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STATE FEEDBACK CONTROLLER DESIGN OF NETWORKED SAMPLED-DATA SYSTEMS WITH PACKET LOSS

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Abstract. *In this paper, the stabilization problem of linear networked sampled-data systems subject to packet loss is studied. First, by discrete-time approach, we establish a discrete-time unified framework. Then, an equivalent stochastic model is constructed in virtue of matrix exponential computation, where the system matrix is characterized by high nonlinearity and randomness with respect to the number of consecutive packet dropouts, which poses difficulties for the expectation operation of a nonlinear and random coupling term in the analysis of the resulting stochastic system. In order to deal with the difficulties, the law of total expectation, confluent Vandermonde matrix and Kronecker product operation are used and then a stabilization controller is designed such that the stochastic stability of the closed-loop stochastic system is guaranteed. Finally, a numerical example and an example using inverted pendulum system are given to show the effectiveness and applicability of the designed algorithm.*

Keywords: Networked sampled-data systems, Packet loss, Discrete-time approach, Stochastic systems, Confluent Vandermonde matrix

1. **Introduction.** With the rapid advancement of the digital hardware technologies and communication technologies, networked systems have aroused considerable attention due to simple installation, high flexibility and reliability, and low maintenance cost compared with the traditional point-to-point wiring control [1-9]. As a result, networked systems have been extensively applied in smart grids, unmanned aerial vehicles and intelligent robot, e.g., [10, 11]. Besides that, the sampling packets in the communication networks may be lost because of the actuator suspension, network congestion and denial-of-service attacks [12-17]. Therefore, it is of critical importance to investigate the stabilization of networked sampled-data systems with packet loss and some efforts have been made on these issues. For example, the fuzzy control of nonlinear systems with packet loss has been investigated in [18] and the robust H_∞ control of uncertain sampled-data systems has been investigated in [19]. For the modeling of networked sampled-data systems, input delay approach [20], discrete-time approach [21] and impulsive modeling approach [22] have been widely used in existing literature. Among the above three methods, the discrete-time method has received special research interest, because the accurate integration on the sampling interval leads to less conservative stability conditions [23]. For the above reason, our first motivation in this paper is modeling the networked sampled-data systems with consecutive packet dropouts by discrete-time approach.

However, by discrete-time approach, the system matrix of the equivalent discrete-time system would be subject to high nonlinearity and randomness with respect to the number

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of successive packet losses, which in turn, gives rise to the difficulties for the controller synthesis of networked sampled-data systems. In [24], some initial efforts have been made on the stabilization problem of networked sampled-data systems under packet losses with the help of discrete-time approach. However, the dimension of resulting linear matrix inequality (LMI) is of very high dimension when the upper bound of successive packet losses is large, which greatly increases the computational complexity. Accordingly, the main difficulties in this article are identified as follows: 1) How to calculate the expectation of a nonlinear and random coupling term such that a stabilization controller is designed? 2) How to obtain a stability condition in the form of LMI such that the dimension of resulting LMI does not increase with the increase of the upper bound of consecutive packet dropouts? Therefore, the second motivation in this paper is to give a favourable reply to the aforementioned issues.

Inspired by the observations above, we aim to study the stabilization of networked sampled-data systems subject to packet loss. The main contributions of the paper are summarized as follows: 1) On the basis of the law of total expectation, confluent Vandermonde matrix and Kronecker product operation, the mathematical expectation of a nonlinear and random coupling term is calculated such that a stabilization controller is designed; 2) Different from [24], the dimension of resulting LMI in this paper does not increase with the increase of the upper bound of consecutive packet dropouts. As a result, the computational complexity of the LMI can be lower for some cases compared with [24].

The structure of this article is organized as follows. First, the problem formulation is presented in Section 2. In Section 3, by the law of total expectation, confluent Vandermonde matrix and Kronecker product operation, a stabilization controller is designed such that the stochastic stability of the considered system is guaranteed. After that, two examples are provided to verify the effectiveness of the designed method in Section 4, and some conclusions are drawn in Section 5.

2. **Problem Formulation.**

Consider the linear networked sampled-data system subject to packet loss as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, and $u(t) \in \mathbb{R}^m$ is the control input. A and B are constant matrices with appropriate dimensions. The initial value is given by $x(0)$.

The system's state of (1) is periodically sampled and the sampling period is h. Denote by $\{t_k\}_{k=0}^{\infty}$ the sampling sequence when the packets are transmitted successfully to the controller with $t_0 = 0$. Therefore, the controller update interval $t_{k+1} - t_k$ satisfies $t_{k+1} - t_k = (\delta_k + 1)h$, where discrete variable δ_k characterizes the number of successive packet losses that might occur in the interval $[t_k, t_{k+1})$. Assume that δ_k has an upper bound $\bar{\delta}$ and the packet drop rate is $\rho \in [0, 1)$. According to Lemma 1 in [25], the probability mass function of discrete-time variable δ_k is

$$\begin{cases} \mathcal{P}\{\delta_k = i\} = \rho^i(1-\rho), \quad i = 0, 1, 2, \ldots, \bar{\delta} - 1, \\ \mathcal{P}\{\delta_k = \bar{\delta}\} = \rho^{\bar{\delta}}. \end{cases} \tag{2}$$

Considering the consecutive packet dropouts occurring in sampler-controller channel, the control input $u(t)$ with a zero-order hold can be represented as

$$u(t) = Kx(t_k), \quad t_k \leq t < t_{k+1} \tag{3}$$

where K is the gain matrix to be determined. By substituting (3) into (1), integrating the equation from t_k to t_{k+1}, the closed-loop system is obtained as follows:

$$x(t_{k+1}) = \left(e^{A(\delta_k+1)h} + \int_0^{(\delta_k+1)h} e^{As} ds BK \right) x(t_k). \tag{4}$$

Let $C = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}$. Note that $e^{Av} = \sum_{i=0}^{\infty} \frac{A^i v^i}{i!}$ and $\int_0^v e^{As} ds B = \sum_{i=0}^{\infty} \frac{A^i v^{i+1}}{(i+1)!} B$, we have

$$e^{Cv} = \sum_{i=0}^{\infty} \frac{C^i v^i}{i!} = \begin{bmatrix} \sum_{i=0}^{\infty} \frac{A^i v^i}{i!} & \sum_{i=0}^{\infty} \frac{A^i v^{i+1}}{(i+1)!} B \\ 0 & I \end{bmatrix} = \begin{bmatrix} e^{Av} & \int_0^v e^{As} ds B \\ 0 & I \end{bmatrix}. \tag{5}$$

It follows from (5) that $e^{A(\delta_k+1)h} + \int_0^{(\delta_k+1)h} e^{As} ds BK$ can be rewritten as

$$e^{A(\delta_k+1)h} + \int_0^{(\delta_k+1)h} e^{As} ds BK = \begin{bmatrix} I & 0 \end{bmatrix} e^{C(\delta_k+1)h} \begin{bmatrix} I \\ K \end{bmatrix}.$$

Then, the discrete-time stochastic (4) can be equivalently expressed as follows:

$$x(t_{k+1}) = \begin{bmatrix} I & 0 \end{bmatrix} e^{C(\delta_k+1)h} \begin{bmatrix} I \\ K \end{bmatrix} x(t_k). \tag{6}$$

The goal of this paper is to design a stabilization controller such that the stochastic system (6) is stochastically stable. More specifically, the stochastic system (6) is said to be stochastically stable if for any initial state $x(0) \in \mathbb{R}^n$, one has $\mathbb{E}\left\{\sum_{k=0}^{\infty} \|x(t_k)\|^2\right\} < \infty$.

3. **Main Results.**

In this section, the stabilization problem of networked sampled-data systems is considered in the presence of successive packet losses.

In order to clarify the issue to be investigated, the following lemma is introduced.

Lemma 3.1. *Let* $\bar{P} = \begin{bmatrix} I \\ 0 \end{bmatrix} P \begin{bmatrix} I & 0 \end{bmatrix}$*, where* P *is specified in (13). Then, the mathematical expectation of the nonlinear and random coupling term* $e^{C^T(\delta_k+1)h} \bar{P} e^{C(\delta_k+1)h}$ *is calculated as*

$$\mathbb{E}\left\{ e^{C^T(\delta_k+1)h} \bar{P} e^{C(\delta_k+1)h} \right\} = \Phi^T (I \otimes P) \Phi \tag{7}$$

where $\Phi = \left((UV_c^{-1}) \otimes \begin{bmatrix} I & 0 \end{bmatrix} \right) \bar{C} e^{Ch}$ *with* V_c *and* $\bar{C}$ *being specified in (8), and* U *being specified in (10).*

Proof: Let $\lambda_1, \lambda_2, \ldots, \lambda_f$ respectively denote as the eigenvalues of matrix $C \in \mathbb{R}^{(n+m)\times(n+m)}$ with their algebraic multiplicities $m_1, m_2, \ldots, m_f$, where $m_1 + m_2 + \cdots + m_f = n + m$. According to analysis in [23, 26], for matrix $C \in \mathbb{R}^{(n+m)\times(n+m)}$ and a scalar v, one has

$$e^{Cv} = \left(\left(\pi(v) V_c^{-1} \right) \otimes I \right) \bar{C} \tag{8}$$

where

$$\pi(v) = \begin{bmatrix} \pi_1(v) & \pi_2(v) & \cdots & \pi_d(v) & \cdots & \pi_f(v) \end{bmatrix}$$

with $\pi_d(v) = \begin{bmatrix} e^{\lambda_d v} & v e^{\lambda_d v} & \cdots & v^{m_d-1} e^{\lambda_d v} \end{bmatrix}$; $V_c = \begin{bmatrix} \Lambda_1 & \Lambda_2 & \cdots & \Lambda_f \end{bmatrix}$ is the confluent Vandermonde matrix generated by $\lambda_1, \lambda_2, \ldots, \lambda_f$, where each Λ_d $(d = 1, 2, \ldots, f)$ is defined as

$$\Lambda_d = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ \lambda_d & 1 & 0 & \cdots & 0 \\ \lambda_d^2 & 2\lambda_d & 2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \lambda_d^{m_d-1} & (m_d-1)\lambda_d^{m_d-2} & \frac{(m_d-1)!}{(m_d-3)!}\lambda_d^{m_d-3} & \cdots & (m_d-1)! \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \lambda_d^{m+n-1} & (m+n-1)\lambda_d^{m+n-2} & \frac{(m+n-1)!}{(m+n-3)!}\lambda_d^{m+n-3} & \cdots & \frac{(m+n-1)!}{(m+n-m_d)!}\lambda_d^{m+n-m_d} \end{bmatrix},$$

and $\bar{C} = \begin{bmatrix} I & C^T & \cdots & C^{(m+n-1)^T} \end{bmatrix}^T$.

According to (8), we have

$$\begin{aligned} & \mathbb{E}\left\{e^{C^T(\delta_k+1)h}\bar{P}e^{C(\delta_k+1)h}\right\} \\ &= \mathbb{E}\left\{e^{C^T h}\bar{C}^T\left(V_c^{-T}\otimes I\right)\left(\pi^T(\delta_k h)\otimes I\right)\bar{P}\left(\pi(\delta_k h)\otimes I\right)\left(V_c^{-1}\otimes I\right)\bar{C}e^{Ch}\right\} \\ &= e^{C^T h}\bar{C}^T\left(V_c^{-T}\otimes I\right)\mathbb{E}\left\{\left(\pi^T(\delta_k h)\pi(\delta_k h)\right)\otimes\bar{P}\right\}\left(V_c^{-1}\otimes I\right)\bar{C}e^{Ch}. \end{aligned} \tag{9}$$

By the law of total expectation and (2), we have

$$\mathbb{E}\left\{\pi^T(\delta_k h)\pi(\delta_k h)\right\} = \pi^T\left(\bar{\delta}h\right)\pi\left(\bar{\delta}h\right)\rho^{\bar{\delta}} + \sum_{s=0}^{\bar{\delta}-1}\pi^T(sh)\pi(sh)\rho^s(1-\rho).$$

It is easily verified that matrix $\mathbb{E}\left\{\pi^T(\delta_k h)\pi(\delta_k h)\right\}$ is positive semidefinite, and hence, there exists a matrix U such that

$$\mathbb{E}\left\{\pi^T(\delta_k h)\pi(\delta_k h)\right\} = U^T U. \tag{10}$$

Therefore, it yields from (9) and (10) that

$$\begin{aligned} & \mathbb{E}\left\{e^{C^T(\delta_k+1)h}\bar{P}e^{C(\delta_k+1)h}\right\} \\ &= e^{C^T h}\bar{C}^T\left(V_c^{-T}\otimes I\right)\left(\left(U^T U\right)\otimes\bar{P}\right)\left(V_c^{-1}\otimes I\right)\bar{C}e^{Ch} \\ &= e^{C^T h}\bar{C}^T\left(V_c^{-T}\otimes I\right)\left(U^T\otimes\begin{bmatrix} I \\ 0 \end{bmatrix}\right)(I\otimes P)\left(U\otimes\begin{bmatrix} I & 0 \end{bmatrix}\right)\left(V_c^{-1}\otimes I\right)\bar{C}e^{Ch} \\ &= e^{C^T h}\bar{C}^T\left(V_c^{-T}U^T\otimes\begin{bmatrix} I \\ 0 \end{bmatrix}\right)(I\otimes P)\left(UV_c^{-1}\otimes\begin{bmatrix} I & 0 \end{bmatrix}\right)\bar{C}e^{Ch} \\ &= \Phi^T(I\otimes P)\Phi \end{aligned} \tag{11}$$

where $\Phi = \left(\left(UV_c^{-1}\right)\otimes\begin{bmatrix} I & 0 \end{bmatrix}\right)\bar{C}e^{Ch}$. Therefore, Equation (7) holds. The proof is complete. □

The following theorem is devoted to designing a stabilization controller for stochastic system (6).

Theorem 3.1. *Let h, ρ and $\bar{\delta}$ be given. Discrete-time stochastic system (6) is stochastically stable if there exist Y and $Q > 0$ such that the following inequality holds:*

$$\begin{bmatrix} -Q & \begin{bmatrix} Q & Y^T \end{bmatrix}\Phi^T \\ * & -(I\otimes Q) \end{bmatrix} < 0 \tag{12}$$

where $\Phi = \left(\left(UV_c^{-1}\right)\otimes\begin{bmatrix} I & 0 \end{bmatrix}\right)\bar{C}e^{Ch}$. Moreover, the desired controller gain is given by $K = YQ^{-1}$ if Inequality (12) is feasible.

Proof: Let us construct the Lyapunov function

$$V(x(t_k)) = x^T(t_k)Px(t_k) \tag{13}$$

where P is a positive definite matrix. Define the difference of the Lyapunov function as

$$\Delta V(x(t_k)) = \mathbb{E}\left\{V(x(t_{k+1}))|x(t_k)\right\} - V(x(t_k)). \tag{14}$$

According to (7), we can have by (6) and (14) that

$$\begin{aligned}
&\mathbb{E}\left\{\Delta V(x(t_k))\right\}\\
&= \mathbb{E}\left\{\mathbb{E}\left\{V(x(t_{k+1}))|x(t_k)\right\} - V(x(t_k))\right\}\\
&= \mathbb{E}\left\{x^T(t_k)\begin{bmatrix} I \\ K \end{bmatrix}^T e^{C^T(\delta_k+1)h}\begin{bmatrix} I \\ 0 \end{bmatrix} P \begin{bmatrix} I & 0 \end{bmatrix} e^{C(\delta_k+1)h}\begin{bmatrix} I \\ K \end{bmatrix} x(t_k) - x^T(t_k)Px(t_k)\right\}\\
&= \mathbb{E}\left\{x^T(t_k)\begin{bmatrix} I \\ K \end{bmatrix}^T \mathbb{E}\left\{e^{C^T(\delta_k+1)h}\bar{P}e^{C(\delta_k+1)h}\right\}\begin{bmatrix} I \\ K \end{bmatrix} x(t_k) - x^T(t_k)Px(t_k)\right\}\\
&= \mathbb{E}\left\{x^T(t_k)\left(\begin{bmatrix} I \\ K \end{bmatrix}^T \Phi^T(I\otimes P)\Phi\begin{bmatrix} I \\ K \end{bmatrix} - P\right)x(t_k)\right\}.
\end{aligned} \tag{15}$$

Remark 3.1. *Since the existence of the variable* δ_k*, the term* $e^{C^T(\delta_k+1)h}\bar{P}e^{C(\delta_k+1)h}$ *in (15) is a coupling matrix subject to high nonlinearity and randomness. Therefore, how to compute the expectation of this term is crucial to the controller design. With the help of confluent Vandermonde matrix approach and Kronecker product operation, the expectation of the nonlinear and random coupling term* $e^{C^T(\delta_k+1)h}\bar{P}e^{C(\delta_k+1)h}$ *is calculated in (7) and then the stabilization controller can be designed.*

By Schur Complement Lemma, $\begin{bmatrix} I \\ K \end{bmatrix}^T \Phi^T(I\otimes P)\Phi\begin{bmatrix} I \\ K \end{bmatrix} - P < 0$ is true if and only if

$$\begin{bmatrix} -P & \begin{bmatrix} I & K^T \end{bmatrix}\Phi^T(I\otimes P) \\ * & -(I\otimes P) \end{bmatrix} < 0. \tag{16}$$

Performing a congruence transformation to Inequality (12) by $\operatorname{diag}\{Q^{-1}, I\otimes Q^{-1}\}$, let $P = Q^{-1}$ and $K = YP$, we obtain (16) immediately, i.e., (12) implies that

$$\begin{bmatrix} I \\ K \end{bmatrix}^T \Phi^T(I\otimes P)\Phi\begin{bmatrix} I \\ K \end{bmatrix} - P < 0$$

holds and then discrete-time stochastic system (6) is stochastically stable. We complete the proof. □

Remark 3.2. *In* [24]*, the stabilization of networked sampled-data systems with packet loss was also studied, where the resulting LMI is of very high dimension when* $\bar{\delta}$ *is large. Different from* [24]*, the dimension of LMI (12) in this paper does not increase with the increase of* $\bar{\delta}$*, which can lead to the LMI (12) with lower computational complexity for some cases. To better illustrate this, two parameters* N_D *and* N_L *are introduced to compare the computational complexity of the LMI (12) with that of LMI in* [24]*. As discussed in* [27, 28]*, the computational complexity of LMI can be estimated as* $N_D^3 N_L$*, where* N_D *is the total number of scalar decision variables and* N_L *is the LMI rows. According to LMI (12), we have* $N_D = n(n+1)/2 + mn$ *and* $N_L = (m+n+1)n$*. As for the LMI in* [24]*, its* $N_D = n(n+1)/2 + mn$ *and* $N_L = \left(\bar{\delta}+2\right)n$*. Therefore, we can conclude that if* $\bar{\delta} > n+m-1$*, the computational complexity of the LMI in this paper could be lower than* *that in* [24]. *For example, for the Example 4.1 ($n = 3$, $m = 2$ and $\bar{\delta} = 8$) in Section 4, the computational complexity of LMI (12) is 31104 and another is 51840.*

4. Illustrative Example.

In this section, two examples are given to verify the effectiveness and applicability of the designed approach in this paper.

Example 4.1. *Consider a linear system with the following parameters:*

$$A = \begin{bmatrix} -5.0 & 0.70 & 0.40 \\ 0 & -5.0 & 0.12 \\ 0 & 0 & 0.10 \end{bmatrix}, \quad B = \begin{bmatrix} 0.20 & 0.30 \\ 0.11 & 0.60 \\ -0.30 & 0.40 \end{bmatrix}.$$

Then, $C = \begin{bmatrix} -5.0 & 0.70 & 0.40 & 0.20 & 0.30 \\ 0 & -5.0 & 0.12 & 0.11 & 0.60 \\ 0 & 0 & 0.10 & -0.30 & 0.40 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ *and the system matrix C has eigenvalues $\lambda_1 = 0.1$, $\lambda_2 = -5$ and $\lambda_3 = 0$ with algebraic multiplicities $m_1 = 1$, $m_2 = 2$ and $m_3 = 2$, respectively. Then, we have*

$$V_c = \begin{bmatrix} 1.0000 & 1 & 0 & 1 & 0 \\ 0.1000 & -5 & 1 & 0 & 1 \\ 0.0100 & 25 & -10 & 0 & 0 \\ 0.0010 & -125 & 75 & 0 & 0 \\ 0.0001 & 625 & -500 & 0 & 0 \end{bmatrix}.$$

The sampling interval is chosen as $h = 0.8$. Suppose that the maximum number of consecutive packet dropouts $\bar{\delta} = 8$ and the packet drop rate $\rho = 0.4$. Therefore, we can get that

$$U = \begin{bmatrix} -0.0006 & 0.0000 & 0.0012 & 0.0005 & 0.0001 \\ 0.0025 & -0.0000 & 0.0025 & -0.0025 & -0.0003 \\ -0.0709 & 0.1762 & -0.0055 & -0.0889 & 0.0989 \\ 0.0583 & 0.5113 & -0.0016 & 0.1437 & -0.7404 \\ 1.0593 & 0.5546 & 0.0034 & 0.9856 & 0.6576 \end{bmatrix}.$$

With the above parameters, we have by solving Inequality (12) that

$$Q = \begin{bmatrix} 62.4387 & -0.0004 & -0.0109 \\ -0.0004 & 62.4394 & -0.0013 \\ -0.0109 & -0.0013 & 56.7997 \end{bmatrix}, \quad Y = \begin{bmatrix} -3.1068 & -5.2048 & 60.2865 \\ -2.2884 & -3.8791 & -32.3023 \end{bmatrix}.$$

Then, the designed controller gain is

$$K = YQ^{-1} = \begin{bmatrix} -0.0496 & -0.0833 & 1.0614 \\ -0.0368 & -0.0621 & -0.5687 \end{bmatrix}.$$

Set the initial value as $x(0) = \begin{bmatrix} 1 & -0.6 & 0.8 \end{bmatrix}^T$. The state trajectories without control input are shown in Figure 1. In Figure 2, the state trajectories with sampled-data control signals are illustrated. From Figure 2, we can see that our proposed design approach is effective.

In the following, we illustrate the influence of $\bar{\delta}$ and ρ on the control performance, respectively. First, given $h = 0.8$ and $\rho = 0.40$, the controller gain matrix K with different $\bar{\delta}$ is given in Table 1. Second, the controller gain matrix K with different ρ is given in Table 2 when $h = 0.8$ and $\bar{\delta} = 11$. From Table 1 and Table 2, we can conclude that when $\bar{\delta} \geq 13$ ($h = 0.8$, $\rho = 0.40$) or $\rho \geq 0.43$ ($h = 0.8$, $\bar{\delta} = 11$), the LMI (12) is unsolvable.

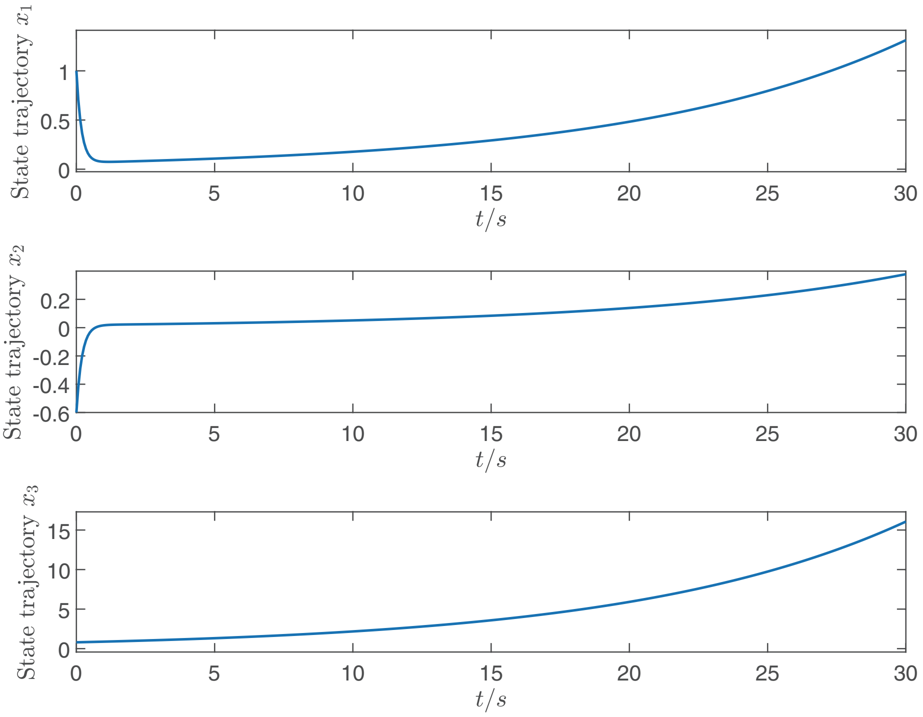

Figure 1. State trajectories of system without control signals

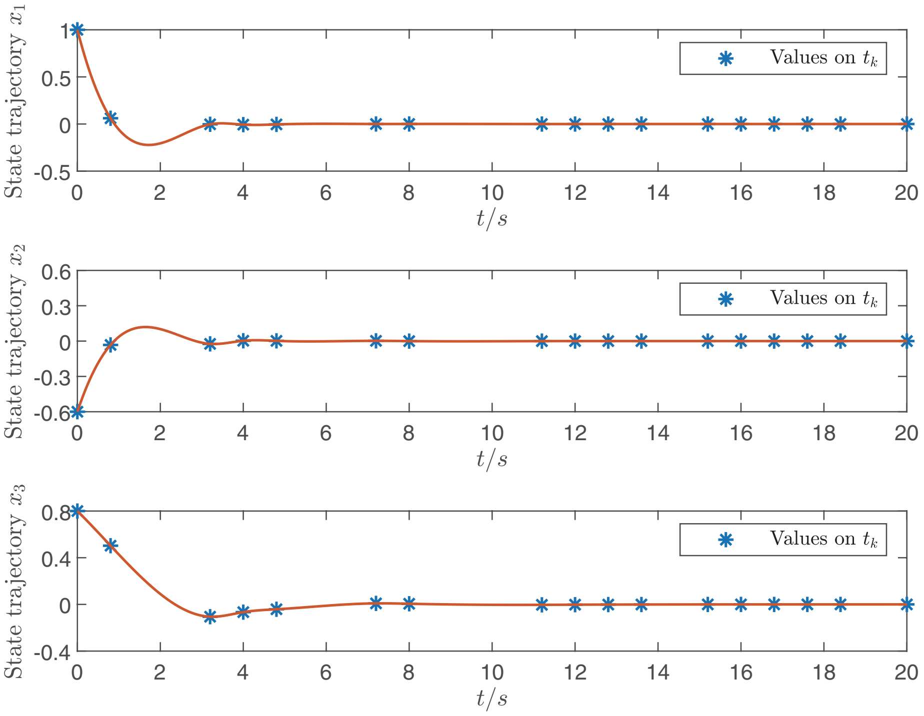

Figure 2. State trajectories of system with control signals

Table 1. The effect of $\bar{\delta}$ on the stability when $h = 0.8$ and $\rho = 0.40$

$\bar{\delta}$	1	$\cdots$	12	13
K	$\begin{bmatrix} -0.0459 & -0.0771 & 1.7678 \\ -0.0339 & -0.0575 & -0.7062 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} -0.0308 & -0.0508 & 0.5534 \\ -0.0230 & -0.0380 & -0.3448 \end{bmatrix}$	–

TABLE 2. The effect of ρ on the stability when $h = 0.8$ and $\bar{\delta} = 11$

ρ	0.01	…	0.42	0.43
K	$\begin{bmatrix} -0.0718 & -0.1195 & 2.0978 \\ -0.0529 & -0.0895 & -0.8420 \end{bmatrix}$	…	$\begin{bmatrix} -0.0299 & -0.0489 & 0.6816 \\ -0.0224 & -0.0367 & -0.2366 \end{bmatrix}$	–

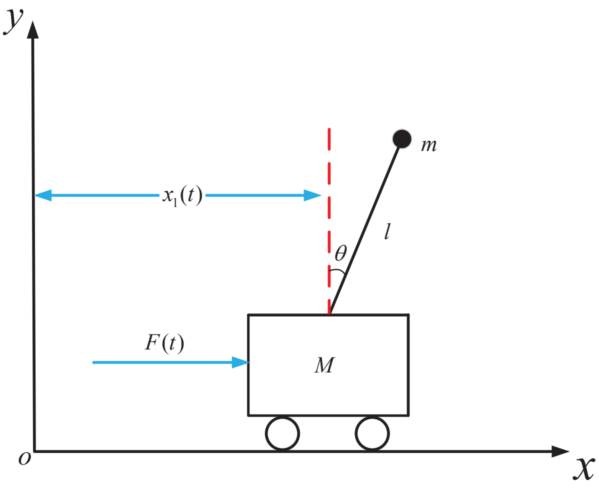

FIGURE 3. Inverted pendulum system

Example 4.2. *Consider the cart and inverted pendulum system in Figure 3 as the controlled plant* [19]*:*

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-3mg}{4M+m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{3(M+m)g}{(4M+m)l} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{4}{4M+m} \\ 0 \\ \frac{-3}{(AM+m)l} \end{bmatrix} u(t)$$

where $M = 8.0$ *kg is the mass of the card,* $m = 2.0$ *kg is the mass of the pendulum,* $l = 0.5$ *m is the half length of the pendulum, and* $g = 9.8$ m/s^2 *is the gravitational acceleration. Then, the matrices* A *and* B *are given by*

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1.7294 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 17.2941 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.1176 \\ 0 \\ -0.1765 \end{bmatrix}.$$

Then, we have

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1.7294 & 0 & 0.1176 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 17.2941 & 0 & -0.1765 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

By calculation, the matrix C *has eigenvalues* $\lambda_1 = 0$, $\lambda_2 = 4.1586$ *and* $\lambda_3 = -4.1586$ *with algebraic multiplicities* $m_1 = 3$, $m_2 = 1$ *and* $m_3 = 1$*, respectively. Then, we can obtain that*

$$V_c = \begin{bmatrix} 1 & 0 & 0 & 1.0000 & 1.0000 \\ 0 & 1 & 0 & 4.1586 & -4.1586 \\ 0 & 0 & 2 & 17.2941 & 17.2941 \\ 0 & 0 & 0 & 71.9197 & -71.9197 \\ 0 & 0 & 0 & 299.0865 & 299.0865 \end{bmatrix}.$$

Assume that the sampling interval $h = 0.2$*, the maximum number of consecutive packet dropouts* $\bar{\delta} = 2$ *and the packet drop rate* $\rho = 0.1$*. According to (10), matrix* U *is obtained as*

$$U = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 \\ -0.0324 & -0.0126 & 0.0161 & 0.0067 & 0.0260 \\ 0.1621 & -0.0632 & -0.0895 & -0.3585 & 0.3198 \\ 0.9862 & 0.0323 & 0.0416 & 1.2349 & 0.9025 \end{bmatrix}.$$

With the above parameters, we can have by solving Inequality (12) that

$$Q = \begin{bmatrix} 192.3267 & -29.8540 & -9.2970 & 0.9109 \\ -29.8540 & 265.4864 & -19.9798 & -60.9751 \\ -9.2970 & -19.9798 & 16.3313 & -38.9339 \\ 0.9109 & -60.9751 & -38.9339 & 188.8119 \end{bmatrix}$$

and

$$Y = \begin{bmatrix} -1.0827 & -3.8974 & 0.8676 & 0.8685 \end{bmatrix} \times 10^3.$$

Then, the designed controller gain is

$$K = YQ^{-1} = \begin{bmatrix} 2.2579 & 5.3658 & 149.5743 & 37.1645 \end{bmatrix}.$$

In this example, the initial value is chosen to be $x(0) = \begin{bmatrix} 1 & -0.8 & 0.5 & -0.3 \end{bmatrix}^T$*. Figure 4 shows the state trajectories of the inverted pendulum system with control signals. From Figure 4, we can see that our proposed design approach is effective.*

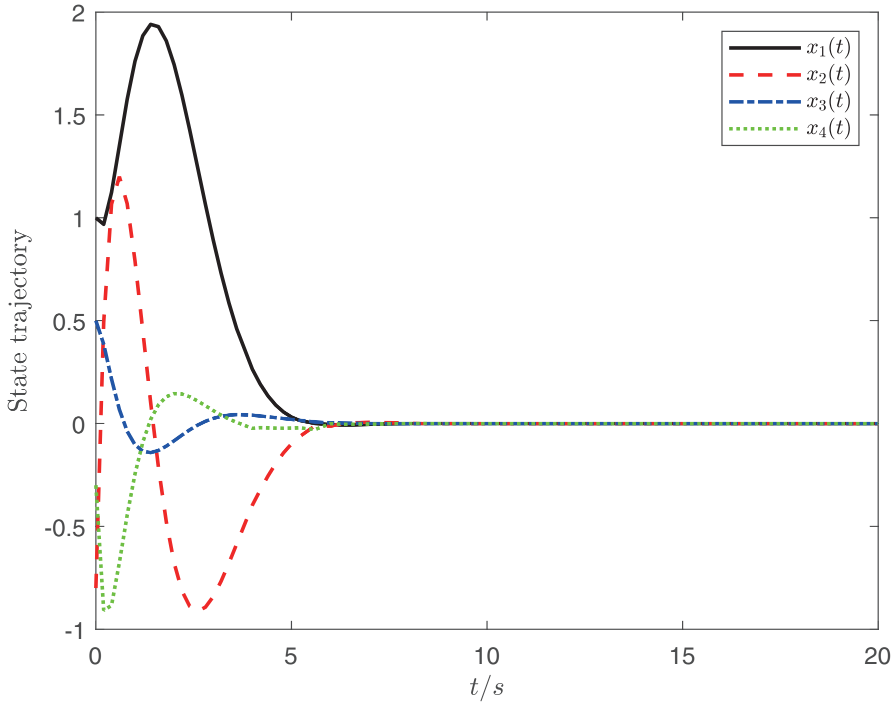

FIGURE 4. State trajectories of the inverted pendulum process with control signals

5. **Conclusions.** In this paper, the stabilization problem of networked sampled-data systems subject to packet loss has been studied. First, we have presented an equivalent discrete-time system with system matrix subject to high-order nonlinearity and randomness. On the basis of the law of total expectation, confluent Vandermonde matrix and Kronecker product operation, the mathematical expectation of a nonlinear and random coupling term has been calculated and then a stabilization controller has been designed. Finally, two examples have been given to show the effectiveness and applicability of the designed algorithm. In this paper, only the effect of packet loss was considered on the stabilization problem of networked sampled-data systems. In the presence of packet loss, further research topics include the stabilization of networked sampled-data systems with time delay by discrete-time approach.

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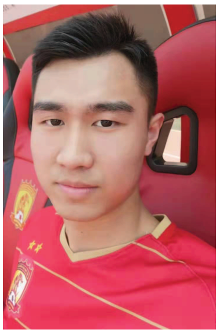
Author Biography



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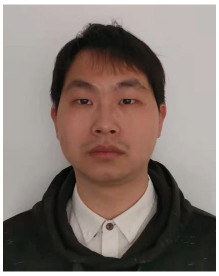

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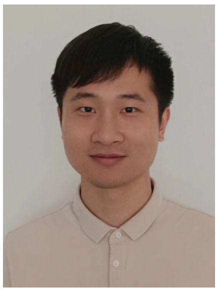

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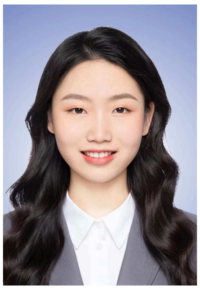

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