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STRATEGIES BASED ON THE FOCUS POINT IN AN ONLINE VICKREY AUCTION OF SECONDHAND GOODS WITH BUYOUT OPTION

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Abstract. *This work studies an online auction process of selling a secondhand good, which cannot be repeated many times. The secondhand goods market is of great significance in optimizing resource allocation, reducing resource consumption and reducing environmental pollution. The development of e-commerce and online auction provides a favorable tool for the prosperity and standardization of the secondhand goods market. Online auctions are widely used since they break the barrier of space. A buyout option allows the bidder to obtain the auction item immediately by accepting the buyout price. Whether a participant wins the auction or not is determined by a specific bidding price rather than the average price of all the others. This bidding price is called a focus point in this paper. A new auction model based on the focus point is proposed to analyze the decision behaviors of the seller and the bidders. It is an application and expansion of one-shot decision theory. By the new model, we obtain the optimal strategy of the bidders, state the necessity of providing the buyout option and suggest the price interval for the seller to increase their revenues. It provides a solution of low adoption rate of buyout price in reality, and provides theoretical support for the secondhand goods auction platform.*
Keywords: Online auction, Vickrey auction, Buyout price, Focus point

1. **Introduction.** An online auction is an auction that is held over the Internet. As a successful application of Internet commerce, an online auction is the combination of a traditional auction and e-commerce [1]. A bidder can take part in an online auction no matter her/his location. The wide application of online auctions provides buyers with chances to purchase more diverse product with potentially lower prices; meanwhile, it provides sellers with a more flexible form to obtain higher returns. As a result, online auctions have received more attention from e-commerce platforms such as Amazon, Yahoo and JD. The research on revenue and procurement management under online auctions is on the rise [2-5].

As a selling format available for the seller, auctions are paid attention to continuously [6,7]. Besides the standard auction formats, Adikari and Dutta [8] proposed an auto pricing strategy approach to determine the bid prices from the advertising agencies' perspective. Bobkova [9] solved a first-price auction for two bidders with asymmetric budget distributions. The efficiency of traditional auctions was partly reduced by the existence of auction participation costs. Online auctions break the limitations of space and time and reduce the participation costs. Different from traditional auctions, an online auction

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usually lasts for a few days. During this period, the bidders arrive sequentially. A buyout option allows the bidders to obtain the lot immediately by accepting the buyout price. The buyout price, as an additional choice, covers the costs of time passing and becomes an important component of online auctions. There are two types of buyout options. The first, which is called a permanent buyout option, is considered in this paper. It remains available throughout the whole auction process. The other is a temporary buyout option, which disappears as soon as a regular bid is submitted, such as eBay's Buy-It-Now option. Gallien and Gupta [10] compared the difference between temporary and permanent buyout options and found the value of the permanent buyout price was higher than the value for a temporary buyout price. Sun et al. [11] demonstrated that regarding the formats for the posted price for the traditional auction and buyout price auction, one format does not dominate the other. They pointed out that the preferred format was determined by a unique cutoff. The fact that a buyout option increased a seller's revenue was also expounded in researches [12,13]. Online auctions are effective ways to sell goods with uncertain value. An appropriate buyout price can provide a reference to avoid the "winner's curse".

An online auction process with a buyout option involves two decision-making parties. For the bidders, they decide to accept the buyout price or bid. For the seller, she/he sets the buyout price to increase her/his revenue. In some academic articles, the buyout price was set according to the Bayesian Nash equilibrium, which was derived from Vickrey's work [14]. In reality, the buyout price was often set too high by an individual seller without enough information. Experimental evidence showed that the bidders did not follow the assumption of risk neutrality and their behavior was not always consistent with rational decision-making theory [15,16]. Simon [17,18] introduced the term bounded rationality to replace the perfect rationality assumption of homo economicus. Shen and Su [19] gave an overview of customer behavior modeling in the auction and revenue management literature. Jiang et al. [20] presented a behavioral choice function to characterize the behaviors of the customers with bounded rationality and then selected the optimal selling format from the posted price, pure auction and buy-price auction mechanisms. Their study concluded that none of three selling formats really dominated the other two, but the buyout-price auction format may outperform the others in most cases of their experiments. Gao and Fan [21] described bidders' bounded rational behaviors using a bidding probability-based function. These methods are lottery-based models which generally follow the Bernoullian framework [22].

We consider a secondhand good which is sold over the Internet through an auction. According to the selling characteristics of secondhand goods, almost every auction item is different and the online auction process cannot be repeated many times. In an auction process, whether a bidder accepts the buyout price or not depends on the highest bidding price offered by the others, and the seller's profit depends on the highest bidding price offered by the bidders. In this situation, we argue that the participant makes a decision based on a particular scenario rather than the weighted average of all possible highest bidding prices, and the chosen scenario is called a focus point. One-shot decision theory was used to build a behavioral model for first-price sealed-bid auctions by Wang and Guo [23]. Different from the previous studies, they built a scenario-based model to reflect the decision-making procedure in the first-price sealed-bid auction. In the proposed method, each bidding price was evaluated by a focus point instead of the weighted average. One-shot decision theory was proposed by Guo [24]. The theory has been used to analyze the newsvendor problem for an original product and multistage one-shot decision-making problems such as optimal stopping [22,25]. Guo [26] put forward the focus theory of choice based on the same fundamental. The idea of our model is similar to the core argument of one-shot decision theory [24] and the fundamental of the model of Wang and Guo [23], but the auction mechanism and the meaning of the focus point are totally different from Wang and Guo's study [23]. They seek the focus point for each bidding price and take the bidding price that generates the best outcome as the optimal strategy for first-price sealed-bid auctions. We study a Vickrey online auction where the focus point of bidder i is the highest bidding price provided by the others in i's opinion. The focus point in our model can reflect the fact that a participant wins the auction or not is determined by the highest bidding price offered by the others.

This paper makes two main contributions to the literature on online auction models. First, we build a behavioral model based on the focus points to describe the decision-making process of the bidders and seller in a Vickrey online auction process. The optimal strategy of the bidder and the buyout price range of the seller are obtained. The influences of the number of participants and the valuation of the bidders on the strategy are analyzed, and the relationship between the different types of focus points is given. Second, insight into online auctions is obtained. The phenomena such as the low adoption rate of a buyout price in the secondhand auction market can be explained. A method for avoiding the problem that bidders' offers are usually concentrated at the end segment is given. A platform such as JD can give the probability distribution of transaction price by the advantage of information to help sellers set bidding prices.

The rest of this paper is organized as follows. in Section 2, online Vickrey auction model based on the focus point is given, and the optimal strategy of the bidder and the buyout price range of the seller is obtained. In Section 3, the relationship between focus point and bidder's valuation, focus point and the number of bidders is analyzed while bidder's valuation follows a normal distribution. In Section 4, concluding remarks are made.

2. **Online Vickrey Auction Model Based on the Focus Point.** We consider an individual seller who intends to sell a secondhand good via auction on the website. The length of the process is T. A typical feature of online auction is that websites often have "proxy bidding" systems that allow bidders to enter the maximum amount and the minimum increment. An online auction with a proxy bidding system can be considered to be a second-price (Vickrey) auction [10,21]. We analyze the decision-making process in an online Vickrey auction, which means that bidders submit their highest bids to the proxy bidding system without knowing the bids of others, and the winner pays the second highest bid. There is a buyout price B, which allows the bidder to obtain the auction item immediately by accepting it. A permanent buyout option that remains available throughout the whole sale process is considered. There are n potential bidders, and their valuations are assumed to follow an independent private value model [27]. Each bidder i $(1 \leq i \leq n)$ has a privately known valuation v_i $(v_i \in [0, \bar{v}])$ and knows the probability density function $f(x)$ of each other bidder j's $(j \neq i)$ valuation. The seller knows every bidder's probability density function of valuation: $f(x)$. Suppose that all bidders' probability density functions are independent and identically distributed, and the cumulative distribution function is $F(x)$.

2.1. **Bidder's optimal strategy.** Whether a bidder obtains the auctioned item or not depends on the highest bidding price. For bidder i, suppose that the highest bidding price of all $j \neq i$ is x^*. The outcomes of bidder i are shown as Table 1, where b_i is the bidding price of i.

From Table 1, we know that without the buyout option, the optimal bidding price of bidder i is still v_i in our model. Therefore, bidder i will not accept the buyout price and will enter v_i as her/his maximum bidding price while $B > v_i$.

TABLE 1. The revenue of bidder i in a Vickrey auction

	$b_i < v_i$	$b_i = v_i$	$b_i > v_i$
$x^* \leq v_i$	0 or $v_i - x^*$	$v_i - x^*$	$v_i - x^*$
$x^* > v_i$	0	0	0 or $v_i - x^* < 0$

If $B \leq v_i$, bidder i will compare the revenue from the buyout price and the revenue from regular bidding. Instead of the average value, for bidder i, a particular state is used to evaluate her/his rivals' highest bidding price. Suppose that in bidder i's opinion, the highest bidding price of all $j \neq i$ is x^*, and then x^* is called a focus point of bidder i. If $x^* > v_i$, which means that bidder i thinks someone's valuation of the item is higher than her/him, she/he should take the buyout price at once. Otherwise, the focus point is selected as follows.

Suppose that the second highest price is x. The profit function of bidder i is denoted as

$$u(x) = \begin{cases} v_i - x, & 0 \leq x \leq v_i \\ 0, & v_i < x \leq \bar{v} \end{cases} \tag{1}$$

The density function of x can be calculated by

$$p(x) = \begin{cases} (n-1)[F(x)]^{n-2} f(x), & 0 \leq x \leq v_i \\ 1 - [F(x)]^{n-1}, & v_i < x \leq \bar{v} \end{cases} \tag{2}$$

In Formula (2), the first item refers to the probability that bidder i wins the auction, and the second item is the probability that at least one person's bid is higher than that of bidder i. Using (1) and (2), the essential concepts for selecting a focus point can be given.

$$\mu(x) = \frac{u(x)}{\max_x u(x)} \tag{3}$$

is called the satisfaction function of bidder i. It is defined over the range of $[0, 1]$, and the satisfaction level $\mu(x)$ is used to measure the relative feeling of the result that x is the second highest bidding price.

$$\pi(x) = \frac{p(x)}{\max_x p(x)} \tag{4}$$

is the degree of bidder i's relative likelihood. It is also defined over the range of $[0, 1]$, and $\pi(x)$ is used to represent the relative position of the probability of event x.

Substituting (1) and (2) into (3) and (4), respectively, we have

$$\mu(x) = 1 - \frac{x}{v_i}, \quad 0 \leq x \leq v_i \tag{5}$$

$$\pi(x) = \frac{[F(x)]^{n-2} f(x)}{\max_x [F(x)]^{n-2} f(x)}, \quad 0 \leq x \leq v_i \tag{6}$$

The focus point is determined by both the satisfaction level and the degree of the relative likelihood. We consider two types of focus points. The one with higher satisfaction level and higher degree of relative likelihood is called an active focus point and is denoted as x_1^*:

$$x_1^* = \arg \max_{x \in [0, v_i)} \min(\pi(x), \mu(x)) \tag{7}$$

Bidder i who chooses an active focus point is called an active bidder. An active focus point reflects the relatively pessimistic attitude of the bidder. She/he has confidence that they can obtain higher profits with a higher probability from the auction process.

The conservative focus point x_2^* is a state with a higher degree of relative likelihood and a lower satisfaction level:

$$x_2^* = \arg \min_{x \in [0, v_i)} \max(1 - \pi(x), \mu(x)) \tag{8}$$

The bidder who chooses a conservative focus point is called a conservative bidder. A conservative focus point reflects the relatively optimistic attitude of the bidder. She/he focuses on avoiding the situation that brings her/him lower profits with a higher probability. Both x_1^* and x_2^* are the Pareto optimal solutions of a bi-objective optimization problem.

We assume that the seller uses a fixed permanent buyout price B. Bidder i can observe a current second highest bidding price x and the buyout price B. She/he should decide to take the buyout price or not. If bidder i takes B, her/his profit is

$$u^{buyout} = v_i - B \tag{9}$$

Each bidder makes his/her optimal strategy by comparing the profits of the regular Vickrey auction process and the buyout price. If $u = u^{buyout}$ holds, the profits from choosing a regular Vickrey auction and purchasing with a buyout price are the same. Since

$$u = v_i - x^* \tag{10}$$

where x^* is a type of focus point, via (9) and (10), if $u = u^{buyout}$, we have

$$x^* = B \tag{11}$$

If $x^* < B$, bidder i will adopt the regular bidding strategy since the profit of the regular bidding is greater than the profit of the buyout option; conversely, if $x^* > B$, bidder i will adopt the buyout option. If $x^* = B$ is satisfied, these two options are indistinguishable to the bidder, and we suppose that she/he will use the regular bidding strategy.

In summary, the optimal strategy of bidder i is obtained as follows.

Theorem 2.1. *In an online Vickrey auction with buyout price B, for bidder i ($i = 1, 2, \ldots, n$), we have the following:*

1) if $B > v_i$, or $x^* \le B \le v_i$, bidder i should participate in the regular auction, and the optimal bidding price $b_i^* = v_i$, where x^* is a type of focus point of bidder i;
2) if $B < x^* < v_i$, or $B < v_i \le x^*$, bidder i should take the buyout price immediately.

From Theorem 2.1, we know bidder i should take the buyout price while $B < v_i$ unless she/he thinks that nobody's valuation of the auction item is higher than B. If bidder i decides to take part in the regular auction, the optimal bidding price is v_i. Therefore, an online Vickrey auction with buyout price B can solve the problem that bidders' offers are usually concentrated at the end of an online auction. The key is to set an appropriate buyout price and to select the focus point of bidder i.

2.2. **Seller's optimal pricing.** The buyout option gives the participants additional information and satisfies the participants' requirements for time sensitivity. If the buyout price is set too low, it will reduce the seller's revenue; meanwhile if it is set too high, it will lose significance since bidders will always participate in the regular bidding process. We argue that the seller evaluates the benefits of the regular bidding format based on the focus point. The seller's profit in a regular bidding can be expressed as $u^s(x)$, where x is the second highest bidding price of all bidders. Without any loss of generality, we suppose that the cost of the auction item is 0. Then,

$$u^s(x) = x, \quad x \in [0, \bar{v}] \tag{12}$$

The probability that x is the second highest price is

$$p^s(x) = n(n-1)[1 - F(x)][F(x)]^{n-2} f(x) \tag{13}$$

Based on the seller's profit function $u^s(x)$ and probability function $p^s(x)$, the satisfaction function $\mu^s(x)$ and the relative likelihood degree function $\pi^s(x)$ can be given as

$$\mu^s(x) = \frac{u^s(x)}{\max_{x\in[0,\bar{v}]} u^s(x)} = \frac{x}{\bar{v}} \tag{14}$$

$$\pi^s(x) = \frac{[1 - F(x)][F(x)]^{n-2} f(x)}{\max_{x\in[0,\bar{v}]} [1 - F(x)][F(x)]^{n-2} f(x)} \tag{15}$$

By substituting (14) and (15) into (7), the active focus point of the seller can be obtained as follows:

$$x_1^{s*} = \arg \max_{x\in[0,\bar{v}]} \min\left(\pi^s(x), \mu^s(x)\right) \tag{16}$$

Similarly, the conservative focus point of the seller can be obtained as

$$x_2^{s*} = \arg \min_{x\in[0,\bar{v}]} \max\left(1 - \pi^s(x), \mu^s(x)\right) \tag{17}$$

With respect to the seller, a buyout price should bring more benefits to her/him than regular bidding. The seller's focus point x^{s*} is the lower bound of the buyout price. The buyout price B makes sense only if there are at least two bidders with a higher valuation than it. Assume that ε is the probability of at least two bidders' valuations being higher than B. The upper bound of the buyout price is set as

$$1 - [F(B)]^n - n[1 - F(B)][F(B)]^{n-1} = \varepsilon \tag{18}$$

Denote $W(B) = [F(B)]^n + n[1 - F(B)][F(B)]^{n-1} - 1 + \varepsilon$. Then, Equation (18) is equivalent to finding the 0 point of $W(B)$. Since $W(0) = \varepsilon - 1 < 0$, $W(\bar{v}) = \varepsilon > 0$, and $W'(B) = n(n-1)[1 - F(B)][F(B)]^{n-2} f(B) > 0$, Equation (18) has a unique solution B^* in interval $[0, \bar{v}]$. ε is used to reflect the probability of the buyout price is accepted, and it is a decreasing function of B. $\varepsilon = 0$ means $B \in [\bar{v}, +\infty)$, and no bidder will choose to accept the buyout price. Meanwhile, $\varepsilon = 1$ means $B = 0$, and the buyout price is certainly accepted. ε is judged by the seller. Using a buyout price should bring more benefits than the regular bidding process. We have

$$x^{s*} \le B \le B^* \tag{19}$$

B is the price where the profit obtained is not less than x^{s*} and the probability of being adopted is not less than ε.

In this section, we analyze an online Vickrey auction with a buyout price and propose a model based on the focus point. The optimal strategy of the bidder and the suggested interval of the buyout price are obtained.

3. **Model Analysis.**

3.1. **The strategies while a bidder's valuation follows a normal distribution.**

If the valuation of bidder i $(1 \le i \le n)$ is a random variable according to a normal distribution $N\left(\frac{\bar{v}}{2}, \sigma^2\right)$ in interval $[0, \bar{v}]$, the probability density function $f(x)$ and the probability distribution function $F(x)$ are expressed as follows.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\frac{\bar{v}}{2})^2}{2\sigma^2}} \tag{20}$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\frac{\bar{v}}{2})^2}{2\sigma^2}} dx \tag{21}$$

3.1.1. *Bidder's optimal strategy while a rival's valuation follows a normal distribution.* The satisfaction function $\mu(x)$ and the relative likelihood degree function $\pi(x)$ can be obtained by substituting (20) and (21) into (5) and (6), respectively:

$$\mu(x) = 1 - \frac{x}{v_i}, \quad 0 \le x \le v_i \tag{22}$$

$$\pi(x) = \frac{[F(x)]^{n-2} f(x)}{\max_{x\in[0,v_i]} [F(x)]^{n-2} f(x)} = \frac{[F(x)]^{n-2} f(x)}{M_1} \tag{23}$$

where $M_1 = [F(m_1)]^{n-2} f(m_1)$, which means that there is an $m_1 \in \left[\frac{\bar{v}}{2}, \bar{v}\right]$ and the maximum of $[F(x)]^{n-2} f(x)$ is at m_1 (see **Appendix A** for the details).

The active focus point x_1^*, which is a solution of $\mu(x) = \pi(x)$, satisfies (24).

$$1 - \frac{x_1^*}{v_i} = \frac{[F(x_1^*)]^{n-2} f(x_1^*)}{M_1} \tag{24}$$

$\mu(x)$ is decreasing in $[0, \bar{v}]$ and the domain of values is $[0, 1]$; meanwhile, $\pi(x)$ is increasing in $[0, m_1]$ and decreasing in the interval $(m_1, \bar{v}]$, and $\pi(m_1) = 1$. There are two intersections of $\mu(x)$ and $\pi(x)$, and they are in $[0, m_1]$ and $(m_1, \bar{v}]$, respectively. The one on the left, that is, $x_1^* \in [0, m_1]$, is chosen as an active focus point since its satisfaction level is higher and an active bidder pursues higher profit.

Similarly, the conservative focus point x_2^*, which is the solution of $\mu(x) = 1 - \pi(x)$, satisfies

$$\frac{x_2^*}{v_i} = \frac{F(x_2^*)^{n-2} f(x_2^*)}{M_1} \tag{25}$$

There are also two intersections of $\mu(x)$ and $1-\pi(x)$, and they are in $[0, m_1]$ and $(m_1, \bar{v}]$, respectively. The one on the right, that is, $x_2^* \in [m_1, \bar{v}]$, is chosen as a conservative focus point since its degree of relative likelihood is higher and a conservative bidder avoids higher possible loss.

3.1.2. *Seller's optimal pricing while a bidder's valuation follows a normal distribution.* When the bidder's valuation follows a normal distribution, the satisfaction function of the seller is

$$\mu^s(x) = \frac{u^s(x)}{\max_{x\in[0,\bar{v}]} u^s(x)} = \frac{x}{\bar{v}} \tag{26}$$

By substituting (20) and (21) into (17), we have that the degree of relative likelihood function of the seller is

$$\pi^s(x) = \frac{[1 - F(x)] F(x)^{n-2} f(x)}{\max_{x\in[0,\bar{v}]} [1 - F(x)][F(x)]^{n-2} f(x)} = \frac{[1 - F(x)] F(x)^{n-2} f(x)}{M_2} \tag{27}$$

where $M_2 = [1 - F(m_2)] F(m_2)^{n-2} f(m_2)$, means that there is an $m_2 \in \left[\frac{\bar{v}}{2}, \bar{v}\right]$ and the maximum of $[1 - F(x)] F(x)^{n-2} f(x)$ is at m_2 (see **Appendix B** for the details).

The active focus point of the seller x_1^{s*} is a solution of $\mu^s(x) = \pi^s(x)$, and it satisfies

$$\frac{x_1^{s*}}{\bar{v}} = \frac{[1 - F(x_1^{s*})] F(x_1^{s*})^{n-2} f(x_1^{s*})}{M_2} \tag{28}$$

$\mu^s(x)$ is decreasing in $[0, \bar{v}]$ and the domain of values is $[0, 1]$, and $\mu^s(x)$ and $\pi^s(x)$ have intersection points in $[0, m_2]$ and $(m_2, \bar{v}]$, respectively. The one on the right, that is, $x_1^{s*} \in (m_2, \bar{v}]$, is chosen as an active focus point of the seller since its satisfaction level is higher and an active seller pursues higher profit.

The conservative focus point of the seller is a solution of $\mu^s(x) = 1 - \pi^s(x)$ that satisfies

$$\frac{x_2^{s*}}{\bar{v}} = 1 - \frac{[1 - F(x_2^{s*})]F(x_2^{s*})^{n-2}f(x_2^{s*})}{M_2} \tag{29}$$

$x_2^{s*} \in [0, m_2]$ is chosen as a conservative focus point of the seller since its degree of relative likelihood is higher and a conservative seller avoids higher opportunity loss.

By (18), (22), (23) and (27), the upper bound of buyout price B^* can be obtained, where B^* satisfies $[F(B^*)]^n + n[1 - F(B^*)][F(B^*)]^{n-1} = 1 - \varepsilon$, and F is the probability according to $N\left(\frac{\bar{v}}{2}, \sigma^2\right)$.

We obtain the focus point of both the bidder and the seller with a valuation following normal distribution. Actually, the degree of relative likelihood function is a normalized version of the probability density function. The focus point can be obtained as long as the probability density function is known, regardless of the specific form.

Theorem 3.1. *In an online Vickrey auction, bidder i's* $(i = 1, 2, \ldots, n)$ *active and conservative focus points can be obtained if* $u(x)$ *and* $p(x)$ *are given. The seller's active and conservative focus points can be obtained while* $u^s(x)$ *and* $p(x)$ *are given. The range of the buyout price is* $[x_t^{s*}, B^*]$, *where* x_t^{s*} $(t = 1, 2)$ *is a type of seller's focus point, and* B^* *is the solution of Equation (18).*

3.2. **Strategy analysis.** We analyze the cases of a bidder's valuation following a normal distribution in the following text. The focus points and buyout price are computed by MATLAB 2016a (We set $\sigma = 10$ to fit the characteristic that the uncertainty of the secondhand good's valuation is relatively large, set $\bar{v} = 100$ since $\int_0^{100} \frac{1}{10\sqrt{2\pi}} e^{-\frac{(x-50)^2}{200}} dx \approx 1$).

Proposition 3.1. *The focus point of bidder i increases while her/his valuation* v_i *increases and the bidder's conservative focus point is not less than the active focus point.*

Proof: The active focus x_1^* satisfies $x_1^* + \frac{x_1^*}{v_i} = 1$. The relative likelihood degree function $\pi(x)$ is a unimodal function, and $x_1^* \in [0, m_1]$. If x_1^* decreases while v_i increases, $\frac{x_1^*}{v_i}$ decreases and $\pi(x_1^*)$ decreases, which is in contradiction with $(x_1^*) + \frac{x_1^*}{v_i} = 1$. Thus, the active focus point increases while v_i increases.

Similarly, we can prove the conservative focus point increases while v_i increases. We have shown that (in **Appendix B**) the conservative focus point $x_2^* \in (m_1, \bar{v}]$, and so $x_1^* \leq x_2^*$. □

Given $\bar{v} = 100$, $\sigma = 10$, and $n = 2$, the active and conservative focus points are as shown in Figure 1.

Proposition 3.1 shows that if an auction item is valuable to bidder i, she/he will think that the item is also valuable to the others. Compared with a conservative bidder, an active bidder is more optimistic about winning the item with a lower bidding price. Actually, Proposition 3.1 holds as long as $\mu(x)$ is a decreasing function and $\pi(x)$ is a unimodal function. Take the active focus point as an example. The graph is as Figure 2.

Proposition 3.2. *The focus point of bidder i increases while the number of bidders n increases.*

Proof: Since $\pi(x) = \frac{F(x)^{n-2}f(x)}{\max_{0 \leq x \leq v_i} F(x)^{n-2}f(x)}$ and $0 < F(x) \leq 1$, $\pi(x)$ decreases while n increases. Similar to the proof of Proposition 3.1, we can obtain that the focus point of bidder i increases while n increases. □

Given $\bar{v} = 100$, $\sigma = 10$, and $n = 100$, the active and conservative focus points are illustrated in Figure 3. Taking Figure 1 and Figure 3 as examples, they show us the result of Proposition 3.2 and the gap between the two types of focus points decreases as the number of bidders increases.

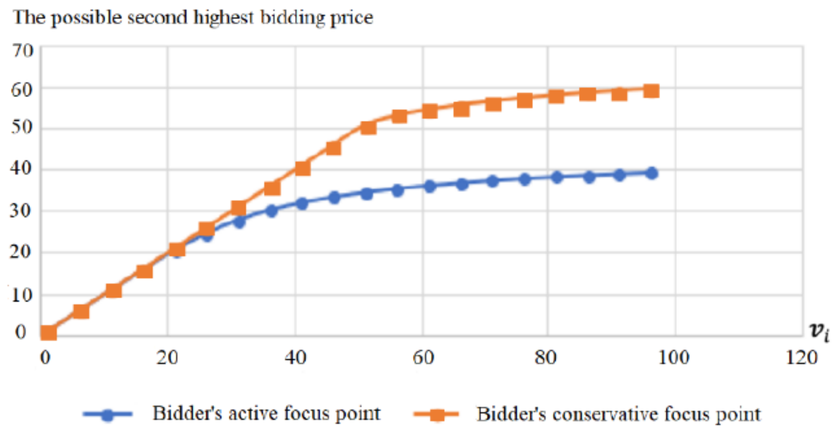

FIGURE 1. The relationship between the focus point and valuation ($n = 2$)

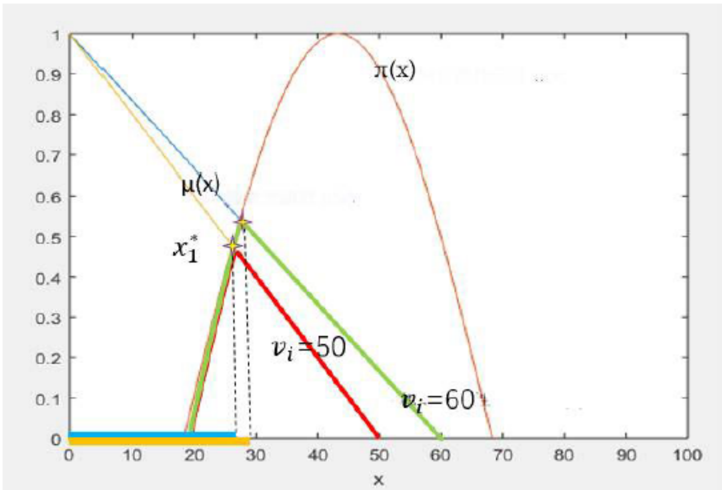

FIGURE 2. The active focus point of bidder i increases with v_i

FIGURE 3. The relationship between the bidder's focus point and valuation ($n = 100$)

As the number of bidders n increases the focus point increases. The greater number of participants lowers the profit from regular bidding and the chance of buyout price adoption increases.

Proposition 3.3. *The focus point of the seller increases as the number of bidders n increases. The active focus point is higher than the conservative one.*

Proof: Similar to the proof process of Proposition 3.2, the focus point of the seller increasing as the number of bidders n increases can be proved. In Section 3.1.2 the seller's optimal pricing, we have shown that $x_1^{s*} \in (m_2, \bar{v}]$ and $x_2^{s*} \in [0, m_2]$, so $x_1^{s*} > x_2^{s*}$. □

Proposition 3.3 shows that the increase of the number of participants can improve the seller's profit, which is intuitive. For a bidder, the active focus point is lower than the conservative one, which means that the active bidder thinks she/he can win the auction with a lower bidding price. For the seller, the active focus point is higher than the conservative one, which means that the active seller thinks she/he can sell the item with a higher bidding price. Both parts are logically consistent since the active person thinks that they can obtain higher profit. Given $\bar{v} = 100$ and $\sigma = 10$, the function of the seller's focus point and the number of participants is illustrated in Figure 4.

By Inequality (19), the range of the buyout price can be obtained. Given $\varepsilon = 0.1$, $\bar{v} = 100$, and $\sigma = 10$, the lower bound (the seller's focus point) and upper bound (B^*) are shown in Figure 5.

The yellow line (highest one with triangle) is the upper bound of the buyout price, which is determined by ε, and it increases while the number of bidders n increases; the green line (the one in the middle with circle) is the lower bound of the buyout price for

FIGURE 4. The relationship between the seller's focus point and n

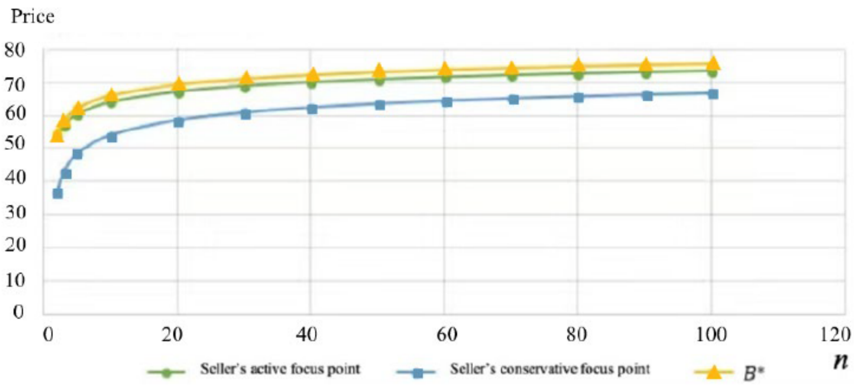

FIGURE 5. The range of the buyout price

an active seller; the blue line (lowest one with square) is the lower bound of the buyout price for a conservative seller. According to the estimated number of bidders, the range of buyout price for each type of seller can be obtained.

3.3. **Optimal strategy.** According to the features of secondhand commodity trading, strategies based on the focus point are used to consistent with the actual decision-making process. If the buyout price B is given, the optimal strategy of bidder i is shown in Theorem 2.1. The key problems are selecting focus points and setting the buyout price. By formulation (7) or (8), the focus point of bidder i can be selected, meanwhile, the focus point of seller can be selected by (16) or (17). Formula (19) gives the range of the buyout price. Theorem 3.1 shows the focus point of bidder and the range of the buyout price can be obtained. Finally, we answered the questions: for seller, how to set the buyout price; for bidder, whether to accept the buyout price. By the propositions, bidders with the same valuation may adopt different strategies due to personality differences. Setting a buyout price requires the seller to provide the bottom probability of acceptance ε. In application, if some seller does not have a clear understanding of this, and has no idea about which type of decision-maker she/he is, the platform can suggest the range of buyout price as $[x_2^{s*}, x_1^{s*}]$, where x_1^{s*}, x_2^{s*} are the active and the conservative focus points of the seller, respectively. Figure 5 shows x_1^{s*} is very close to B^*.

4. **Conclusions.** In this paper, the strategies of an online Vickrey auction with a buyout price are studied. The decision-making approach based on the focus point is provided. The proposed model can be utilized to reflect the risk preferences of bidders and the seller, directly, by choosing different types of focus points. The focus point is based on the comprehensive consideration of the satisfaction level and the degree of relative likelihood. Compared with existing methods, the proposed model emphasizes that whether an auction is won or not is determined by the highest bidding price, and the participant estimates the highest bidding price of the others (focus point) as a strategy reference. We have obtained the optimal strategy of different types of bidders and shown that the seller's profit can be improved and the phenomenon of bids being densely distributed at end segment can be avoided by setting an appropriate buyout price. The buyout price range of the seller is suggested according to the focus point and the probability that the buyout price would be accepted. The relationships between different types of participants are analyzed. For bidders with the same valuation, an active bidder is more likely to adopt a regular bidding strategy and a conservative bidder is more likely to accept the buyout price. Regarding the seller and the setting of the buyout price B, an active seller will set a higher buyout price than a conservative seller when they face the same lot and bidders. As the number of bidders n increases, the focus point of the participant increases, which means that the greater the number of participants is, the more valuable the item. Meanwhile, the difference between the two types of focus points decreases. When n is large enough, the active and conservative participants' estimates tend to be consistent.

We analyze a common case that the bidder's valuation follows a normal distribution. This model can be applied while the probability density function is known. The case that the valuation follows other probability distribution with different auction mechanisms is a direction worth exploring. After that, specific suggestions on the appropriate auction form can be given according to the characteristics of the platform. A more noteworthy direction in the future is to seek cooperation with the platform such as JD. The model will evolve into a data-driven decision-making method based on the focus point by using the historical transaction data of the platform to describe and update the probability density function of auction item's valuation. Take a secondhand mobile phone that is sold through the platform as an example. The platform can calculate the valuation's probability density function of the mobile phone through the similar ones that have been sold. The similarity is determined by the key attributes such as brand, type and the length of time it has been in use.

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REFERENCES

[1] G. I. Doukidis, K. Pramatari and G. Lekakos, OR and the management of electronic services, *European Journal of Operational Research*, vol.187, no.3, pp.1296-1309, 2008.
[2] H. K. Bhargava, G. Csapo and R. Muller, On optimal auctions for mixing exclusive and shared matching in platforms, *Management Science*, vol.66, no.6, pp.2653-2676, 2020.
[3] S. Bose and A. Daripa, Optimal sale across venues and auctions with a buy-now option, *Economic Theory*, vol.38, no.1, pp.137-168, 2009.
[4] Y. Zhang, X. Hu, Y. Li and Y. Gao, A model of refundable oder concellation problem with fulfillment cost, *International Journal of Innovative Computing, Information and Control*, vol.16, no.4, pp.1165-1181, 2020.
[5] Y. Wang, T. Li and W. Su, Research on the bi-level programming of underground logistics project based on public-private partnership mode, *ICIC Express Letters*, vol.13, no.6, pp.505-511, 2019.
[6] R. Wang, Auctions versus posted-price selling, *American Economic Review*, vol.83, no.4, pp.838-851, 1993.
[7] E. Pinker, A. Seidmann and Y. Vakrat, Using bid data for the management of sequential, multi-unit, online auctions with uniformly distributed bidder valuations, *European Journal of Operational Research*, vol.202, no.2, pp.574-583, 2010.
[8] S. Adikari and K. Dutta, A new approach to real-time bidding in online advertisements: Auto pricing strategy, *Informs Journal on Computing*, vol.31, no.1, pp.66-82, 2018.
[9] N. Bobkova, Asymmetric budget constraints in a first-price auction, *Journal of Economic Theory*, vol.186, 104975, 2020.
[10] J. Gallien and S. Gupta, Temporary and permanent buyout prices in online auctions, *Management Science*, vol.53, no.5, pp.814-833, 2007.
[11] D. Sun, E. Li and J. C. Hayya, The optimal format to sell a product through the Internet: Posted price, auction, and buy-price auction, *International Journal of Production Economics*, vol.127, no.1, pp.147-157, 2010.
[12] E. B. Budish and L. N. Takeyama, Buy prices in online auctions: Irrationality on the Internet?, *Economics Letters*, vol.72, no.3, pp.325-333, 2001.
[13] T. Mathews, The impact of discounting on an auction with a buyout option: A theoretical analysis motivated by E-bay's buy-it-now feature, *Journal of Economics*, vol.81, no.1, pp.25-52, 2004.
[14] W. Vickrey, Counters peculation, auctions, and competitive sealed tenders, *The Journal of Finance*, vol.16, no.1, pp.8-37, 1961.
[15] X. Gabaix, D. Laibson, G. Moloche and S. Weinberg, Costly information acquisition: Experimental analysis of a boundedly rational model, *American Economic Review*, vol.96, no.4, pp.1043-1068, 2006.
[16] E. Garbarino and R. Slomin, Preferences and decision errors in the winner's curse, *Journal of Risk and Uncertainty*, vol.34, no.3, pp.241-257, 2007.
[17] H. A. Simon, A behavioral model of rational choice, *The Quarterly Journal of Economics*, vol.69, pp.99-118, 1955.
[18] H. A. Simon, *Models of Man: Social and Rational; Mathematical Essays on Rational Human Behavior in a Social Setting*, John Wiley and Sons, New York, 1957.
[19] Z. J. M. Shen and X. Su, Customer behavior modeling in revenue management and auctions: A review and new research opportunities, *Production and Operations Management*, vol.16, no.6, pp.713-728, 2007.

[20] Z. Z. Jiang, S. C. Fang, Z. P. Fan and D. Wang, Selecting optimal selling format of a product in B2C online auctions with boundedly rational customers, *European Journal of Operational Research*, vol.226, no.1, pp.139-153, 2013.
[21] G. Gao and Z. Fan, Seller's revenue in online temporary buyout-price auctions considering bidders' bounded rationality behavior, *Chinese Journal of Management Science*, vol.25, no.7, pp.102-112, 2017.
[22] P. Guo and Y. Li, Approaches to multistage one-shot decision making, *European Journal of Operational Research*, vol.236, no.2, pp.612-623, 2014.
[23] C. Wang and P. Guo, Behavioral models for first-price sealed-bid auctions with the one-shot decision theory, *European Journal of Operational Research*, vol.261, pp.994-1000, 2017.
[24] P. Guo, One-shot decision theory, *IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans*, vol.41, no.5, pp.917-926, 2011.
[25] P. Guo and X. Ma, Newsvendor models for innovative products with one-shot decision theory, *European Journal of Operational Research*, vol.239, no.2, pp.523-536, 2014.
[26] P. Guo, Focus theory of choice and its application to resolving the St. Petersburg, Allais, and Ellsberg paradoxes and other anomalies, *European Journal of Operational Research*, vol.276, pp.1034-1043, 2019.
[27] P. Klemperer, Auction theory: A guide to the literature, *Journal of Economic Surveys*, vol.13, no.3, pp.227-286, 1999.

Appendix A.

We just need to show there is an $m_1 \in \left[\frac{\bar{v}}{2}, \bar{v}\right]$, such that $p(m_1) = F(m_1)^{n-2} f(m_1) = \max_x [F(x)]^{n-2} f(x)$. Since $p(x) = [F(x)]^{n-2} \cdot f(x)$, by $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left(x-\frac{\bar{v}}{2}\right)^2}{2\sigma^2}}$ and $F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left(x-\frac{\bar{v}}{2}\right)^2}{2\sigma^2}} dx$, we have $p'(x) = [F(x)]^{n-3} \cdot f(x) \cdot \left[(n-2)f(x) - \frac{x-\frac{\bar{v}}{2}}{\sigma^2} F(x)\right]$. While $0 \le x \le \frac{\bar{v}}{2}$, $p'(x) \ge 0$, so $p(x)$ is monotonically increasing in $\left[0, \frac{\bar{v}}{2}\right]$. While $\frac{\bar{v}}{2} < x \le \bar{v}$, let $h(x) = (n-2)f(x) - \frac{x-\frac{\bar{v}}{2}}{\sigma^2} F(x)$, then $h'(x) = -(n-1)\frac{x-\frac{\bar{v}}{2}}{\sigma^2} f(x) - F(x) \le 0$, $h(x)$ is monotonically decreasing in $\left(\frac{\bar{v}}{2}, \bar{v}\right]$. Since $h\left(\frac{\bar{v}}{2}\right) > 0$, $h(\bar{v}) < 0$, according to the Zero Existence theorem, there must be a unique zero point $m_1 \in \left[\frac{\bar{v}}{2}, \bar{v}\right]$. So $M_1 = \max_x p(x) = p(m_1)$.

Appendix B.

Substituting $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left(x-\frac{\bar{v}}{2}\right)^2}{2\sigma^2}}$ and $f(x)' = -\frac{x-\frac{\bar{v}}{2}}{\sigma^2} f(x)$ into $p(x) = n(n-1) \cdot (1 - F(x)) \cdot [F(x)]^{n-2} \cdot f(x)$, the first derivative of $p^s(x)$ versus x is $p^{s'}(x) = n(n-1) \cdot [F(x)]^{n-3} \cdot f(x) \left[(n-2)(1-F(x))f(x) - \frac{x-\frac{\bar{v}}{2}}{\sigma^2} F(x)(1-F(x)) - F(x)f(x)\right]$. Let $q(x) = (n-2)(1 - F(x))f(x) - \frac{x-\frac{\bar{v}}{2}}{\sigma^2} F(x)(1-F(x)) - F(x)f(x)$, $0 \le x \le \frac{\bar{v}}{2}$, $q(x) \ge 0$, so $p^{s'}(x) \ge 0$ and $p^s(x)$ is increasing in $\left[0, \frac{\bar{v}}{2}\right]$. Denote m_2 is the solution of $q(x) = 0$, then $m_2 \in \left[\frac{\bar{v}}{2}, \bar{v}\right]$ and $M_2 = [1 - F(m_2)][F(m_2)]^{n-2} f(m_2) = \max_{x\in[0,\bar{v}]} [1 - F(x)][F(x)]^{n-2} f(x)$.

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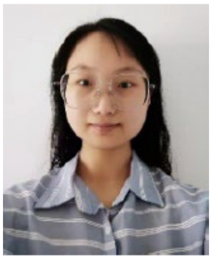

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