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MULTI-AGENT EVENT-TRIGGERED CONTAINMENT CONTROL WITH JOINT CONNECTED SWITCHING TOPOLOGY

Wenjun Wei[1,2] and Jiahui Lv[1]

[1]School of Automation and Electrical Engineering
[2]The Key Laboratory of Opto-Technology and Intelligent Control, Ministry of Education
Lanzhou Jiaotong University
No. 88, Anning West Road, Anning District, Lanzhou 730070, P. R. China
weiwenjun@mail.lzjtu.cn; 12201419@stu.lzjtu.edu.cn

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Abstract. *The problem of event-triggered containment control of multi-agent systems (MAS) with joint connected switching topology is studied in this paper. In actual communication, the network topology may be switched and disconnected due to interference. In order to make the system still be able to realize containment control in these emergencies and to minimize communication consumption and save energy, an output regulation control strategy based on event-triggered is proposed. The leaders are considered as external systems and the method of output adjustment is used to work out the containment control problem. Then, a novel event-triggered control protocol with joint connectivity topology based on the state observer is designed to complete the output regulation control while excluding the Zeno behavior. We present proof that under the designed control strategy, the followers will asymptotically converge to the convex hull formed by multiple leaders if the communication topology is jointly connected in a limited time. Finally, the simulation results show the validity of the conclusion.*

Keywords: Multi-agent systems, Event-triggered, Joint connection, Switching topology, Containment control

1. **Introduction.** The containment control of MAS is closely related to social life and has been widely applied in many fields. Many scholars have done a lot of related researches, such as the formation of UAVs [1,2], aircraft formation control [3,4], synchronous control and tracking problems [5,6], and multi-agent simulation methods to solve practical problems [7,8]. The essence of these researches is to ensure a cluster of self-governing agents to accomplish many specific missions through distributed control strategies. As a research direction of multi-agent containment control, the leader-follower multi-agent system has gradually developed from a single leader in the early stage to multiple leaders later, and the containment control problem is a typical situation of multiple leaders. In 2008, Ji et al. first proposed the containment control problem of MAS in [7]. Containment control refers to a group of followers achieving and sustaining moving in the minimum convex hull formed by leaders under the guidance of leaders.

There are many applications of containment control in practice that have been studied more and more widely in [8-14]. The problem of containment control of MAS at a predetermined time using the observation function of distributed observers was studied in [8]. In [9], the authors considered the adaptive mechanism and a control protocol that uses only information exchanged between agents to implement containment control is designed. The containment control problem of low-order linear continuous MAS in the presence of

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time delay was studied in [10]. In [11,12], the authors further studied the nonlinear multi-agent containment control, and the time delay based on nonlinearity was considered in [12]. Containment control with doped random noise achieved the mean-square containment control utilizing properties of analytic semigroups and fixed point theorem in [13]. The formation containment control problem of surface warships under hierarchical sliding mode control strategy was studied in [14].

The above researches on containment control all consider the continuous transmission of signals between agents, which requires the communication network to communicate correctly in a constant time and high requirements for communication, and aggravates the battery energy consumption of the agents that some agents are battery-powered. The containment control under the event-triggered transmission mechanism [15-23] solves this problem. In the event-triggered transmission mechanism, only when the gap between the actual state and the reference level of the system exceeds a certain threshold, the system can communicate with each other to update the current state, thus effectively decreasing system communication frequency, decreasing the occupation of communication resources and saving energy. The event-triggered containment control (ETCC) problem of familiar linear MAS was studied in [15,16]. ETCC for heterogeneous MAS using an output-based regulation approach was investigated in [15]. The trigger mechanism in [16] was separated from the eigenvalue of the Laplacian matrix correlated with the information interaction topology. In [17], the authors' centralized and decentralized triggering rules were compared and analyzed for the ETCC problem of high-order nonlinear MAS. ETCC of the heterogeneous MAS was studied in [18,19]. The backstepping method, Lyapunov function method, and neural network were used to study the ETCC problem of high-order heterogeneous nonlinearity in [19]. Some scholars studied the ETCC problem in combination with the time-delay problems [20,21]. In [20], the sum-of-squares method was applied to the stability analysis of containment control problems, and ETCC for the low-order MAS with fixed time delay was studied. In [21], the observer-based ETCC problem for MAS with time delay was studied. Both [22] and [23] have investigated the ETCC problem of second-order MAS. The former is studied under the condition of MIMO multi-agent systems with aperiodic communication under external disturbance, while the latter by having sampled location data.

The communication topology considered in the above references is connected graphs, which have strict requirements for the topology. In reality, since the interaction area of information communication is limited, many dynamical systems may be subject to some unpredictable structural adjustments such as accidental failures and burst environmental perturbations [24,25], resulting in the communication topology that is disconnected and may switch at some time. At present, there is relatively little research on the joint-connected switching topology of multi-agent ETCC. To solve this problem, we study and propose a novel ETCC strategy of MAS with the condition of joint connected switching topology. There are three main contributions in this paper.

1) A multi-agent ETCC strategy with jointly connected switching topologies is proposed, and the containment error and state feedback control law with distributed observers are designed to transform the containment control problem into an output regulation problem. Different from [9], [10], and [11], to effectively utilize network resources and energy and avoid continuous monitoring, this paper adopts a multi-agent ETCC strategy to study the enveloping control problem.

2) The state-observer-based event-triggered control rule is designed with a jointly connected topology that has smaller communication constraints, which theoretically and experimentally excludes Zeno behavior. Unlike the fixed communication topology assumed in [15] and [16], this paper considers the joint connected switching topology, which has

weak requirements for the communication topology between agents. And the designed state observer can estimate the immeasurable state and still complete the containment control.

3) It is proved that the designed triggering strategy can make the followers converge asymptotically to the convex hull formed by multiple leaders, and the effectiveness of the state feedback control law and the event-triggered control rule is verified by simulation.

The remaining components of this article are as follows: Section 2 introduces the related knowledge of algebraic graph theory and joint connected switching topology and required theorems and lemmas; Section 3 describes the research problem and the design of the control law; the main results are given in Section 4; in Section 5, experimental simulations are carried out to prove the correctness of the theory; the last section draws conclusions.

Notations: $\boldsymbol{N}$ represents the set of positive integers. $\boldsymbol{R}^n$ represents the vector space on the n-dimensional real body R, $\boldsymbol{R}^{n\times n}$ represents the dimensional real matrix space. $\boldsymbol{M}\otimes\boldsymbol{N}$ is the *Kronecker* product of $\boldsymbol{M}$ and $\boldsymbol{N}$. $\text{diag}\{\boldsymbol{M}_1, \boldsymbol{M}_2, \ldots, \boldsymbol{M}_n\}$ represents the diagonal matrix with elements $\boldsymbol{M}_i$, $i = 1, 2, \ldots, n$. $\mathbf{1}_n$ is an n-dimensional column vector with all ones. $\boldsymbol{I}_n$ is an n-dimensional square matrix whose elements on the main diagonal are all ones and the rest are all zeros. $\mathbf{0}$ represents all zero matrices with proper dimension. $\boldsymbol{\rho}(A)$ is the minimum nonzero eigenvalue of matrix A. $\mathcal{H} > 0$ represents the matrix $\mathcal{H}$ is positive definite. $col(x_1, \ldots, x_m)$ is the column vector of elements.

2. Mathematic Preliminaries.

The communication topology of MAS is abstracted into a simple directed graph for description. The following introduces the related concepts and symbols of graph theory and the related definitions and lemmas of joint connected topology and enveloping control. These are the basis for solving the problem.

2.1. Graph theory.

Suppose that the Nth order directed graph $G = \{\boldsymbol{V}, \boldsymbol{\varepsilon}, \boldsymbol{A}\}$ is the information interaction topology among the agents, where $\boldsymbol{V} = \{1, 2, \ldots, N\}$ represents the set of vertices of N agents, and a single agent can be understood as a vertex of the weighted graph $G = \{\boldsymbol{V}, \boldsymbol{\varepsilon}, \boldsymbol{A}\}$. $\boldsymbol{\varepsilon} \in \boldsymbol{V} * \boldsymbol{V}$ denotes the set of edges formed between the nodes in the graph. $\boldsymbol{A} = (a_{ij}) \in \boldsymbol{R}^{n\times n}$ denotes the neighbor matrix of graph G, and i, j denotes the corresponding i, jth agent. The neighbors of the ith agent can be denoted by $N_i = \{j \in \boldsymbol{V} : (i, j) \in \boldsymbol{\varepsilon}, i \neq j\}$. Suppose there is no loop of its own nodes in the graph, i.e., $(i, i) \notin \boldsymbol{\varepsilon}$, if $(i, j) \in \boldsymbol{\varepsilon}$, then $a_{ij} > 0$, which means that point j is an adjacent node of point i, otherwise $a_{ij} = 0$. If $(i, j) \in \boldsymbol{\varepsilon}$ and $(j, i) \in \boldsymbol{\varepsilon}$, then the graph G is known as an undirected graph, otherwise it is a directed graph.

The information interaction topology graph G is composed of followers and leaders, which is a disconnected graph under the condition of switching topology. $d_{in}(v_n) = \sum_{i=1}^{n} a_{ij}$ is the in-degree of node i, and define the in-degree matrix of graph G as $\boldsymbol{D}_{in} = \text{diag}\{d_{in}(v_1), \ldots, d_{in}(v_n)\}$, and the Laplacian matrix as

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j \\ \sum_{i=1}^{n} a_{ij}, & i = j \end{cases}. \tag{1}$$

2.2. Definitions and related lemmas of joint-connected topology and containment control.

Definition 2.1. [26] *Assume the topological graphs $G_1, G_2, \ldots, G_\phi$ have the same set of vertices V, and $G_{1-\phi}$ represents the union of the set of edges of $G_1, G_2, \ldots, G_\phi$, and $G_{1-\phi}$ is jointly connected if $G_1, G_2, \ldots, G_\phi$ is connected. ϕ is the number of all possible topologies.*

Consider a set of bounded, nonempty and continuous time spans $[t_k, t_{k+1})$, $k = 0, 1, \ldots$, where $t_0 = 0$, $t_{k+1} - t_k \leq T_1$, $T_1 > 0$. There exist finite small time subintervals $\left[t_k^r, t_k^{r+1}\right)$, $r = 0, 1, \ldots, m_k-1$, $t_k^0 = t_k$, $t_k^{m_k} = t_{k+1}$, $m_k \geq 0$ within the time span $[t_k, t_{k+1})$, $k = 0, 1, \ldots$ and it satisfies $t_k^{r+1} - t_k^r \geq T_2$, $T_2 > 0$. In addition, when the information interaction topology graphs of the MAS are dynamically switched, we consider the switching signal ξ_t when $t \in [0, +\infty)$, $\xi_t \in \aleph$, $\aleph = \{1, 2, \ldots, \phi\}$, G_{ξ_t} denotes the topology graph of the multi-agent system at moment t, and $\mathcal{L}^{\xi_t}$ is defined as the corresponding Laplace matrix.

Definition 2.2. [27] *Define $\boldsymbol{Q}$ be the set of real vector space $\boldsymbol{W} \subseteq \boldsymbol{R}^n$. If there is a point $(1-c)x + cy \in \boldsymbol{Q}$ for any c $(0 \leq c < 1)$ and any x, y in the set $\boldsymbol{Q}$, then $\boldsymbol{Q}$ is convex. The convex hull of point set $Y = \{y_1, y_2, \ldots, y_n\}$ in $\boldsymbol{W}$ is the smallest convex set containing all points in Y, and expressed by* $\mathrm{Co}(Y)$, *then*

$$\mathrm{Co}(Y) = \left\{ \sum_{i=1}^{n} \boldsymbol{\beta}_i y_i | y_i \in Y, \boldsymbol{\beta}_i \in R, \boldsymbol{\beta}_i \geq 0, \sum_{i=1}^{n} \boldsymbol{\beta}_i = 1 \right\}. \tag{2}$$

Definition 2.3. [28] *In the topology graphs, n followers and m leaders are defined, and the leader's in-degree is zero. Considering the dynamic switching topology, the Laplacian matrix of G can be expressed as*

$$\mathcal{L}^{\xi_t} = \begin{bmatrix} \mathcal{L}_1^{\xi_t} & \mathcal{L}_2^{\xi_t} \\ \mathbf{0}_{m\times n} & \mathbf{0}_{m\times m} \end{bmatrix}, \tag{3}$$

where $\mathcal{L}_1^{\xi_t} \in \boldsymbol{R}^{n\times n}$ describes information interactions between followers, and $\mathcal{L}_2^{\xi_t} \in \boldsymbol{R}^{n\times m}$ describes information interactions between leaders and followers.

Assumption 2.1. [10] *The information interaction topology graph G_{ξ_t} of the multi-agent system switches at time t_k, and then its topology structure remains unchanged during span $[t_k, t_{k+1})$. For the followers in the connected subgraph, there exists at least one leader who communicates with the followers.*

Lemma 2.1. [19] *Under Assumption 2.1, all eigenvalues of $\mathcal{L}_1$ have positive real parts, every element of $-\mathcal{L}_1^{-1}\mathcal{L}_2$ is nonnegative, and the summary of every row element of matrix $-\mathcal{L}_1^{-1}\mathcal{L}_2$ is one.*

Definition 2.4. *For each subgraph G_i, $i \in \aleph$, $\delta_i = \boldsymbol{\rho}(L_P)$, since $\aleph$ is finite, the set $\{\delta_i \colon i \in \aleph\}$ is finite. Define*

$$\delta_{\min} = \min\{\delta_i : i \in \aleph\}, \tag{4}$$

which are positive and independent of time.

Lemma 2.2. [26] *During $[t_k, t_{k+1})$, the topological graph is jointly connected, if and only if*

$$\bigcup_{t\in[t_k,t_{k+1})} \zeta(\xi_t) = \{1, \ldots, N\}, \tag{5}$$

where $\zeta(\xi_t) = \{i\colon$ all nonzero eigenvalues λ_i of matrix $\mathcal{L}_1$ corresponding to $G_{\xi_t}, i \in 1, \ldots, N\}$.

3. **Problem Description and Design of Control Laws.**

Based on the knowledge in Section 2, we introduce the problem studied, and design a distributed state observer control law and event triggering mechanism for the system to solve the problem.

3.1. **Problem description.**

A system of $n + m$ agents is considered in this paper, which is composed of n followers represented as $\mathcal{F} = \{1, 2, \ldots, n\}$ and m leaders represented as $\mathcal{R} = \{n + 1, n + 2, \ldots, n + m\}$. The system description is as follows:

$$\begin{cases} \dot{\boldsymbol{x}}_i(t) = \boldsymbol{A}_i \boldsymbol{x}_i(t) + \boldsymbol{B}_i \boldsymbol{u}_i(t), & t \geq 0,\ i \in \mathcal{F}, \\ \dot{\boldsymbol{v}}_k(t) = S \boldsymbol{v}_k(t), & t \geq 0,\ i \in R, \end{cases} \tag{6}$$

where $\boldsymbol{x}_i(t) \subseteq \boldsymbol{R}^n$ indicates the state input of the ith follower and $\boldsymbol{u}_i(t) \in \boldsymbol{R}^p$ indicates the control input of the ith follower, $\boldsymbol{A}_i \in \boldsymbol{R}^{n \times n}$, $\boldsymbol{B}_i \in \boldsymbol{R}^{n \times m}$ are the known control gain matrices. $\boldsymbol{v}_k(t) \in \boldsymbol{R}^n$ is the state of the kth leader.

Definition 3.1. [29] *The multi-agent system is said to realize containment control if each follower converges to the convex hull formed by leaders under the action of the designed control law. That is*

$$\lim_{t \to \infty} dist\{x_i(t), \mathrm{Co}\{x_j(t) | i \in F, j \in R\}\} = 0. \tag{7}$$

Define $\boldsymbol{x} = col(x_1, \ldots, x_n)$, $\boldsymbol{v} = col(v_{n+1}, \ldots, v_{n+m})$, then the error of the followers as

$$\boldsymbol{e}(t) = (\mathcal{L}_1 \otimes \boldsymbol{I}_n)\boldsymbol{x} + (\mathcal{L}_2 \otimes \boldsymbol{I}_n)\boldsymbol{v}. \tag{8}$$

According to Lemma 2.1 and Definition 3.1, the multi-agent system (6) can realize containment control if and only if $\lim_{t \to \infty} \boldsymbol{e}(t) = 0$; thus,

$$\lim_{t \to \infty} \boldsymbol{x} = \left(-\mathcal{L}_1^{-1}\mathcal{L}_2 \otimes \boldsymbol{I}_n\right) \boldsymbol{v}. \tag{9}$$

3.2. **Design of distributed state observer control law.** In practical application, since some followers in MAS cannot obtain the leader's information, the state of the leader needs to be estimated by the followers' observer so that the followers can quickly follow up with the leaders in a certain time. The distributed state feedback control protocol of the ith follower is as follows [28]:

$$\begin{cases} \boldsymbol{u}_i(t) = \boldsymbol{K}_{i,1}\boldsymbol{x}_i(t) + \boldsymbol{K}_{i,2}\boldsymbol{\varsigma}_i(t), \\ \dot{\boldsymbol{\varsigma}}_i(t) = S\boldsymbol{\varsigma}_i(t) + \gamma \left[\sum_{j \in N_i} - a_{ij}(\boldsymbol{\varsigma}_i(t) - \boldsymbol{\varsigma}_j(t)) + \sum_{k=n+1}^{n+m} b_{k,i}(\boldsymbol{v}_k(t) - \boldsymbol{\varsigma}_i(t))\right], \end{cases} \tag{10}$$

where $\boldsymbol{K}_{i,1}, \boldsymbol{K}_{i,2} \in \boldsymbol{R}^{p \times N}$ are controller gain matrices to be found, which will be given below. γ is some positive constant. $\boldsymbol{\varsigma}_i(t) \in \boldsymbol{R}^N$ represents the state of dynamic compensator of the ith follower. $b_{k,i}$ is the weighting between the followers and the leaders.

Combine event-triggered control rule based on the control protocol (10), define $\left\{k_{w_i}^i\right\}$, $i \in \mathcal{F}$ as the w_i event-triggered moment of the ith follower and let $\hat{\boldsymbol{\varsigma}}_i(t) = \boldsymbol{\varsigma}_i\left(k_{w_i}^i\right)$ denote the marker data at the moment of the ith observer state trigger, and then protocol (10) can be rewritten as

$$\begin{cases} \boldsymbol{u}_i(t) = \boldsymbol{K}_{i,1}\boldsymbol{x}_i(t) + \boldsymbol{K}_{i,2}\hat{\boldsymbol{\varsigma}}_i(t), \\ \dot{\boldsymbol{\varsigma}}_i(t) = S\boldsymbol{\varsigma}_i(t) + \gamma \left[\sum_{j \in N_i} - a_{ij}(\hat{\boldsymbol{\varsigma}}_i(t) - \hat{\boldsymbol{\varsigma}}_j(t)) + \sum_{k=n+1}^{n+m} b_{k,i}(\boldsymbol{v}_k(t) - \hat{\boldsymbol{\varsigma}}_i(t))\right]. \end{cases} \tag{11}$$

The closed-loop system composed of system (6), containment error (8) and control law (11) can be formed as

$$\begin{cases} \dot{\boldsymbol{x}}_c(t) = \boldsymbol{A}_c\boldsymbol{x}_c(t) + \boldsymbol{B}_c\bar{\boldsymbol{v}}_k(t), \\ \dot{\bar{\boldsymbol{v}}}_k(t) = \bar{S}\bar{\boldsymbol{v}}(t), \\ \boldsymbol{e}(t) = \boldsymbol{D}_c\boldsymbol{x}_c(t) + \boldsymbol{E}_c\bar{\boldsymbol{v}}_k(t), \end{cases} \tag{12}$$

where $\boldsymbol{x}_c(t) = \left[\boldsymbol{x}_i(t)^T, \hat{\boldsymbol{\varsigma}}_i(t)^T\right]^T$ is the closed-loop state, $\bar{S} = \boldsymbol{I}_n \otimes S$, $\boldsymbol{D}_c = [\mathcal{L}_1 \otimes \boldsymbol{I}_n\ 0]$, $\boldsymbol{E}_c = \mathcal{L}_2 \otimes \boldsymbol{I}_n$. $\bar{\boldsymbol{v}}_k(t) = \boldsymbol{1}_n \otimes \boldsymbol{v}_k(t)$, $\boldsymbol{A}_c = \begin{bmatrix} A + BK_1 & BK_2 \\ 0 & \bar{S} - \mathcal{L}_1 \otimes \gamma \end{bmatrix}$, $\boldsymbol{B}_c = \begin{bmatrix} 0 \\ -\mathcal{L}_2 \otimes \gamma \end{bmatrix}$.

We transform the containment control problem (6) into an output regulation problem (12).

Definition 3.2. [30] *According to the control law (11), the closed-loop system (12) will meet the following two requirements.*

1) The matrix $\boldsymbol{A}_c$ has Hurwitz stability.

2) For a randomly given original condition $\boldsymbol{x}_c(0)$ and $\boldsymbol{v}_k(0)$, we have

$$\lim_{t\to\infty} \boldsymbol{e}(t) = \lim_{t\to\infty} \boldsymbol{D}_c\boldsymbol{x}_c(t) + \boldsymbol{E}_c\bar{\boldsymbol{v}}_k(t) = 0. \tag{13}$$

The following assumptions are made for Definition 3.2.

Assumption 3.1. *The matrix pair $(\boldsymbol{A}_i, \boldsymbol{B}_i)$, $i \in \mathcal{F}$ is stabilizable.*

Assumption 3.2. *The linear matrix equation*

$$S = \boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{U}_i, \quad i \in \mathcal{F} \tag{14}$$

has a solution $\boldsymbol{U}_i$.

Remark 3.1. *It should be noted that Assumptions 3.1 and 3.2 are standard in the literature on output regulation problems. Assumptions 3.1 and 3.2 also appear in [31,32]. In particular, the solvability of (14) is a necessary condition for the output regulation problem.*

Let $\mathcal{H} > 0$, and $\mathcal{H}$ is one answer of the Riccati equation (15) and Lyapunov inequality (16).

$$\mathcal{H}S + S^T\mathcal{H} - 2\delta_{\min}\mathcal{H}^2 + \delta_{\min}\boldsymbol{I} < 0, \tag{15}$$

$$\mathcal{H}S + S^T\mathcal{H} \leq 0. \tag{16}$$

Under Assumption 3.1, we can choose $\boldsymbol{K}_i^1$, $i \in \mathcal{F}$ such that $\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_i^1$ would be satisfied with Hurwitz condition. Let $\boldsymbol{K}_i^2$ be given by

$$\boldsymbol{K}_i^2 = \boldsymbol{U}_i - \boldsymbol{K}_i^1, \quad i \in \mathcal{F}, \tag{17}$$

where U_i is the solution of (14). From (14) we have

$$\bar{S} = \boldsymbol{A} + \boldsymbol{B}\boldsymbol{K}_1 + \boldsymbol{B}\boldsymbol{K}_2, \tag{18}$$

where $\bar{S} = \boldsymbol{I}_n \otimes S$, $\boldsymbol{A} = \mathrm{diag}\{\boldsymbol{A}_1, \boldsymbol{A}_2, \ldots, \boldsymbol{A}_n\}$, $\boldsymbol{B} = \mathrm{diag}\{\boldsymbol{B}_1, \boldsymbol{B}_2, \ldots, \boldsymbol{B}_n\}$, $\boldsymbol{K}_1 = \mathrm{diag}\{\boldsymbol{K}_1^1, \boldsymbol{K}_2^1, \ldots, \boldsymbol{K}_n^1\}$, $\boldsymbol{K}_2 = \mathrm{diag}\{\boldsymbol{K}_1^2, \boldsymbol{K}_2^2, \ldots, \boldsymbol{K}_n^2\}$.

3.3. **Design of event trigger rule.** The distributed event-triggered algorithms for follower multi-agents are designed as follows:

$$\|\Xi\| > \frac{\sigma\delta_{\min}}{2\mu\varphi}\sqrt{\frac{1}{\beta}}\|\mathrm{Z}\|, \tag{19}$$

where $0 \leq \sigma < 1$, $\varphi = \|(\boldsymbol{I}_n \otimes \mathcal{H}^2)\|$, $\beta = \|(\mathcal{L}_1 \otimes \boldsymbol{I}_{n+m})^2\|$, $\mathrm{Z} = col(\mathrm{Z}_1, \ldots, \mathrm{Z}_n)$, $\Xi = col(\Xi_1, \ldots, \Xi_n)$, $\boldsymbol{I}_{n+m} \otimes \gamma = \mu\mathcal{H}$.

$$\mathrm{Z}_i = \sum_{j\in N_i} - a_{ij}(\boldsymbol{\varsigma}_i(t) - \boldsymbol{\varsigma}_j(t)) + \sum_{k=n+1}^{n+m} b_{k,i}(\boldsymbol{v}_k(t) - \boldsymbol{\varsigma}_i(t)), \tag{20}$$

$$\Xi_i = \sum_{j\in N_i} a_{ij}(\hat{\boldsymbol{\varsigma}}_j(t) - \hat{\boldsymbol{\varsigma}}_i(t)) + \sum_{k=n+1}^{n+m} b_{k,i}(\boldsymbol{v}_k(t) - \hat{\boldsymbol{\varsigma}}_i(t)) - \mathrm{Z}_i. \tag{21}$$

The follower agents update their information at the trigger moment $\{k_{w_i}^i\}$, $i \in \mathcal{F}$ when the conditions for the event trigger are satisfied.

4. **Main Result.** In this study, an event-triggering control protocol based on state observer is used to ensure the containment control of multi-agent systems under the condition of joint connected switching topology. In Section 3, we give the dynamics equation of the system, and the designed state observer protocol and event-triggering control mechanism. The advantage of this method is that it can reduce communication energy consumption while ensuring the completion of enveloping control of the system even when the communication topology is constantly changing. According to the above analysis, the following theorems are obtained.

Theorem 4.1. *Considering the multi-agent system (6), according to Assumption 3.1 and Assumption 3.2, the switching topology form exhibited each time the switching signal* ξ_t *occurs satisfies Assumption 2.1. According to Equations (15), (16), and (17), obtaining* P *and* K*, under the control law (11) and the trigger control algorithm (19), each follower can enter the convex hull formed by the leaders under any original condition.*

Proof: From (6), (11), (20) and (21), we have

$$\dot{\varsigma}_i(t) = S\varsigma_i(t) + \gamma \left[\sum_{j \in N_i} - a_{ij}(\varsigma_i(t) - \varsigma_j(t)) + \sum_{k=n+1}^{n+m} b_{k,i}(\boldsymbol{v}_k(t) - \varsigma_i(t)) \right] + \gamma \Xi_i. \tag{22}$$

It follows from (6), (8), (11) and (22) that

$$\dot{\boldsymbol{x}}_c(t) = \boldsymbol{A}_c \boldsymbol{x}_c(t) + \sum_{k=n+1}^{n+m} \boldsymbol{B}_c \bar{\boldsymbol{v}}_k(t) + (\boldsymbol{I}_{3n} \otimes \gamma) \begin{pmatrix} 0 \\ 0 \\ \Xi \end{pmatrix}. \tag{23}$$

Let $\varpi = \boldsymbol{\varsigma} - (-\mathcal{L}_1^{-1}\mathcal{L}_2 \otimes \boldsymbol{I}_N)\boldsymbol{v}$, then the derivative of l with respect to time is as follows:

$$\begin{aligned} \dot{\varpi} &= (\boldsymbol{I}_n \otimes S)\boldsymbol{\varsigma} - (\mathcal{L}_1 \otimes \gamma)\boldsymbol{\varsigma} - (\mathcal{L}_2 \otimes \gamma)\boldsymbol{v} + (\boldsymbol{I}_n \otimes \gamma)\Xi + \left(\mathcal{L}_1^{-1}\mathcal{L}_2 \otimes S\right)\boldsymbol{v} \\ &= ((\boldsymbol{I}_n \otimes S) - (\mathcal{L}_1 \otimes \gamma))\varpi + (\boldsymbol{I}_n \otimes \gamma)\Xi. \end{aligned} \tag{24}$$

We adopt the Lyapunov function $V = \varpi^T(\boldsymbol{I}_n \otimes \mathcal{H})\varpi$, then

$$\begin{aligned} \dot{V} &= 2\varpi^T(\boldsymbol{I}_n \otimes \mathcal{H})(((\boldsymbol{I}_n \otimes S) + (\boldsymbol{I}_n \otimes \gamma)\Xi - (\mathcal{L}_1 \otimes \gamma))\varpi) \\ &= \varpi^T\left(\boldsymbol{I}_n \otimes \left(\mathcal{H}S + S^T\mathcal{H}\right) - \left(\mathcal{L}_1 \otimes 2\mu\mathcal{H}^2\right)\right)\varpi + 2\varpi^T\left(\boldsymbol{I}_n \otimes \mu\mathcal{H}^2\right)\Xi. \end{aligned} \tag{25}$$

Let $\tilde{\varpi} = (\mathfrak{J} \otimes \boldsymbol{I}_N)\varpi$, where $\mathfrak{J}$ is an orthogonal matrix and meets the condition $\mathfrak{J}\mathcal{L}_1\mathfrak{J}^T = \mho = \operatorname{diag}\{\lambda_1, \ldots, \lambda_n\}$, and λ_n is the nth eigenvalue of $\mathcal{L}_1$.

$$\begin{aligned} \dot{V} &= \tilde{\varpi}^T\left(\boldsymbol{I}_n \otimes \left(\mathcal{H}S + S^T\mathcal{H}\right) - \left(\mho \otimes 2\mu\mathcal{H}^2\right)\right)\tilde{\varpi}\frac{1}{2} + 2\mu\varpi^T\left(\boldsymbol{I}_n \otimes \mathcal{H}^2\right)\Xi \\ &\le \sum_{n}^{i=1} \tilde{\varpi}_i^T\left(\mathcal{H}S + S^T\mathcal{H} - 2\lambda_i\mu\mathcal{H}^2\right)\tilde{\varpi}_i + \varepsilon\mu\varpi^T\left(\boldsymbol{I}_n \otimes \mathcal{H}^2\right)\left(\boldsymbol{I}_n \otimes \mathcal{H}^2\right)^T\varpi + \frac{\mu}{\varepsilon}\Xi^T\Xi \\ &\le -\delta_{\min}\sum_{n}^{i=1}\tilde{\varpi}_i^T\tilde{\varpi}_i + \varepsilon\mu\varphi^2\varpi^T\varpi + \frac{\mu}{\varepsilon}\Xi^T\Xi \\ &\le -\left(\delta_{\min} - \varepsilon\mu\varphi^2\right)\varpi^T\varpi + \frac{\mu\left(\frac{\sigma\delta_{\min}}{2\mu\varphi}\sqrt{\frac{1}{\beta}}\right)^2}{\varepsilon}\mathrm{Z}^T\mathrm{Z}. \end{aligned} \tag{26}$$

Let $\varepsilon = \frac{\delta_{\min}}{2\mu\varphi^2} > 0$ and $\tilde{\varpi}^T\tilde{\varpi} = \varpi^T\left(\mathfrak{J}\mathfrak{J}^T \otimes \boldsymbol{I}_N\right)\varpi = \varpi^T\varpi$, and one has

$$\dot{V} \le -\left(\delta_{\min} - \frac{\delta_{\min}}{2}\right)\varpi^T\varpi + \frac{\sigma^2\delta_{\min}}{2\beta}\mathrm{Z}^T\mathrm{Z}$$

$$\leq -\frac{\delta_{\min}}{2}\varpi^T\varpi + \frac{\sigma^2\delta_{\min}}{2\beta}\varpi^T(\mathcal{L}_1\otimes \boldsymbol{I}_N)^T(\mathcal{L}_1\otimes \boldsymbol{I}_N)\varpi$$
$$\leq -\frac{(1-\sigma^2)\,\delta_{\min}}{2}\varpi^T\varpi$$
$$\leq 0. \tag{27}$$

Considering the infinite sequences $V(t_i)$, $i=0,1,\dots$ and using Cauchy's convergence criteria, we have that, for any $\forall\vartheta>0$, there exists a positive number M_ϑ, such that $\forall K\geq M_\vartheta$,

$$\left|\int_{t_k}^{t_{k+1}}\dot{V}(t)\mathrm{d}t\right| < \vartheta. \tag{28}$$

Rewrite (28) into the following sum of integrals

$$\int_{t_k^0}^{t_k^1}\left[-\dot{V}(t)\right]\mathrm{d}t+\cdots+\int_{t_k^{t_k-1}}^{t_k^{m_k}}\left[-\dot{V}(t)\right]\mathrm{d}t<\vartheta, \tag{29}$$

for each integral

$$\int_{t_k^i}^{t_k^{i+1}}\left[-\dot{V}(t)\right]\mathrm{d}t \geq \delta_{\min}\int_{t_k^i}^{t_k^{i+1}}\sum_{i\in\zeta\left(\xi\left(t_k^i\right)\right)}\tilde{\varpi}_i^T\tilde{\varpi}_i\mathrm{d}t \geq \delta_{\min}\int_{t_k^i}^{t_k^i+\tau}\sum_{i\in\zeta\left(\xi\left(t_k^i\right)\right)}\tilde{\varpi}_i^T\tilde{\varpi}\mathrm{d}t, \tag{30}$$

$i=1,2,\dots,N$. Thus,

$$\vartheta>\delta_{\min}\left[\int_{t_k^0}^{t_k^0+T_2}\sum_{i\in\zeta\left(\xi\left(t_k^0\right)\right)}\tilde{\varpi}_i^T\tilde{\varpi}_i\mathrm{d}t+\cdots+\int_{t_k^{m_k-1}}^{t_k^{m_k-1}+T_2}\sum_{i\in\zeta\left(\xi\left(t_k^{m_k-1}\right)\right)}\tilde{\varpi}_i^T\tilde{\varpi}_i\mathrm{d}t\right]. \tag{31}$$

Noting that there are finite switches in the span $[t_k,t_{k+1})$, the number m_k is finite for each $k=0,1,\dots,m_k$. Thus, for $K>M_\vartheta$, there is

$$\vartheta>\delta_{\min}\int_{t_k^i}^{t_k^i+T_2}\sum_{i\in\zeta\left(\xi\left(t_k^0\right)\right)}\tilde{\varpi}_i^T\tilde{\varpi}_i\mathrm{d}t,\quad i=0,1,\dots,m_k-1, \tag{32}$$

or

$$\lim_{t\to\infty}\int_t^{t+T_2}\sum_{i\in\zeta\left(\xi\left(t_k^0\right)\right)}\tilde{\varpi}_i^T(s)\tilde{\varpi}_i(s)\mathrm{d}s=0,\quad i=0,1,\dots,m_k-1. \tag{33}$$

Thus,

$$\lim_{t\to\infty}\int_t^{t+T_2}\left[\sum_{i\in\zeta\left(\xi\left(t_k^0\right)\right)}\tilde{\varpi}_i^T(s)\tilde{\varpi}_i(s)+\cdots+\sum_{i\in\zeta\left(\xi\left(t_k^{m_k-1}\right)\right)}\tilde{\varpi}_i^T(s)\tilde{\varpi}_i(s)\right]\mathrm{d}s=0. \tag{34}$$

From Lemma 2.2, since $\bigcup_{t\in[t_k,t_{k+1})}\zeta(\xi_t)=\{1,\dots,N\}$ due to the joint connectivity of the graphs during $[t_k,t_{k+1})$, (34) can be written as

$$\lim_{t\to\infty}\int_t^{t+T_2}\left[\sum_{i=1}^N a_i\tilde{\varpi}_i^T(s)\tilde{\varpi}_i(s)\right]\mathrm{d}s=0, \tag{35}$$

where $a_1,a_2,\dots,a_N$ are some positive integers. $\dot{V}(t)<0$ implies $\varpi(t)$ is bounded and so is $\dot{\varpi}(t)$ according to Formula (24). Thus, $\sum_{i=1}^N a_i\tilde{\varpi}_i^T(s)\tilde{\varpi}_i(s)$ is uniformly continuous.

Invoking Barbalat's Lemma, we can conclude that $\lim_{t\to\infty}\sum_{i=1}^{N} a_i\tilde{\varpi}_i^T(t)\tilde{\varpi}_i(t) = 0$. This implies that

$$\lim_{t\to\infty} \varpi_i(t) = 0, \quad i = 0, 1, \ldots, N. \tag{36}$$

Therefore, each follower can enter the convex hull formed by the leaders.

After proving that the event-triggered control law can achieve the containment control under the joint connectivity topology, we study the bounds of event-triggered conditions. The presence of Zeno behavior in event triggering would make the above proof impossible. The Zeno behavior of the event trigger mechanism is that the system event trigger is fired an infinite number of times per unit of time. This does not meet the purpose of this article to save resources by designing event triggering control mechanisms. Theorem 4.2 calculates the minimum time interval between event triggers and proves that the system does not have Zeno behavior.

Theorem 4.2. *Consider the heterogeneous linear multi-agent (6) with the protocol (11). Suppose all assumptions and conditions in Theorem 4.1 hold, and the triggering condition is given as (19). Then for any initial condition, the event-triggered span* $\left[k_{w_{i+1}}^i - k_{w_i}^i\right)$ *is lower bounded by* τ, $\tau > 0$ *and is given by the following equation:*

$$\begin{aligned}
&\tau = \frac{\sigma\delta_{\min}}{2\mu\varphi\sqrt{\beta}(\|(\boldsymbol{I}_n \otimes S)\| + \|(\mathcal{L}_1 \otimes \mu\mathcal{H})\|) + \sigma\delta_{\min}\|(\mathcal{L}_1 \otimes \mu\mathcal{H})\|},\\
&k_{w_{i+1}}^i - k_{w_i}^i \geq \tau,\ i \in \{1, 2, \ldots\}.
\end{aligned} \tag{37}$$

Proof: It can be concluded from (19)-(21) and (24) that

$$\begin{aligned}
\frac{\mathrm{d}\|\Xi\|}{\mathrm{d}t} &= \frac{\Xi^T\dot{\Xi}}{\|\Xi\|} \leq \left\|-\dot{\mathrm{Z}}\right\| = \|(\mathcal{L}_1 \otimes \boldsymbol{I}_N)\dot{\boldsymbol{\varsigma}} + (\mathcal{L}_2 \otimes \boldsymbol{I}_N)\dot{\boldsymbol{v}}\| = \|(\mathcal{L}_1 \otimes \boldsymbol{I}_N)\dot{\varpi}\|\\
&= \|(\mathcal{L}_1 \otimes \boldsymbol{I}_N)(((\boldsymbol{I}_n \otimes S) - (\mathcal{L}_1 \otimes \gamma))\varpi + (\boldsymbol{I}_n \otimes \gamma)\Xi)\|\\
&\leq ((\boldsymbol{I}_n \otimes S) + (\mathcal{L}_1 \otimes \mu\mathcal{H}))\|\mathrm{Z}\| + (\mathcal{L}_1 \otimes \mu\mathcal{H})\|\Xi\|\\
&\leq \left((\boldsymbol{I}_n \otimes S) + (\mathcal{L}_1 \otimes \mu\mathcal{H}) + \frac{\sigma\delta_{\min}}{2\mu\varphi}\sqrt{\frac{1}{\beta}}(\mathcal{L}_1 \otimes \mu\mathcal{H})\right)\|\mathrm{Z}\|, \quad t \in \left[k_{w_i}^i, k_{w_{i+1}}^i\right).
\end{aligned} \tag{38}$$

As a result,

$$\begin{aligned}
\tau = k_{w_{i+1}}^i - k_{w_i}^i &\geq \frac{\frac{\sigma\delta_{\min}}{2\mu\varphi}\sqrt{\frac{1}{\beta}}\|\mathrm{Z}\|}{\left((\boldsymbol{I}_n \otimes S) + (\mathcal{L}_1 \otimes \mu\mathcal{H}) + \frac{\sigma\delta_{\min}}{2\mu\varphi}\sqrt{\frac{1}{\beta}}(\mathcal{L}_1 \otimes \mu\mathcal{H})\right)\|\mathrm{Z}\|}\\
&= \frac{\sigma\delta_{\min}}{2\mu\varphi\sqrt{\beta}((\boldsymbol{I}_n \otimes S) + (\mathcal{L}_1 \otimes \mu\mathcal{H})) + \sigma\delta_{\min}(\mathcal{L}_1 \otimes \mu\mathcal{H})} > 0.
\end{aligned} \tag{39}$$

Therefore, the lower limit of event-triggered span $\left\{k_{w_{i+1}}^i - k_{w_i}^i\right\}$ are τ, $\tau > 0$, and the Zeno behavior is ruled out.

5. **Simulation Example.** Based on theoretical analysis, the ETCC of multi-agent with joint connected switching topology is verified utilizing a simulation example. The system consists of seven agents, as shown in Figure 1, where 1, 2, 3, and 4 represent followers and 5, 6, and 7 represent leaders. The choice of switching topology of the multi-agent system should satisfy the requirement that its union should be a joint-connected graph. Secondly, according to the lemma of the second part, the appropriate coefficient matrix and gain matrix are selected. The relevant characteristics of the Laplacian matrix of the communication topology graph meet the requirements of the lemma, which is also the key to selecting a specific example.

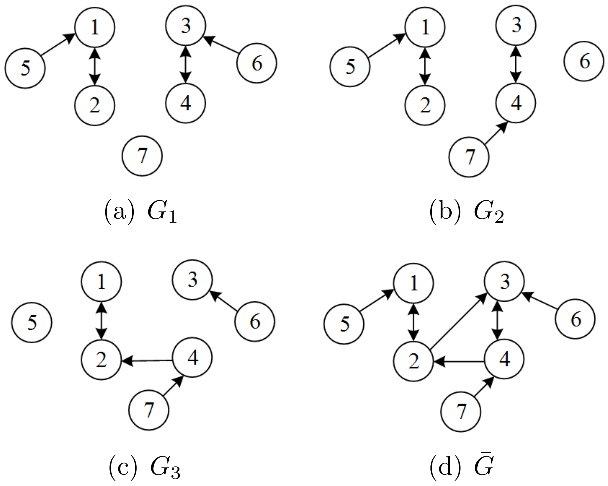

FIGURE 1. The communication topology of MAS

The matrix of the dynamic equation corresponding to the system (6) is given as follows: $S = \begin{bmatrix} 1 & -3 \\ 1 & -1 \end{bmatrix}$, $\boldsymbol{A}_1 = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$, $\boldsymbol{B}_1 = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$, $\boldsymbol{A}_2 = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$, $\boldsymbol{B}_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$, $\boldsymbol{A}_3 = \begin{bmatrix} 2 & -1 \\ 3 & 3 \end{bmatrix}$, $\boldsymbol{B}_3 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\boldsymbol{A}_4 = \begin{bmatrix} -1 & -4 \\ 3 & 0 \end{bmatrix}$, $\boldsymbol{B}_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

According to Equation (14), we can get $\boldsymbol{U}_1 = [0 \;\; 1]$, $\boldsymbol{U}_2 = [1 \;\; 3]$, $\boldsymbol{U}_3 = [-1 \;\; -2]$, $\boldsymbol{U}_4 = [-2 \;\; -1]$. Let $\boldsymbol{K}_{1,1} = [1 \;\; 1]$, $\boldsymbol{K}_{2,1} = [-0.5 \;\; 9]$, $\boldsymbol{K}_{3,1} = [2 \;\; -5]$, $\boldsymbol{K}_{4,1} = [2 \;\; 0]$. From Equation (17), one has $\boldsymbol{K}_{1,2} = [-1 \;\; 0]$, $\boldsymbol{K}_{2,2} = [1.5 \;\; -6]$, $\boldsymbol{K}_{3,2} = [-3 \;\; 3]$, $\boldsymbol{K}_{4,2} = [-4 \;\; -1]$. From Laplacian matrix, one has $\delta_{\min} = 0.328$, and choose $\mu = 0.75$. According to Equation (15), we can get $\mathcal{H} = \begin{bmatrix} 0.2694 & 0.1741 \\ 0.1741 & 0.1906 \end{bmatrix}$.

From Figure 1, the possible information interaction topologies of the MAS are $\{G_1, G_2, G_3\}$. The topology is switched in order of preference $G_1 \to G_2 \to G_3 \to G_1 \to \cdots$, and every graph is active for 0.3 s, then 0.9 s is taken as a cycle, and the topology in each cycle constitutes a joint connected topology of a graph. The process of topology switching is shown in Figure 2.

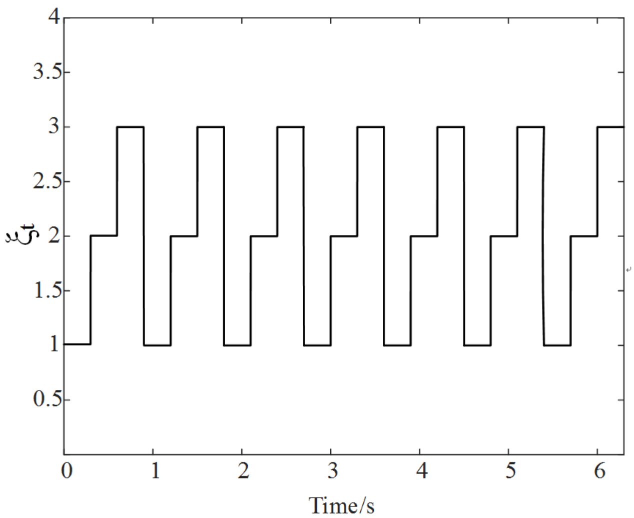

FIGURE 2. Process of switching topologies

Consider the three joint connected graphs G_1, G_2, and G_3 in Figure 1. Its union is connected. The Laplacian matrix of each topological graph is as follows:

$$\mathcal{L}_1^1 = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad \mathcal{L}_1^2 = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix},$$

$$\mathcal{L}_1^3 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathcal{L}_2^1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{L}_2^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \mathcal{L}_2^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

Denote the four eigenvalues of $\mathcal{L}_1^1$ as $\lambda_1 = 0.328$, $\lambda_2 = 0.328$, $\lambda_3 = 2.618$, $\lambda_4 = 2.618$; $\zeta(1) = \{1, 2, 3, 4\}$. Denote the four eigenvalues of $\mathcal{L}_1^2$ as $\lambda_1 = 0.328$, $\lambda_2 = 0.328$, $\lambda_3 = 2.618$, $\lambda_4 = 2.618$; $\zeta(2) = \{1, 2, 3, 4\}$. Denote the four eigenvalues of $\mathcal{L}_1^3$ as $\lambda_1 = 0.328$, $\lambda_2 = 2.618$, $\lambda_3 = 1$, $\lambda_4 = 1$; $\zeta(3) = \{1, 2, 3, 4\}$. Thus, $\zeta(1) \bigcup \zeta(2) \bigcup \zeta(3) = \{1, 2, 3, 4\}$, which illustrates the result in Lemma 2.2.

Figure 3 and Figure 4 show the state trajectories of the multi-agents under two groups of different initial values. For the convenience of description, we named the simulation result generated by the first group of data and the simulation result generated by the second group of data as *group1* and *group2*, respectively. It can be seen that under different initial conditions, all subsequent agents of the system can quickly converge to the convex hull formed by the leaders. From the trigger moments of the follower agents shown in Figure 5, the communication between the agent and its neighbor is not continuous but occurs when the event trigger condition is met. This greatly reduces the loss of power and network space and also verifies that there is no Zeno behavior in the system.

According to Figure 6, under different initial states, the errors between the real-time states and the trigger time states of all the following agents converge to zero rapidly and finally reach the convex hull of the leaders. Figure 7 and Figure 8 show the state error between the observed value and the actual value of the follower agents states under the

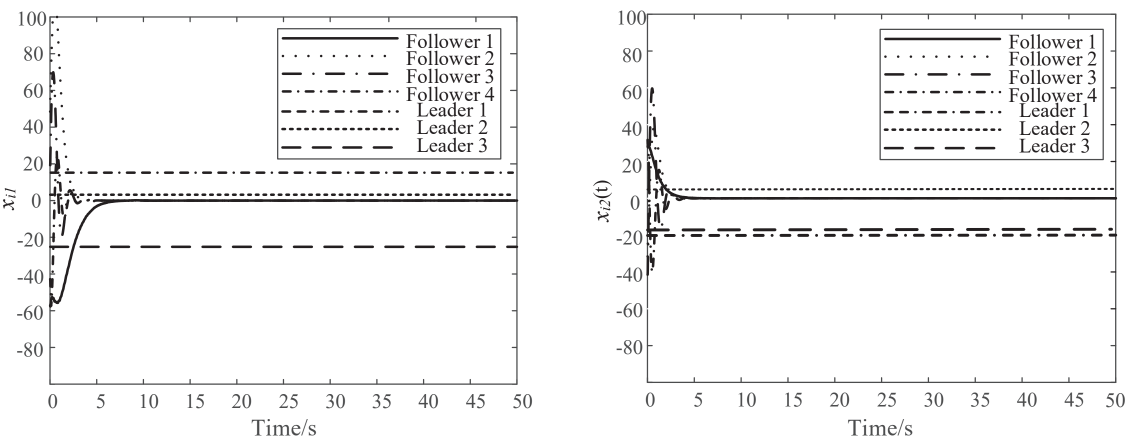

FIGURE 3. State trajectories $x_{i1}(t)$, $x_{i2}(t)$ of multi-agents of *group1*

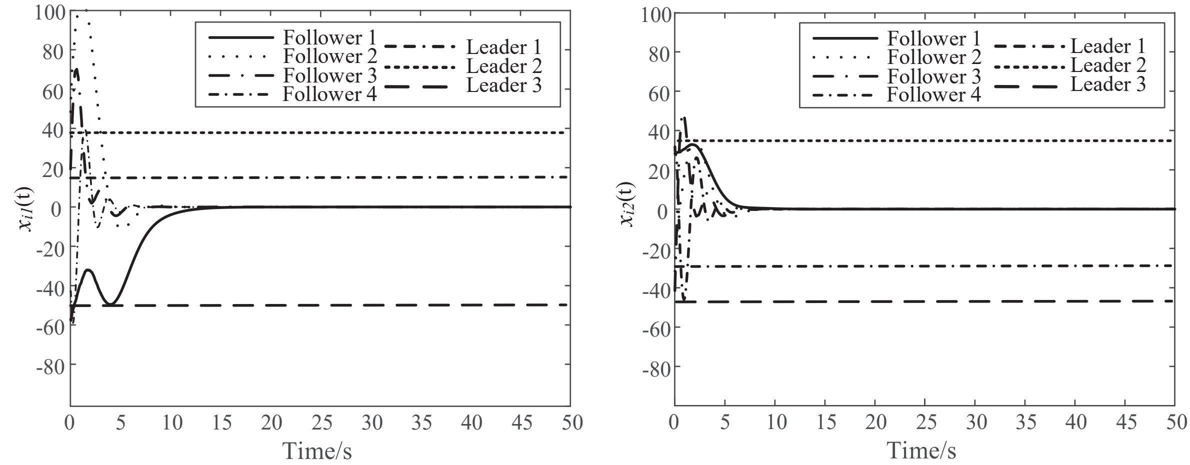

FIGURE 4. State trajectories $x_{i1}(t)$, $x_{i2}(t)$ of multi-agents of *group2*

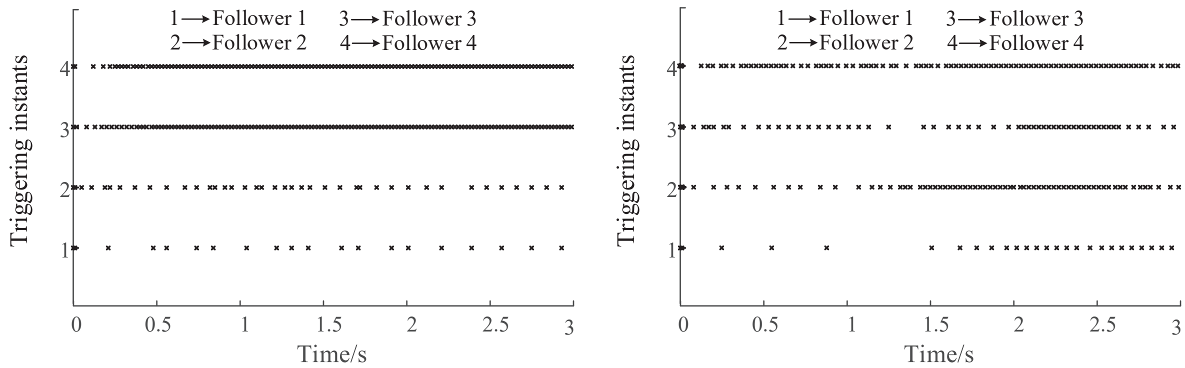

FIGURE 5. Event-triggered times (*group1* on the left)

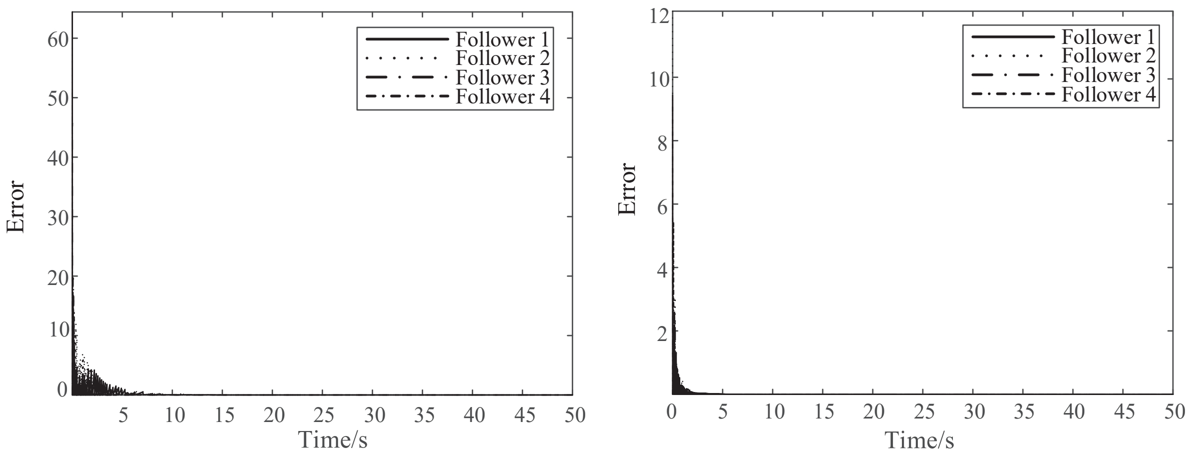

FIGURE 6. Error between real-time states and trigger time states (*group1* on the left)

two groups of initial values. It can be seen from the figure that the error between the observed value and the actual value of the follower agents state converges rapidly to zero under the designed control law. Therefore, simulation experiments verify the correctness of the theoretical results.

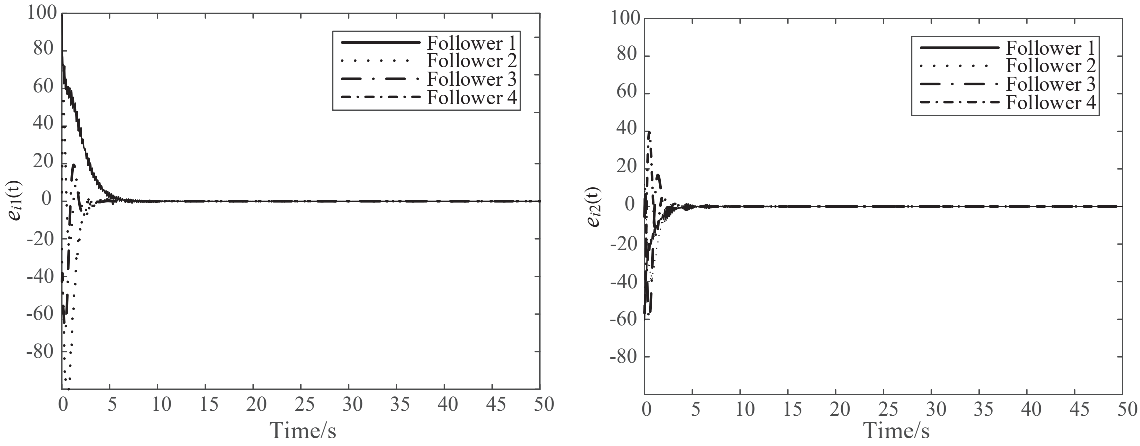

FIGURE 7. The error $e_{i1}(t)$ and $e_{i2}(t)$ between the observed and actual values of *group1*

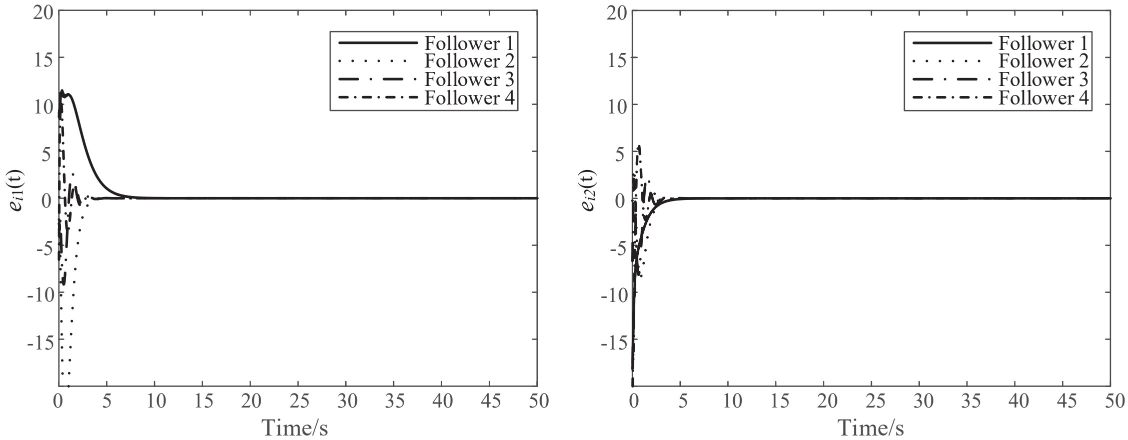

FIGURE 8. The error $e_{i1}(t)$ and $e_{i2}(t)$ between the observed and actual values of *group2*

6. **Conclusions.** In this paper, we study the ETCC problem for the joint connectivity switching topology. The event-triggered joint connectivity topology is more realistic and has smaller communication constraints. We transform the containment control problem into an output regulation problem by designing the state feedback control law with containment errors and distributed observers. The designed event-triggered control law based on the state observer is achieved of containment control of multi-agent, excluding Zeno behavior, and effectively reduces the communication load while reducing power consumption. Finally, we verify the feasibility of the designed control mechanism and event-triggered rule by numerical simulations. In the future, the ETCC problem of joint connected switching topology under random disturbance can be considered to further improve the control algorithm and optimize the simulation parameters. In addition, communication delays are common in the system, so how to design the controller to complete the containment control is also a problem to be solved.

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Author Biography

Wenjun Wei is a doctor of engineering, professor and graduate tutor. He is an important member of the innovation team of "Changjiang Scholars and Innovation Team Development Plan" of the Ministry of Education, "Research on Complete Set of Coating Equipment and Coating Process of Computer Fully Automatic Control of Ultra-Large Volume Automobile Lamps and Lanterns" of the national key problem solving plan, and "Research, Development and Transformation of Railway Station Full Electronic Intelligent Control Series Module" of the national innovation fund.

Jiahui Lv received the B.Sc. degree in rail transit signal from Lanzhou Jiaotong University, Jinzhou, China, in 2020, where she is currently pursuing the M.Sc. degree with the College of Traffic Information Engineering and Control. Her current research interests include multi-agent containment control, event-trigged control.