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# ADAPTIVE EVENT-TRIGGERED ASYMPTOTIC TRACKING FOR A CLASS OF NONLINEAR SYSTEMS WITH ACTUATOR FAILURES AND TIME-DELAYS

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Abstract. *In this paper, event-trigger based adaptive control scheme is developed to address the robust fault-tolerant compensation control problem for Lipschitz nonlinear system with actuator failures and multiple state time delays. While both eventual faults and delayed-state perturbations are unknown, the adaptive laws are constructed to estimate the unknown controller paraments online. Then, by using a positive control gain function and an integrable auxiliary signal, a class of robust adaptive event-triggered controllers is designed to compensate the effects of time-delay functions, compound disturbances, and actuator faults. In addition, by introducing the event-trigger control strategy can reduce the system redundancy greatly. On the basis of Lyapunov-Krasovskii theory, it is shown that the proposed control method can ensure all the closed-loop signals of the system are bounded and the system states converge zero asymptotically. Finally, the simulation results are considered to illustrate the efficiency of the adaptive event-triggered control scheme.*

**Keywords:** Adaptive fault-tolerant control, Unknown time delay function, Event-triggered control, Asymptotic stability, Lipschitz nonlinear systems

1. **Introduction.** In recent years, with the development of industrial technology, the complexity of the actual systems model is getting higher and higher. The analysis of the corresponding control systems and the design of adaptive fault-tolerant controller are also widely considered in various fields, such as network systems [1, 2, 3], power systems [4, 5], and aircraft systems [6, 7, 8]. On the one hand, in the actual industrial systems, due to components aging, failure and other factors, the actuator or sensor in the process of high load operation is inevitable failure, if the systems cannot be maintained timely and effectively, it may lead to systems performance decline, even instability phenomenon. Therefore, many adaptive fault-tolerant control (AFTC) schemes have been studied to solve this problem [9, 10, 11, 12]. In [9], a robust adaptive fault-tolerant control was considered for a class of linear systems with actuator faults and parameter uncertainties. Then, a class of robust adaptive state feedback controllers was designed in [10] to solve the problem of robust fault-tolerant control for linear systems with mismatched parameters. In [11], a neural networks-based fault tolerant controller was designed for fault models with unknown control inputs. The work in [12] provided a robust adaptive fault-tolerant controller for a closed-loop reference model with actuator faults. However, the problems they considered were limited to linear systems and cannot be applied to nonlinear systems studied in this paper.

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On the other hand, the above problems are all considered without time delay. As we all know, time delay often makes the systems unstable or performance degradation in the actual control systems, so it is necessary to study the systems with time delay. Zhang et al. [13] considered a robust adaptive control scheme for a class of nonlinear systems with time delays and dead zones and guaranteed the accurate tracking of reference signals. In [14], Qi et al. studied the finite-time adaptive control problem for a class of nonlinear systems with quantized input signals and unknown time delays. However, there exist a few works to address the FTC problem with time delay such as [15, 16]. Li and Yang [15] studied a robust adaptive control problem of nonlinear systems with time-varying delay. Wu and Park [16] addressed the FTC problem for a class of uncertain switched nonlinear systems with time delay and actuator failure.

In reality, continuous signals in the control systems will lead to waste of resources, and increase the burden and loss of the system. Therefore, in order to reduce unnecessary redundancy of the system and save system resources, the control method based on event-triggered control (ETC) scheme came into being. In recent years, a large number of event-triggered control strategies have been proposed. For example, two kinds of event-triggered cooperative controller strategies were proposed in [17] for strict feedback nonlinear systems, and the proposed cooperative control strategy canceled the assumption of the input-to-state stability (ISS). For a class of linear systems with time delay and disturbance, Chen et al. [18] designed an event-trigger based adaptive controller to guarantee the asymptotic convergence of the system state to zero. In [19], Li et al. designed an ETC method for a class of nonlinear systems with gain function. Then, the problem of event-triggered control was concerned for stochastic nonlinear systems in [20]. The work in [21] studied a class uncertain nonlinear systems development events triggered by the adaptive tracking controller problem. In [22], an event-triggered controller which used dynamic gain was designed to solve the problem of adaptive output feedback control for a class of switched stochastic nonlinear systems. Moreover, the authors in [23] designed an event-triggered controller based on the model for a continuous time nonlinear systems.

Motivated by the observations above, this paper tries to solve the problem of adaptive event-triggered control for a class of Lipschitz nonlinear systems with actuator faults and multiple state-time delays. The main contributions of this paper can be summarized as follows.

(i) The system considered in this paper contains unknown external disturbances, unpredictable actuator faults and multiple state-time delays which are different from recent articles [9, 10, 11, 12, 18, 26]. Based on a positive gain function, the effects of compound disturbances and actuator faults were compensated perfectly. Meanwhile, the system states asymptotically converge to zero.

(ii) This paper expanded on the work of [15], the problem of the adaptive event-triggered control co-design was considered, and reduced the systems redundancy effectively.

(iii) Different from the results in [16, 18], the unknown time-varying delay was allowed in this paper. The display expression for the control gain function was provided to address the errors which were caused by time-varying delay.

The paper is organized as follows. The system model and control objective are proposed in Section 2. In Section 3, the adaptive event-triggered fault-tolerant controller with adaptation updated laws is designed. Section 4 gives the main result and the analysis of the closed-loop system, and two simulation results are presented in Section 5. Finally, conclusions are drawn in Section 6.

## 2. Problem Formulation and Problem Statement.

Consider a class of nonlinear systems as follows:

$$\dot{x}(t) = Ax(t) + g(x(t)) + B(u(t) + f(x_\tau)) + B_d d(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input and $d(t) \in \mathbb{R}^l$ is the bounded unknown disturbance. In addition, $x_\tau = [x(t-\tau_1(t)), x(t-\tau_2(t)), \ldots, x(t-\tau_n(t))]^T \in \mathbb{R}^n$ is a delayed-state vector, $\tau_i(t)$, $i = 1, 2, \ldots, n$, is the time-varying time delay satisfying $\tau_i(t) \le \tau_i^*$, $\dot{\tau}_i^* \le \bar{\tau}_i < 1$, $x(t) = \psi(t)$, $t \in [-\tau^*, 0]$, $\psi(t)$ is the initial condition, and $\tau^* = \max_{1\le i\le n} \tau_i^*$. Matrices $A$ and $B$ are known constant matrices. Matrix $B_d$ satisfies $B_d = BD$ with $D \in \mathbb{R}^m$ being a known matrix. $f(\cdot)$ represents the system uncertainty, which contains the multiple delayed states. $g(\cdot) \in \mathbb{R}^m$ denotes the mismatched nonlinearity satisfying the Lipschitz condition, namely, there exists a Lipschitz constant $\gamma > 0$, that is

$$||g(x) - g(y)|| \le \gamma ||x - y|| \tag{2}$$

where $\|\cdot\|$ denotes the Euclidean norm.

**Remark 2.1.** *The fault-tolerant control problem based on event triggering in* [18] *is only applied to linear systems and cannot be applied to more general nonlinear systems, so its method will be limited in actual systems. Moreover, it does not consider the influence of time-varying delay on system stability, which poses a challenge to the design of event-triggered fault-tolerant controllers.*

The fault models in [9] are considered as follows:

$$\begin{aligned} u_{ij}^{\mathrm{F}}(t) &= \rho_i^j u_i(t) + \kappa_i^j u_{si}(t), \\ \rho_i^j \kappa_i^j &= 0, \quad i = 1, 2, \ldots, n_u,\ j = 1, 2, \ldots, L \end{aligned} \tag{3}$$

where $\rho_i^j \in [0, 1)$ is the unknown actuator efficiency factor, the index $j$ represents the $j$th faulty mode and $L$ is the number of total faulty modes. $u_{si}(t)$ is the unparametrisable bounded time-varying stuck fault in the $i$th actuator.

It is worth mentioning that (3) implies the following four cases:

1) $\rho_i^j \ne 0$ and $\kappa_i^j = 0$: In this case, $u_{ij}^{\mathrm{F}}(t) = \rho_i^j u_i(t)$, where $0 < \rho_i^j < 1$. This indicates partial loss of effectiveness.
2) $\rho_i^j = 0$ and $\kappa_i^j \ne 0$: $\rho_i^j = 0$ indicates that $u_{ij}^{\mathrm{F}}(t)$ can no longer be influenced by the control inputs $u_i(t)$. This means that $u_i(t)$ is stuck at unparametrisable bounded time-varying function $u_{si}(t)$.
3) $\rho_i^j = 0$ and $\kappa_i^j = 0$: This case corresponds to outage.
4) $\rho_i^j \ne 0$ and $\kappa_i^j \ne 0$: This means there are no errors in the systems.

For the convenience of description, the above fault model is expressed as follows:

$$u^{\mathrm{F}}(t) = \rho u(t) + \kappa u_s(t) \tag{4}$$

where $\rho \in \{\rho^1, \rho^2, \ldots, \rho^L\}$ and $\kappa \in \{\kappa^1, \kappa^2, \ldots, \kappa^L\}$.

Then the system (1) with actuator faults (4) can be written by

$$\dot{x} = Ax + g(x) + B(\rho u + f(x_\tau) + \kappa u_s) + B_d d \tag{5}$$

Before starting the process, the following assumptions and lemma are made.

**Assumption 2.1.** *There exist unknown constants* $\eta$ *and* $\bar{d}$ *such as* $||u_s|| \le \eta$, $||d|| \le \bar{d}$, *respectively. This means that the actuator stuck fault and the external disturbance are bounded by unknown constants.*

**Assumption 2.2.** $rank[B\rho] = rank[B]$.

**Assumption 2.3.** [13] *There exist unknown positive constants* $\nu$, $\varpi$, *such that the following inequality holds:*

$$||f(x_\tau)|| \le \sum_{i=0}^{n} \varpi_i \alpha_i(|x(t-\tau_i)|) + \nu \tag{6}$$

*where* $\alpha(\zeta)$ *satisfying* $\zeta \ge 0$ *is a known and continuously differentiable class-*$\mathcal{K}$ *function.*

**Lemma 2.1.** [9] *The following inequality holds:*

$$x^T PB\rho B^T Px \ge \mu \left\|x^T PB\right\|^2 \tag{7}$$

*where* $\mu$ *is an unknown positive constant.*

**Remark 2.2.** *According to Assumption 2.2 and matrices* $A$ *and* $B$, *there exists a constant matrix* $K \in R^{m\times n}$ *such that* $A+B\rho K$ *is stable. Therefore, we can select a positive definite matrix* $P$ *satisfying the following inequality*

$$P(A+B\rho K) + (A+B\rho K)^T P + \gamma^2 P^2 + I < 0 \tag{8}$$

*where* $\gamma$ *is the same positive constant as in Lipschitz condition (2).*

Based on the above description, the control objective of this paper is to design an adaptive event-triggered fault-tolerant controller for a class of Lipschitz nonlinear systems with time-varying delays. The presented method not only can ensure that all closed-loop signals are bounded but also can guarantee the system states converge to zero asymptotically under the condition of time-varying delays, actuator failures and external disturbances. To this end, the controller design is given in the next section.

## 3. Adaptive Event-Triggered Fault-Tolerant Control Co-Design.

In this part, the adaptive FTC controller is designed as follows

$$\alpha(t) = \hat{K}x - \frac{\hat{\theta}_1^2 \psi(S) B^T Px}{||B^T Px||\hat{\theta}_1 + \sigma} - \frac{1}{2}\hat{\theta}_2 \psi(S) B^T Px \tag{9}$$

where $\hat{K}$ is the estimate of $K$ satisfying $\hat{K} = K + \tilde{K}$ and $\hat{K} = \left[\hat{K}_1, \hat{K}_2, \ldots, \hat{K}_m\right]^T \in R^{m\times n}$, with the following adaptive control laws:

$$\dot{\hat{K}}_i = -\Gamma_i \psi(S) x x^T P b_i - \Gamma_i \sigma(t) \hat{K}_i \tag{10}$$

$$\dot{\hat{\theta}}_1 = -l_1 \sigma(t) \hat{\theta}_1 + 2 l_1 \psi(S) \left\|B^T Px\right\| \tag{11}$$

$$\dot{\hat{\theta}}_2 = -l_2 \sigma(t) \hat{\theta}_2 + 2 l_2 \psi(S)^2 \left\|B^T Px\right\|^2 \tag{12}$$

where $S = x^T Px$, $\Gamma_i$, $i = 1, 2, \ldots, m$ are positive parameter matrices. $b_i$ is the $i$th column of the matrix $B$, such that $B = [b_1, b_2, \ldots, b_m]$. $l_1$ and $l_2$ are any positive constants. $\theta_1$ and $\theta_2$ are all unknown constants, and $\hat{\theta}_1$ and $\hat{\theta}_2$ are the estimates of $\theta_1$ and $\theta_2$ satisfying $\hat{\theta}_1 = \theta_1 + \tilde{\theta}_1$ and $\hat{\theta}_2 = \theta_2 + \tilde{\theta}_2$. In addition, $\psi(\chi) = c_1 + c_2 \sum_{i=0}^{n} \overline{\alpha}^2 \left(\sqrt{(\chi/\lambda_{\min}(P))}\right)$, $\chi \ge 0$, $c_1$, $c_2$ are positive constants to be designed. $\sigma(t) > 0$ is a positive uniform continuous and bounded function satisfying $\lim_{t\to\infty} \int_{t_0}^{t} \sigma(\tau) d\tau \le \bar{\sigma} < \infty$, where $\bar{\sigma}$ is a positive constant.

The event-triggered mechanism is defined as follows:

$$\begin{aligned} u(t) &= \alpha(t_k),\ \forall t \in [t_k, t_{k+1}), \\ t_{k+1} &= \inf\{t \in R||e_i(t)| \ge E_i\} \end{aligned} \tag{13}$$

where $e_i(t) = \alpha_i(t) - u_i(t)$ denotes the measurement error. $E_i \in \{E_1, E_2, \ldots, E_m\}$ are the positive parameters to be designed. $t_k$, $k \in Z^+$ is the controller update time, that is the time $t_k$ will be updated as $t_{k+1}$ whenever (13) is triggered, and the controller $u(t)$ will be applied to the system.

In addition, for all $t \in [t_k, t_{k+1})$, if $|e_i(t)| \leq E_i$, there exist continuous time-varying parameters $\lambda_i$, $i = 1, 2, \ldots, m$ satisfying $\lambda_i(t_k) = 0$, $\lambda_i(t_{k+1}) = 1$ and $|\lambda_i(t)| \leq 1$, such that

$$\alpha(t) = u(t) + \lambda(t) \tag{14}$$

where $\lambda(t) = [\lambda_1(t)E_1, \lambda_2(t)E_2, \ldots, \lambda_m(t)E_m]^T$ and $||\lambda(t)|| \leq \bar{E}^*$, $\bar{E}^*$ is an unknown constant.

## 4. **Stability Analysis.**

**Theorem 4.1.** *Consider the nonlinear systems (1) with fault-tolerant model (4) satisfying Assumptions 2.1 and 2.2, the adaptive event-triggered fault-tolerant controller and the adaptive laws (9)-(13). Then all the signals of closed-loop systems are bounded. And the system states converge to zero asymptotically. In addition, the Zeno behavior can be avoided.*

**Proof:** According to [13], we consider the Lyapunov-Krasovskii function such that

$$\begin{aligned} V &= V_1 + V_2 + V_3 + V_4 \\ V_1 &= \int_0^S \psi(\xi) d\xi \\ V_2 &= \sum_{i=0}^{n} \int_{t-\tau_i}^{t} \delta\alpha_i^2(||x(\xi)||) d\xi \\ V_3 &= \frac{1}{2}\mu \left( l_1^{-1}\tilde{\theta}_1^2 + l_2^{-1}\tilde{\theta}_2^2 \right) \\ V_4 &= \psi(S) \sum_{i=0}^{n} \rho_i \tilde{K}_i^T \Gamma_i^{-1} \tilde{K}_i \end{aligned} \tag{15}$$

where $\delta > 0$ is a constant. $\tilde{\theta}_1$, $\tilde{\theta}_2$ and $\tilde{K}_i$ are parameter estimation errors.

From (8), one has

$$2x^T P g \leq 2\gamma \left\| x^T P \right\| ||x|| \leq x^T \left( \gamma^2 P^2 + I \right) x \tag{16}$$

According to (5), (6), (15) and (16), the first derivative of $V_1$ is

$$\begin{aligned} \dot{V}_1 &= 2\psi(S) x^T P (Ax + g(x) + B(\rho u + \kappa u_s + f(x_\tau)) + B_d d) \\ &\leq \psi(S) x^T \left( PA + A^T P + \gamma^2 P^2 + I \right) x + 2\psi(S) x^T P B \rho \alpha \\ &\quad + 2\psi(S) x^T P B (\kappa u_s + f(x_\tau) + \rho\lambda + Dd) \end{aligned} \tag{17}$$

By noting Assumption 2.3 and Young's inequality, it is shown that

$$\begin{aligned} 2\psi(S) x^T P B f(x_\tau) &\leq 2\psi \left\| B^T P x \right\| \left( \sum_{i=1}^{n} \varpi_i \alpha_i(||x(t-\tau_i)|| + \nu) \right) \\ &\leq \sum_{i=0}^{n} \delta(1 - \bar{\tau}_i)\alpha_i^2(||x(t-\tau_i)||) + 2\psi(S) \left\| B^T P x \right\| \nu \\ &\quad + \sum_{i=1}^{n} \frac{\varpi}{\delta(1-\bar{\tau}_i)} \psi^2(S) \left\| B^T P x \right\|^2 \end{aligned} \tag{18}$$

According to Assumptions 2.1 and 2.2, there exists a positive constant $\theta_1$ defined as follows

$$\begin{aligned} & 2\psi(S)x^TPB(\rho\lambda+\kappa u_s+Dd)+2\psi(S)\left\|B^TPx\right\|\nu \\ \leq & 2\psi(S)\left\|x^TPB\right\|\,||\rho\lambda+\kappa u_s+Dd+\nu|| \\ \leq & 2\mu\theta_1\psi(S)\left\|B^TPx\right\| \end{aligned} \tag{19}$$

It is easy to see that, $\varpi$ and $\bar{\tau}_i$ are both bounded constants, so there also exists a positive constant $\theta_2$ satisfying

$$\sum_{i=1}^{n}\frac{\varpi}{\delta(1-\bar{\tau}_i)}\psi^2(S)\left\|B^TPx\right\|^2\leq\mu\theta_2\psi^2(S)\left\|B^TPx\right\|^2 \tag{20}$$

Substituting (18)-(20) into (17), it can be obtained as follows

$$\begin{aligned} \dot{V}_1\leq & \psi(S)x^T\left(PA+A^TP+\gamma^2P^2+I\right)x+2\psi(S)x^TPB\rho\alpha \\ & +\sum_{i=0}^{n}\delta(1-\bar{\tau}_i)\alpha_i^2(||x(t-\tau_i)||)+2\mu\theta_1\psi(S)\left\|B^TPx\right\| \\ & +\mu\theta_2\psi^2(S)\left\|B^TPx\right\|^2 \end{aligned} \tag{21}$$

According to event-triggered controller (9) and Lemma 2.1, that is

$$\begin{aligned} 2\psi(S)x^TPB\rho\alpha\leq & 2\psi(S)x^TPB\rho\hat{K}x-\mu\hat{\theta}_2\psi^2(S)\left\|B^TPx\right\|^2 \\ & -\frac{2\mu\hat{\theta}_1^2\psi^2(S)\left\|B^TPx\right\|^2}{\left\|B^TPx\right\|\hat{\theta}_1+\sigma} \end{aligned} \tag{22}$$

And then we can get

$$\begin{aligned} \dot{V}_1\leq & \psi(S)x^T\left(PA+A^TP+\gamma^2P^2+I\right)x \\ & +2\psi(S)x^TPB\rho\hat{K}x-\frac{2\mu\hat{\theta}_1^2\psi^2(S)\left\|B^TPx\right\|^2}{\left\|B^TPx\right\|\hat{\theta}_1+\sigma} \\ & -\mu\hat{\theta}_2\psi^2(S)\left\|B^TPx\right\|^2+\sum_{i=0}^{n}\delta(1-\bar{\tau}_i)\alpha_i^2(||x(t-\tau_i)||) \\ & +2\mu\theta_1\psi(S)\left\|B^TPx\right\|+\mu\theta_2\psi^2(S)\left\|B^TPx\right\|^2 \end{aligned} \tag{23}$$

From (15), we can also obtain that

$$\dot{V}_2\leq\sum_{i=1}^{n}\delta\alpha_i(||x||)-\sum_{i=0}^{n}\delta(1-\bar{\tau}_i)\alpha_i^2(||x(t-\tau_i)||) \tag{24}$$

$$\dot{V}_3\leq\mu\left(l_1^{-1}\bar{\theta}_1\dot{\theta}_1+l_2^{-1}\bar{\theta}_2\dot{\theta}_2\right) \tag{25}$$

$$\dot{V}_4\leq 2\psi(S)\sum_{i=1}^{n}\rho_i\tilde{K}_i^T\Gamma_i^{-1}\dot{\tilde{K}}_i \tag{26}$$

Meanwhile, it is worth noting that

$$\begin{aligned} & \mu\theta_1\psi(S)\left\|B^TPx\right\|-\frac{2\mu\hat{\theta}_1^2\psi(S)^2\left\|B^TPx\right\|^2}{\left\|B^TPx\right\|\hat{\theta}_1+\sigma}+\mu\tilde{\theta}_1\psi(S)\left\|B^TPx\right\| \\ = & \frac{2\mu\sigma\hat{\theta}_1\psi(S)\left\|B^TPx\right\|}{\left\|B^TPx\right\|\hat{\theta}_1+\sigma} \end{aligned} \tag{27}$$

$$-\mu\hat{\theta}_2\psi(S)^2\left\|B^TPx\right\|^2+\mu\theta_2\psi(S)^2\left\|B^TPx\right\|^2+\mu\tilde{\theta}_2\psi(S)^2\left\|B^TPx\right\|^2=0 \tag{28}$$

$$2\psi(S)x^TPB\rho\hat{K}x+2\psi(S)\sum_{i=1}^{n}\rho_i\tilde{K}_i^T\Gamma_i^{-1}\dot{\hat{K}}_i\le\mu\sigma\psi(S)\tilde{K}^T\hat{K} \tag{29}$$

So that is

$$\begin{aligned}\dot{V}\le{}&\psi(S)x^T\left(P(A+B\rho K)+(A+B\rho K)^TP+\gamma^2P^2+I\right)x\\&+\mu\sigma\left(\tilde{\theta}_1^2-\tilde{\theta}_1\theta_1+\tilde{\theta}_2^2-\tilde{\theta}_2\theta_2+\tilde{K}^2-\tilde{K}K\right)+\sum_{i=1}^{n}\delta\alpha_i(||x||)\\&+\frac{2\mu\sigma\hat{\theta}_1\psi(S)\left\|B^TPx\right\|}{\left\|B^TPx\right\|\hat{\theta}_1+\sigma}\end{aligned} \tag{30}$$

For Inequality (30), which is equivalent to $P(A+B\rho K)+(A+B\rho K)^TP+\gamma^2P^2+I<0$ in light of Schur complements. Define $-Q=P(A+B\rho K)+(A+B\rho K)^TP+\gamma^2P^2+I<0$, it follows from (30) that

$$\begin{aligned}\dot{V}\le&-\psi(S)x^TQx+\sum_{i=1}^{n}\delta\alpha_i(||x||)+\frac{2\mu\sigma\hat{\theta}_1\psi(S)\left\|B^TPx\right\|}{\left\|B^TPx\right\|\hat{\theta}_1+\sigma}\\&+\mu\sigma\left(\tilde{\theta}_1^2-\tilde{\theta}_1\theta_1+\tilde{\theta}_2^2-\tilde{\theta}_2\theta_2+\tilde{K}^2-\tilde{K}K\right)\end{aligned} \tag{31}$$

According to the inequality $0\le\frac{ab}{a+b}\le a$, $\forall a,b>0$, we can obtain

$$\begin{aligned}\dot{V}&\le-\psi(S)x^TQx+\sum_{i=1}^{n}\delta\alpha_i(||x||)+\mu\sigma\left(\frac{\theta_1^2}{4}+\frac{\theta_2^2}{4}+\frac{||K||^2}{4}\right)\\&\le-\lambda_{\min}(Q)\psi(S)||x||^2+\sum_{i=1}^{n}\delta\alpha_i(||x||)+\sigma\eta\end{aligned} \tag{32}$$

where $\lambda_{\min}(Q)$ denotes the minimum eigenvalue of $Q$, and $\eta=\mu\left(2+\theta_1^2/4+\theta_2^2/4+K^2/4\right)$.

The following facts hold: $x^TQx\le\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)S}$ and $\alpha_i^2(||x||)\le\frac{S}{\lambda_{\min}(P)}\left(\bar{\alpha}^2\sqrt{(S/\lambda_{\min}(P))}\right)$. So we have

$$\begin{aligned}\dot{V}&\le S\left(\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\psi(S)+\sigma\eta+\frac{\delta}{\lambda_{\min}(P)}\sum_{i=1}^{n}\bar{\alpha}^2\left(\sqrt{\frac{S}{\lambda_{\min}(P)}}\right)\right)\\&=S\left(-\bar{c}_1-\bar{c}_2\sum_{i=1}^{n}\bar{\alpha}^2\left(\sqrt{\frac{S}{\lambda_{\min}(P)}}\right)\right)+\sigma\eta\end{aligned} \tag{33}$$

where $\bar{c}_1=\frac{c_1\lambda_{\min}(Q)}{\lambda_{\max}(P)}$, $\bar{c}_2=\frac{c_2\lambda_{\min}(Q)}{\lambda_{\max}(P)}-\frac{\delta}{\lambda_{\min}(P)}$. Here, for any given $c_1$, $c_2$, $P$, $Q$, we can choose $\delta$ sufficiently small such that $c_1>0$, $c_2>0$. Therefore, the derivative of $V$ becomes as follows:

$$\dot{V}\le-\bar{c}_1x^TPx+\sigma\eta\le-c_1\lambda_{\min}(Q)||x||^2+\sigma\eta \tag{34}$$

Let $\bar{x}(t)=\left[x^T,\tilde{\theta}_1^T,\tilde{\theta}_2^T,\tilde{K}^T\right]^T$, and then integrating (15) over $[t_0,t]$ yields

$$\begin{aligned}V(t)&\le V(t_0)-\int_{t_0}^{t}\dot{V}(s)ds\\&\le V(t_0)-\int_{t_0}^{t}c_1\lambda_{\min}(Q)||x(s)||^2ds+\int_{t_0}^{t}\sigma(s)\eta ds\\&\le V(t_0)+\bar{\sigma}\eta\end{aligned} \tag{35}$$

which means $\tilde{x}$ is uniformly bounded; furthermore, $x(t)$ is uniformly bounded and uniformly continuous. Furthermore, by (33), we also obtain that

$$\lim_{t\to\infty}\int_{t_0}^{t} c_1\lambda_{\min}(Q)||x(s)||^2 ds \le V(\tilde{x}(t_0))+\bar{\sigma}\eta \tag{36}$$

which implies that $\lambda_{\min}(Q)||x(s)||^2$ is uniformly bounded.

According to (34) and the Barbalat Lemma [24], we have

$$\lim_{t\to\infty}||x(t)|| = 0 \tag{37}$$

That concludes the proof.

**Remark 4.1.** *The work of* [18] *was extended in this paper, where the FTC problem did not consider the influences of multiple state time-varying delays on the systems. In this paper, the time delay is allowed unknown and delayed-state perturbation satisfies a very general growth condition. By introducing an integrable auxiliary signal, the effects of time-varying delay can be counteracted successfully.*

Finally, we show that there exists a $t^* > 0$, such that $\forall k \in z^+$, $\{t_{k+1} - t_k\} \ge t^*$. To this end, according to $e_i(t) = \alpha_i(t) - u_i(t)$, $\forall t \in [t_k, t_{k+1})$, we have

$$\frac{d}{dt}||e_i|| \le \frac{d}{dt}(e_i * e_i)^{\frac{1}{2}} = sign(|e_i|)\dot{e}_i \le |\dot{\alpha}_i| \tag{38}$$

According to (9), we obtain that

$$\dot{\alpha} = \frac{\partial\alpha}{\partial x}\dot{x} + \sum_{i=1}^{m}\frac{\partial\alpha}{\partial\hat{K}_i}\dot{\hat{K}}_i + \frac{\partial\alpha}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 + \frac{\partial\alpha}{\partial\hat{\theta}_2}\dot{\hat{\theta}}_2 + \frac{\partial\alpha}{\partial\sigma}\dot{\sigma} \tag{39}$$

Since the $x$, $\hat{K}$, $\hat{\theta}_1$, $\hat{\theta}_2$ are continuous and uniformly bounded, we can obtain that $\sigma$ and $\dot{\alpha}$ are also continuous and bounded functions. So there exists a constant $\phi > 0$, such that $\|\dot{\alpha}\| \le \phi$. By noting that $|e_i(t_k)| = 0$, and $\lim_{t\to t_{k+1}}|e_i(t)| = |E_i|$, we conclude that there exist the inter-execution intervals $t_i^*$ that must satisfy $t_i^* \ge E_i/\phi$, namely, the Zeno behavior [25] is successfully excluded.

## 5. Simulation Examples.

In this section, two simulation examples are investigated to show the effectiveness of the proposed method.

### 5.1. Example 1.

In this section, a time delay nonlinear system with external disturbances and actuator faults is considered as follows

$$\dot{x}(t) = Ax(t) + g(x(t)) + B(u(t) + f(x_\tau)) + B_d d(t) \tag{40}$$

where

$$A = \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

The unknown time delay $f(x_\tau)$ and nonlinear function $g(x,t)$ are given as

$$f(x_\tau) = \begin{bmatrix} \varsigma_1 x^T(t)x(t-\tau_1) + \varsigma_1 x^T(t-\tau_1)x(t-\tau_2) \\ \varsigma_2 \\ \varsigma_3 x^T(t)x(t-\tau_2) + \varsigma_3 x^T(t-\tau_1)x(t-\tau_2) \end{bmatrix}$$

$$g(x,t) = \begin{bmatrix} 0.33\sin(x_1) & 0 \end{bmatrix}^T$$

where the nonlinear function $g(x)$ satisfies Lipschitz condition and Lipschitz constant $\gamma = 0.33$, the unknown time delay function satisfying $||f(x_\tau)\| \le \sum_{j=0}^{2}\varpi_j\alpha_j(|x(t-\tau_j)|)+\nu$ where $\varpi_j$ $(j = 0,1,2)$, $\nu$ are unknown positive constants. We choose $\alpha_0(\zeta) = \alpha_1(\zeta) =$

$\alpha_2(\zeta) = \zeta$, $\bar{\alpha}_0(\zeta) = \bar{\alpha}_1(\zeta) = \bar{\alpha}_2(\zeta) = 1$, $\zeta \geq 0$, $\varsigma_1 = \sin(t)$, $\varsigma_2 = 0.1\sin(t) + 0.1$, $\varsigma_3 = \sin(t)$, $\tau_1 = 0.1(1 + \sin(t))$, $\tau_2 = 0.1(1 + \sin(t))$, and the external disturbance $\omega(t) = \sin(t)$. Some simulation parameters are chosen as $x(0) = [0.2, -0.3]^T$, $\hat{\theta}_1(0) = 0.5$, $\hat{\theta}_2(0) = 0.5$, $\hat{K}_i(0) = [0, 0]^T$, $i = 1, 2, 3$, $\sigma(t) = 0.1 + 0.5e^{-0.2t}$, $c_1 = 5$, $c_2 = 10$, $\Gamma_i = 25$, $i = 1, 2, 3$, $l_1 = 100$, $l_2 = 100$, $u_s = \sin(t)$, $\rho = diag(0.65, 1, 0)$, $\kappa = diag(0, 0, 1)$, $E_1 = 1.2$, $E_2 = 1.5$, $E_3 = 1.8$.

By solving the matrix inequality (8) with LMI toolbox of MATLAB, the matrix $P$ is given as

$$P = \begin{bmatrix} 1.4448 & -0.0000 \\ -0.0000 & 1.4448 \end{bmatrix}$$

We consider the following fault model: before 10s, the system does not show faults. After 10s, the first actuator loses 35% of its effectiveness and the third actuator is stuck at $\sin(t)$.

The results of the simulation are as follows. Figure 1 shows that all the system states can converge to a neighborhood of origin with the controller. Figure 2 and Figure 3 show the estimates of the control parameters $\hat{K}$ and $\hat{\theta}_i$, $i = 1, 2$. From Figure 2 and Figure 3, one can know that all the control signals are bounded, which indicates that the parameters can be used to design the controller. Finally, the controllers signals $u_i$, $i = 1, 2, 3$ and the corresponding event-triggered events are presented in Figures 4-6. Meanwhile, the number of event triggers is shown in Table 1.

TABLE 1. The numbers of triggering events

| Control input | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ |
|---|---|---|---|
| Number | 83 | 96 | 110 |

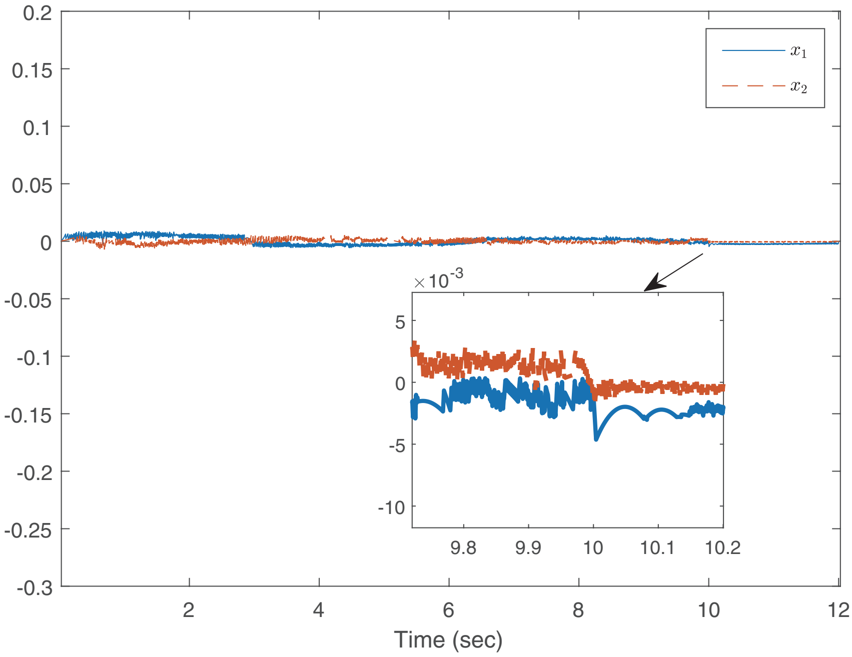

FIGURE 1. Response of system states $x_1$, $x_2$

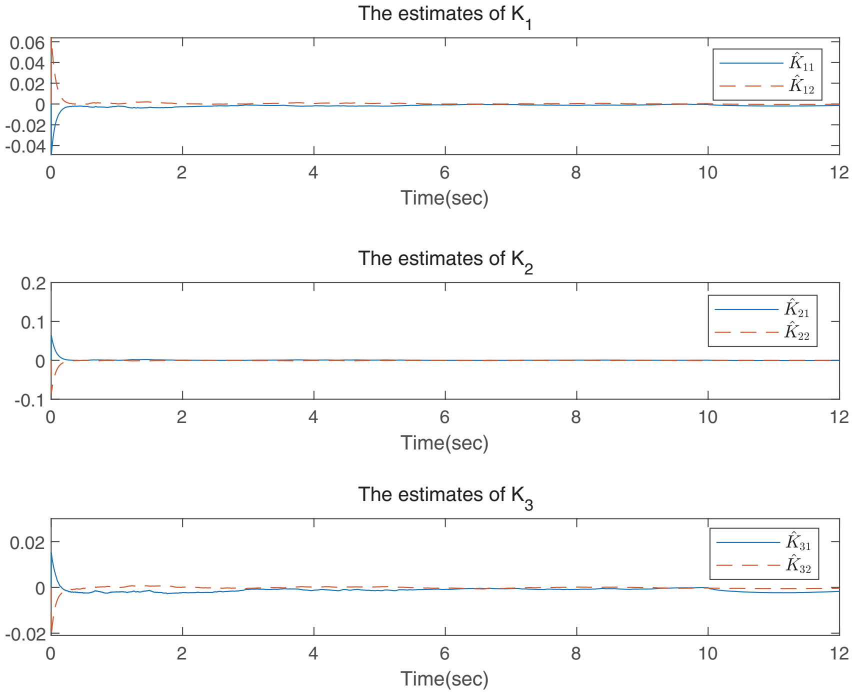

Figure 2. The estimates of the matrix $K$

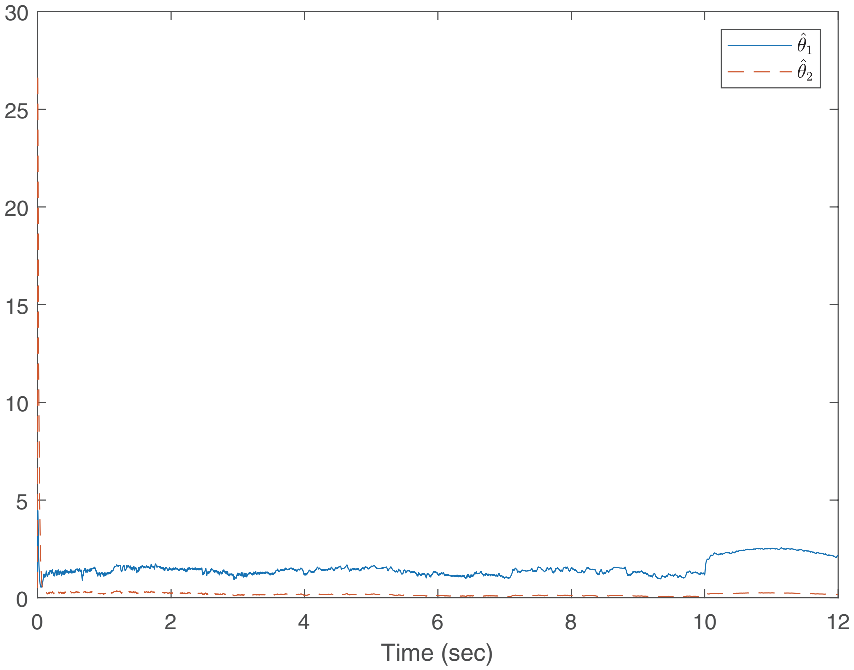

Figure 3. Adaptive parameters $\hat{\theta}_1$, $\hat{\theta}_2$

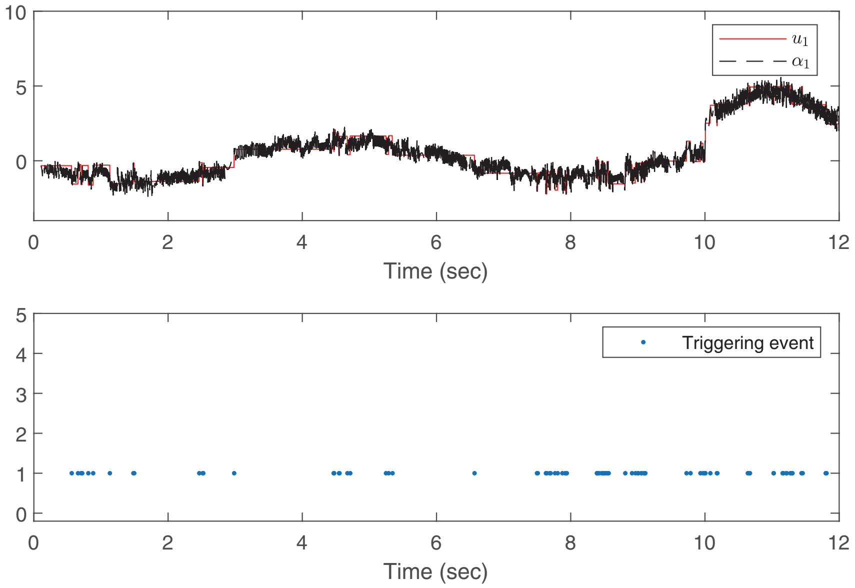

FIGURE 4. The control input signals $u_1$, $\alpha_1$ and the corresponding triggering event

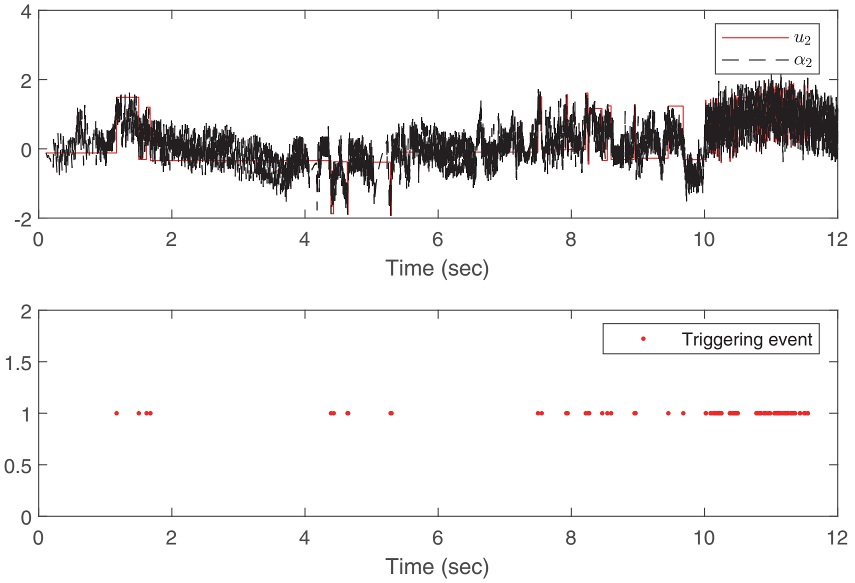

FIGURE 5. The control input signals $u_2$, $\alpha_2$ and the corresponding triggering event

5.2. **Example 2.** In this section, we consider an FTC model in [15], where the system and input distribution matrices are

$$A = \begin{bmatrix} -2.98 & 0.93 & 0 & -0.0340 \\ -0.99 & -0.21 & 0.035 & -0.0011 \\ 0 & 0 & -2 & 1 \\ 0.39 & -5.55 & 0 & -1.89 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.032 & 0.5 & 1.55 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1.6 & 1.8 & -2 \end{bmatrix} \qquad D = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

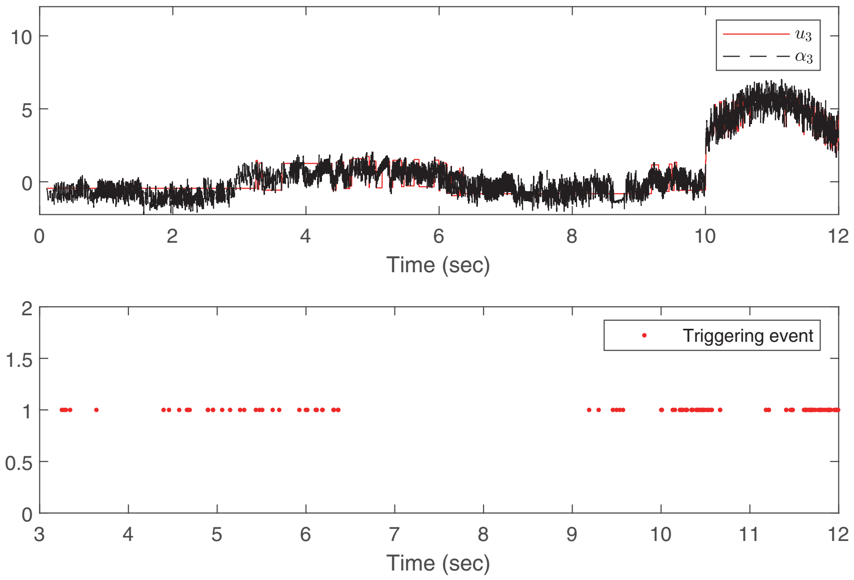

FIGURE 6. The control input signals $u_3$, $\alpha_3$ and the corresponding triggering event

The unknown time delay $f(x_\tau)$ and nonlinear function $g(x,t)$ are given as

$$f(x_\tau) = \begin{bmatrix} \varsigma_1 x^T(t)x(t-\tau_1) + \varsigma_1 x^T(t-\tau_1)x(t-\tau_2) \\ \varsigma_2 \\ \varsigma_3 x^T(t)x(t-\tau_2) + \varsigma_3 x^T(t-\tau_1)x(t-\tau_2) \end{bmatrix}$$

$$g(x,t) = \begin{bmatrix} \sin(x_1) & 0 & 0 & 0 \end{bmatrix}^T$$

where $\varsigma_1$, $\varsigma_2$ and $\varsigma_3$ are unknown parameters, and $\tau_1$ and $\tau_2$ are unknown time varying delay. It is observed that the function $g(x,t)$ satisfies Lipschitz condition with $\gamma = 1$, and the unknown time delay function satisfies $||f(x_\tau)|| \leq \sum_{j=0}^{2} \varpi_j \alpha_j(|x(t-\tau_j)|) + \nu$, where $\varpi_j$ $(j = 0,1,2)$, $\nu$ are unknown positive constants. Let $\alpha_0(\zeta) = \alpha_1(\zeta) = \alpha_2(\zeta) = \zeta$, $\bar{\alpha}_0(\zeta) = \bar{\alpha}_1(\zeta) = \bar{\alpha}_2(\zeta) = 1$, $\zeta \geq 0$, and then Assumption 2.3 is satisfied. For the simulation purpose, the unknown parameters are chosen as $\varsigma_1 = \sin(t)$, $\varsigma_2 = 0.1\sin(t) + 0.1$, $\varsigma_3 = \sin(t)$, the time delay parameters are chosen as $\tau_1 = 0.1(1+\sin(t))$, $\tau_2 = 0.1(1+\sin(t))$, and $\omega(t) = 0.1\sin(t)$ is the disturbance.

By solving the matrix inequality (8) with LMI toolbox of MATLAB, the matrix $P$ is given as

$$P = \begin{bmatrix} 0.0142 & -0.0620 & -0.0011 & 0.0000 \\ -0.0620 & 4.8217 & 0.0842 & 0.0011 \\ -0.0011 & 0.0842 & 0.3818 & 0.0058 \\ 0.0000 & 0.0011 & 0.0058 & 0.0157 \end{bmatrix}$$

We consider the following fault model: before 10s, the system works in fault free case. After 10s, the first actuator loses 20% of its effectiveness and the third actuator is stuck at $0.1\sin(t)$. The following parameters are chosen in the simulation: $x(0) = [-4,-1,2,-1]^T$, $\hat{\theta}_1(0) = 5$, $\hat{\theta}_2(0) = 5$, $\hat{K}_i(0) = [0,0,0,0]^T$, $i = 1,2,3$, $\sigma(t) = 0.1e^{-0.1t}$, $c_1 = 10$, $c_2 = 5$, $\Gamma_i = 50, i = 1,2,3$, $l_1 = 25$, $l_2 = 25$, $u_s = 0.1\sin(t)$, $\rho = diag(0.8,1,0)$, $\kappa = diag(0,0,1)$, $E_1 = 0.6, E_2 = 0.8, E_3 = 0.8$.

The results of the simulation are shown in Figures 7-12. It is shown in Figure 7 that all the system states can converge to a neighborhood of origin. The estimates of $\hat{K}$ are revealed in Figure 8. Furthermore, the boundedness of parameter estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ is shown in Figure 9. Finally, the controllers signals $u_i$, $i = 1, 2, 3$ and the corresponding event-triggered events are presented in Figures 10-12.

Different from the results [15, 16, 18], in this paper, we consider the problem of the event-triggered FTC under time-varying conditions. By the detailed simulation comparison, we conclude that the computation burden of the communication procedure is effectively alleviated.

TABLE 2. The numbers of triggering events

| Control input | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ |
|---|---|---|---|
| Number | 33 | 46 | 63 |

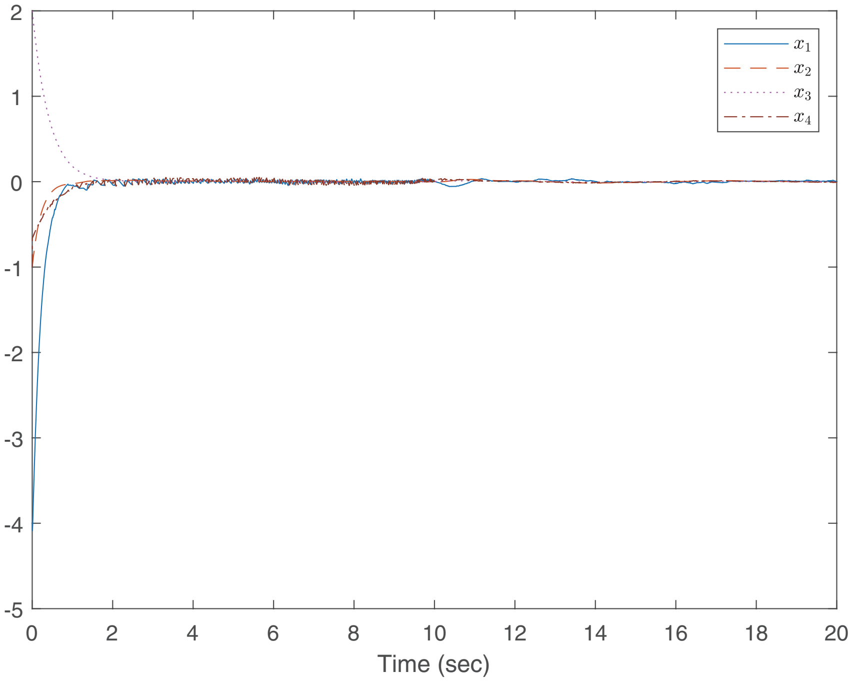

FIGURE 7. Response of system states $x_1$, $x_2$, $x_3$ and $x_4$

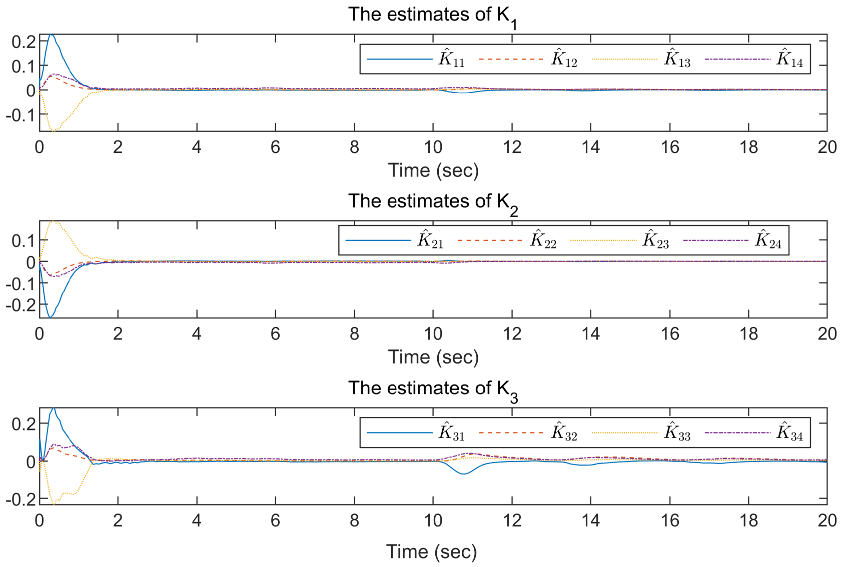

FIGURE 8. The estimates of the matrix $K$

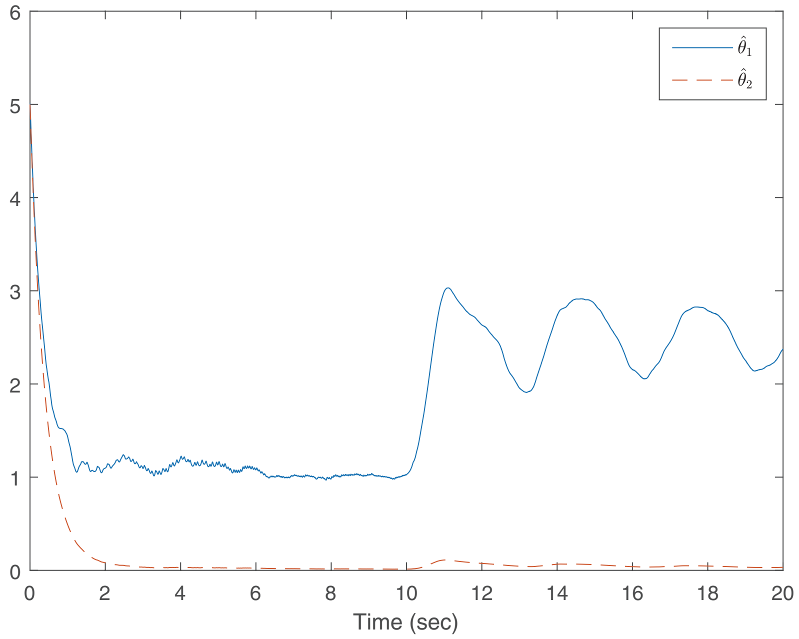

FIGURE 9. Adaptive parameters $\hat{\theta}_1$, $\hat{\theta}_2$

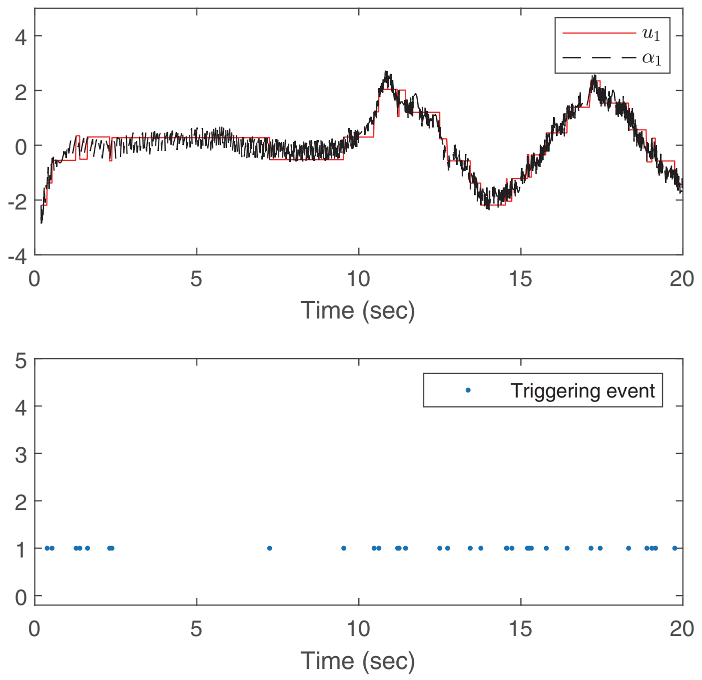

FIGURE 10. The control input signals $u_1$, $\alpha_1$ and the corresponding triggering event

6. **Conclusion.** In this paper, we have considered an adaptive event-triggered control co-design for a class of nonlinear systems with actuator faults, multiple time-varying delays and external disturbances. To compensate the effects of unknown systems parameters and multiple delay perturbation functions and reduce the systems redundancy, the adaptive event-triggered fault-tolerant control co-design and the adaptive laws are investigated. It can not only effectively eliminate the effects of faults, time-delays and disturbances, but also significantly alleviate for the communication burden. Finally, two simulation

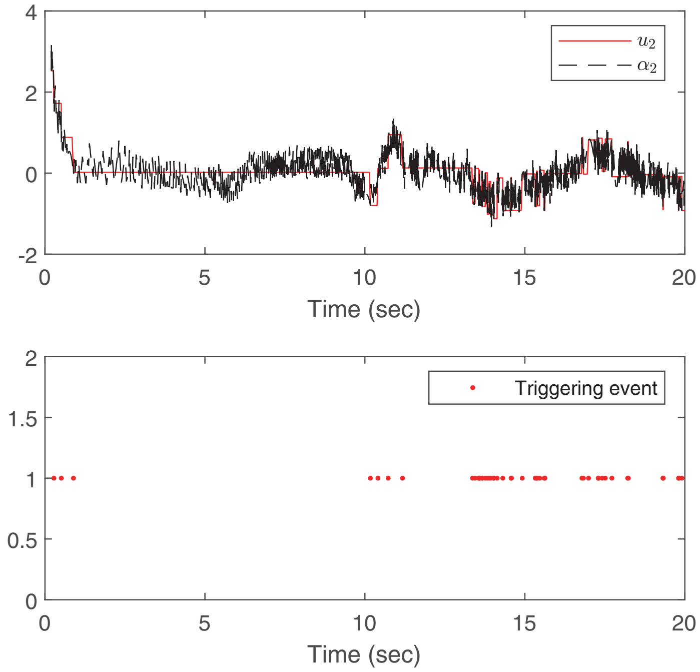

FIGURE 11. The control input signals $u_2$, $\alpha_2$ and the corresponding triggering event

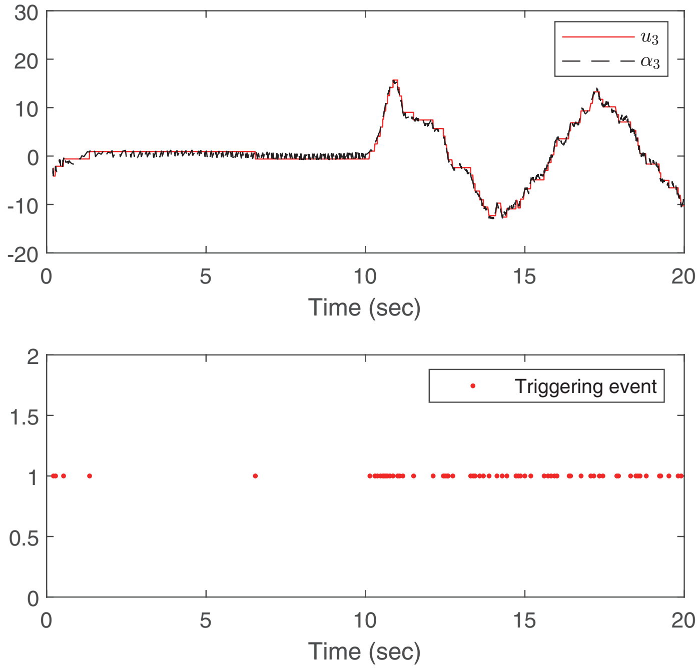

FIGURE 12. The control input signals $u_3$, $\alpha_3$ and the corresponding triggering event

results show that the presented control scheme can guarantee that all the system states are asymptotically stable and all the closed-loop signals are globally bounded.

## REFERENCES

[1] J. Gong, B. Jiang and Q. Shen, Adaptive fault-tolerant neural control for large-scale systems with actuator faults, *International Journal of Control, Automation and Systems*, vol.17, pp.1421-1431, 2019.

[2] L. Wang, G. Chen, X. Wang and W. K. Tang, Controllability of networked MIMO systems, *Automatica*, vol.69, pp.405-409, 2016.

[3] N. Dong, B. Yuan and P. Lu, Fault-tolerant control for multiple networked spacecraft under actuator saturation, *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, vol.231, pp.558-569, 2016.

[4] B. Kaviarasan, R. Sakthivel and O. Kwon, Robust fault-tolerant control for power systems against mixed actuator failures, *Nonlinear Analysis: Hybrid Systems*, vol.22, pp.249-261, 2016.

[5] W. Liu and C. Zhou, Fault-tolerant finite-time fuzzy control for nonlinear power systems with time delays and actuator faults, *ISA Transactions*, vol.118, pp.44-54, 2021.

[6] Z. Peng, R. Qi and B. Jiang, Adaptive fault tolerant control for hypersonic flight vehicle system with state constraints, *Journal of the Franklin Institute*, vol.357, pp.9351-9377, 2020.

[7] J. P. Koeln, H. C. Pangborn, M. A. Williams, M. L. Kawamura and A. G. Alleyne, Hierarchical control of aircraft electro-thermal systems, *IEEE Transactions on Control Systems Technology*, vol.28, pp.1218-1232, 2020.

[8] L. Liu, Y. Liu, S. Tong and Z. Gao, Relative threshold-based event-triggered control for nonlinear constrained systems with application to aircraft wing rock motion, *IEEE Transactions on Industrial Informatics*, vol.18, pp.911-921, 2022.

[9] X. J. Li and G. H. Yang, Robust adaptive fault-tolerant control for uncertain linear systems with actuator failures, *IET Control Theory & Applications*, vol.6, pp.1544-1551, 2012.

[10] L. Wu, G. Yang and D. Ye, Robust adaptive fault-tolerant control for linear systems with actuator failures and mismatched parameter uncertainties, *IET Control Theory & Applications*, vol.8, pp.441-449, 2014.

[11] Q. Shen, B. Jiang and P. Shi, Adaptive fault tolerant control against actuator faults, *International Journal of Adaptive Control and Signal Processing*, vol.31, pp.147-162, 2016.

[12] S. Xiao and J. Dong, Robust adaptive fault-tolerant tracking control for uncertain linear systems with actuator failures based on the closed-loop reference model, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol.50, pp.3448-3455, 2020.

[13] Z. Zhang, S. Xu and B. Zhang, Exact tracking control of nonlinear systems with time delays and dead-zone input, *Automatica*, vol.52, pp.272-276, 2015.

[14] X. Qi, W. Liu, Y. Yang and J. Lu, Adaptive finite-time fuzzy control for nonlinear systems with input quantization and unknown time delays, *Journal of the Franklin Institute*, vol.357, pp.7718-7742, 2020.

[15] Y. Li and G. Yang, Robust adaptive fault-tolerant control for a class of uncertain nonlinear time delay systems, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol.47, pp.1554-1563, 2017.

[16] L. Wu and J. H. Park, Adaptive fault-tolerant control of uncertain switched nonaffine nonlinear systems with actuator faults and time delays, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol.50, pp.3470-3480, 2020.

[17] L. Xing, C. Wen, Z. Liu, H. Su and J. Cai, Event-triggered output feedback control for a class of uncertain nonlinear systems, *IEEE Transactions on Automatic Control*, vol.64, pp.290-297, 2019.

[18] X. Chen, Y. Wu, Y. Yan, R. Zhang, Q. Yu and L. Wu, Event-triggered adaptive fault-tolerant control of uncertain linear time-delayed systems with actuator failures, *2020 Chinese Control And Decision Conference (CCDC)*, 2020.

[19] B. Li, J. Xia, H. Zhang, H. Shen and Z. Wang, Event-triggered adaptive fuzzy tracking control for nonlinear systems, *International Journal of Fuzzy Systems*, vol.22, pp.1389-1399, 2020.

[20] B. Li, J. Xia, H. Zhang, H. Shen and Z. Wang, Event-triggered adaptive fuzzy tracking control for stochastic nonlinear systems, *Journal of the Franklin Institute*, vol.357, pp.9505-9522, 2020.

[21] W. Chen, J. Wang, K. Ma, B. He, X. Huang and T. Wang, Novel adaptive neural networks control with event-triggered for uncertain nonlinear system, *Journal of the Franklin Institute*, vol.357, pp.10410-10431, 2020.

[22] M. Yuan and J. Zhai, Adaptive output feedback control for a class of switched stochastic nonlinear systems under event-triggered mechanism, *Asian Journal of Control*, 2021.

[23] Z. Chen, B. Niu, X. Zhao, L. Zhang and N. Xu, Model-based adaptive event-triggered control of nonlinear continuous-time systems, *Applied Mathematics and Computation*, vol.408, 126330, 2021.

[24] J. J. Slotine and W. Li, *Applied Nonlinear Control*, Prentice Hall, 1990.

[25] K. H. Johansson, M. Egerstedt, J. Lygeros and S. Sastry, On the regularization of Zeno hybrid automata, *Systems & Control Letters*, vol.38, pp.141-150, 1999.

[26] Z. Liu, J. Yu, L. Zhao, Y. Ma, B. Xue and S. Cheng, Adaptive $H_\infty$ sliding mode control for a class of uncertain Markovian jump systems with time-delay, *ICIC Express Letters*, vol.14, no.4, pp.319-327, 2020.

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