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ADAPTIVE DIRECT DATA DRIVEN DESIGN FOR TWO DEGREES OF FREEDOM CONTROLLERS

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Abstract. *This paper develops an adaptive direct data driven control scheme for one closed loop system with two degrees of freedom controllers, i.e., forward controller and feedback controller simultaneously, and the plant-model perfect matching conditions are satisfied. To apply direct data driven control to designing forward controller and feedback controller without the mathematical model of plant, virtual input and virtual disturbance are constructed to derive one optimization problem, whose decision variables are the unknown controller parameters. Through using the idea of adaptation, one parameter adjustment loop is added as the outer loop in such a way the unknown controller parameters are changed with environment varying, according to our constructed parameter adjustment law. Furthermore, after giving some considerations on the parameterized controllers, Lyapunov's stability theory is also used to derive the parameter adjustment law such that stability can be guaranteed for the whole adaptive system. Finally, two simulation examples confirm our theoretical results.*

Keywords: Direct data driven control, Adaptive mechanism, Two degrees of freedom, Lyapunov stability

1. **Introduction.** Adaptation means to change oneself so that one's behavior will confirm to new or changed circumstances, and the area of adaptive control has grown to be one of the richest in terms of algorithms, design techniques, analytical tools and modifications. An adaptive controller is thus a controller that can modify its behavior in response to changes in the dynamics of the process and the character of the disturbances. Intuitively an adaptive controller is a controller with adjustable parameters and a mechanism for adjusting the parameters. Moreover, an adaptive control system can be thought of as having two loops. One loop is a normal feedback with the process and the controller. The other loop is the parameter adjustment loop. As adaptive systems have useful properties, which can be profitably used to design control systems with improved performance and functionality, so a control engineer should know about adaptive systems.

The main advantage of direct data driven control is that the controller is designed directly by using only input-output measured data without identifying the plant, i.e., the mathematical model for the plant is unknown. Because of some safety and production restrictions, the open loop system is not used widely in many industrial production process. So in such situation, it is very urgent to design controller in closed loop system. However,

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the difficulty about the controller design in closed loop system is that the correlation is considered between the input and external disturbance, induced by the feedback loop. If considering the problem of designing controller in closed loop system, the controller is always designed on the basis of a known mathematical model for the plant. However, the mathematical model for the plant cannot be easily determined in the industrial process, as the cost of developing the mathematical model for the plant is very high.

To avoid the identification process for unknown model for the plant and to design the controller directly in closed loop system, direct data driven control scheme is studied deeply here, where the feedback controller and forward controller exist in the same closed loop system simultaneously. Adaptive control can deal with external disturbance [1], which adjusts the parameter estimates of controller based on feedback signal. When the plant is affected by external disturbance such that the output prediction deviates away our desired trajectory, then the generated error signal is used to adjust the parameters of controller until the output prediction corresponds to the designed trajectory. Many parameter adaptation algorithms are proposed in [2], whatever in deterministic environment or stochastic environment. In [3], a new multi variable adaptive control (MRAC) scheme is designed for the plants with arbitrary relative degree, which does not require a stringent symmetry assumption related with the plat high frequency gain matrix. An adaptive controller makes use of a closed loop reference model as an observer [4], and guarantees global stability or asymptotic output tracking. A reference model robust adaptive controller and a linear quadratic regulator are combined to obtain a high performance and robust control system [5]. An extremum seeking algorithm is proposed for reference tracking and stabilization of unstable discrete time systems without control directions [6]. [7] establishes a novel result for adaptive asymptotic tracking control of uncertain switched linear systems. One commonly used adaptive mechanism is named as model reference adaptive control [8], where direct and indirect model reference adaptive control strategies are proposed for multi variable piecewise affine systems. These piecewise affine systems constitute a popular tool to model hybrid systems or nonlinear systems. [9] proposes a novel combined model reference adaptive control for unknown multi input multi output systems with guaranteed parameter convergence. To control the time delay system, some novel modifications are given for a predictor based model reference adaptive controller [10], so that input delays are compensated in uncertain nonlinear system. When considering stability and convergence for adaptive control system, Mazenc construction is used to design a simple strict Lyapunov function in a rather intuitive manner [11]. Multiple model adaptive control is applied to improving the transient response of nonlinear system [12], by using several switched models. Generally the related literature on adaptive control is very high, and here we cannot enumerate all of them. The detailed interplay of industry, applications, technology, theory and research on adaptive control is discussed in [13]. Furthermore, the author proposes one zonotope parameter identification algorithm to calculate a set that contains the unknown controller parameters consistent with the measured output and the given bound of external disturbance [14].

Based on our previous works on direct data driven control scheme, this new paper develops a new idea of adaptation for direct data driven control scheme through using only input-output measured data directly without the mathematical model for the considered plant, i.e., we combine the adaptation and direct data driven control scheme to design or identify the unknown controller parameters. To the best of our knowledge, in all published papers on direct data driven control scheme, the obtained controller parameters are time-invariant, i.e., parameter estimates are constants. However, the special property of this constant condition for the controller parameters is not realistic in reality, as the environment is changing, so the controller parameters must be time-varying with the environment changes. However, how can we describe this varying property of the unknown controller parameters with environment changes? The answer is to apply the adaptation into adjusting the controller parameters. The key problem in this paper is to determine the adjustment mechanism, so that the parameter adjustment mechanism is constructed to adapt the controller parameters automatically. As a consequence, the controller parameters are changed on the basis of adjustment mechanism from error signal, which is the difference between the actual output of the closed loop system and the output of the reference model. Here this error signal is similar to the concept from model reference adaptive control. The main essence in direct data driven control, the problem of designing feedback controller and forward controller simultaneously can be transformed into one complex optimization problem, whose decision variables are the unknown controller parameters.

Generally, the main contributions of this paper consist of two folds: one is to introduce that direct data driven control scheme in one closed loop system with two degrees of freedom controllers, i.e., forward controller and feedback controller, exist in the same closed loop system, and the other is to combine the idea of adaptation and direct data driven control, and then a new parameter adaptation algorithm is developed to adjust the unknown controller parameters in the constructed parameter adjustment mechanism.

This paper is organized as follows. In Section 2, the structure of our considered closed loop system is presented. A short introduction of the classical model reference control is used to design the unknown parameterized controllers, and the deficiency of classical model reference control is pointed out. Here, the name of two degrees of freedom means that forward controller and feedback controller are all in the same closed loop system simultaneously. In Section 3, direct data driven control scheme is proposed to design the unknown parameterized controllers by minimizing one optimization problem. Section 4 develops the idea of adaptation into direct data driven control to form our proposed adaptive direct data driven control scheme, and furthermore, Lyapunov's stability theory is applied to constructing the parameter adaptation algorithm for adjusting unknown controller parameters in the sense of adaptation. Section 5 presents some simulation results to prove the control performance for our proposed adaptive direct data driven control scheme. Section 6 ends the paper with final conclusion and points out the future work.

2. **Problem Description.**

2.1. **Closed loop system with two degrees of freedom controllers.**

Assume that the plant is a linear time invariant discrete time process, it is denoted by a rational transfer function form $P(z)$, and $P(z)$ is unknown. Throughout the whole closed loop experimental process, only a sequence of input-output measured data, corresponding to the plant $P(z)$, are collected. The input-output relation between the input and output is described as follows:

$$y(t) = P(z)u(t) + d(t) \tag{1}$$

where z is a time shift operator, i.e., $zu(t) = u(t-1)$, $P(z)$ is one transfer function of the plant, $u(t)$ is the measured input, $y(t)$ is the measured output corresponding to the plant $P(z)$, and $d(t)$ is the external noise. When $d(t)$ in Equation (1) is unknown but has known bound, we regard the uncertainty associated with $d(t)$ as additive noise because of the way it enters the input-output relation in Equation (1).

Consider the above simple closed loop system with two degrees of freedom controllers in Figure 1, the input-output relations in the whole closed loop system are written as

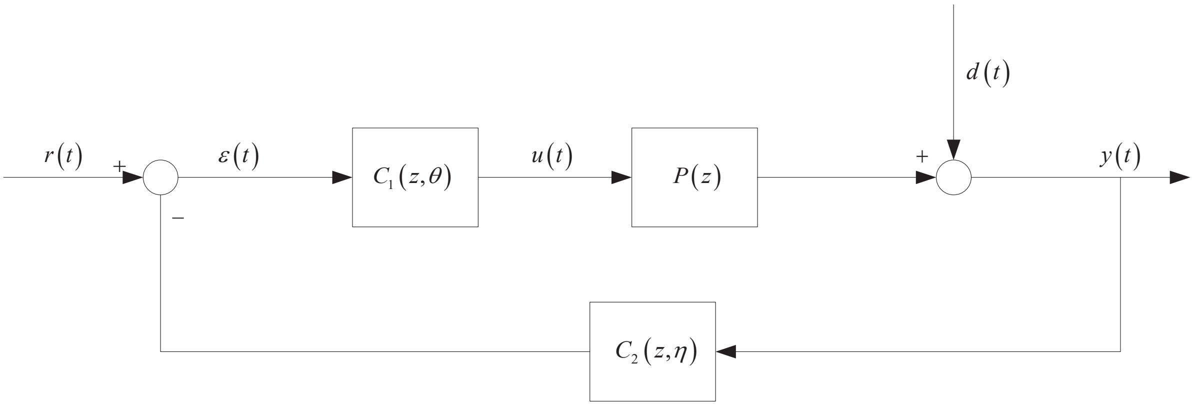

FIGURE 1. Closed loop system with two degrees of freedom controllers

follows:

$$\begin{cases} y(t) = P(z)u(t) + d(t) \\ u(t) = C_1(z,\theta)\varepsilon(t) = C_1(z,\theta)[r(t) - C_2(z,\eta)y(t)] \\ \varepsilon(t) = r(t) - C_2(z,\eta)y(t) \end{cases} \tag{2}$$

where $r(t)$ is the excited signal, $C_1(z,\theta)$ and $C_2(z,\eta)$ are two degrees of freedom controllers, which are parametrized by two unknown parameter vectors θ and η respectively, i.e., these two controllers $C_1(z,\theta)$ and $C_2(z,\eta)$ are independently parametrized as the following linear affine forms:

$$\begin{aligned} &C_1(z,\theta) = \alpha^T(z)\theta, \quad C_2(z,\eta) = \beta^T(z)\eta \\ &\alpha(z) = [\alpha_1(z), \alpha_2(z), \ldots, \alpha_n(z)]^T, \quad \beta(z) = [\beta_1(z), \beta_2(z), \ldots, \beta_n(z)]^T \\ &\theta = [\theta_1, \theta_2, \ldots, \theta_n]^T, \quad \eta = [\eta_1, \eta_2, \ldots, \eta_n]^T \end{aligned} \tag{3}$$

where $\alpha(z)$ and $\beta(z)$ denote two known basis function vectors, θ and β are two unknown parameter vectors with dimension n. The choices of these two basic function vectors $\alpha(z)$ and $\beta(z)$ can be as orthogonal basis functions or kernel functions, and from practical perspective, $\alpha(z)$ and $\beta(z)$ are always chosen as follows:

$$\alpha(z) = \beta(z) = \begin{bmatrix} 1 & z & z^2 & \cdots & z^n \end{bmatrix}$$

These parametrized forms for two controllers $C_1(z,\theta)$ and $C_2(z,\eta)$ can be chosen as the common PID controllers, i.e., the common PID controllers can be rewritten as our considered parametrized forms in Equation (3).

Formulating Equation (2) again, the new input-output relations in terms of the excited input $r(t)$ and external noise $d(t)$ are given as

$$y(t) = \frac{P(z)C_1(z,\theta)}{1 + P(z)C_1(z,\theta)C_2(z,\eta)} r(t) + \frac{1}{1 + P(z)C_1(z,\theta)C_2(z,\eta)} d(t) \tag{4}$$

2.2. **Classical model reference control.** In the above closed loop system with two degrees of freedom controllers, the first transfer function from the excited signal $r(t)$ to the measured output $y(t)$ is named as the closed loop transfer function, and the second transfer function from the external signal $d(t)$ to the measured output $y(t)$ is called as the sensitivity function. From Equation (4), the control task of classical model reference control is to tune the unknown parameter vectors θ and η corresponding to two degrees of freedom controllers $(C_1(z,\theta), C_2(z,\eta))$ in order to achieve the expected closed loop transfer function and expected sensitivity function.

Given the closed loop transfer function $M(z)$ and expected sensitivity function $S(z)$, we want to guarantee that the closed loop transfer function approximates to its expected function $M(z)$, and the sensitivity function tends to its expected function $S(z)$, too. The expected closed loop transfer function $M(z)$ and expected sensitivity function $S(z)$ can be chosen in priori, based on the goal of the control designer. For example, if the goal of designing the closed loop controllers is to achieve the zero tracking error, then the expected closed loop transfer function $M(z)$ can be chosen as one constant value 1.

The problem of tuning these two unknown parameter vectors θ and η is formulated as the following classical model reference control problem:

$$\begin{aligned}&\min_{\theta,\eta} J_{MR}(\theta,\eta)\\ &= \left\|\frac{P(z)C_1(z,\theta)}{1+P(z)C_1(z,\theta)C_2(z,\eta)} - M(z)\right\|_2^2 + \left\|\frac{1}{1+P(z)C_1(z,\theta)C_2(z,\eta)} - S(z)\right\|_2^2 \end{aligned} \tag{5}$$

where in above optimization problem, $\|.\|_2^2$ is the common Euclidean norm. In Equation (5), before solving this optimization problem with respect to two unknown parameter vectors θ and η, the priori knowledge about the plant $P(z)$ may be needed, for example, its transfer function form or state space form. So in this classical model reference control, as the plant $P(z)$ is unknown, then firstly the identification strategy is needed to identify $P(z)$. To avoid the identification process of the plant $P(z)$, virtual reference feedback tuning control is proposed to directly identify the unknown parameter vectors θ and η in two controllers $(C_1(z,\theta), C_2(z,\eta))$ from the measured input-output data $Z^N = \{u(t), y(t)\}_{t=1}^N$, where N is the number of data points.

3. **Direct Data Driven Control.**

Given two controllers $\{(C_1(z,\theta), C_2(z,\eta))\}$, as the closed loop transfer function from $r(t)$ to $y(t)$ is $M(z)$, we apply one arbitrary signal $r(t)$ to exciting the formal closed loop system (2), and the output of the closed loop system is described as $y(t) = M(z)r(t)$.

Consider one special excited input $\bar{r}(t)$, the necessary condition about that the closed loop transfer function is $M(z)$ is that the two closed loop systems have the same output $y(t)$ under a given input. During classical model reference control, this necessary condition holds in case of choosing suitable controller and excited signal. However, above description does not hold, due to unknown plant. The idea of direct data driven control means that virtual input $\bar{r}(t)$ and virtual disturbance $\bar{d}(t)$ need to be constructed firstly, so we give the detailed process of constructing virtual input $\bar{r}(t)$ and virtual disturbance $\bar{d}(t)$, respectively.

3.1. **Virtual input.**

Suppose that a pair of controllers $\{C_1(z,\theta), C_2(z,\eta)\}$ result in a closed loop system, whose transfer function is $M(z)$. Then if the closed loop system is fed by one reference signal $r(t)$, its output equals $M(z)r(t)$. Hence, a necessary condition for the closed loop system to have the same transfer function as the reference model is that the outputs of the two systems for a given $\bar{r}(t)$ are equal to each other. Classical model reference design method tries to impose such a necessary condition by first selecting a reference $\bar{r}(t)$ and then chooses $\{C_1(z,\theta), C_2(z,\eta)\}$ such that the condition is satisfied. However, for a general selection of $\bar{r}(t)$, our task is difficult to accomplish if a model of the plant is not available. The basic idea of the direct data driven control is to perform a wise selection of $\bar{r}(t)$.

Collecting input-output data $\{u(t), y(t)\}_{t=1}^N$ corresponding to the plant $P(z)$, then for every measured output $y(t)$, define one virtual input $\bar{r}(t)$ such that

$$y(t) = M(z)\bar{r}(t) \tag{6}$$

This virtual input $\bar{r}(t)$ does not exist in reality, so it cannot be used to generate actual measured output $y(t)$. However, virtual input $\bar{r}(t)$ can be obtained by equation $y(t) = M(z)\bar{r}(t)$, i.e., $y(t)$ is the measured output of the closed loop system, when the excited signal $\bar{r}(t)$ is applied with no disturbance $d(t) = 0$.

Due to the unknown plant $P(z)$, and when $P(z)$ is excited by $u(t)$, its output is $y(t)$. If the closed loop system is excited by virtual input $\bar{r}(t)$ and $y(t)$ simultaneously, then we choose two suitable controllers $\{(C_1(z,\theta), C_2(z,\eta))\}$ to obtain one expected signal $u(t)$. The construction of virtual input $\bar{r}(t)$ can be seen in Figure 2, and the tracking error $\varepsilon(t)$ is defined as

$$\varepsilon(t) = \bar{r}(t) - C_2(z,\eta)y(t) = \left(M^{-1}(z) - C_2(z,\eta)\right) y(t) \tag{7}$$

From Figure 2, we see that when the closed loop system is excited by $(\bar{r}(t), y(t), d(t) = 0)$, the expression of $u(t)$ is given as

$$u(t) = C_1(z,\theta)\varepsilon(t) = C_1(z,\theta)\left(M^{-1}(z) - C_2(z,\eta)\right) y(t) \tag{8}$$

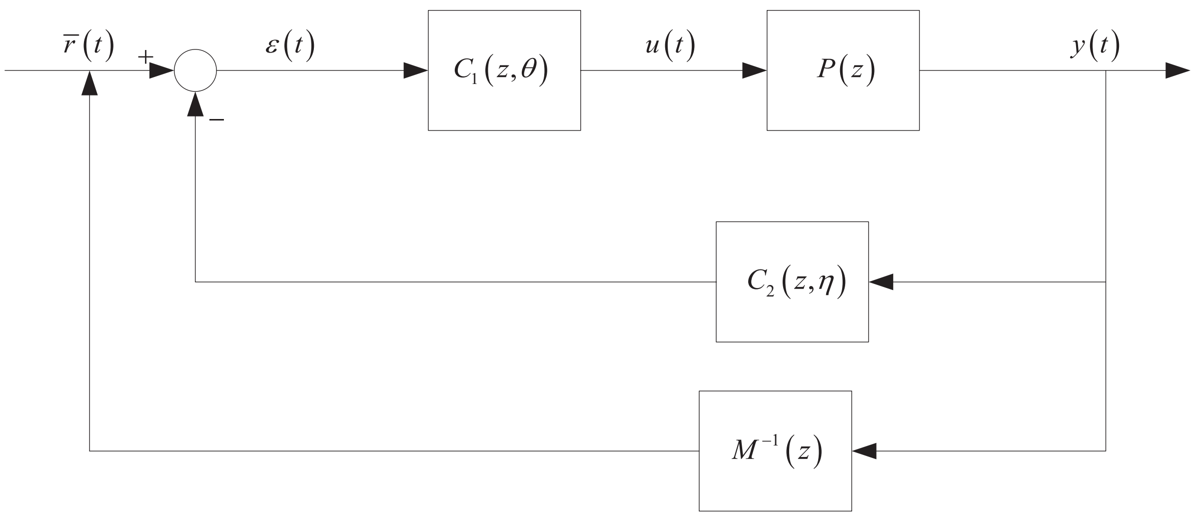

Figure 2. Construction of virtual input

3.2. **Virtual disturbance.** The construction of virtual disturbance is similar to virtual input. Given the measured output $y(t)$, consider a signal $\bar{d}(t)$ such that $y(t) + \bar{d}(t)$ is the desired output, when the reference signal is zero $(r(t) = 0)$ and the disturbance signal is $\bar{d}(t)$. Thus, the signal $\bar{d}(t)$ is such that $y(t) + \bar{d}(t) = S(z)\bar{d}(t)$ and this equation can be used for the computation of $\bar{d}(t)$.

Here the construction of $\bar{d}(t)$ is proposed. Given the measured output $y(t)$, define one virtual disturbance $\bar{d}(t)$ to guarantee that when the closed loop system is excited by virtual disturbance $\bar{d}(t)$, the obtained output $\bar{y}(t)$ is defined as

$$\bar{y}(t) = y(t) + \bar{d}(t) \tag{9}$$

The construction of virtual disturbance $\bar{d}(t)$ is shown in Figure 3, and then this virtual disturbance $\bar{d}(t)$ satisfies

$$\bar{y}(t) = y(t) + \bar{d}(t) = S(z)\bar{d}(t) \tag{10}$$

Equation (10) is used to generate virtual disturbance $\bar{d}(t)$, which means that when the closed loop system is excited by the following signals $(r(t) = 0,\ \bar{d}(t),\ \bar{y}(t) = y(t) + \bar{d}(t))$,

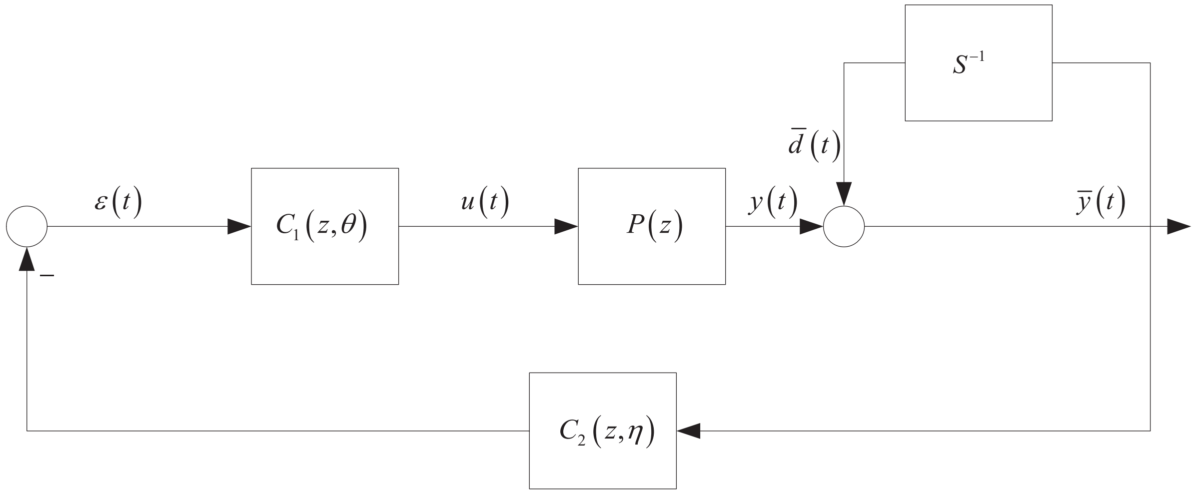

FIGURE 3. Construction of virtual disturbance

the input signal $u(t)$ for the plant is obtained as

$$u(t) = C_1(z,\theta)\varepsilon(t) = -C_1(z,\theta)C_2(z,\eta)\bar{y}(t) \tag{11}$$

As $\bar{y}(t)$ in Equation (11) is not the true output, and instead the true output is $y(t)$, so $\bar{y}(t)$ in Equation (11) must be changed into $y(t)$.

Combining two equations (9) and (11), we get

$$\begin{aligned} y(t) &= \bar{y}(t) - \bar{d}(t) = S(z)\bar{d}(t) - \bar{d}(t) = (S(z)-1)\bar{d}(t) = (S(z)-1)S^{-1}(z)\bar{y}(t) \\ \bar{y}(t) &= \frac{S(z)}{S(z)-1}y(t) \end{aligned} \tag{12}$$

where two relations are used in above derivations:

$$\begin{cases} \bar{y}(t) = y(t) + \bar{d}(t) = S(z)\bar{d}(t) \\ \bar{d}(t) = S^{-1}(z)\bar{y}(t) \end{cases}$$

Substituting Equation (12) into (11), we obtain

$$u(t) = -C_1(z,\theta)C_2(z,\eta)\frac{S(z)}{S(z)-1}y(t) \tag{13}$$

Using Equations (8) and (13), two unknown parameter vectors θ and η in two controllers $\{(C_1(z,\theta), C_2(z,\eta))\}$ can be identified by solving the following optimization problem.

$$\begin{aligned} \min_{\theta,\eta} J_{VR}^N(\theta,\eta) = &\frac{1}{N}\sum_{t=1}^{N}\left[u(t) - C_1(z,\theta)\left(M^{-1}(z) - C_2(z,\eta)\right)y(t)\right]^2 \\ &+ \frac{1}{N}\sum_{t=1}^{N}\left[u(t) + C_1(z,\theta)C_2(z,\eta)\frac{S(z)}{S(z)-1}y(t)\right]^2 \end{aligned} \tag{14}$$

where $M^{-1}(z)$ and $S(z)$ appear in above optimization Equation (14). These two inverse transfer functions always exist in some special cases, for example, minimum phase system or linear causal system. However, if these two inverse transfer functions do not exist in reality, we can use their pseudo inverse forms in Equation (14).

Observing optimization problem (14), all variables are known except for those two parametrized controllers $\{(C_1(z,\theta), C_2(z,\eta))\}$. More specifically, input-output data $\{u(t), y(t)\}_{t=1}^N$ can be collected by sensors, these two expected transfer functions $M(z)$ and $S(z)$ are priori known. Roughly speaking, plant $P(z)$ is not in that optimization problem (14), and Equation (14) is the main contribution in direct data driven control. Furthermore, the optimization problem (14) embodies that the problem of designing two unknown controllers (forward controller and feedback controller) can be transformed to identify those two unknown parameter vectors. This transformation will simplify the latter computational complexity about designing controllers with the parameterized forms.

3.3. **Comment.** From the theoretical perspective on this direct data driven control, the most important conditions are that these two inverse transfer functions $M^{-1}(z)$ and $S^{-1}(z)$ exist, as they are used to compute their corresponding virtual input and virtual disturbance. From above construction processed for virtual input and virtual disturbance, i.e., Figure 2 and Figure 3, the complete structure for direct data driven control is plotted in Figure 4, where two inverse transfer functions $M^{-1}(z)$, $S^{-1}(z)$, the constructed virtual input and virtual disturbance, etc. are all included. Roughly speaking, these two inverse transfer functions $M^{-1}(z)$ and $S^{-1}(z)$ embody two relations between the input and output, i.e., it holds

$$\begin{cases} u(t) = C_1(z,\theta)\big(M^{-1}(z) - C_2(z,\eta)\big)y(t) \\ u(t) = -C_1(z,\theta)C_2(z,\eta)\dfrac{S(z)}{S(z)-1}y(t) \end{cases} \tag{15}$$

Equation (15) will be used in latter designing controller for our proposed adaptive direct data driven control scheme.

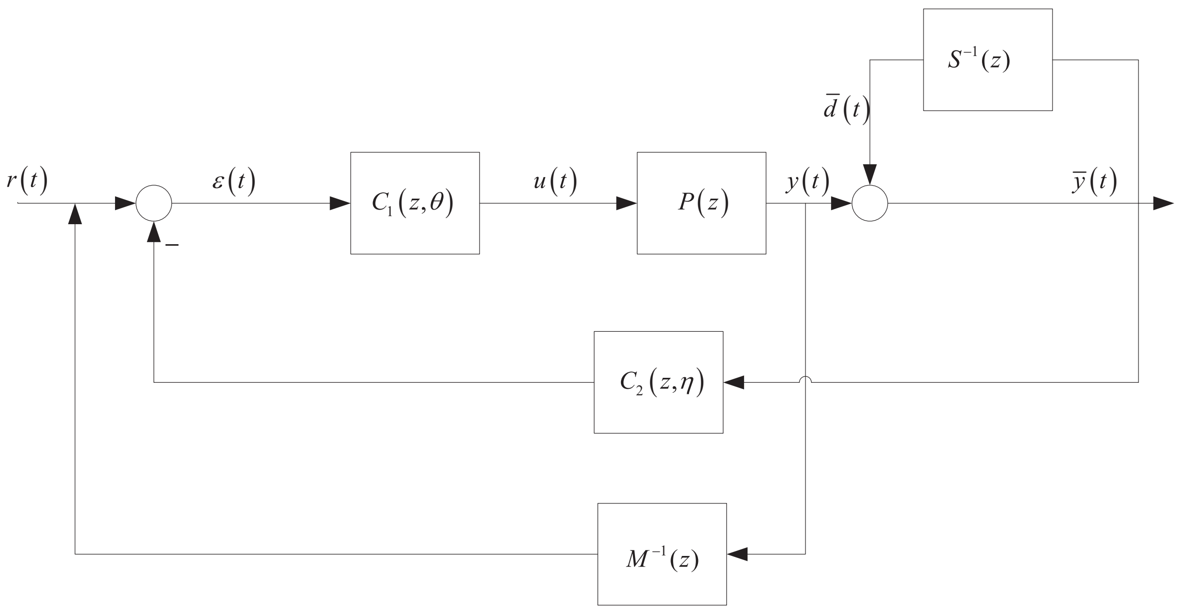

FIGURE 4. Structure for direct data driven control

Observing Figure 4 again and introducing adaptation into direct data driven control, from optimizaiton problem (14), the measured data sets $\{u(t), y(t)\}_{t=1}^{N}$ and $\Big\{u(t), \frac{S(z)}{S(z)-1}y(t)\Big\}_{t=1}^{N}$ are applied to identifying the unknown controller parameters $\{\theta, \eta\}$. Through minimizing the optimization problem (14), parameter estimates of controller parameters $\{\theta, \eta\}$ are obtained by differentiation with respect to θ and η, then by setting the derivative equal to zero. However, the obtained parameter estimates are fixed constants, and these constants keep invariant with environment changes. To describe the time variant property about the changing environment, one adjustment or adaptive mechanism is added in the considered closed loop system, so that the parameter estimates will be changed on the basis of adjustment mechanism. Based on the main process of direct data driven control, the adjustment mechanism must be added about measured data sets $\{u(t), y(t)\}_{t=1}^{N}$ and $\left\{u(t), \frac{S(z)}{S(z)-1}y(t)\right\}_{t=1}^{N}$, but not data set $\{y(t), M(z)r(t)\}_{t=1}^{N}$.

4. **Adaptive Direct Data Driven Control.** An adaptive controller is formed by combining an on-line parameter estimator, which provides estimates of unknown parameters at each instant, with a control law that is motivated from the known parameter case. The way the parameter estimator, also referred to as adaptive law, is combined with the control law.

4.1. **Basic idea.** A block diagram of the adaptive direct data driven control scheme is shown in Figure 5. This adaptive system has one ordinary feedback loop composed of the plant and the controller and other feedback loop that changes the controller parameters. The parameters are changed on the basis of feedback from the error. The ordinary feedback loop is called the inner loop, and the parameter adjustment loop is called the outer loop.

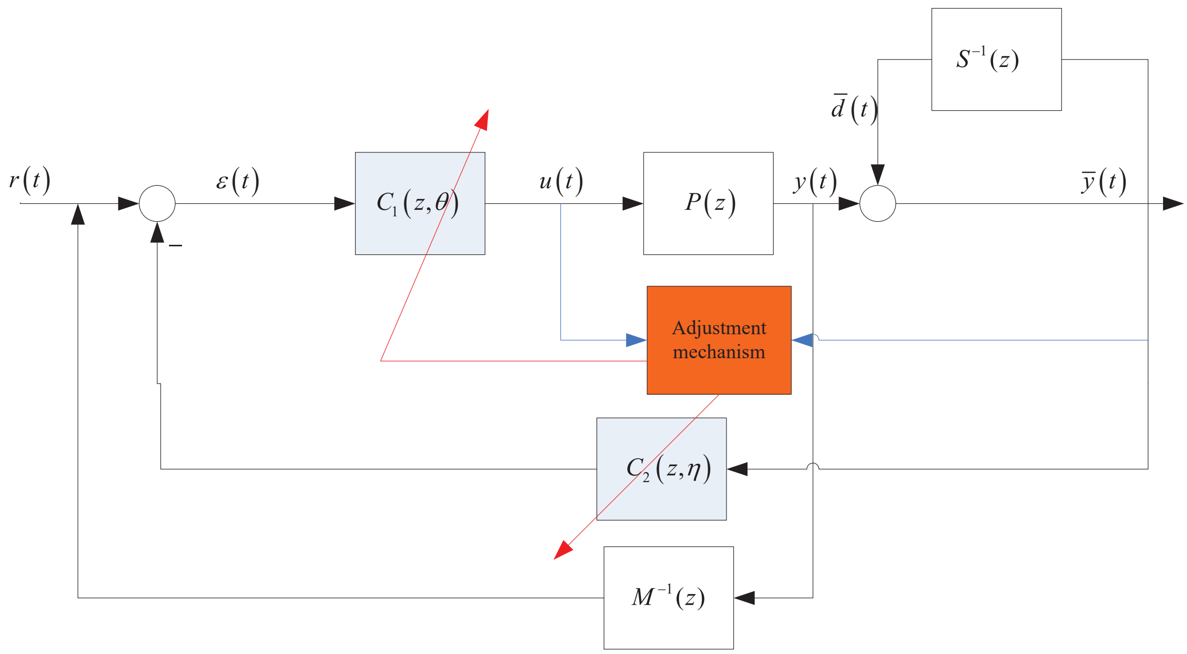

FIGURE 5. Block diagram of adaptive direct data driven control scheme

As in adjustment mechanism for Figure 5, the goals of designing forward controller $C_1(z, \theta)$ and feedback controller $C_2(z, \eta)$ are to make the output of forward controller $C_1(z, \theta)$ equal to the input $u(t)$ of the plant $P(z)$, while guaranteeing the output of feedback controller $C_2(z, \eta)$ approaches to the error signal.

Observing optimization problem (14) again, then define

$$\begin{cases} e_1(t) = u(t) - C_1(z, \theta)\big(M^{-1}(z) - C_2(z, \eta)\big)y(t) \\ e_2(t) = u(t) + C_1(z, \theta)C_2(z, \eta)\dfrac{S(z)}{S(z) - 1}y(t) \end{cases} \tag{16}$$

Then optimization problem (14) is rewritten as

$$\min_{\theta,\eta} J_{VR}^N(\theta,\eta) = \frac{1}{N}\sum_{t=1}^{N} e_1^2(t) + \frac{1}{N}\sum_{t=1}^{N} e_2^2(t) \tag{17}$$

In case of linear affine forms for those two parameterized controllers, substitute Equation (3) into (17) to get

$$\begin{cases} e_1(t) = u(t) - \alpha^T(z)\theta\left(M^{-1}(z) - \beta^T(z)\eta\right)y(t) \\ e_2(t) = u(t) + \alpha^T(z)\theta\beta^T(z)\eta\dfrac{S(z)}{S(z)-1}y(t) \end{cases} \tag{18}$$

Taking the partial derivative with respect to θ and η, we obtain

$$\begin{cases} \dfrac{\partial e_1(t)}{\partial\theta} = -\alpha^T(z)\left(M^{-1}(z) - \beta^T(z)\eta\right)y(t) \\ \dfrac{\partial e_1(t)}{\partial\eta} = \alpha^T(z)\theta\beta(z)y(t) \end{cases} \tag{19}$$

and

$$\begin{cases} \dfrac{\partial e_2(t)}{\partial\theta} = \alpha^T(z)\beta^T(z)\eta\dfrac{S(z)}{S(z)-1}y(t) \\ \dfrac{\partial e_2(t)}{\partial\eta} = \alpha^T(z)\theta\beta^T(z)\dfrac{S(z)}{S(z)-1}y(t) \end{cases} \tag{20}$$

To adjust those controller parameters (θ,η) in such way the loss function $J_{VR}^N(\theta,\eta)$ is sufficiently small, it is reasonable to change the controller parameters in the direction of the negative gradient (θ,η). That idea of adaptation is applied to solving the parameter estimators (θ,η), i.e.,

$$\begin{bmatrix} \dfrac{d\theta}{dt} \\ \dfrac{d\eta}{dt} \end{bmatrix} = -\gamma \begin{bmatrix} \dfrac{dJ_{VR}^N(\theta,\eta)}{dt} \\ \dfrac{dJ_{VR}^N(\theta,\eta)}{dt} \end{bmatrix} \tag{21}$$

where parameter γ determines the adaptation rate, and

$$\begin{aligned} \frac{dJ_{VR}^N(\theta,\eta)}{dt} &= \frac{2}{N}\sum_{t=1}^{N} e_1(t)\left[-\alpha^T(z)\big(M^{-1}(z) - \beta^T(z)\eta\big)y(t)\right] \\ &\quad + \frac{2}{N}\sum_{t=1}^{N} e_2(t)\left[\alpha^T(z)\beta^T(z)\eta\frac{S(z)}{S(z)-1}y(t)\right] \\ \frac{dJ_{VR}^N(\theta,\eta)}{dt} &= \frac{2}{N}\sum_{t=1}^{N} e_1(t)\left[\alpha^T(z)\theta\beta(z)y(t)\right] \\ &\quad + \frac{2}{N}\sum_{t=1}^{N} e_2(t)\left[\alpha^T(z)\theta\beta^T(z)\frac{S(z)}{S(z)-1}y(t)\right] \end{aligned} \tag{22}$$

The partial derivatives $\frac{dJ_{VR}^N(\theta,\eta)}{dt}$ and $\frac{dJ_{VR}^N(\theta,\eta)}{dt}$ are called the sensitivity derivatives of the adaptive system. Equation (21) is called as the MIT (Massachusetts Institute of Technology) rule in the parameter adjustment mechanism, and the MIT rule can be regarded as a gradient scheme to minimize the squared error.

4.2. **Design controller using Lyapunov theory.** The basic idea of adaptive direct data driven control is introduced in above Section 4.1. As we all know, stability is one important issue in all control theory, so in this section we give how to use Lyapunov's theory to construct algorithm for adjusting controller parameters in adaptive system. To achieve this goal, we need to derive a differential equation for the tracking error. This differential equation contains the adjustment parameters. We try to construct a Lyapunov function and adaptive mechanism such that the tracking will go to zero. When using the Lyapunov theory for adaptive systems, partial derivative $\frac{dV}{dt}$ of one constructed Lyapunov function V must be negative semidefinite. Then the latter procedure is to determine the error equation and a Lyapunov function with a bounded second partial derivative.

From the theoretical perspective, firstly we transform the problem of identifying unknown parameter vectors in virtual reference feedback tuning control into two linear regression models, which are suitable for zonotope parameter identification algorithm.

Observing the optimization problem (14) in virtual reference feedback tuning control, our goal is to identify two optimal parameter vectors such that the following ideal forms hold:

$$\begin{cases} u(t) = C_1(z,\theta)\big(M^{-1}(z) - C_2(z,\eta)\big)y(t) \\ u(t) = -C_1(z,\theta)C_2(z,\eta)\dfrac{S(z)}{S(z)-1}y(t) \end{cases}$$

Let

$$C_1(z,\theta)\left(M^{-1}(z) - C_2(z,\eta)\right)y(t) = -C_1(z,\theta)C_2(z,\eta)\frac{S(z)}{S(z)-1}y(t) \tag{23}$$

Then

$$\left(M^{-1}(z) - C_2(z,\eta) + C_2(z,\eta)\frac{S(z)}{S(z)-1}\right)y(t) = 0 \tag{24}$$

It means that

$$\begin{cases} \big((S(z)-1)M^{-1}(z) + C_2(z,\eta)\big)y(t) = 0 \\ C_2(z,\eta)y(t) - (S(z)-1)M^{-1}(z)y(t) = 0 \end{cases} \tag{25}$$

Substituting the linear affine form (3) of parameterized controller $C_2(z,\eta)$ into above Equation (25), then we have

$$\begin{bmatrix} 1 & z & z^2 & \cdots & z^n \end{bmatrix} \begin{bmatrix} 1 \\ \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{bmatrix} y(t) = (S(z)-1)M^{-1}(z)y(t) \tag{26}$$

where one special basic function vector is

$$\alpha(z) = \beta(z) = \begin{bmatrix} 1 & z & z^2 & \dots & z^n \end{bmatrix}^T$$

Rewrite Equation (26) as one linear regression model:

$$y(t) = y(t-1)\eta_1 + y(t-2)\eta_2 + \cdots + y(t-n)\eta_n + (S(z)-1)M^{-1}(z)y(t)$$

Hence

$$y(t) = \frac{M(z)}{M(z)-S(z)+1} \times \begin{bmatrix} y(t-1) & y(t-2) & \cdots & y(t-n) \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{bmatrix} \tag{27}$$

Introduce the regression vector $\varphi_1(t)$ as

$$\varphi_1(t) = \frac{M(z)}{M(z) - S(z) + 1} \left[\begin{array}{cccc} y(t-1) & y(t-2) & \cdots & y(t-n) \end{array} \right]$$

Then Equation (27) will be a linear regression model:

$$y(t) = \varphi_1(t)\eta \tag{28}$$

After unknown parameter vector η is identified and substituting its estimator $\hat{\eta}$ into the parametrized controller, then controller $C_2(z, \hat{\eta})$ is obtained. Applying this obtained controller $C_2(z, \hat{\eta})$ into Equation (26), we have

$$u(t) = -C_1(z, \theta)C_2(z, \hat{\eta}) \frac{S(z)}{S(z) - 1} y(t) \tag{29}$$

Similarly based on special basic function vector $\alpha(z)$, Equation (29) can be written as another linear regression model:

$$u(t) = \varphi_2(t)\theta \tag{30}$$

where the regressor vector $\varphi_2(t)$ is defined as

$$\varphi_2(t) = -\frac{S(z)}{S(z) - 1} C_2(z, \hat{\eta})\, y(t)$$

From these two linear regression models (28) and (30), we see that firstly unknown parameter vector η can be identified on the basis of linear regression model (28), and then after substituting its estimator $\hat{\eta}$ into regressor vector $\alpha(z)$, another unknown parameter vector θ is obtained from linear regression model (30).

Comment: As adaptive control problem characterizes the desired behavior of the closed loop system, we need to find a mechanism for adjusting the parameters, i.e., determine a suitable control law with adjustable parameters. Considering the problem for Equation (28), $y(t) = \varphi_1(t)\eta$, the simple gradient algorithm can be used to identify the unknown controller parameters η. Then the estimation error is simplified as $\varepsilon(t) = \frac{y(t) - \hat{y}(t)}{m^2(t)} = \frac{y(t) - \varphi_1(t)\eta(t-1)}{m^2(t)}$, where $m^2(t) \leq c > 0$ for some constant c is the normalizing signal, and $\eta(t-1)$ corresponds to the parameter estimators at time instant $t-1$. The estimation error $\varepsilon(t)$ at time instant t depends on the most recent estimate of η, which at time instant t assumed to be $\eta(t-1)$. The adaptive law for parameters η is of the following form: $\eta(t) = \eta(t-1) + \varepsilon(t)\varphi_1(t)$, $\eta(0) = 0.1$.

Combining those two equations (28) and (30), two error signals exist in the whole designing process, so we consider them as

$$e_3(t) = y(t) - \varphi_1(t)\eta + u(t) - \varphi_2(t)\theta \tag{31}$$

Assume two true parameter vectors θ_0 and η_0 exist, such that the perfect matching can be achieved, i.e., $y(t) = \varphi_1(t)\eta_0$, $u(t) = \varphi_2(t)\theta_0$. Then the total tracking error $e_3(t)$ is

$$\begin{aligned} e_3(t) &= \varphi_1(t)\eta_0 - \varphi_1(t)\eta + \varphi_2(t)\theta_0 - \varphi_2(t)\theta = \varphi_1(t)(\eta_0 - \eta) + \varphi_2(t)(\theta_0 - \theta) \\ &= \left[\begin{array}{cc} \varphi_1(t) & \varphi_2(t) \end{array} \right] \left[\begin{array}{c} \eta_0 - \eta \\ \theta_0 - \theta \end{array} \right] = \varphi(t)(\xi_0 - \xi) \end{aligned} \tag{32}$$

where regressor vector and parameter vector are defined as follows:

$$\varphi(t) = \left[\begin{array}{cc} \varphi_1(t) & \varphi_2(t) \end{array} \right]; \quad \xi_0 = \left[\begin{array}{c} \eta_0 \\ \theta_0 \end{array} \right]; \quad \xi = \left[\begin{array}{c} \eta \\ \theta \end{array} \right]$$

To construct one Lyapunov function, we firstly introduce a state space representation between the parameters ξ and the tracking error $e_3(t)$ as

$$\begin{cases} \dfrac{dx}{dt} = Ax + B(\xi_0 - \xi) \\ e_3(t) = Cx \end{cases} \tag{33}$$

where in Equation (33) variable t is neglected for notational clarity. From model control theory, it holds

$$e_3(t) = \left[C(sI - A)^{-1}B\right](\xi_0 - \xi) = \varphi^T(t)(\xi_0 - \xi)$$

It means that

$$\left[C(sI - A)^{-1}B\right] = \varphi^T(t) = \begin{bmatrix} \varphi_1(t) & \varphi_2(t) \end{bmatrix}$$

If the homogeneous system $\frac{dx}{dt} = Ax$ is asymptotically stable, then there exist positive definite matrices P and Q such that

$$A^T P + PA = -Q$$

Then one Lyapunov function is constructed as follows:

$$V = \frac{1}{2}\left(\gamma x^T P x + (\xi_0 - \xi)^2\right) \tag{34}$$

Taking the partial derivation with respect to time, then it holds

$$\frac{dV}{dt} = \frac{\gamma}{2}\left(\frac{dx^T}{dt} Px + x^T P \frac{dx}{dt}\right) - (\xi_0 - \xi)\frac{d\xi}{dt} \tag{35}$$

Substituting Equation (33) into (35), we obtain

$$\begin{aligned} \frac{dV}{dt} &= \frac{\gamma}{2}(\xi_0 - \xi)^T B^T P x + \frac{\gamma}{2}\left(x^T A^T P x + x^T P A x\right) + \frac{\gamma}{2} x^T P B(\xi_0 - \xi) - (\xi_0 - \xi)\frac{d\theta}{dt} \\ &= \frac{\gamma}{2} x^T \left(A^T P + PA\right) x + (\xi_0 - \xi)\left(-\frac{d\xi}{dt} + \frac{\gamma}{2} B^T P x + \frac{\gamma}{2} x^T P B\right) \\ &= -\frac{\gamma}{2} x^T Q x + (\xi_0 - \xi)\left(-\frac{d\theta}{dt} + \gamma B^T P x\right) \end{aligned} \tag{36}$$

The parameter adjustment law is chosen as

$$\frac{d\xi}{dt} = \gamma B^T P x = \gamma B^T P (sI - A)^{-1} B(\xi_0 - \xi) \tag{37}$$

It means that

$$\begin{bmatrix} \dfrac{d\eta}{dt} \\ \dfrac{d\theta}{dt} \end{bmatrix} = \gamma B^T P (sI - A)^{-1} B \begin{bmatrix} \eta_0 - \eta \\ \theta_0 - \theta \end{bmatrix} \tag{38}$$

Then the partial derivative of the constructed Lyapunov function will be negative as long as $x \neq 0$. The state vector x and the tracking error $e_3(t) = Cx$ will go to zero as t goes to infinity. The parameter adjustment law (37) can be applied to replacing Equation (21) in the adaptive direct data driven control scheme.

5. **Simulation Examples.** Here in this section, two simulation examples are used to prove the efficiency of our proposed theories about adaptive direct data driven control scheme without mathematics of the plant.

1) Firstly consider the following linear discrete time system, whose model of the plant is

$$P(z) = \frac{1 - 0.2z^{-1}}{1 - 0.6z^{-1}}$$

where true plant $P(z)$ is unknown, and the expected closed loop transfer function $M(z)$ is

$$M(z) = \frac{11.306(z - 0.53)}{z^2 - 0.706z + 0.32}$$

Applying direct data driven control to dealing with the measured data, then the initial values for controller parameters are given as

$$\theta_0 = \begin{bmatrix} K_{p0} & K_{i0} & K_{d0} \end{bmatrix}^T = \begin{bmatrix} 0.4021 & 0.3504 & 0.2475 \end{bmatrix}^T$$

In simulation, sampled period is chosen as 0.5s. From the closed loop step response curve in Figure 6, we see that after initial parameter values of the controller are tuned by off-line direct data driven control, the closed loop step response starts to run without any substantial oscillation, and the overshot is very small. During the first 80 seconds, the initial parameter values of the controller keep running smoothly. After 80 seconds, some parameter values of the plant are changed as follows:

$$P(z) = \frac{1 - 1.8z^{-1}}{1 - 0.1z^{-1}}$$

Due to the insufficient stability of the PID controller, the considered closed loop system will divergence, which means that the initial parameter values corresponding to direct data driven control cannot adapt to the time varying property. Closed loop step response

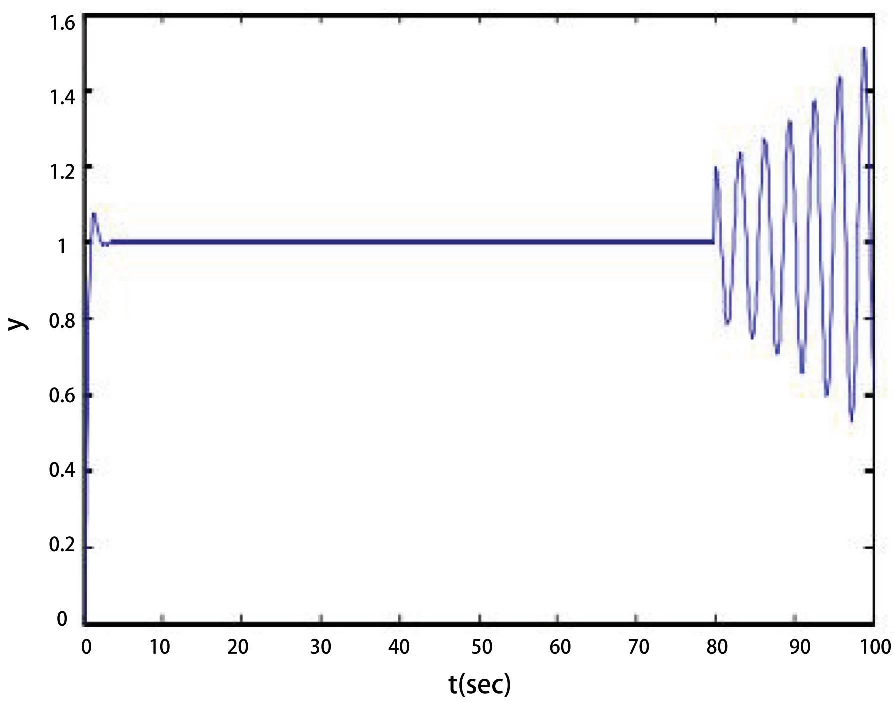

Figure 6. Step response curve for direct data driven control

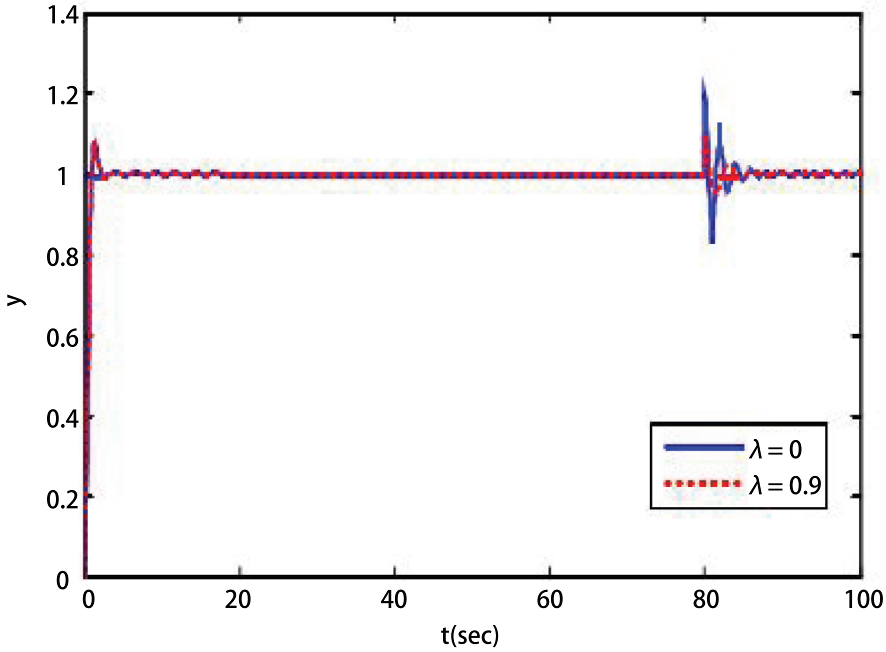

FIGURE 7. Step response curve for adaptive direct data driven control

obtained by our proposed adaptive direct data driven control is plotted in Figure 7, where at 80 seconds, the whole closed loop system fluctuates. However, after updating new measured data, the controller parameters are continuously corrected, so that the whole closed loop system will be stable in 4 seconds. Meanwhile, the simulation curves show when the forgetting factor is closed to the value of 1, the closed loop system's fluctuation is gentle, but the correction time is long. On the contrary, when the forgetting factor approaches to zero, the system starts to fluctuate greatly, and the correction process is relatively fast. Such above facts mean the choice of forgetting factor will affect the fluctuation size and the speed for correction.

2) Secondly considering one discrete time linear system, its transfer function form is described as follows:

$$P(z) = \frac{(z-1.2)(z-0.4)}{z(z-0.3)(z-0.8)}$$

One classical PID controller is used here in this second simulation.

$$C_1(z,\theta) = \alpha^T(z)\theta = \begin{bmatrix} \frac{z^2}{z^2-z} & \frac{z}{z^2-z} & \frac{1}{z^2-z} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

The true PID controller is given as

$$C_1(z,\theta) = \alpha^T(z)\theta = \begin{bmatrix} \frac{z^2}{z^2-z} & \frac{z}{z^2-z} & \frac{1}{z^2-z} \end{bmatrix} \begin{bmatrix} 0.86 \\ 0.2 \\ 0.1 \end{bmatrix}$$

The expected closed loop transfer function is chosen as

$$M(z) = \frac{z(z-1)\left(0.86z^4 - 1.1z^3 + 3.9z^2 + 0.8z + 0.48\right)}{z^7 - 3z^6 - 0.96z^5 - 0.72z^4 - 0.93z^3 + 3.9z^2 + 0.8z + 0.48}$$

The input-output measured data $\{u(t), y(t)\}_{t=1,2,\ldots,1000}$ are collected in the closed loop environment, and the number of data points is set 2000. To use the idea of virtual

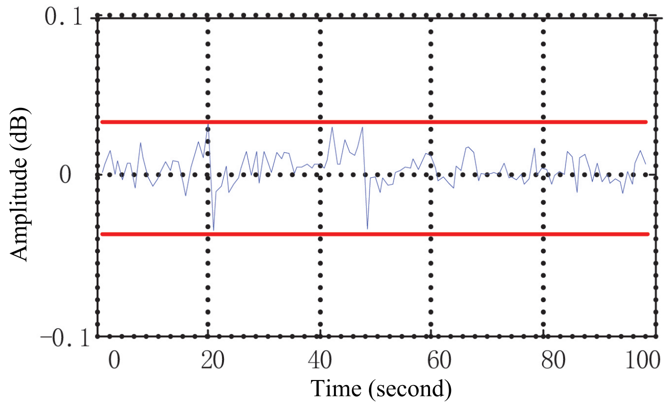

FIGURE 8. The input signal as a white noise

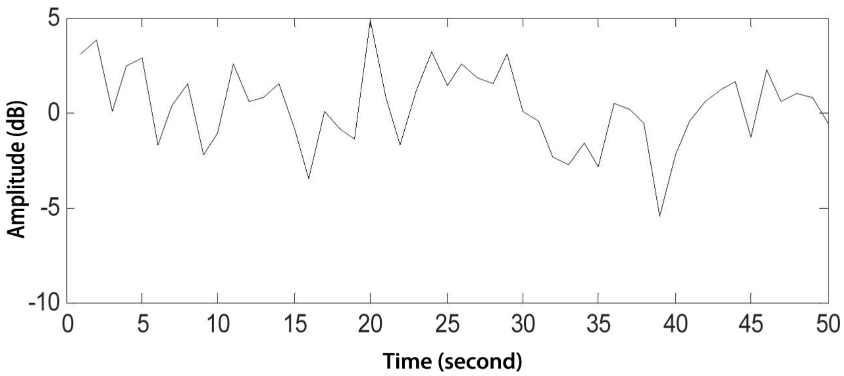

FIGURE 9. The measured output signal

reference feedback tuning control in designing parameter vectors, the plant model $P(z)$ is excited by zero mean Gaussian white noise, which is plotted in Figure 8 with the number of data points beging 1000, and the measured output data is seen in Figure 9. The adaptive parameter algorithm is used to solve the optimization problem (17). Before starting this iteration algorithm, the initial values of the unknown parameter vector are selected as

$$\theta = \begin{bmatrix} 0.75 & 0.25 & 0.15 \end{bmatrix}^T$$

As the considered adaptive parameter algorithm is also one iterative algorithm, after 80 iterative steps we define the relative error for the obtained parameter estimators as $\frac{\left\|\hat{\theta}_N - \hat{\theta}_{N-1}\right\|}{\left\|\hat{\theta}_N\right\|}$. The tendency of cost function is shown in Figure 10, with the iterative steps increase. From Figure 10, we see that cost function is decreased with the iterative steps, and after 80 iterative steps, the cost function will approach to zero value. It means the iterative parameter estimator $\hat{\theta}_{80}$ can be used as the final parameter estimator.

Step 1: Before designing this PID controller or estimating these three unknown parameters $\{k_p, k_i, k_d\}$, their initial values are chosen as $\{9.8, 3.0, 2.0\}$.

Step 2: Compute $J_1\left(\hat{\theta}_i^k\right)$ and $\nabla J_1\left(\hat{\theta}_i^k\right)$

Step 3: Construct $\hat{\theta}_i^{k+1} = \hat{\theta}_i^k - h\nabla J_1\left(\hat{\theta}_i^k\right)$, where $h = 0.6$.

Step 4: Choose $\epsilon = 0.1$, if $\left\|\hat{\theta}_i^{k+1} - \hat{\theta}_i^k\right\| \leq \epsilon$, then terminate the first order gradient algorithm, or go back to step 1.

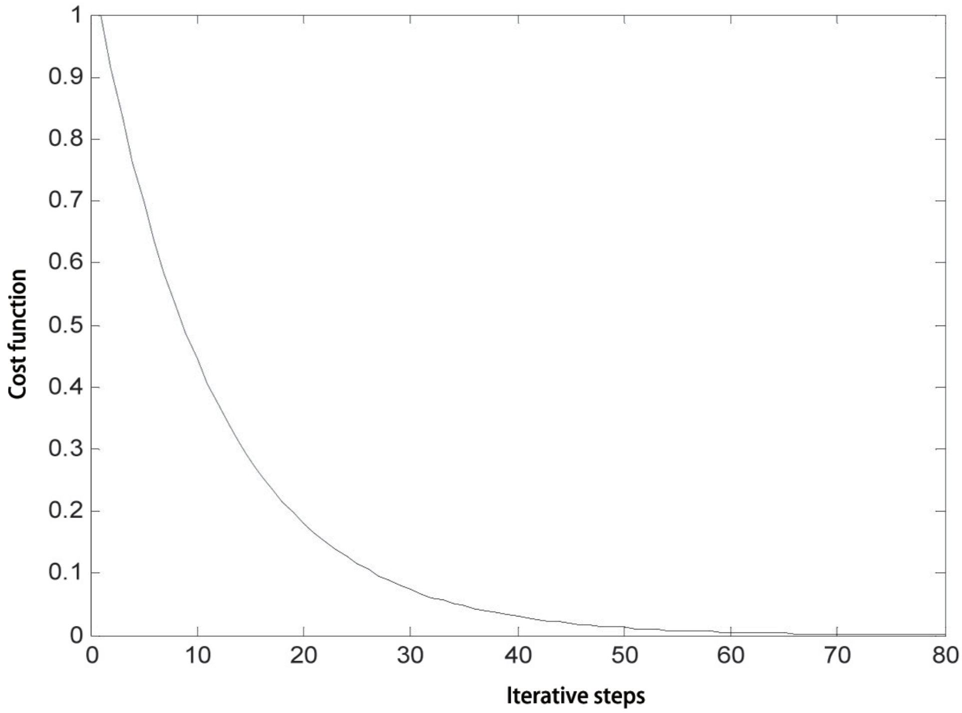

Figure 10. The decreased cost function

Step 5: The final controller parameters are obtained as $\{17, 3.8, 2.8\}$, whose iterative convergence curves are shown in Figure 10.

6. **Conclusion.** The paper connects adaptive control and direct data driven control to adjust the controller parameters adaptively for one closed loop system with two degrees of freedom controllers, i.e., forward controller and feedback controller. After the main process of the direct data driven control is given more detailed, one adjustment mechanism is constructed by using measured data, and the parameter adjustment law is proposed to achieve the adaptation. Furthermore, Lyapunov's stability theory is also used to derive parameter adjustment law such that stability can be guaranteed. Such adaptive direct data driven control scheme expands the ability of the traditional adaptive control to deal with the changing environment, and can be applied to more complex network system, which is our next idea.

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