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PERIODIC EVENT-TRIGGERED TRACKING CONTROL FOR NONHOLONOMIC WHEELED MOBILE ROBOTS

Sijie Wan[1], Yueyuan Zhang[1,2,*], Zhiyong He[1] and Liang Chen[1]

[1]School of Mechanical and Electrical Engineering
Soochow University
No. 8, Jixue Road, Xiangcheng District, Suzhou 215131, P. R. China
sjwanwansijie@stu.suda.edu.cn; { hezhiyong; ChenL }@suda.edu.cn
*Corresponding author: zhangyueyuan@suda.edu.cn

[2]Zhejiang Key Laboratory of Robotics and Intelligent Manufacturing Equipment Technology
Ningbo Institute of Material Technology and Engineering, Chinese Academy of Sciences
No. 1219, West Zhongguan Road, Zhenhai District, Ningbo 315201, P. R. China

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Abstract. *This paper proposes a periodic event-triggered control (PETC) method for tracking problems of nonlinear mobile robotic systems. Periodic state measurement is adopted as the relaxation of continuous measurement of system states is conductive to the application for real physical systems. Further, event-triggered control method is proposed to save the network communication resources. In combination of these two advantages, PETC is applied for nonholonomic wheeled mobile robots with Zeno phenomenon avoided. Finally, experimental comparative results show that PETC can provide suitable accuracy with a significant reduction of 98.75% of the controller updates.*

Keywords: Periodic event-triggered control, Nonholonomic wheeled mobile robot, Tracking control

1. **Introduction.** Wheeled mobile robots are widely used in many fields, such as urban rescue and search [1], space exploration [2] and automatic material handling [3]. As a basic problem of mobile robot applications, trajectory tracking problems have been studied by many scholars, and many control methods have been proposed to improve the system performance, such as adaptive sliding mode control [4], reinforcement learning neural network-based control [5] and model predictive control [6]. The implementation of the physical systems is based on digital systems and the design of digital controller is usually driven by sampling time. However, time-triggered scheme requires to define sampling period when emulating, which means a large amount of data will be transformed among the system. Therefore, in the case of limited network bandwidth and high communication requirements, it is challenging to realize the trajectory tracking control of wheeled mobile robots.

In order to reduce the utilization of communication resources in physical systems, event-triggered control (ETC) strategy has been proposed in last decades [7-9]. The controller and the sensor are physically separated; the control task is executed when an external event occurs. These events are defined by the event-triggered mechanism depending on the state or the output of the system, which can be both dynamic or static [10,11]. So far, ETC strategies have been divided into two mainstream methods: continuous event-triggered control (CETC) and periodic event-triggered control (PETC). In CETC

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schemes, it is necessary to detect the triggered signal instantly which enhances the requirement of hardware devices. Moreover, the system has a positive lower bound on the interevent times to avoid Zeno phenomenon. Thus, it takes much work to prevent the occurrence of Zeno phenomenon. To make up this limitation, PETC is proposed to set a sampling interval in advance and only detect the triggered condition periodically to judge whether to transmit the measured signal, which avoids Zeno phenomenon [12]. Therefore, PETC is more suitable for digital control systems, such as network control systems, embedded systems and robot control systems [13].

At present, researchers have shown a growing interest in periodic event-triggered control method in theory. In 2011, PETC strategy is first proposed for linear system [14], which inherits advantages of periodic sampling control and event-triggered control to make the plant achieve a better control performance. With the consideration of external disturbances, Li et al. develops a distributed PETC strategy and a detailed distributed receding horizon control for decoupled nonlinear systems [15]. Under stochastic effects, a global stabilization via PETC output-feedback is concerned in [16]. Further, the stabilization of nonlinear systems following incrementally quadratic nonlinear system is investigated in [17]. Although there are many theoretical advantages and related works on PETC strategy, more and more researchers are paying more attention to PETC method for practical systems. The PETC control for the hand position centroid formulation problems of multiple mobile robots is investigated in [18]. For $(3, 0)$ mobile robots, PETC strategy is developed to the consensus problems of a multi-vehicle autonomous system [19]. With the application to nonlinear robotic systems for joint space, control signal is updated based on PETC mechanism [13]. However, most of the existing results concentrate on the stability and stabilization problems. Hence, we design a periodic event-triggered control for trajectory tracking of wheeled mobile robots in this paper. The main contributions of this paper are as follows: i) The PETC scheme is designed under the condition that the tracking error of the system is uniformly bounded; ii) Tracking control problem of wheeled mobile robots is solved by using PETC and the experimental results indicate a 98.75% reduction of the number of the control updates in comparison with a time triggered control scheme.

The remaining part is arranged as follows. The robotic system of a nonholonomic wheeled mobile robot and related assumptions and lemmas are stated in Section II. Section III proposes a periodic event-triggered control method for nonholonomic wheeled mobile robots and proves the stability of the system. In Section IV, experimental results are presented and analyzed based on the comparative examples between PETC and TTC. Summaries are proposed last in Section V.

2. **Problem Statement and Preliminaries.**

2.1. **Robotic system.** The system model of the two wheeled mobile robot is shown in Figure 1, and its driving mode can be differential drive or synchronous drive. (x, y) represents the centroid position of a nonholonomic wheeled mobile robot in Cartesian coordinate system, θ is the angle between the robot's forward direction and the horizontal axis of the coordinate axis, ω is the angular velocity of the mobile robot's steering, and v is its forward velocity.

There exists nonholonomic constraint (1) for robots when the wheeled mobile robot does not have universal wheels:

$$\dot{x} \sin \theta = \dot{y} \cos \theta \tag{1}$$

Based on the kinematic principle of a mobile robot, the model is built with ω and v as control variables, and the kinematic equation of the nonholonomic mobile robot can be

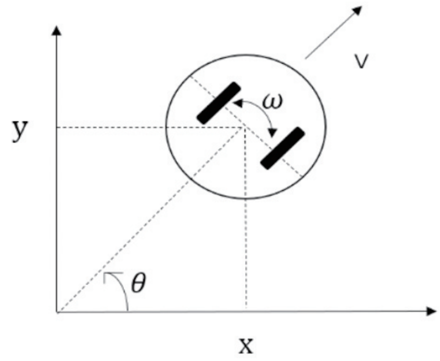


FIGURE 1. Model of a wheeled mobile robot

expressed as follows:

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = \omega \end{cases} \tag{2}$$

Given that x_r, y_r, θ_r are the desired coordinate of the mobile robot, v_r and ω_r are the desired control quantity, the reference trajectory kinematics equation can be obtained:

$$\begin{cases} \dot{x}_r = v_r\cos\theta_r \\ \dot{y}_r = v_r\sin\theta_r \\ \dot{\theta}_r = \omega_r \end{cases} \tag{3}$$

Let x_e, y_e and θ_e be the tracking error, and the coordinate transformation matrix is as follows:

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{4}$$

According to Formulas (2)-(4), we can deduce the tracking error state equation of the nonholonomic wheeled mobile robot:

$$\begin{cases} \dot{x}_e = \omega y_e - v + v_r\cos\theta_e \\ \dot{y}_e = -\omega x_e + v_r\sin\theta_e \\ \dot{\theta}_e = \omega_r - \omega \end{cases} \tag{5}$$

The problem of tracking control in this paper can be attributed to setting appropriate control law u and event-triggered conditions, so that the tracking error of the mobile robot system converges to zero in finite time, which means $\lim_{t\to\infty} \|x_e, y_e, \theta_e\|^T = 0$.

2.2. **Related assumptions and lemmas.** Before the main theorems, some assumptions and lemmas are necessary to be given first.

Assumption 2.1. *v_r, ω_r, $\dot{v}_r$ and $\dot{\omega}_r$ are bounded continuous functions. Let their upper bounds be Δ_r. In addition, $v_r^2 + \omega_r^2 > 0$.*

Lemma 2.1. *$g(t)$: $\mathbb{R}_+ \to \mathbb{R}$ is a continuous differentiable function and* $\lim_{t\to\infty} g(t) = 0$. *If $\dot{g}(t) = g_0(t) + \eta(t)$, $g_0(t)$ is uniformly continuous and* $\lim_{t\to\infty} \eta(t) = 0$, *then* $\lim_{t\to\infty} g_0(t) = 0$.

3. Periodic Event-Triggered Control Design for Wheeled Mobile Robots.

3.1. Stability and stabilization. In this part, we design controllers law based on Lyapunov function, and the stabilization of the wheeled mobile robot system is verified.

Theorem 3.1. *Consider the tracking error of the wheeled mobile robot system converges to zero with time under the control law (6).*

$$u = \begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} v_r \cos\theta_e + K_1 x_e \\ \omega_r + K_2 y_e v_r \sin\theta_e/\theta_e + K_3\theta_e \end{bmatrix} \tag{6}$$

where K_1, K_2, K_3 are controller gains with positive constants. v, ω are defined as before and $v_r > 0$. Then, the wheeled mobile robot system can achieve the trajectory tracking correctly.

Proof: Let Lyapunov alternate function:

$$V = \frac{1}{2}x_e^2 + \frac{1}{2}y_e^2 + \frac{1}{2K_2}\theta_e^2 \tag{7}$$

The derivation of Lyapunov function (7) is given below by combining (5) and (6):

$$\begin{aligned} \dot{V} &= x_e\dot{x}_e + y_e\dot{y}_e + \frac{1}{K_2}\theta_e\dot{\theta}_e \\ &= x_e\left(\omega y_e - v + v_r\cos\theta_e\right) + y_e\left(-\omega x_e + v_r\sin\theta_e\right) + \frac{1}{K_2}\theta_e(\omega_r - \omega) \\ &= -v x_e + x_e v_r\cos\theta_e + y_e v_r\sin\theta_e + \frac{1}{K_2}\theta_e(\omega_r - \omega) \\ &= -x_e(v_r\cos\theta_e + K_1x_e) + x_e v_r\cos\theta_e + y_e v_r\sin\theta_e \\ &\quad + \frac{1}{K_2}\theta_e(\omega_r - (\omega_r + K_2y_ev_r\sin\theta_e/\theta_e + K_3\theta_e)) \\ &= -K_1x_e^2 - \frac{K_3}{K_2}\theta_e^2 \le 0 \end{aligned} \tag{8}$$

When x_e and θ_e are both zero, the equal sign holds.

From Lyapunov stability theorem we can get

$$\lim_{t\to\infty}\left(K_1x_e^2 + \frac{K_3}{K_2}\theta_e^2\right) = 0$$

Further we get

$$\lim_{t\to\infty} x_e = 0, \quad \lim_{t\to\infty}\theta_e = 0 \tag{9}$$

Let

$$\begin{cases} g(t) = \theta_e(t) \\ \eta(t) = K_3\theta_e \end{cases}$$

According to Lemma 2.1, we get

$$\lim_{t\to\infty} y_e v_r = 0 \tag{10}$$

In the same way, let

$$\begin{cases} g(t) = x_e(t) \\ \eta(t) = K_1x_e \end{cases}$$

We get

$$\lim_{t\to\infty} y_e\omega_r = 0 \tag{11}$$

Then

$$\lim_{t\to\infty}\sqrt{v_r^2 + \omega_r^2}y_e = 0 \tag{12}$$

According to Assumption 2.1, we get

$$\lim_{t\to\infty} y_e = 0 \tag{13}$$

Therefore, the system is stable under the control law u designed as (6).

3.2. **Periodic event-triggered control design.** The mobile robot is connected with the controller through the network. When the event-triggered condition is satisfied, the network will be closed-loop, and a series of triggering times t_k ($k \in \mathbb{Z}_{\geq 0}$) is generated. For any $t \in [t_k, t_{k+1})$, the control quantity is constant, that is, $\hat{v} = v(t_k)$, $\hat{\omega} = \omega(t_k)$. In this case, the tracking error state equation of the system can be expressed as

$$\begin{cases} \dot{x}_e = \hat{\omega} y_e - \hat{v} + v_r \cos\theta_e \\ \dot{y}_e = -\hat{\omega} x_e + v_r \sin\theta_e \qquad t \in [t_k, t_{k+1}) \\ \dot{\theta}_e = \omega_r - \hat{\omega} \end{cases} \tag{14}$$

The event-triggered control system is a hybrid dynamic system, which is expressed as

$$\begin{cases} \dot{q} = f(q), & q \in C \\ q^+ = g(q), & q \in D \end{cases} \tag{15}$$

where $q = (x_e, y_e, \theta_e, \hat{v}, \hat{\omega}, \tau)^T$

$$f(q) = \begin{bmatrix} \hat{\omega} y_e - \hat{v} + v_r \cos\theta_e \\ -\hat{\omega} x_e + v_r \sin\theta_e \\ \omega_r - \hat{\omega} \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad g(q) = \begin{bmatrix} x_e \\ y_e \\ \theta_e \\ v \\ \omega \\ \tau \end{bmatrix} \tag{16}$$

q is the state of the hybrid dynamic system, q^+ is the pulse value of q at this time, flow set C represents that the event-triggered condition is not satisfied, jump set D represents that the event-triggered condition is satisfied, v and ω are defined in Equation (2), and τ is the time variable.

Define the error term $e_v = \hat{v} - v$, $e_\omega = \hat{\omega} - \omega$. In this case, we can set the event-triggered conditions as follows:

$$\begin{aligned} C &= \left\{ q \in Q \,\middle\|\, \sqrt{e_v^2 + e_\omega^2} < \max\left\{ \sigma\sqrt{K_1 x_e^2 + K_3 \theta_e^2}, \varepsilon \right\} \right\} \\ D &= \left\{ q \in Q \,\middle\|\, \sqrt{e_v^2 + e_\omega^2} \geq \max\left\{ \sigma\sqrt{K_1 x_e^2 + K_3 \theta_e^2}, \varepsilon \right\} \right\} \end{aligned} \tag{17}$$

where $Q = \mathbb{R}^5 \times \mathbb{R}_{\geq 0}$, $\sigma \in (0, 1)$ is the real number set, and ε is the static threshold.

4. **Experimental Results.** In this section, the effectiveness of the designed controller for the wheeled mobile robot system is verified. Referring to Kim and Oh's article [20], we set controller parameters as $K_1 = K_2 = K_3 = 2$, the parameters of the desired trajectory are set as $v_r = 1$ m/s, $\omega_r = 1$ rad/s, the initial pose of the wheeled mobile robot is $(x_0, y_0, \theta_0) = (0, -2.5, 1)$, and the initial pose of the desired trajectory can be set as $(x_r, y_r, \theta_r) = (0, 0, 0)$. The parameters in the event-triggered condition are $\sigma = 0.8$, $\varepsilon = 0.05$, and the sampling interval of the system is $h = 0.2$ s. The tracking responses of the mobile robot are presented in Figures 2-5.

It is observed from Figure 2 and Figure 3 that the wheeled mobile robot can remain stable and tracks the preset trajectory under the proposed PETC. Simultaneously, the system state error converges to the neighborhood of zero in about 2 seconds which implies the validity of our PETC method. In Figure 4 and Figure 5 the number of event

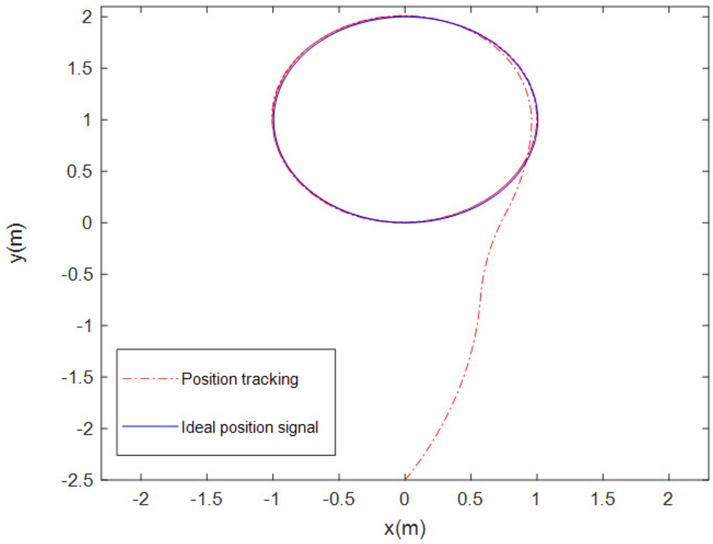


FIGURE 2. Tracking trajectory

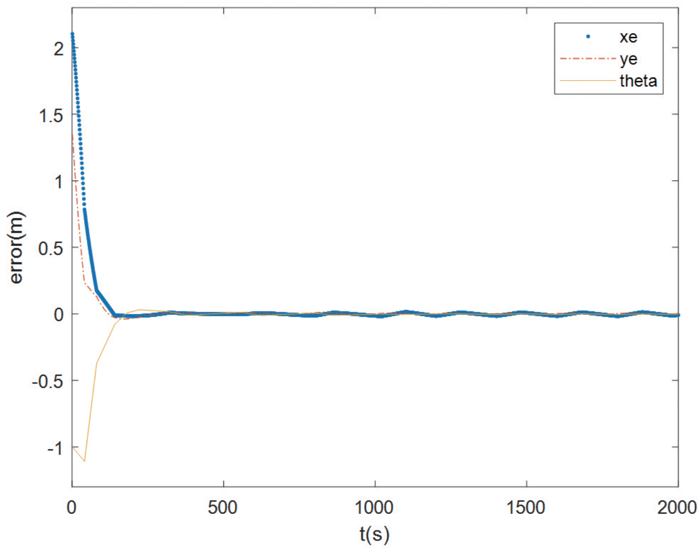


FIGURE 3. System states error

triggering is shown, taking the sampling period of $\Delta t = 1$ ms and the required total time in experiments of $T = 20$ s, the total updates number required by sampling TTC (time triggered control) is 2000 times in contrast to the PETC method that only requires 22 updates.

In order to further verify the performance of the controller, this section conducts simulation research on the influence of different parameter changes in the event-triggered mechanism, defines that the performance of the controller is evaluated by the tracking accuracy e, the total simulation time is T, and the number of event triggering is k.

$$e = \max \sqrt{x_e^2 + y_e^2 + \theta_e^2}, \quad t \in \left[\frac{1}{2}T, T\right]$$

It can be seen from Table 1 and Table 2 that with the increase of ε and σ, the system tracking error increases, and the number of events decreases slightly, but in the long-term simulation, the decrease will be more obvious. The change of σ is inversely proportional to k, but has little effect on the tracking accuracy of the system.

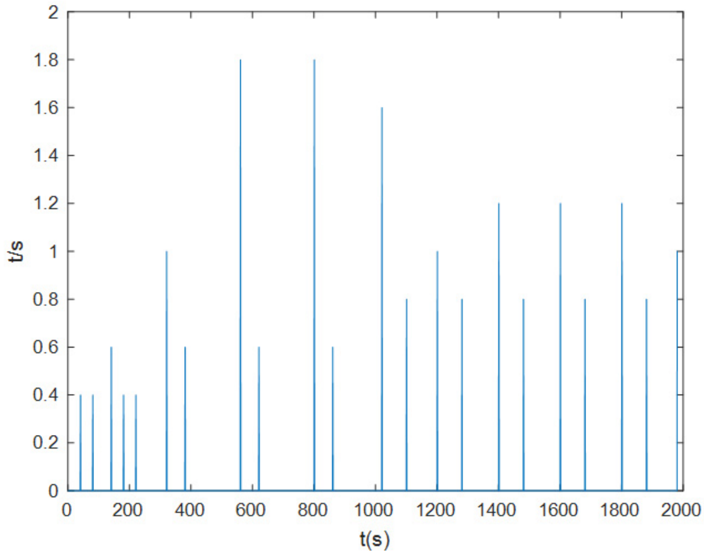


Figure 4. Number of triggers (PETC)

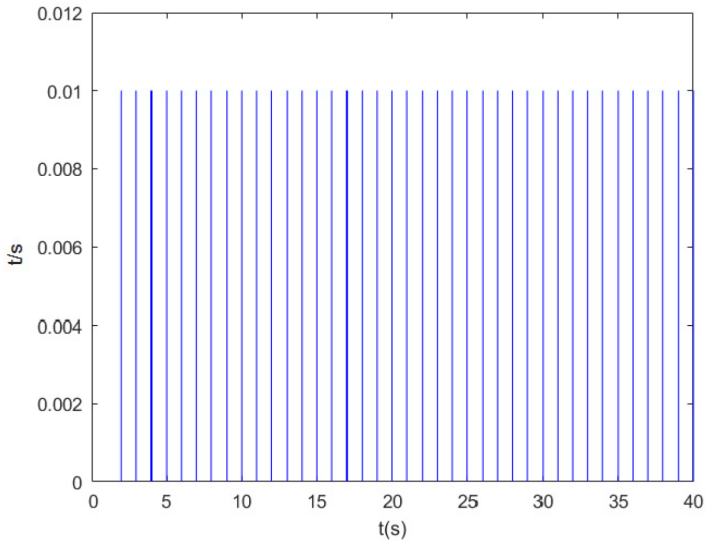


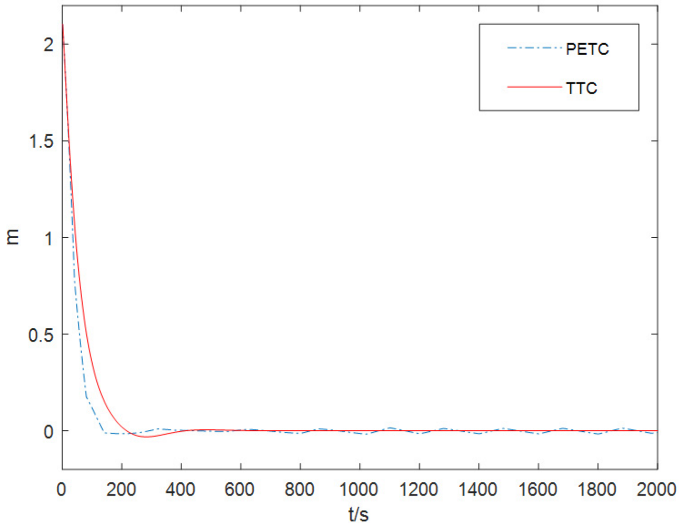
Figure 5. Number of triggers (TTC)

Table 1. The value of e and k under different ε ($\sigma = 0.8$, $h = 0.2$ s)

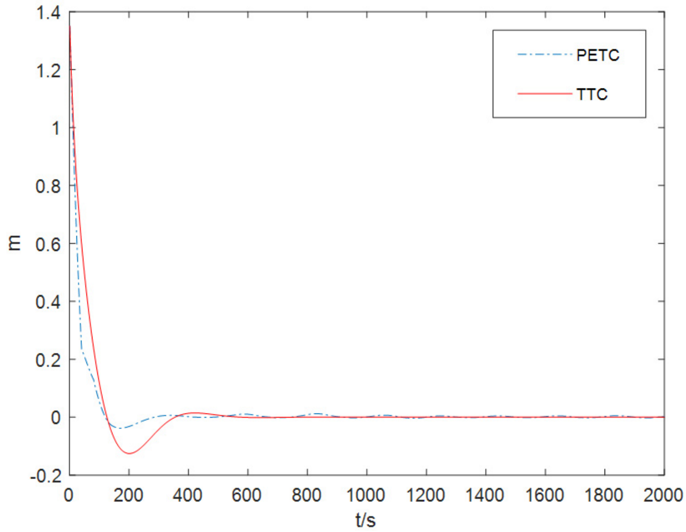
ε	e	k
0.01	0.0034	25
0.05	0.0178	22
0.1	0.0342	23
0.5	0.1621	22

Table 2. The value of e and k under different σ ($\varepsilon = 0.05$, $h = 0.2$ s)

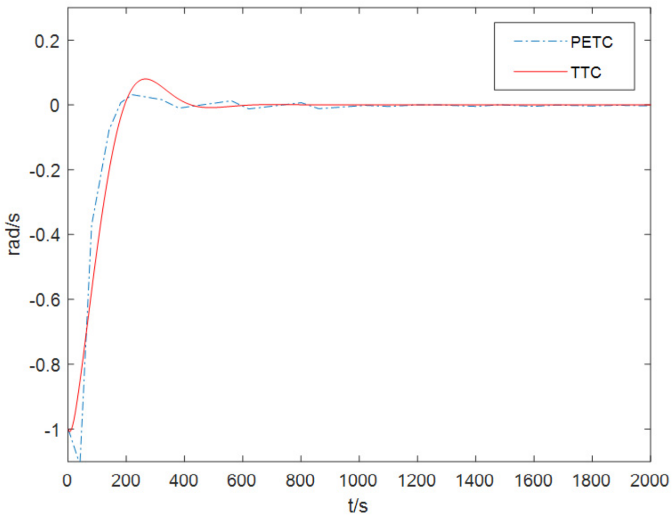
σ	e	k
0.1	0.0176	30
0.5	0.0158	23
0.8	0.0178	22



(a) Comparison of system lateral error x_e



(b) Comparison of system longitudinal error y_e



(c) Comparison of system advance angle error θ_e

FIGURE 6. Comparison of convergence effect of x_e, y_e, θ_e

In order to verify the effectiveness of the controller, a discussion about the periodic event-triggered control contrasted with a sampling time triggered control is presented, and the results are shown in Figures 6(a)-6(c).

It is not difficult to see from the simulation results that the periodic event-triggered control strategy significantly reduces the number of control updates. Thus, the occupation of network resources can be reduced. Moreover, as shown in Figure 6, the change speed of the systematic error is faster under the PETC strategy, which can converge to the neighborhood near the zero point in a shorter time. In addition, through the calculation of system tracking error, it can be seen that PETC belongs to approximate control, which has a larger error than sampling TTC, and is more suitable for the occasions with low precision requirements.

Consequently, compared with 2000 triggered times by sampling TTC, PETC strategy only needs to update 25 times within the same real-time experiments and the system can still maintain good control performance. Therefore, the experimental results show that for wheeled mobile robots, PETC strategy can reduce the update times of control quantity by at least 98.75% compared with sampling time triggered control method. Moreover, under the periodic event-triggered strategy, the error of the system can converge to the neighborhood of zero faster.

5. **Conclusions.** In this work, periodic event-triggered control for trajectory tracking problem of mobile robots is developed and experimentally validated for a study case of a two wheeled mobile robot. By relaxing continuous measurement of system states, a PETC mechanism is designed to realize the trajectory tracking of wheeled mobile robots. The experimental results of PETC method indicate that for wheeled mobile robots, the controller updates can be reduced at least 98.75% with suitable accuracy. The decrease of control signal communication implies the decrement of data transmission between the controller and the implement, thus reducing the burden on the network.

In future research, the event-triggered conditions can be optimized to reduce the communication of the system. Further, by considering the complex practical systems with uncertainty and time-delays, the event-triggered strategy can be applied to enhancing the practicability.

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Author Biography

Sijie Wan obtained the bachelor's degree in electrical engineering and its automation from Soochow University, China, in 2021. He is currently working towards the master's degree in Soochow University. His main research interests include robot control, artificial intelligence and machine vision.

Yueyuan Zhang received the B.S. degree and the Ph.D. degree in control science and engineering from Jiangnan University, Wuxi, China, in 2015 and 2020, respectively. From 2016 to 2018, she was a Visiting Student with the School of Electrical and Electronic Engineering, The University of Adelaide, SA, Australia. Her research interests include stochastic semi-Markov jump systems, control theory and applications, model predictive control, event-triggered control scheme and robust control.

Zhiyong He received the bachelor's and master's degrees in mechatronics engineering from the Hefei University of Technology, China, in 1998 and 2003, respectively, and the Ph.D. degree in optical engineering from Soochow University. He is currently an Associate Professor in School of Mechanical and Electrical Engineering, Soochow University, China. His research interests are machine vision and image processing.

Liang Chen received the Ph.D. degree in control engineering from a joint Ph.D. program at Zhejiang University & TU Berlin in 2009. He is now an Associate Professor in the department of automation engineering with the School of Mechanical and Electric Engineering, Soochow University. His research interests include intelligent control and deep learning based intelligent sensoring and fault diagnosis.