.98 | 216.19 | 212.42 | 204.43 | 256.64 | 244.81 | 112.19 |
| $a_{swa}$ (cm) | 24.09 | 23.27 | 23.73 | 24.78 | 20.57 | 20.28 | 37.67 |
| $a_{sw}$ (cm) | 27.36 | 27.36 | 27.36 | 27.36 | 27.36 | 27.36 | 22.06 |
| $a_{swc}$ (cm) | 25.14 | 25.51 | 26.02 | 27.19 | 21.09 | 22.20 | 42.07 |
| $A_{swa}$ (cm$^2$) | 25.47 | 24.59 | 25.08 | 26.20 | 21.74 | 21.44 | 53.89 |
| $A_{sw}$ (cm$^2$) | 28.92 | 28.92 | 28.92 | 28.92 | 28.92 | 28.92 | 31.56 |
| $A_{sf}$ (cm$^2$) | 15.79 | 15.79 | 15.79 | 15.79 | 15.79 | 15.79 | 0.00 |
| $A_{swc}$ (cm$^2$) | 26.58 | 26.96 | 27.51 | 28.74 | 22.29 | 23.47 | 60.18 |
| $C_t$ ($) | $13.53C_c$ | $13.50C_c$ | $13.56C_c$ | $13.70C_c$ | $13.04C_c$ | $13.09C_c$ | $16.10C_c$ |

4. **Results.** Table 1 shows the seven cases of the numerical example. For the cases 1 to 6, the values of $b_f$, $b_w$, $t_f$, $d$, $a_{sw}$, $A_{sw}$, $A_{sf}$ are constant. For the case 1, the values of $d_a$, $a_{swa}$, and $A_{swa}$ are less than $d_c$, $a_{swc}$, and $A_{swc}$, respectively. For the cases 2 to 4 (symmetrical section, but not in reinforcing steel areas), the values $d_a = d_c$, $a_{swa}$ is less than $a_{swc}$ and $A_{swa}$ is less than $A_{swc}$. For the case 5, the values of $d_a$, $a_{swa}$, and $A_{swa}$ are less than $d_c$, $a_{swc}$, and $A_{swc}$, respectively. For the case 6 (symmetrical section, but not in reinforcing steel areas), the values $d_a = d_c$, $a_{swa}$ is less than $a_{swc}$ and $A_{swa}$ is less than $A_{swc}$. For the case 7, the section is constant and rectangular, but not in reinforcing steel areas, the values $d_a = d = d_c$, $a_{swa}$ is less than $a_{swc}$, $A_{swa}$ is less than $A_{swc}$, $a_{sw}$ is less than $a_{swa}$ and $a_{swc}$, and $A_{sw}$ is less than $A_{swa}$ and $A_{swc}$. $C_t$ lower for all cases occurs in case 5 with a total cost of $13.04C_c$, and considering the symmetric beam for aesthetics, it is shown in case 6 with a total cost of $13.09C_c$.

Now, the optimal model is applied to rectangular section beam with straight haunches, setting $b_f = b_w$ in Equation (39), removing Equation (40), and setting $t_f = 0$ Equation (41).

Table 2 presents the comparison between T-B (T-beams) and R-B (rectangular beams) with straight haunches (optimal solution) for the first four cases of Table 1.

TABLE 2. Comparison between T-B and R-B for the cases 1, 2, 3 and 4

| Concept | Case 1 | | Case 2 | | Case 3 | | Case 4 | |
|---|---|---|---|---|---|---|---|---|
| | T-B | R-B | T-B | R-B | T-B | R-B | T-B | R-B |
| $L_a$ (m) | 4.55 | 4.55 | 4.36 | 4.36 | 4.55 | 4.55 | 5.00 | 5.00 |
| $L_c$ (m) | 4.36 | 4.36 | 4.36 | 4.36 | 4.55 | 4.55 | 5.00 | 5.00 |
| $b_f$ (cm) | 49.64 | 28.10 | 49.64 | 28.10 | 49.64 | 28.10 | 49.64 | 28.10 |
| $b_w$ (cm) | 24.87 | 28.10 | 24.87 | 28.10 | 24.87 | 28.10 | 24.87 | 28.10 |
| $t_f$ (cm) | 15.00 | 0.00 | 15.00 | 0.00 | 15.00 | 0.00 | 15.00 | 0.00 |
| $d_a$ (cm) | 209.58 | 197.09 | 216.19 | 203.25 | 212.42 | 199.73 | 204.43 | 192.23 |
| $d$ (cm) | 82.91 | 93.67 | 82.91 | 93.67 | 82.91 | 93.67 | 82.91 | 93.67 |
| $d_c$ (cm) | 218.98 | 205.93 | 216.19 | 203.25 | 212.42 | 199.73 | 204.43 | 192.23 |
| $a_{swa}$ (cm) | 24.09 | 22.68 | 23.27 | 21.91 | 23.73 | 22.34 | 24.78 | 23.33 |
| $a_{sw}$ (cm) | 27.36 | 35.13 | 27.36 | 35.13 | 27.36 | 35.13 | 27.36 | 35.13 |
| $a_{swc}$ (cm) | 25.14 | 23.66 | 25.51 | 24.02 | 26.02 | 24.50 | 27.19 | 25.60 |
| $A_{swa}$ (cm$^2$) | 25.47 | 27.08 | 24.59 | 26.16 | 25.08 | 26.68 | 26.20 | 27.86 |
| $A_{sw}$(cm$^2$) | 28.92 | 41.95 | 28.92 | 41.95 | 28.92 | 41.95 | 28.92 | 41.95 |
| $A_{sf}$ (cm$^2$) | 15.79 | 0.00 | 15.79 | 0.00 | 15.79 | 0.00 | 15.79 | 0.00 |
| $A_{swc}$ (cm$^2$) | 26.58 | 28.26 | 26.96 | 28.68 | 27.51 | 29.26 | 28.74 | 30.57 |
| $C_t$ ($) | $13.53C_c$ | $13.73C_c$ | $13.50C_c$ | $13.70C_c$ | $13.56C_c$ | $13.76C_c$ | $13.70C_c$ | $13.89C_c$ |

Table 2 shows for all cases the following: $b_f$ is greater and $b_w$ is less for T-beams than rectangular beams, $d_a$ and $d_c$, $a_{swa}$ and $a_{swc}$ are less for rectangular beams than for T-beams, $d$ and $a_{sw}$ are greater for rectangular beams than for T-beams, $A_{swa}$ and $A_{swc}$ are greater for rectangular beams than for T-beams, $A_{sw} + A_{sf}$ is greater for T-beams than for rectangular beams ($A_{sf}$ is equal to zero for rectangular beams), and $C_t$ is less for T-beams than for rectangular beams.

Table 3 shows the volumes of concrete and reinforced steel for T-B (T-beams) for the seven cases of Table 1.

TABLE 3. Volumes of concrete and reinforced steel for T-beams for the seven cases

| Concept | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 | Case 6 | Case 7 |
|---|---|---|---|---|---|---|---|
| $V_t$ (m$^3$) | 6.72 | 6.71 | 6.73 | 6.78 | 6.50 | 6.57 | 8.09 |
| $V_c$ (m$^3$) | 6.64 | 6.63 | 6.65 | 6.70 | 6.43 | 6.50 | 8.00 |
| $V_s$ (cm$^3$) | 76480.62 | 76241.26 | 76712.87 | 77778.82 | 72851.74 | 73236.32 | 90024.31 |

Table 3 clearly shows that the best option is presented in case 5 because it has the lowest volumes (total, concrete and reinforcing steel), and also lowest total cost, and the worst option is presented in case 7 because it has the highest volumes (total, concrete and reinforcing steel), and also highest total cost.

Table 4 shows the comparison between the volumes of the concrete and reinforced steel for T-B (T-beams) and R-B (rectangular beams) for the four cases of Table 2.

TABLE 4. Comparison between the volumes for T-B and R-B

| Concept | Case 1 | | Case 2 | | Case 3 | | Case 4 | |
|---|---|---|---|---|---|---|---|---|
| | T-B | R-B | T-B | R-B | T-B | R-B | T-B | R-B |
| $V_t$ (m$^3$) | 6.72 | 7.06 | 6.71 | 7.06 | 6.73 | 7.07 | 6.78 | 7.10 |
| $V_c$ (m$^3$) | 6.64 | 6.99 | 6.63 | 6.99 | 6.65 | 6.99 | 6.70 | 7.02 |
| $V_s$ (cm$^3$) | 76480.62 | 74884.24 | 76241.26 | 74645.24 | 76712.87 | 75143.61 | 77778.82 | 76271.37 |

Table 4 clearly shows that the T-beams with respect to the R-beams for all cases present the lowest volumes (total and concrete), and highest reinforcing steel volume, and also lowest total cost.

Now, substituting the concrete and reinforcing steel volumes into Equation (45) to obtain the total weight of the T-beam and rectangular beam (concrete weight = 23.54 kN/m$^3$ and reinforcing steel weight = 76.94 kN/m$^3$), these values are shown in Figure 6.

Figure 6 clearly shows that the T-beams present the lowest weight with respect to the rectangular beams for all cases.

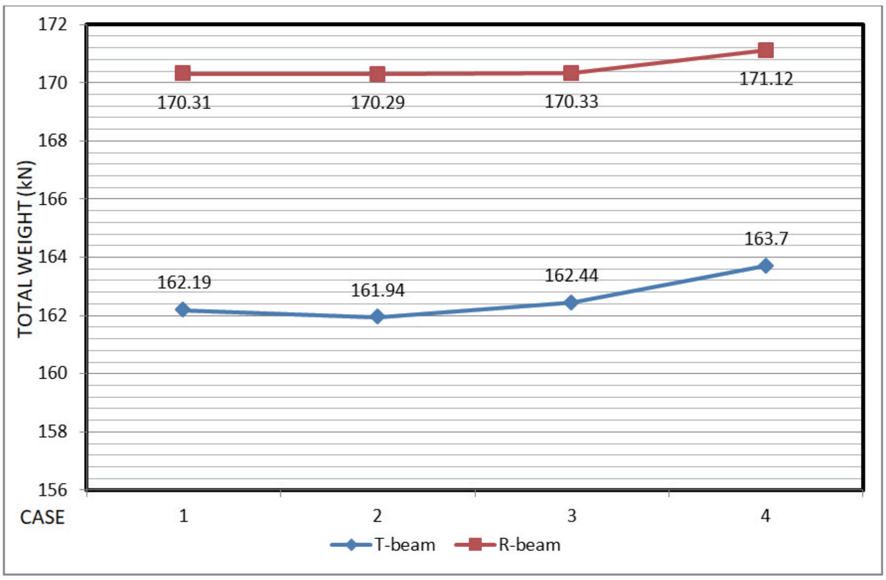


FIGURE 6. Total weight of the T-beam and rectangular beam

5. **Conclusions.** In the present research a model has been presented to obtain the minimum cost for reinforced concrete T-beams subjected to any type of load, because it is shown as a function of the negative moments in support A and support B and the maximum positive moment with straight haunches (symmetrical or non-symmetrical beam).

The proposed optimal model assumes that the constant parameters are $M_{uAB}$, $M_{uBA}$, $M_{umax}$, $L_1$, $L_2$, $L_a$, $L_c$, $L$, $f'_c$, $\emptyset_f$, $f_y$, $\alpha$, $r$, and the decision variables are $b_f$, $b_w$, $t_f$, $d_a$, $d$, $d_c$, $a_{swa}$, $a_{sw}$, $a_{swc}$, $A_{swa}$, $A_{sw}$, $A_{sf}$, $A_{swc}$.

This work shows a practical design of a T-section reinforced concrete beam for a road bridge with the uniformly distributed dead load $w = 20$ kN/m, and the live load for a truck according to AASHTO (American Association of State and Highway Transportation Officials) specifications for bridge design [32].

The main conclusions are as the following.

1) The minimal cost design of reinforced concrete T-beams for a road bridge (practical design) is presented in case 5.

2) The minimal cost design of reinforced concrete T-beams for a road bridge (practical design) for symmetrical section is presented in case 6.

3) The T-beams are more economical with respect to rectangular beams (see Table 2).

4) The T-beams have less volume than the rectangular beams (see Table 4).

5) The T-beams have less weight with respect to rectangular beams (see Figure 6).

The model presented can be adapted to optimize simply supported reinforced concrete T-beams taking account of the critical moments that occur in the intervals of $0 \leq x \leq L/4$ and $3L/4 \leq x \leq L$, to obtain more reduced sections closer to the supports.

The main advantage of the reinforced concrete T-section beams with straight haunches over constant section is that if the reinforced concrete T-section beams with straight haunches are lighter than constant section beams, the structural members that support the beams such as columns and footings can be cheaper and have greater savings in materials.

The suggestions for future research are

1) Optimization of reinforced concrete T-section beams with parabolic haunches (non-prismatic beams).

2) Optimization of reinforced concrete I-section beams with straight and parabolic haunches (non-prismatic beams).

3) Optimization of the complete structure taking into account the beams, slabs, columns and footings.

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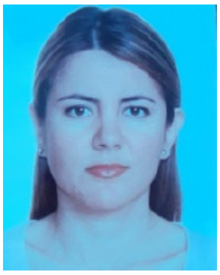

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