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H_∞ CONTROL OF NETWORKED SYSTEMS WITH NOISY SAMPLING INTERVALS AND ACTUATORS SATURATION

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Abstract. *In practical engineering, the sampling intervals of networked control systems are often subject to undesirable physical constraints, which results in noisy sampling intervals. Furthermore, due to the physical limitations of components, saturation often occurs in actuators. In view of these, we focus on the stabilization of networked control systems with noisy sampling intervals and actuators saturation. First, a closed-loop stochastic system model, whose system matrix is characterized by high nonlinearity and randomness, is obtained by considering the noisy sampling intervals and actuators saturation in a unified framework. In order to obtain a sufficient condition for the mean-square stability of resulting closed-loop discrete-time stochastic system with a prescribed* H_∞ *performance, the confluent Vandermonde matrix approach and Kronecker product operation are utilized in the mathematical expectation of a nonlinear and random coupling matrix, and then a saturated* H_∞ *controller is designed. Finally, two examples are provided to verify the effectiveness of the designed method.*
Keywords: Networked control systems, Noisy sampling intervals, Actuators saturation, Discrete-time approach, H_∞ control

1. **Introduction.** With the rapid development of digital hardware technologies in the past few decades, networked control systems have garnered particular attention due to their applications in the practical engineering, e.g., intelligent manufacturing, aerospace and industrial network [1-5]. Compared to the point-to-point systems, networked control systems connect controlled plants, sensors, controllers and actuators in different geographical locations through communication network and have the advantages of high flexibility, strong reliability, easy maintenance and convenient installation [6-9]. Due to their important theoretical value and engineering practicability, the stabilization problems of networked control systems have received a wide range of research interests [10-15].

However, some new challenging issues have inevitably emerged when we take advantage of networked systems, for example, saturation effect. Due to the physical limitations of components, saturation often occurs in actuators, which will produce nonlinear characteristics and may lead to poor oscillation behavior or even instability. Therefore, it is of importance to study the stabilization of networked control systems subject to saturation effect, and fruitful research results have been achieved [16-21]. For example, the stabilization problem of networked control systems under asynchronous samplings and actuators saturation was studied in [18]. The authors in [19] studied local stabilization of linear discrete-time systems subject to control saturation. Moreover, in [20], H_∞ synthesis problems were investigated for networked control systems subject to input saturation.

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In the above-mentioned literature, the sampling intervals of the considered networked sampled-data systems (both periodic and aperiodic) are usually supposed to be bounded and deterministic. In the engineering practice, however, the sampling intervals of networked control systems often happen in a random way as a result of the undesirable physical constraints (such as the clock error drift, the performance degradation of signal transmission device and the random failure of sampler) [22, 23]. In other words, the sampling intervals may be subject to noisy perturbations and fluctuate around an ideal sampling period with a certain probability distribution. Recently, some results about the control of sampled-data systems with noisy sampling intervals have been reported. For example, in [24], the stabilization of networked control systems with noisy sampling intervals and stochastic time-varying delays was investigated.

In existing literature, three main approaches have been used in the stabilization problem of networked control systems, i.e., input delay approach [25], discrete-time approach [26] and impulsive modeling approach [27]. Since the exact integration over a sampling interval leads to less conservative stability conditions [29], the discrete-time approach has received particular research interests. For example, in [30], the discrete-time approach was used to investigate the stabilization and control problems of sampled-data systems with noisy sampling intervals and packet dropouts. In this article, the approach is also based on the discrete-time modeling of sampled-data systems but we consider disturbance input and actuators saturation in communication networks. To the best of the authors' knowledge, few results were reported on the H_∞ control problem of networked control systems subject to noisy sampling intervals and actuators saturation by discrete-time approach, which motivates this study.

In this paper, the authors are concerned with the saturated H_∞ control problem for a class of networked control systems with noisy sampling intervals. In Section 2, a discrete-time system is established for the networked control system with noisy sampling intervals and actuators saturation. On the basis of the confluent Vandermonde matrix approach and Kronecker product operation, the mean-square stable criteria and saturated H_∞ controller are then obtained in Section 3. Finally, two examples are presented to show the effectiveness and applicability of the proposed design approach in Section 4, and some conclusions are drawn in Section 5.

2. **Problem Formulation.** Consider the following networked control system subject to noisy sampling intervals and actuators saturation:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bs(u(t)) + E\omega(t), \\ y(t) = Cx(t) + D\omega(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the controlled output and $\omega(t) \in l_2\,[0,\infty)$ is a disturbance input vector. A, B, C, D, E are constant matrices with appropriate dimensions. $x(0)$ is the initial value of system (1). The nonlinear function $s(\cdot)$ is a saturation mapping, which is defined as

$$s(u) = \begin{bmatrix} s_1(u_1) & s_2(u_2) & \cdots & s_m(u_m) \end{bmatrix}^T$$

where $s_j(u_j) = \text{sign}(u_j)\min\{u_{j,\max}, |u_j|\}$ for each $j = 1, 2, \ldots, m$.

The system's state of (1) is aperiodically sampled and the sampling interval h_k under consideration is made up of two parts, i.e., $h_k = h + \varepsilon_k$, where constant h stands for the nominal sampling interval and stochastic variable ε_k characterizes the sampling errors resulting from unpredictable environmental phenomena. The probability function of the stochastic variable ε_k is denoted as $f(\varepsilon)$, where $f(\varepsilon)$ is known a priori with ε satisfying $h + \varepsilon > 0$.

Sampling the plant (1) with period h_k and then the $u(t)$ with a zero-order hold is

$$u(t) = Kx(t_k), \quad t_k \le t < t_{k+1} \tag{2}$$

where $\{t_k\}_{k=0}^{\infty}$ is the sampling sequence and K is the gain matrix to be determined. By substituting (2) into (1), integrating the equation from t_k to t_{k+1}, denoting $x(t_k)$, $\omega(t_k)$, $y(t_k)$ by x_k, ω_k, y_k, and noting that $h_k = t_{k+1} - t_k$, the closed-loop system is obtained as follows:

$$\begin{cases} x_{k+1} = e^{Ah_k}x_k + \int_0^{h_k} e^{As}dsBs(Kx_k) + \int_0^{h_k} e^{As}dsE\omega_k, \\ y_k = Cx_k + D\omega_k. \end{cases} \tag{3}$$

Remark 2.1. *In* [31]*, the H_∞ control of active suspension systems subjected to constant sampling and saturated control input was studied. Under the time-varying sampling intervals, the stabilization of sampled-data systems under actuators saturation was investigated in* [18]*. In this note, we further deal with the H_∞ control of networked control systems with noisy sampling intervals and actuators saturation.*

The goal of this paper is to design a stabilization controller such that the stochastic system (3) is exponentially mean-square stable with a prescribed H_∞ performance:

1) The discrete-time stochastic system (3) with $\omega_k = 0$ is said to be exponentially mean-square stable if there exit $\vartheta > 0$ and $\mu \in (0, 1)$ such that

$$\mathbb{E}\left\{\|x_k\|^2\right\} \le \vartheta\mu^\sigma \mathbb{E}\left\{\|x_0\|^2\right\}$$

holds for all $x_0 \in \mathbb{R}^n$ and $\sigma > \kappa$, where κ is a sufficiently large positive integer;

2) Under the assumption of zero initial condition, the controlled output y_k satisfies

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\|y_k\|^2\right\} < \gamma^2\mathbb{E}\left\{\sum_{k=0}^{\infty}\|\omega_k\|^2\right\}$$

for any nonzero ω_k, where $\gamma > 0$ is a given disturbance attenuation level.

3. Main Results.

In this section, a sufficient condition for the mean-square stability of discrete-time stochastic system (3) with a prescribed H_∞ performance is obtained in Theorem 3.1 and then a saturated H_∞ controller is designed in Theorem 3.2.

Theorem 3.1. *Given positive parameters γ, τ and controller gain matrix K. Discrete-time stochastic system (3) is exponentially stable in the mean-square sense with a prescribed H_∞ performance γ if there exists matrix $P > 0$ such that the following condition holds:*

$$\begin{bmatrix} \Pi_{11} + \tilde{C} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0 \tag{4}$$

where

$$\Pi_{11} = \begin{bmatrix} -P & \tau K^T(I-H)/2 \\ * & -\tau I \end{bmatrix} + \begin{bmatrix} I & K^THB^T \\ 0 & B^T \end{bmatrix} \Phi^T(I\otimes P)\Phi \begin{bmatrix} I & 0 \\ HKB & B \end{bmatrix},$$

$$\tilde{C} = \begin{bmatrix} C^TC & 0 \\ 0 & 0 \end{bmatrix}, \quad \Pi_{12} = \begin{bmatrix} I & K^THB^T \\ 0 & B^T \end{bmatrix} \Phi^T(I\otimes P)\Phi \begin{bmatrix} 0 \\ E \end{bmatrix} + \begin{bmatrix} C^TD \\ 0 \end{bmatrix},$$

$$\Pi_{22} = \begin{bmatrix} 0 & E^T \end{bmatrix} \Phi^T(I\otimes P)\Phi \begin{bmatrix} 0 \\ E \end{bmatrix} + D^TD - \gamma^2 I$$

with $\Phi = \left(UV_z^{-1}\otimes\begin{bmatrix} I & 0 \end{bmatrix}\right)\bar{Z}e^{Zh}$, $\bar{Z}$ and V_z are defined in (12), U is defined in (13), and H is defined in (5).

Proof: According to analysis in [32], the saturation function $s(u)$ can be expressed as

$$s(u) = Hu + \psi(u) \tag{5}$$

where H is a diagonal matrix satisfying $0 < H < I$, and the nonlinear function $\psi(u)$ satisfies

$$\psi^T(u)\left(\psi(u) - (I - H)u\right) \leq 0. \tag{6}$$

Let us construct the Lyapunov function

$$V(x_k) = x_k^T P x_k \tag{7}$$

where P is a positive definite matrix. Define the difference of the Lyapunov function as

$$\Delta V(x_k) = \mathbb{E}\left\{V(x_{k+1})|x_k\right\} - V(x_k). \tag{8}$$

Then, it yields from (3) and (8) that

$$\begin{aligned}
&\mathbb{E}\left\{\Delta V(x_k) + \|y_k\|^2 - \gamma^2\|\omega_k\|^2\right\} \\
&= \mathbb{E}\left\{\left(e^{Ah_k}x_k + \int_0^{h_k} e^{As}dsBs(Kx_k)\right)^T P\left(e^{Ah_k}x_k + \int_0^{h_k} e^{As}dsBs(Kx_k)\right)\right. \\
&\quad + 2\left(e^{Ah_k}x_k + \int_0^{h_k} e^{As}dsBs(Kx_k)\right)^T P\int_0^{h_k} e^{As}dsE\omega_k \\
&\quad \left. + \omega_k^T E^T \int_0^{h_k} e^{A^T s}dsP\int_0^{h_k} e^{As}dsE\omega_k - x_k^T P x_k + \|y_k\|^2 - \gamma^2\|\omega_k\|^2\right\}.
\end{aligned} \tag{9}$$

Inspired by [33], let $Z = \begin{bmatrix} A & I \\ 0 & 0 \end{bmatrix}$. Noting that $e^{Av} = \sum_{i=0}^{\infty} \frac{A^i v^i}{i!}$ and $\int_0^v e^{As}ds = \sum_{i=0}^{\infty} \frac{A^i v^{i+1}}{(i+1)!}$, we have

$$e^{Zv} = \sum_{i=0}^{\infty} \frac{C^i v^i}{i!} = \begin{bmatrix} \sum\limits_{i=0}^{\infty} \frac{A^i v^i}{i!} & \sum\limits_{i=0}^{\infty} \frac{A^i v^{i+1}}{(i+1)!} \\ 0 & I \end{bmatrix} = \begin{bmatrix} e^{Av} & \int_0^v e^{As}ds \\ 0 & I \end{bmatrix}. \tag{10}$$

It follows from (10) that $e^{Ah_k} + \int_0^{h_k} e^{As}dsBK$ and $\int_0^{h_k} e^{As}dsE$ can be rewritten as

$$e^{Ah_k} + \int_0^{h_k} e^{As}dsBK = \begin{bmatrix} I & 0 \end{bmatrix} e^{Zh_k} \begin{bmatrix} I \\ BK \end{bmatrix}$$

and $\int_0^{h_k} e^{As}dsE = \begin{bmatrix} I & 0 \end{bmatrix} e^{Zh_k} \begin{bmatrix} 0 \\ E \end{bmatrix}$, respectively. Accordingly, one has

$$\begin{aligned}
&\mathbb{E}\left\{\Delta V(x_k) + \|y_k\|^2 - \gamma^2\|\omega_k\|^2\right\} \\
&= \mathbb{E}\left\{\begin{bmatrix} x_k^T & \psi^T(Kx_k) \end{bmatrix} \begin{bmatrix} I & K^T H B^T \\ 0 & B^T \end{bmatrix} e^{Z^T h_k} \begin{bmatrix} I \\ 0 \end{bmatrix} P \begin{bmatrix} I & 0 \end{bmatrix} e^{Zh_k} \begin{bmatrix} I & 0 \\ BHK & B \end{bmatrix} \begin{bmatrix} x_k \\ \psi(Kx_k) \end{bmatrix}\right. \\
&\quad + 2\begin{bmatrix} x_k^T & \psi^T(Kx_k) \end{bmatrix} \begin{bmatrix} I & K^T H B^T \\ 0 & B^T \end{bmatrix} e^{Z^T h_k} \begin{bmatrix} I \\ 0 \end{bmatrix} P \begin{bmatrix} I & 0 \end{bmatrix} e^{Zh_k} \begin{bmatrix} 0 \\ E \end{bmatrix} \omega_k + 2x_k^T C^T D\omega_k \\
&\quad \left. + \omega_k^T\left(\begin{bmatrix} 0 & E^T \end{bmatrix} e^{Z^T h_k} \begin{bmatrix} I \\ 0 \end{bmatrix} P \begin{bmatrix} I & 0 \end{bmatrix} e^{Zh_k} \begin{bmatrix} 0 \\ E \end{bmatrix} + D^T D - \gamma^2 I\right)\omega_k + x_k^T\left(C^T C - P\right)x_k\right\}.
\end{aligned} \tag{11}$$

According to analysis in [29], let $\lambda_1, \lambda_2, \ldots, \lambda_f$ respectively denote as the eigenvalues of matrix $Z \in \mathbb{R}^{2n\times 2n}$ with their algebraic multiplicities $n_1, n_2, \ldots, n_f$, where $n_1 + n_2 + \cdots + n_f = 2n$. Then for a scalar v, one has

$$e^{Zv} = \left(\left(\pi(v)V_z^{-1}\right) \otimes I\right)\bar{Z} \tag{12}$$

where

$$\pi(v) = \left[\begin{array}{cccccc} \pi_1(v) & \pi_2(v) & \cdots & \pi_d(v) & \cdots & \pi_f(v) \end{array} \right]$$

with $\pi_d(v) = \left[\begin{array}{cccc} e^{\lambda_d v} & v e^{\lambda_d v} & \cdots & v^{n_d-1} e^{\lambda_d v} \end{array} \right]$, $\bar{Z} = \left[\begin{array}{cccc} I & Z^T & \cdots & Z^{(2n-1)T} \end{array} \right]^T$ and V_z is the confluent Vandermonde matrix introduced in [34, 35].

It is easily verified that matrix $\mathbb{E}\left\{\pi^T(\varepsilon_k)\pi(\varepsilon_k)\right\}$ is positive semidefinite, and hence, there exists a matrix U such that

$$\mathbb{E}\left\{\pi^T(\varepsilon_k)\pi(\varepsilon_k)\right\} = U^T U. \tag{13}$$

Let $\bar{P} = \left[\begin{array}{c} I \\ 0 \end{array} \right] P \left[\begin{array}{cc} I & 0 \end{array} \right]$. Then, according to (12) and (13), the expectation of a product of three matrices including the matrix exponential e^{Zh_k} and its transpose can be calculated as

$$\begin{aligned} \mathbb{E}\left\{e^{Z^T h_k}\bar{P}e^{Zh_k}\right\} &= e^{Z^T h}\mathbb{E}\left\{e^{Z^T\varepsilon_k}\bar{P}e^{Z\varepsilon_k}\right\}e^{Zh} \\ &= e^{Z^T h}\mathbb{E}\left\{\bar{Z}^T\left(V_z^{-T}\otimes I\right)\left(\pi^T(\varepsilon_k)\otimes I\right)\bar{P}\left(\pi(\varepsilon_k)\otimes I\right)\left(V_z^{-1}\otimes I\right)\bar{Z}\right\}e^{Zh} \\ &= e^{Z^T h}\bar{Z}^T\left(V_z^{-T}\otimes I\right)\mathbb{E}\left\{\pi^T(\varepsilon_k)\pi(\varepsilon_k)\otimes\bar{P}\right\}(V_z^{-1}\otimes I)\bar{Z}e^{Zh} \\ &= e^{Z^T h}\bar{Z}^T\left(V_z^{-T}\otimes I\right)\left(U^T U\otimes\bar{P}\right)\left(V_z^{-1}\otimes I\right)\bar{Z}e^{Zh} \\ &= e^{Z^T h}\bar{Z}^T\left(V_z^{-T}U^T\otimes\left[\begin{array}{c} I \\ 0 \end{array} \right]\right)(I\otimes P)\left(UV_z^{-1}\otimes\left[\begin{array}{cc} I & 0 \end{array} \right]\right)\bar{Z}e^{Zh} \\ &= \Phi^T(I\otimes P)\Phi. \end{aligned} \tag{14}$$

Remark 3.1. *Since the existence of random variable ε_k, the term $e^{Z^T h_k}\left[\begin{array}{c} I \\ 0 \end{array} \right] P \left[\begin{array}{cc} I & 0 \end{array} \right] e^{Zh_k}$ in (11) is a coupling matrix subject to high nonlinearity and randomness. With the help of confluent Vandermonde matrix approach and Kronecker product operation, the expectation in (14) is calculated and then saturated H_∞ controller can be designed.*

Considering (6), for a given positive scalar τ, it is obtained from (11) and (14) that

$$\mathbb{E}\left\{\Delta V(x_k) + \|y_k\|^2 - \gamma^2\|\omega_k\|^2\right\} \le \mathbb{E}\left\{\zeta_k^T \left[\begin{array}{cc} \Pi_{11}+\tilde{C} & \Pi_{12} \\ * & \Pi_{22} \end{array} \right] \zeta_k\right\} \tag{15}$$

where

$$\zeta_k = \left[\begin{array}{cc} \left[\begin{array}{cc} x_k^T & \psi^T(Kx_k) \end{array} \right] & \omega_k^T \end{array} \right]^T, \quad \tilde{C} = \left[\begin{array}{cc} C^T C & 0 \\ 0 & 0 \end{array} \right],$$

$$\Pi_{11} = \left[\begin{array}{cc} -P & \tau K^T(I-H)/2 \\ * & -\tau I \end{array} \right] + \left[\begin{array}{cc} I & K^T H B^T \\ 0 & B^T \end{array} \right] \Phi^T(I\otimes P)\Phi \left[\begin{array}{cc} I & 0 \\ HKB & B \end{array} \right],$$

$$\Pi_{12} = \left[\begin{array}{cc} I & K^T H B^T \\ 0 & B^T \end{array} \right] \Phi^T(I\otimes P)\Phi \left[\begin{array}{c} 0 \\ E \end{array} \right] + \left[\begin{array}{c} C^T D \\ 0 \end{array} \right],$$

$$\Pi_{22} = \left[\begin{array}{cc} 0 & E^T \end{array} \right] \Phi^T(I\otimes P)\Phi \left[\begin{array}{c} 0 \\ E \end{array} \right] + D^T D - \gamma^2 I.$$

If $\left[\begin{array}{cc} \Pi_{11}+\tilde{C} & \Pi_{12} \\ * & \Pi_{22} \end{array} \right] < 0$, we can conclude that

$$\mathbb{E}\left\{\Delta V(x_k) + \|y_k\|^2 - \gamma^2\|\omega_k\|^2\right\} < 0. \tag{16}$$

For all $k \in \mathbb{N}_0$, we have by summing up (16) that

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\|y_k\|^2\right\} - \gamma^2\mathbb{E}\left\{\sum_{k=0}^{\infty}\|\omega_k\|^2\right\} \le \mathbb{E}\left\{x^T(0)Px(0)\right\} - \mathbb{E}\left\{x^T(\infty)Px(\infty)\right\}. \quad (17)$$

Notice that $\mathbb{E}\left\{x^T(\infty)Px(\infty)\right\} > 0$, under zero initial condition, it is obtained from (17) that $\mathbb{E}\left\{\sum_{k=0}^{\infty}\|y_k\|^2\right\} < \gamma^2\mathbb{E}\left\{\sum_{k=0}^{\infty}\|\omega_k\|^2\right\}$ holds for any nonzero ω_k.

When $\omega_k = 0$, it yields from (7) and (15) that $\mathbb{E}\{\Delta V(x_k)\} \le \mathbb{E}\left\{x_k^T\Pi_{11}x_k\right\}$. Therefore, $\begin{bmatrix} \Pi_{11}+\tilde{C} & \Pi_{12} \\ * & \Pi_{22}\end{bmatrix} < 0$ implies $\Pi_{11} < 0$, i.e., $\mathbb{E}\{\Delta V(x_k)\} < 0$. Hence, one has

$$\mathbb{E}\{\Delta V(x_k)\} \le \mathbb{E}\left\{x_k^T\Pi_{11}x_k\right\} \le -\lambda_{\min}(-\Pi_{11})\mathbb{E}\left\{x_k^Tx_k\right\} \le -\frac{\lambda_{\min}(-\Pi_{11})}{\lambda_{\max}(P)}\mathbb{E}\{V(x_k)\}.$$

By defining ϖ with $0 < \varpi < \min\{\lambda_{\min}(-\Pi_{11}), \lambda_{\max}(P)\}$, one has

$$\mathbb{E}\{\Delta V(x_k)\} < -\frac{\varpi}{\lambda_{\max}(P)}\mathbb{E}\{V(x_k)\}. \quad (18)$$

Then, in terms of the Lyapunov function $V(x_k) = x_k^TPx_k$, one has

$$\lambda_{\min}(P)\|x_k\|^2 \le V(x_k) \le \lambda_{\max}(P)\|x_k\|^2. \quad (19)$$

According to the Lemma 1 in [36], it yields from (18) and (19) that

$$\mathbb{E}\left\{\|x_k\|^2\right\} \le \vartheta\mu^{\sigma}\mathbb{E}\left\{\|x_0\|^2\right\}$$

where $\vartheta = \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}$ and $\mu = 1 - \frac{\varpi}{\lambda_{\max}(P)} \in (0,1)$.

Accordingly, if $\begin{bmatrix} \Pi_{11}+\tilde{C} & \Pi_{12} \\ * & \Pi_{22}\end{bmatrix} < 0$, stochastic system (3) is exponentially mean-square stable with a prescribed H_∞ performance γ. The proof is thus completed. □

The following theorem is devoted to designing a saturated H_∞ controller for stochastic system (3).

Theorem 3.2. *Let $\gamma > 0$ and $\tau > 0$ be given. Discrete-time stochastic system (3) is exponentially stable in the mean-square sense with a prescribed H_∞ performance γ if there exist matrices $Q > 0$ and Y such that the following inequality holds:*

$$\begin{bmatrix} -Q & \tau Y^T(I-H)/2 & 0 & \begin{bmatrix} Q & Y^THB^T \end{bmatrix}\Phi^T & QC^T \\ * & -\tau I & 0 & \begin{bmatrix} 0 & B^T \end{bmatrix}\Phi^T & 0 \\ * & * & -\gamma^2 I & \begin{bmatrix} 0 & E^T \end{bmatrix}\Phi^T & D^T \\ * & * & * & -(I\otimes Q) & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \quad (20)$$

Furthermore, if Inequality (20) is feasible, the desired saturated H_∞ controller gain matrix is given by $K = YQ^{-1}$.

Proof: By using the Schur complement formula, $\begin{bmatrix} \Pi_{11}+\tilde{C} & \Pi_{12} \\ * & \Pi_{22}\end{bmatrix} < 0$ holds if and only if

$$\begin{bmatrix} -P & \tau K^T(I-H)/2 & 0 & \begin{bmatrix} I & K^THB^T \end{bmatrix}\Phi^T & C^T \\ * & -\tau I & 0 & \begin{bmatrix} 0 & B^T \end{bmatrix}\Phi^T & 0 \\ * & * & -\gamma^2 I & \begin{bmatrix} 0 & E^T \end{bmatrix}\Phi^T & D^T \\ * & * & * & -\left(I\otimes P^{-1}\right) & 0 \\ * & * & * & * & -I \end{bmatrix} < 0. \quad (21)$$

Performing a congruence transformation to Inequality (20) by $\mathrm{diag}\{Q^{-1}, I, I, I, I\}$, let $P = Q^{-1}$ and $Y = KQ$, we obtain (21) immediately, i.e., (20) implies that $\begin{bmatrix} \Pi_{11} + \tilde{C} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0$ holds and then the stochastic system (3) is exponentially mean-square stable with a prescribed H_∞ performance γ. □

4. **Illustrative Example.** In this section, we give two examples to prove the effectiveness of the designed approach proposed in this paper.

Example 4.1. *Consider a continuous-time networked system with the following parameters:*

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -0.5 & 0.8 & 0.4 \\ 0 & -0.5 & 0.12 \\ 0 & 0 & 0.3 \end{bmatrix} x(t) + \begin{bmatrix} 0.2 \\ 0.11 \\ -0.3 \end{bmatrix} s(u(t)) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \omega(t), \\ y(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x(t). \end{cases}$$

Set the nominal sampling interval as $h = 0.4$ and the sampling errors obey the uniform distribution of -0.2 to 0.2. The parameters of the saturation function are chosen as $u_{1,\max} = 1$ and $H = 0.19$. Suppose that $\tau = 1$ and $\gamma = 0.3$. Solve Inequality (20) by using the LMI toolbox on MATLAB, the following feasible solutions are obtained:

$$Q = \begin{bmatrix} 4.2101 & 0.0004 & 0.0002 \\ 0.0004 & 0.0003 & -0.0000 \\ 0.0002 & -0.0000 & 0.0001 \end{bmatrix} \times 10^3, \quad Y = \begin{bmatrix} -1.5759 & -0.1125 & 0.4187 \end{bmatrix}.$$

According to Theorem 3.2, the desired saturated H_∞ controller gain is

$$K = YQ^{-1} = \begin{bmatrix} -0.0007 & 0.0627 & 7.2625 \end{bmatrix}.$$

Set $x(0) = \begin{bmatrix} 0.4 & -0.3 & 0.5 \end{bmatrix}^T$ and $\omega(t) = \sin(0.1t)e^{-0.1t}$. The sampling errors with uniform distribution are shown in Figure 1. Figure 2 shows the state trajectories of closed-loop systems with noisy sampling intervals and actuators saturation. By calculation,

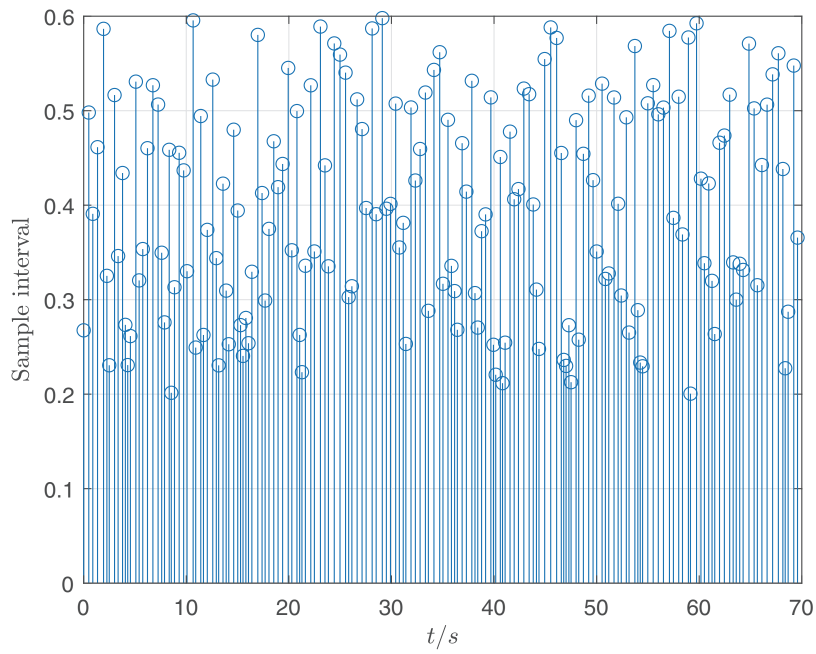

FIGURE 1. The sampling errors with uniform distribution

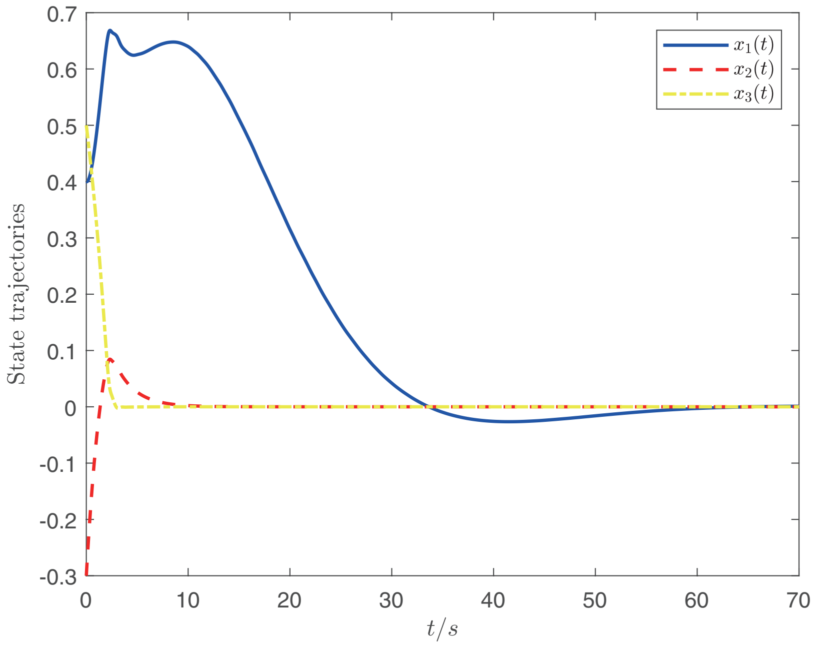

FIGURE 2. State trajectories of system with control inputs

$\frac{\|y\|_2}{\|\omega\|_2} = \frac{0.3966}{1.8199} = 0.2179 < \gamma = 0.3$, *which shows the effectiveness of the proposed saturated* H_∞ *controller design approach.*

Example 4.2. *Consider a separately excited DC motor without load* [33, 37]*, and the dynamics are described as*

$$u = e_a = L\frac{di_a}{dt} + Ri_a + e_b \tag{22}$$

and

$$J\frac{dw}{dt} + \bar{B}\omega = K_a i_a \tag{23}$$

where $u = e_a$ *is the armature winding input voltage;* $e_b = K_b\omega$ *is the back-electromotive-force voltage;* L *is the armature winding inductance;* i_a *is the armature winding current;* R *is the armature winding resistance;* J *is the system moment of inertia;* $\bar{B}$ *is the system damping coefficient;* K_a *and* K_b *are the torque constant and the back-electromotive-force constant, respectively; and* ω *is the rotor angular speed.*

In this example, we select $x_1 = i_a$*,* $x_2 = \omega$ *and* $x_3 = \theta$*, where* θ *is the angle of rotor. Accordingly, (22) and (23) can be rewritten as the following state-space model:*

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_b}{L} & 0 \\ \frac{K_a}{J} & -\frac{\bar{B}}{J} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} u.$$

The parameters of the motor used in this article are $J = 4.26 \times 10^{-5}$ kgm^2*,* $L = 0.17$ H*,* $R = 4.67$ Ω*,* $\bar{B} = 4.73 \times 10^{-5}$ Nms/rad*,* $K_a = 1.47 \times 10^{-2}$ Nm/A*,* $K_b = 1.47 \times 10^{-2}$ Vs/rad*, and then, the matrixes* A *and* B *are calculated as*

$$A = \begin{bmatrix} -27.4706 & -0.0865 & 0 \\ 0.0865 & -1.1103 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 5.8824 \\ 0 \\ 0 \end{bmatrix}.$$

We are interested in the armature winding current and the angle of rotor; therefore, the matrix of the output equation is $C = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$. *Moreover, we suppose that* $E = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$ *and* $D = 3$. *Assume that the nominal sampling interval* $h = 0.5$, *and the sampling errors obey the uniform distribution of* -0.2 *to* 0.2. *According to (13), matrix* U *is obtained as*

$$U = \begin{bmatrix} 0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & 0.0000 \\ 0.0000 & -0.0000 & -0.0000 & 0.0001 & 0.0000 & -0.0006 \\ -0.0016 & 0.0000 & 0.0016 & -0.0011 & -0.0075 & -0.0008 \\ 0.0488 & -0.0002 & -0.0445 & -0.0874 & -0.0066 & -0.0017 \\ -0.8766 & 0.0160 & -0.9032 & -0.0287 & -0.0088 & -0.0009 \\ -0.5123 & -51.8877 & -0.4269 & 0.0698 & -0.0120 & 0.0021 \end{bmatrix}.$$

In addition, the parameters of the saturation function are chosen as $u_{1,\max} = 1$ *and* $H = 0.1$. *Suppose that* $\tau = 1$ *and* $\gamma = 3.2$. *With the above parameters, we can have by solving Inequality (20) that*

$$Q = \begin{bmatrix} 0.3408 & 0.2492 & -0.2236 \\ 0.2492 & 0.2760 & -0.2443 \\ -0.2236 & -0.2443 & 0.2185 \end{bmatrix}$$

and

$$Y = \begin{bmatrix} 0.5244 & 0.5927 & -0.5540 \end{bmatrix}.$$

Then, the designed controller gain is

$$K = YQ^{-1} = \begin{bmatrix} -0.4459 & -9.5184 & -13.6350 \end{bmatrix}.$$

In this example, the initial value is chosen to be $x(0) = \begin{bmatrix} 1 & -0.8 & 0.5 \end{bmatrix}^T$, *and* $\omega(t) = \cos(t)e^{-0.5t}$. *Figure 3 shows the state trajectories of the closed-loop system. By calculation,* $\frac{\|y\|_2}{\|\omega\|_2} = \frac{1.5835}{1.2149} = 1.3034 < \gamma = 3.2$, *which shows the effectiveness of the proposed saturated* H_∞ *controller design approach.*

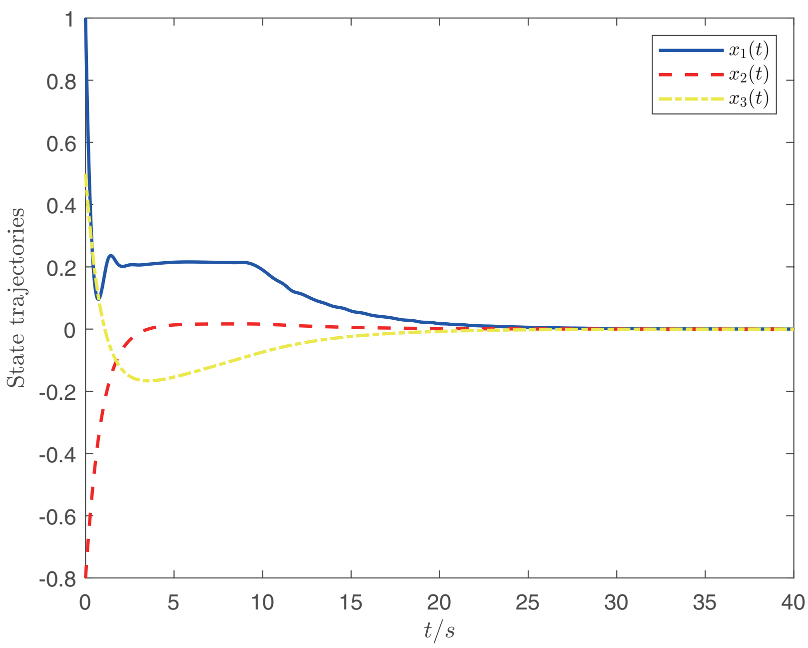

Figure 3. State trajectories of system with control inputs

5. **Conclusions.** In this paper, the stabilization problem of networked systems with noisy sampling intervals and actuators saturation was studied. By using the discrete-time approach, we first established an equivalent stochastic system model. By means of the confluent Vandermonde matrix approach and Kronecker product operation, the mathematical expectation of a nonlinear and random coupling matrix was calculated and then a sufficient condition for the mean-square stability of resulting closed-loop discrete-time stochastic system with a prescribed H_∞ performance was obtained. Based on this, a saturated H_∞ controller was designed. Finally, two examples were provided to show the effectiveness of the designed approach. In the future work, the H_∞ control problem of networked control systems subject to noisy sampling intervals, packet losses and actuators saturation will be considered.

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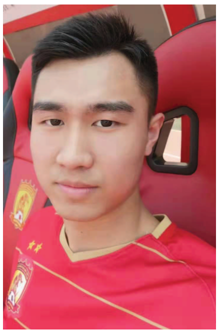
Author Biography



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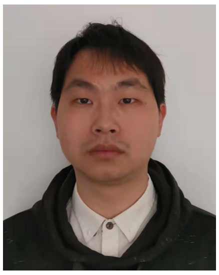

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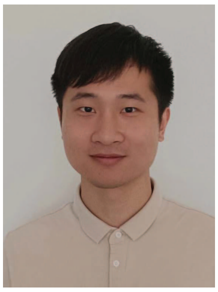

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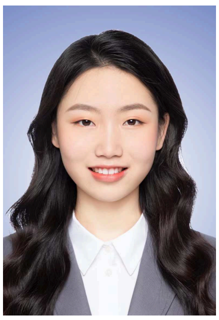

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