International Journal of Innovative Computing, Information and Control
Volume **18**, Number **5**, October **2022**
 ISSN 1349-4198
pp. 1633–1644

CHANNEL IDENTIFICATION OF BAND LIMITED COMMUNICATION SYSTEM WITH MULTI-PATH INTERFERENCES

Lianming Sun[1,*], Jianing Liu[2] and Junbiao Zeng[2]

[1]Faculty of Environmental Engineering
[2]Graduate School of Environmental Engineering
The University of Kitakyushu
1-1 Hibikino, Wakamatsu-ku, Kitakyushu 808-0135, Japan
{ c1dca401; c1mca401 }@eng.kitakyu-u.ac.jp
*Corresponding author: sun@kitakyu-u.ac.jp

Received March 2022; revised June 2022

Abstract. *Channel models are fundamental channel state information for communication system design and information transceiving. In orthogonal frequency division multiplexing (OFDM) communication systems, the signal bandwidth is severely limited within an active frequency band; as a result, little information on the channel dynamics beyond the signal band can be extracted from the transceiving data directly in most of the existing methods. Consequently, the channel model obtained by conventional identification algorithms mainly describes the channel dynamics within the frequency band of active carriers, whereas large uncertainty remains beyond the signal band. The uncertainty often leads to significant performance degradation so it should be removed for adaptive system design. In this paper, a novel channel identification algorithm is developed, where the spectra over the full frequency band are extracted from some specified signals reconstructed by the received signals, and then the channel dynamics beyond the signal band is estimated through sparse identification. The numerical examples of time-varying channel and performance comparison of the adaptive filters designed by using the proposed and conventional methods show the effectiveness of the proposed algorithm.*
Keywords: Channel identification, OFDM communication, Spectral analysis, Sparse identification

1. **Introduction.** Communication is an essential information transceiving tool in information society [1]. Among various communication technologies, orthogonal frequency division multiplexing (OFDM) is known as high transmission capacity, less ghost effects, less obstructions from the adjacent channels; consequently, it has been adopted as a core technology for terrestrial digital broadcasting, wireless local area network (LAN), the latest Wi-Fi standard (IEEE 802.11ax), and the 4th and 5th-generation mobile systems [2, 3]. Due to the mobility of transmitter, receiver or scattering objects, the response of the communication channel varies with time, adaptive processing techniques are required to maintain high communication performance, especially under time-varying fading channel. Therefore, channel identification becomes one of major issues to obtain channel state information (CSI) in OFDM systems. Channel identification can be performed by training based methods such as least squares methods (LS), minimum mean square error (MMSE), maximum likelihood (ML) from training data [4], or blind channel identification techniques based on second-order statics, properties of signal space [5], or the semi blind technique by introducing a small number of training symbols into channel identification [6]. Moreover, some deep learning algorithms have also been applied [7]. It has

DOI: 10.24507/ijicic.18.05.1633

been demonstrated that such techniques obtain good estimates of OFDM channels at the active carriers.

Channel identification requires sufficient frequency components to extract the dynamics dominating the adaptive system. In OFDM systems, however, in order not to interfere the adjacent channels and to simplify the physical circuit of transmitter, the carriers far away from the central frequency are assigned as null carriers that do not convey any information data. It means that the transmitted signal is restricted within a specified frequency band, and the transceived data contain less information at the null carriers beyond the signal band. As a result, channel identification cannot give confidential CSI in the frequency band of null carriers, and causes performance degradation in convergence or stability of the conventional adaptive processing methods, whereas some filter coefficients are left unaltered since the dynamic modes are not excited at the null carriers [8].

Band-pass filtering techniques, information supplement and numerical optimization techniques are three possible solutions for this problem. Band-pass filter with very steep cut-off characteristics may reduce the affection of dynamic uncertainty beyond the signal band, but it is difficult to implement such a filter since its long impulse response may cause extra inter-carrier interference. Information supplement scheme extracts information as much as possible to supplement information deficiency in the frequency band of null carriers, for example, dummies beyond the signal band, which are generally not allowed in many practical applications, are discussed in [9]. Alternatively, by using the OFDM signal property, a time domain approach to extracting the dynamics beyond the signal band was proposed without any dummies [10], and a low computational complexity for real-time implementation was also developed in frequency domain [11], but the extracted information beyond the signal band may be weak against the interferences or noises. On the other hand, some numerical algorithms such as extrapolation or regularization methods may estimate the dynamic characteristics beyond the signal band to some extent [12], whereas the sparse algorithms are used for band-limited signal reconstruction or data compression [13, 14]. Nevertheless, their performance degrades with increasing band width of null carriers. Therefore, combination of information supplement scheme with numerical optimization techniques is strongly desired to obtain high quality of CSI using the information extracted from transceived data.

In this paper, a novel OFDM channel identification algorithm where sparse channel identification is supplemented by extra information extracted from OFDM property and the transceived data, and the applications to design adaptive filters are considered for multi-path interference in OFDM communication systems. The highlight of this work is the combination of information supplement scheme and sparse identification. Information extraction beyond the signal band is illustrated, and the sparse identification is investigated for the signal band limited channels. Furthermore, the application of identified channel model to adaptive multi-path compensator is demonstrated in time-varying channels. Performance comparison of adaptive filters designed by the proposed and conventional algorithms is executed and the numerical results demonstrate the effectiveness of the proposed algorithm to remove the uncertainty beyond signal band and its applicability in adaptive signal processing design.

2. OFDM Channel Model.

The transceived signals in OFDM communication systems are obtained by fast Fourier transform (FFT) and its inverse (IFFT) [1]. Let the length of FFT/IFFT be denoted as M. For the simplicity of notation, the normalized sampling instants are denoted as integer k. Correspondingly, the transmitted and received baseband signals are $d(k)$, $r(k)$, respectively. Indicate the guard interval (GI) of OFDM symbol as m_{gi}, then the symbol length becomes $M_{\text{tx}} = M + m_{\text{gi}}$, and $d(k)$ can also be represented by the m-th sample in l-th symbol period $d_l(m)$, where

$$l = \lfloor k/M_{\text{tx}} \rfloor + 1,\ m = \text{rem}(k, M_{\text{tx}}) - m_{\text{gi}}. \tag{1}$$

Here $\lfloor k/M_{\text{tx}} \rfloor$ is the largest integer no more than k/M_{tx}, $\text{rem}(k, M_{\text{tx}})$ is the remainder of k/M_{tx}. Then the m-th sample of transmitted baseband signal in l-th symbol period

$$d_l(m) = \sum_{n=-N}^{N} D_l(n) e^{j\omega_n m},\ m = -m_{\text{gi}}, \ldots, M-1, \tag{2}$$

where the values of $v_l(m)$ for $m = -m_{\text{gi}}, \ldots, -1$ are the copies of $v_l(m)$ for $m = M - m_{\text{gi}}, \ldots, M-1$, $\omega_n = 2\pi n/M$ is the the normalized frequency of n-th carrier that conveys information symbol $D_l(n)$ given as follows:

$$D_l(n) = \begin{cases} \text{Information symbols } (\neq 0), & \text{for } 0 < |n| \leq N \\ 0, & \text{for } n = 0,\ |n| > N \end{cases}. \tag{3}$$

Here the number of active carriers is $2N$, $2N < M$. It can be seen that the spectrum of transmitted signal $d_l(m)$ in an effective symbol period, i.e., the FFT window $m = 0, \ldots, M-1$ is limited to $0 < |n| \leq N$ with the signal band, whereas the spectral density beyond signal band for $|n| > N$ is 0. Assume that the received signal under multi-path interference is expressed by

$$r(k) = \sum_{\tau=0}^{L_h} h_\tau d(k-\tau) + w(k), \tag{4}$$

where $w(k)$ is assumed as an additive white Gaussian noise (AWGN) with zero mean and finite variance, and independent of the source signal $d(k)$. Similarly as $d_l(m)$, $r(k)$ and $w(k)$ are also arranged in the form of $r_l(m)$, $w_l(m)$. h_τ is the coefficient corresponding to the τ-th delay interference wave, and L_h is the largest effective tap of multi-path. In the communication system, the channel model is often a sparse one where some parameters in $\{h_0, \ldots, h_{L_h}\}$ are close to 0. Clearly, if the sparse model can be obtained, it will help users to design an effective adaptive filter.

Moreover, the channel dynamics in frequency domain is expressed by

$$H\left(e^{-j\omega}\right) = h_0 + h_1 e^{-j\omega} + \cdots + h_{L_h} e^{-jL_h\omega}. \tag{5}$$

It implies that $H\left(e^{-j\omega}\right)$ for $|\omega| > \omega_N$ is hard to be obtained directly from the transceiving signals for $m = 0, \ldots, M-1$ where the signals' steady states do hold little information on the channel dynamics beyond the signal band. In the next section, the virtual signals associated with transient response are re-constructed to supplement the information deficiency.

3. **Information Supplement Scheme.**

Notice that the transmitted signals in guard interval $d_l(-m_{\text{gi}}), \ldots, d_l(-1)$ are the copies of $d_l(M - m_{\text{gi}}), \ldots, d_l(M-1)$, and then some information may be extracted from the subtraction of the received signals, which are treated as the transient responses of channel dynamics.

3.1. **Signal construction for channel identification.**

In order to extract information of channel dynamics beyond the signal band, two virtual signals are constructed from the original transmitted and received signals as follows:

$$x(k) = d\left(k + m_{\text{gi}}\right) - d\left(k + m_{\text{gi}} - M\right), \tag{6}$$

$$y(k) = r\left(k + m_{\text{gi}}\right) - r\left(k + m_{\text{gi}} - M\right). \tag{7}$$

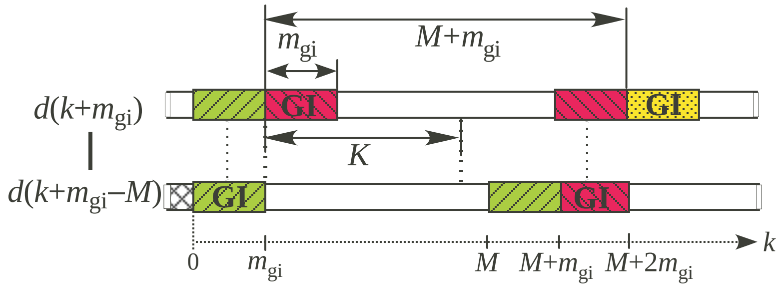

FIGURE 1. Illustration of signal $x(k)$

As an example, the signal $x(k)$ defined in (6) is illustrated in Figure 1, where K is an integer satisfying $m_{\text{gi}} < K + m_{\text{gi}} \leq M$. Moreover, the formulation of $x_l(m)$ and $y_l(m)$ can also be constructed similarly as $d_l(m)$ or $r_l(m)$.

From the feature of GI, $x_l(m) = 0$ holds for $-m_{\text{gi}} \leq m < 0$. Then the data blocks in $m = 0, \ldots, L_h$ and $m = M, \ldots, M + L_h$ can be treated as a transient response of $x_l(m)$

$$\boldsymbol{y}_l = \boldsymbol{X}\boldsymbol{h} + \boldsymbol{v}_l, \tag{8}$$

where

$$\boldsymbol{X} = \begin{bmatrix} x_l(0) & 0 & \cdots & 0 \\ x_l(1) & x_l(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ x_l(L_h) & \cdots & x_l(1) & x_l(0) \\ x_l(M) & x_l(M-1) & \cdots & x_l(M-L_h) \\ 0 & x_l(M) & \cdots & x_l(M-L_h+1) \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & x_l(M) \end{bmatrix}, \ \boldsymbol{h} = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{L_h} \end{bmatrix},$$

$$\boldsymbol{y}_l = [y_l(0), y_l(1), \ldots, y_l(L_h), y_l(M), y_l(M+1), \ldots, y_l(M+L_h)]^T,$$

$$\boldsymbol{v}_l = [v_l(0), v_l(1), \ldots, v_l(L_h), v_l(M), v_l(M+1), \ldots, v_l(M+L_h)]^T,$$

and $v_l(m)$ is the noise term defined as $x_l(m)$ and $y_l(m)$. It is seen that (8) provides some information on the channel dynamics; however, as illustrated in [4], LS or MMSE algorithms in time domain are time consuming. Next, the spectral properties of $x_l(m)$ and $y_l(m)$ are investigated.

3.2. **Spectral properties of re-constructed signals.** Let the spectral estimation of $\overline{XX}_l(n)$ be defined by

$$\overline{XX}_l(n) = \frac{1}{l} \sum_{l_1=1}^{l} X_{l_1}^*(n) X_{l_1}(n), \tag{9}$$

where

$$X_l(n) = \sum_{m=0}^{K-1} x_l(m) e^{-j\omega_n m} \tag{10}$$

for $n = -M/2+1, \ldots, M/2$. If the symbol number l is large enough, then following (2) $\overline{XX}_l(n)$ can be approximated by

$$\overline{XX}_l(n) \approx XX(n) = 2\overline{D}^2 \sum_{\bar{n} \in \bar{N}(n)} \frac{1 - \cos(\omega_n - \omega_{\bar{n}}) K}{1 - \cos(\omega_n - \omega_{\bar{n}})} + 2K^2 \overline{D}^2 \tag{11}$$

for $0 < |n| \leq N$ where $\overline{D}^2$ is a constant determined by the constellation of modulation, $\overline{N}(n)$ is a set of $\{\bar{n}\,|\mathrm{rem}\,(n-\bar{n}, M) \neq 0$ and $0 < |\bar{n}| \leq N\}$, and

$$\overline{XX}_l(n) \approx XX(n) = 2\overline{D}^2 \sum_{\bar{n}=-N,\bar{n}\neq 0}^{N} \frac{1-\cos(\omega_n-\omega_{\bar{n}})\,K}{1-\cos(\omega_n-\omega_{\bar{n}})} \tag{12}$$

for $N < |n| < M/2$ or $n = 0$.

Give the analogue definition for cross-spectral estimation of $x_l(m)$ and $y_l(m)$ as

$$\overline{XY}_l(n) = \frac{1}{l}\sum_{l_1=1}^{l} X_{l_1}^*(n)Y_{l_1}(n). \tag{13}$$

Then the channel dynamics can be approximated by [11]

$$H\left(e^{-j\omega_n}\right)\overline{XX}_l(n) \approx \overline{XY}_l(n) + \sum_{\tau} h_\tau\left(XE_{\tau,1}(n)+XE_{\tau,2}(n)\right), \tag{14}$$

where $XE_{\tau,1}(n)$ and $XE_{\tau,2}(n)$ are leakage terms from other carriers to n-th carrier, the inter-symbol leakage at n-th carrier due to the partial FFT window $m = 0, \ldots, K-1$. $XE_{\tau,1}(n)$ can be approximated by

$$XE_{\tau,1}(n) = \overline{D}^2 \sum_{\bar{n}\in\overline{N}(n)} \left(\frac{1-e^{-j(\omega_n-\omega_{\bar{n}})K}}{1-\cos(\omega_n-\omega_{\bar{n}})}\left(e^{-j\omega_n\tau} - e^{-j\omega_{\bar{n}}\tau}\right)\right). \tag{15}$$

Furthermore, the spectral leakage $XE_{\tau,2}(n)$ is

$$XE_{\tau,2}(n) = 2e^{-j\omega_n\tau}\overline{D}^2 K\tau \tag{16}$$

for $0 < |n| \leq N$, while for $N < |n| < M/2$ or $n = 0$ it turns to

$$XE_{\tau,2}(n) = 0. \tag{17}$$

In (14), $\overline{XX}_l(n)$ and $\overline{XY}_l(n)$ can be estimated from $x(k)$ and $y(k)$ directly, while the spectral distortion terms $XE_{\tau,1}(n)$ and $XE_{\tau,2}(n)$ can be calculated beforehand without any observation data and the information of channel dynamics. Consequently, it is possible to estimate the channel property beyond the signal band if $\overline{XY}_l(n)$ is compensated by $XE_{\tau,1}(n)$ and $XE_{\tau,2}(n)$.

On the other hand, when $H\left(e^{-j\omega}\right)$ is time-varying, a forgetting factor can be used to estimate the spectra as follows.

$$\overline{XX}_l(n) = \lambda\overline{XX}_{l-1}(n)+X_l^*(n)X_l(n), \tag{18}$$

$$\overline{XY}_l(n) = \lambda\overline{XY}_{l-1}(n)+X_l^*(n)Y_l(n) \tag{19}$$

respectively, where the forgetting factor λ is a constant such that $0 < \lambda \leq 1$. In the next section, estimation of frequency response using the spectral properties is illustrated.

4. Channel Identification.

The channel state information (CSI) inside signal band is revealed first in channel identification.

4.1. Identification inside signal band.

Generally the data of $d(k)$ or $D_l(n)$ are unavailable at the receivers except the training symbols or pilot symbols. Provided that some pilot symbols are scattered among the carriers P_n with normalized frequency ω_{P_n}, so the information symbol $D_l(P_n)$ at carrier P_n is known, consequently

$$\hat{H}\left(e^{-j\omega_{P_n}}\right) = \frac{R_l(P_n)}{D_l(P_n)} \tag{20}$$

is obtained at pilot carrier frequency ω_{P_n}. At non-pilot carriers inside the signal band where $|n| < N$, linear or spline interpolation is effective choice to estimate channel dynamics $H\left(e^{-j\omega_n}\right)$, for example, a linear interpolation

$$\hat{H}\left(e^{-j\omega_n}\right) = \hat{H}\left(e^{-j\omega_{P_{n,1}}}\right) + \frac{n - P_{n,1}}{P_{n,2} - P_{n,1}}\left(\hat{H}\left(e^{-j\omega_{P_{n,2}}}\right) - \hat{H}\left(e^{-j\omega_{P_{n,1}}}\right)\right), \tag{21}$$

yields $\hat{H}\left(e^{-j\omega_n}\right)$ where $P_{n,1}$ and $P_{n,2}$ are the number of two adjacent pilot sub-carriers, $P_{n,1} \le n \le P_{n,2}$. Moreover, the source symbols $\hat{D}_l(n)$ can be given by

$$\hat{D}_l(n) = \begin{cases} \dfrac{R_l(n)}{\hat{H}\left(e^{-j\omega_n}\right)}, & \text{for } 0 < |n| \le N \\ 0, & \text{for } n = 0, \text{ or } N < |n| < \dfrac{M}{2} \end{cases}, \tag{22}$$

and then $\hat{d}_l(m)$ can be calculated by IFFT easily.

4.2. **Channel identification beyond signal band.** Following (14), the channel frequency response beyond the signal band satisfies that

$$H\left(e^{-j\omega_n}\right) = \overline{H}\left(e^{-j\omega_n}\right) + \sum_{\tau} h_\tau^{(i)} A_\tau(n), \text{ for } n = 0 \text{ or } N < |n| < \frac{M}{2}, \tag{23}$$

for l is large enough, where h_τ is the coefficients of channel model, and $\overline{H}\left(e^{-j\omega_n}\right)$, $A_\tau(n)$ are given by

$$\overline{H}\left(e^{-j\omega_n}\right) = \begin{cases} \hat{H}\left(e^{-j\omega_n}\right), & 0 < |n| < N \\ \dfrac{\overline{XY}_l(n)}{\overline{XX}_l(n)}, & n = 0 \text{ or } N < |n| < \dfrac{M}{2} \end{cases}, \tag{24}$$

$$A_\tau(n) = \frac{XE_{\tau,1}(n) + XE_{\tau,2}(n)}{XX(n)}. \tag{25}$$

It is seen that $A_\tau(n)$ can be pre-calculated, and spectra $\overline{XX}_l(n)$, $\overline{XY}_l(n)$ of $x_l(m)$ and $y_l(m)$ can be estimated from the constructed signals $x(k)$ and $y(k)$, then channel identification can be implemented by solving the following optimization problem

$$\hat{\boldsymbol{h}} = \arg \min_{\hat{h}_0, \hat{h}_1, \ldots, \hat{h}_{L_h}} \left(\left\| (\boldsymbol{\Phi} - \boldsymbol{A})\hat{\boldsymbol{h}} - \overline{\boldsymbol{H}} \right\|_2^2 + \lambda_1 \left\| \hat{\boldsymbol{h}} \right\|_0 + \lambda_2 \left\| \boldsymbol{y}_l - \boldsymbol{X}\hat{\boldsymbol{h}} \right\|_2^2 \right), \tag{26}$$

where $\boldsymbol{X}$ and $\boldsymbol{y}_l$ are given in (8), the term of L^2 norm evaluates the model accuracy, whereas the term of L^0 norm considers the sparse model to simplify the design of adaptive filter, and

$$\hat{\boldsymbol{h}} = \begin{bmatrix} \hat{h}_0 \\ \hat{h}_1 \\ \vdots \\ \hat{h}_{L_h} \end{bmatrix}, \quad \overline{\boldsymbol{H}} = \begin{bmatrix} \overline{H}\left(e^{-j\omega_{-\frac{M}{2}+1}}\right) \\ \overline{H}\left(e^{-j\omega_{-\frac{M}{2}+2}}\right) \\ \vdots \\ \overline{H}\left(e^{-j\omega_{\frac{M}{2}}}\right) \end{bmatrix}, \quad \boldsymbol{\Phi} = \begin{bmatrix} 1 & e^{-j\omega_{-\frac{M}{2}+1}} & \cdots & e^{-j\omega_{-\frac{M}{2}+1} L_h} \\ 1 & e^{-j\omega_{-\frac{M}{2}+2}} & \cdots & e^{-j\omega_{-\frac{M}{2}+2} L_h} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & e^{-j\omega_{\frac{M}{2}-1}} & \cdots & e^{-j\omega_{\frac{M}{2}-1} L_h} \\ 1 & e^{-j\omega_{\frac{M}{2}}} & \cdots & e^{-j\omega_{\frac{M}{2}} L_h} \end{bmatrix},$$

$$
\boldsymbol{A} = \begin{bmatrix}
A_0\left(-\frac{M}{2}+1\right) & A_1\left(-\frac{M}{2}+1\right) & \cdots & A_{L_h}\left(-\frac{M}{2}+1\right) \\
\vdots & \vdots & \cdots & \vdots \\
A_0(-N-1) & A_1(-N-1) & \cdots & A_{L_h}(-N-1) \\
0 & 0 & 0 & 0 \\
A_0(N+1) & A_1(N+1) & \cdots & A_{L_h}(N+1) \\
\vdots & \vdots & \cdots & \vdots \\
A_0\left(\frac{M}{2}\right) & A_1\left(\frac{M}{2}\right) & \cdots & A_{L_h}\left(\frac{M}{2}\right)
\end{bmatrix}.
$$

The problem in (26) can be solved by some L^0 optimization algorithms for sparse identification such as iterative hard thresholding algorithm [15], orthogonal matching pursuit [16], compressive sampling matching pursuit [17]. In (26), $\boldsymbol{\Phi} - \boldsymbol{A}$ can be pre-calculated, and $\overline{H}\left(e^{-j\omega_n}\right)$ is given by (24) from the data. Then the optimization algorithms yield an estimation of a sparse model for the communication channel. Next, some distortion and noise terms that associate with channel identification will be discussed.

4.3. **Affections of distortion and noise terms.** The spectrum of the ideal transmitted OFDM signal is limited in the frequency band where the active carriers convey information data. However, in the received signal some distortion may appear due to the additive noise, inter-carrier interference (ICI) or inter-symbol interference (ISI) caused by long multi-path interference.

Consider the affection of noise to the proposed channel identification algorithm. If the additive observation noise $w(k)$ in (4) is independent of the transmitted signal $d(k)$, for the cross-spectrum $\overline{XV}_l(n)$ of $x_l(m)$ and $v_l(m)$ defined in (8)

$$\overline{XV}_l(n) = \frac{1}{l}\sum_{l_1=1}^{l} X^*(l_1, n)V(l_1, n) \to 0 \tag{27}$$

holds true for $l \to \infty$. It implies that with increasing of symbol periods l the affection of additive noise can be reduced even the received signal contains strong noise.

On the other hand, if the delay tap L_h of multi-path interference is beyond GI, i.e., $L_h \geq m_{\mathrm{gi}}$, ICI and ISI arise in the received signal and collapse the carriers' orthogonality; consequently distortion appears beyond the signal band. This problem can be handled by extracting information from the distortion caused by long delay taps [18].

Using CSI over the full frequency band obtained in channel identification, adaptive filter can be designed [19]. In the next section, a simple frequency domain method will be investigated.

5. **Design of Adaptive Filters Based on Channel Identification.** Consider a simple case where the channel model has minimum phase, the adaptive filter

$$F\left(z^{-1}\right) = f_0 + f_1 z^{-1} + \cdots + f_{L_f} z^{-L_f} \tag{28}$$

as shown in Figure 2 to remove the multi-path interferences in real time for recovery of information symbols, where the filter output $s(k)$ is desired to be close to $d(k)$ at the receiver. It can be updated iteratively by minimizing the following criterion

$$Q\left(e^{-j\omega_n}\right) = \frac{1}{2}\left(E^{(l)}\left(e^{-j\omega_n}\right)\right)^* E^{(l)}\left(e^{-j\omega_n}\right) \tag{29}$$

where $E^{(l)}\left(e^{-j\omega_n}\right)$ is the error given by

$$E^{(l)}\left(e^{-j\omega_n}\right) = 1 - F^{(l-1)}\left(e^{-j\omega_n}\right)\hat{H}^{(l)}\left(e^{-j\omega_n}\right). \quad (30)$$

Here the supper subscript (l) indicates the iteration number, whereas it is replaced by $e^{-i\omega_n k_0} - F^{(l-1)}\left(e^{-j\omega_n}\right)\hat{H}^{(l)}\left(e^{-j\omega_n}\right)$ for non-minimum phase channel model, i.e., $s(k) \to d(k-k_0)$ with k_0 delay taps of $d(k)$. If the antenna array can be used, the non-minimum phase may also be handled by antenna diversity. Denote antenna number as P, and the filter cascading the p-th antenna as $F_p(z^{-1})$, then the error $E^{(l)}\left(e^{-j\omega_n}\right)$ becomes

$$E^{(l)}\left(e^{-j\omega_n}\right) = 1 - \sum_{p=1}^{P} F_p^{(l)}\left(e^{-j\omega_n}\right)\hat{H}_p^{(l)}\left(e^{-j\omega_n}\right). \quad (31)$$

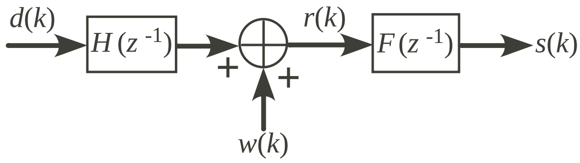

FIGURE 2. Diagram of adaptive filter $F\left(z^{-1}\right)$, the output $s(k)$ is a recovered signal of $d(k)$

Then in the l-th iteration, the following frequency response

$$\Delta F^{(l)}\left(e^{-j\omega_n}\right) = \mu\frac{\left(\hat{H}^{(l)}\left(e^{-j\omega_n}\right)\right)^*}{\left|\hat{H}^{(l)}\left(e^{-j\omega_n}\right)\right|^2}E^{(l)}\left(e^{-j\omega_n}\right) = \mu\frac{1}{\hat{H}^{(l)}\left(e^{-j\omega_n}\right)}E^{(l)}\left(e^{-j\omega_n}\right), \quad (32)$$

or for the filters in antenna array

$$\Delta F_p^{(l)}\left(e^{-j\omega_n}\right) = \mu\frac{\left(\hat{H}_p^{(l)}\left(e^{-j\omega_n}\right)\right)^* E^{(l)}\left(e^{-j\omega_n}\right)}{\sum_p\left|\hat{H}_p^{(l)}\left(e^{-j\omega_n}\right)\right|^2} \quad (33)$$

yields the update $\Delta F^{(l)}\left(e^{-j\omega_n}\right)$. Therefore, the update of filter weights of $F\left(z^{-1}\right)$ can be given by inverse Fourier transform of $\Delta F\left(e^{-j\omega_n}\right)$, where μ is the step size. Correspondingly the weight coefficients of $F\left(z^{-1}\right)$ can be updated by

$$\Delta \boldsymbol{f}^{(l)} = \left(\boldsymbol{\Psi}^H\boldsymbol{\Lambda}\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi}^H\boldsymbol{\Lambda}\Delta\boldsymbol{F}^{(l)}\left(e^{-j\omega}\right) \quad (34)$$

where

$$\boldsymbol{\Psi} = \begin{bmatrix} 1 & e^{-j\omega_{-\frac{M}{2}+1}} & \cdots & e^{-j\omega_{-\frac{M}{2}+1}L_f} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & e^{-j\omega_{\frac{M}{2}}} & \cdots & e^{-j\omega_{\frac{M}{2}}L_f} \end{bmatrix}, \quad \Delta\boldsymbol{F}^{(l)}\left(e^{-j\omega}\right) = \begin{bmatrix} \Delta F^{(l)}\left(e^{-j\omega_{-\frac{M}{2}+1}}\right) \\ \vdots \\ \Delta F^{(l)}\left(e^{-j\omega_{\frac{M}{2}}}\right) \end{bmatrix}$$

Here $\boldsymbol{\Lambda}$ is a diagonal weight matrix with respect to the identification accuracy at corresponding carrier frequencies.

Following (29), $Q\left(e^{-j\omega_n}\right)$ has minimum when $F\left(e^{-j\omega_n}\right)H\left(e^{-j\omega_n}\right) = 1$, and the optimal weights of $F\left(z^{-1}\right)$ can be obtained when $H\left(z^{-1}\right)$ has minimum phase. For non-minimum phase, the delayed approximation or the diversity of antenna array can be used for design of adaptive filters.

6. **Numerical Simulation Examples.** The examples of a time-varying channel identification and an adaptive filter to remove the multi-path interferences are considered to demonstrate the effectiveness of the channel identification and adaptive filter design.

6.1. **Simulation conditions.** In the example, the FFT and GI lengths are $M = 256$, $m_{\text{gi}} = 64$, respectively, and $2N = 150$ active carriers are used for transceiving, it means that about 2/5 carriers are null. 1/4 of carriers are pilot carriers used for identification. The modulation is 16 quadrature amplitude modulation (QAM). The delay profile of multi-path interference is illustrated in Figure 3, where $|h_\tau|$ for $\tau = 10 \sim 20$ varies within the shadow range and their phases change randomly in $[-\pi, \pi]$ every 10 symbol periods. A sparse model can be used to approximate the channel dynamics. It is seen that the multi-path interferences are strong so 3 antennas followed by adaptive filter $F_p(z^{-1})$, $p = 1, 2, 3$ are used to remove the multi-path interferences.

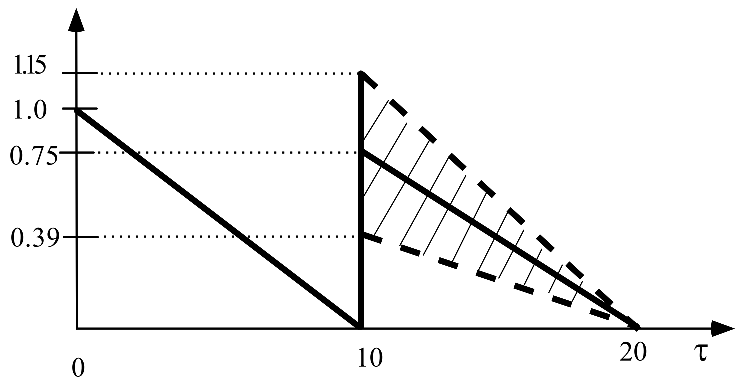

FIGURE 3. Delay profile of multi-path interferences

6.2. **Simulation results.** The magnitude of the estimated channel model's frequency response is shown in the left of Figure 4. By using the information extracted from the reconstructed signals, the frequency response beyond the signal band is close to the true one. Using the estimated sparse channel model, the adaptive filter is designed to mitigate the multi-path interferences, and the compensated channel in the right of Figure 4 shows that the interferences are mitigated well over the full frequency band. It is seen that with the aid of CSI obtained by the proposed algorithm, the compensated frequency response is close to 1 even in the frequency band of null carriers.

As a comparison, the common conventional methods based on extrapolation of the information inside the signal band to the null carriers are also used in the example. Though

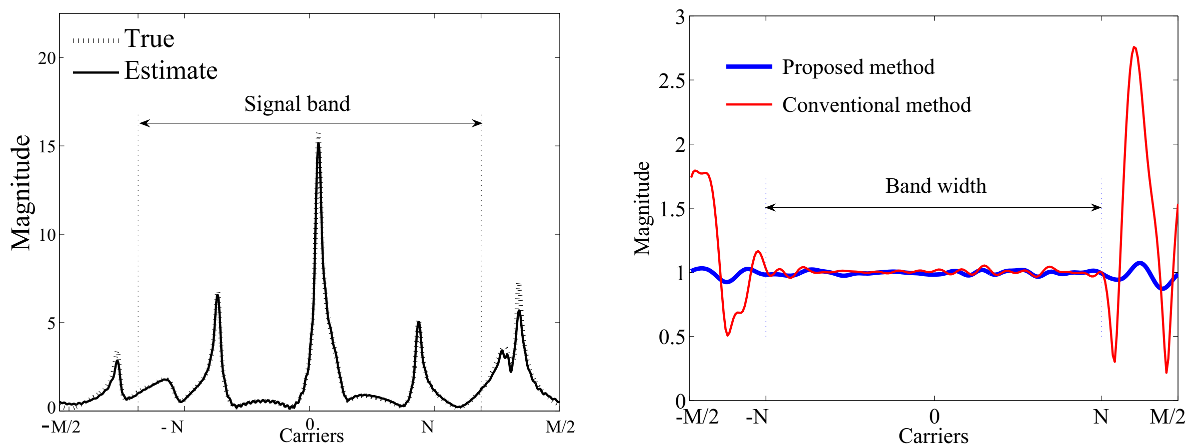

FIGURE 4. The frequency response of the channel before and after removing multi-path interference

they have the similar CSI inside the signal band as the proposed method, extrapolation cannot obtain high quality of CSI on carriers $|n| > N$ due to the fact that the frequency response is not a smooth function and the bandwidth of null carriers is too wide. The large uncertainty beyond the signal band degrades the adaptive filter's performance since it may have long impulse response and cause inter symbol interferences.

Figure 5 illustrates bit error rate (BER) of the filter output signal $s(k)$ under various signal to noise ratio (SNR) where the true CSI varies every 10 symbols. The proposed method uses CSI of full frequency band to mitigate the inferences, so it converges to optimal performance after large variation of channels faster than the conventional method with large uncertainty beyond the signal band. Moreover, the adaptive filter has superior BER performance in the proposed method where the interferences at the frequencies of null carriers are also removed, so BER is almost 0 when the noise is not too strong.

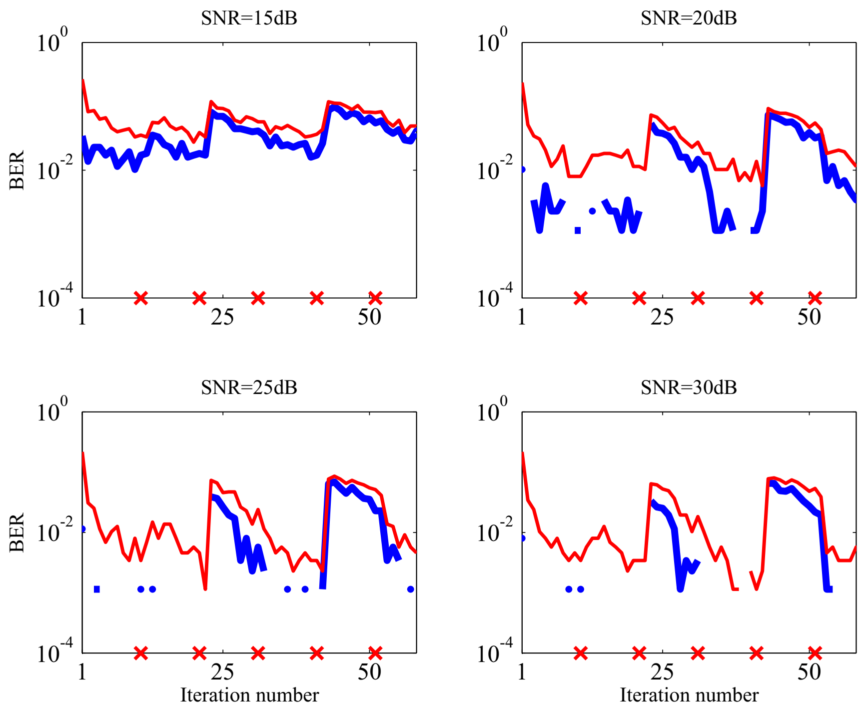

FIGURE 5. BER vs. various noise levels. Thick line: Proposed method; Thin line: Conventional method; X: varying point

In low SNR situations, the adaptive performance of convergence and BER is also influenced by the errors of recovered $\hat{D}_l(n)$ in (22). If the recovery of source symbols is combined with some error correcting techniques, the proposed method can reduce the affection of noise.

7. Conclusions. The algorithm of channel identification for a communication system with severely limited signal band and the design of adaptive processing filters have been developed in this work. By using the re-constructed signals, channel identification can extract important information from the transient response of band-limited signals even in time-varying channels, and the model is further simplified by sparse identification. It is shown that the estimated model with information of full frequency band can be used for designing stable adaptive filters with fast convergence. Some meaningful issues, for example, channel identification for system with very long multi-path interferences or strong noise, and stability of adaptive systems in the new generation of communication systems, will be considered in the future works.

REFERENCES

[1] J. G. Proakis and M. Salehi, *Digital Communication*, 5th Edition, McGraw-Hill, 2008.
[2] S. Avallone, P. Imputato, G. Redieteab, C. Ghosh and S. Roy, Will OFDMA improve the performance of 802.11 Wifi networks?, *IEEE Wireless Communications*, vol.28, no.3, pp.100-107, 2021.
[3] K. Pahlavan and P. Krishnamurthy, Evolution and impact of Wi-Fi technology and applications: A historical perspective, *International Journal of Wireless Information Networks*, vol.28, pp.3-19, 2021.
[4] P. Sure and C. M. Bhuma, A survey on OFDM channel estimation techniques based on denoising strategies, *Engineering Science and Technology, An International Journal*, vol.20, pp.629-636, 2017.
[5] L. Sun and X. Liu, Algorithm of signal source localization via sensor networks, *ICIC Express Letters*, vol.12, no.5, pp.441-447, 2018.
[6] R. Govil, Different types of channel estimation techniques used in MIMO-OFDM for effective communication systems, *Int. Journal of Engineering Research & Technology*, vol.7, no.7, 2018.
[7] M. Soltani, V. Pourahmadi, A. Mirzaei and H. Sheikhzadeh, Deep-learning based channel estimation, *IEEE Commun. Lett.*, vol.23, no.4, pp.652-655, 2019.
[8] G. Ysebaert, F. Pisoni, M. Bonavetura, R. Hug and M. Moonen, Echo cancellation in DMT-receivers: Circulant decomposition canceller, *IEEE Trans. Signal Processing*, vol.52, no.9, pp.2612-2624, 2004.
[9] J. Coiffi and J. Bingham, A data driven multitone echo canceller, *IEEE Trans. Commun.*, vol.42, no.10, pp.2853-2869, 1994.
[10] L. Sun and A. Sano, Coupling channel identification for OFDM relay station in SFN, *Proc. of the 14th European Signal Processing Conference (EUSIPCO'06)*, Florence, Italy, 2006.
[11] L. Sun, Channel identification for OFDM communication system in frequency domain, *Fourier Transform – Signal Processing*, pp.81-110, DOI: 10.5772/36191, 2012.
[12] G. Pillonetto, System identification using kernel-based regularization: New insights on stability and consistency issues, *Automatica*, vol.93, pp.321-332, 2018.
[13] M. Nagahara, *Sparse Modeling – Fundamentals and Its Applications to Dynamical Systems*, Corona Publishing, Tokyo, Japan, 2017.
[14] S. Sun, B. Sun, Q. Wang and C. Zhou, A sparse representation data compression algorithm for power plant, *International Journal of Innovative Computing, Information and Control*, vol.18, no.1, pp.41-56, 2022.
[15] T. Blumensath and M. E. Davies, Iterative thresholding for sparse approximations, *Journal of Fourier Analysis and Applications*, vol.14, no.5, pp.629-654, 2008.
[16] G. M. Davis, S. G. Mallat and Z. Zhang, Adaptive time-frequency decompositions, *Optical Engineering*, vol.33, no.7, pp.2183-2191, 1994.
[17] D. Needell and J. A. Tropp, CoSaMP: Iterative signal recovery from incomplete and inaccurate samples, *Appl. Comput. Harmonic Anal.*, vol.26, no.3, pp.301-321, 2008.
[18] L. Sun, A. Sano, W. Sun and A. Kijiwara, Channel identification and interference compensation for OFDM system in long multipath environment, *Signal Processing*, vol.89, pp.1589-1601, 2009.
[19] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, Upper Saddle River, NJ, 1996.

Author Biography

Lianming Sun received the B.E. degree in automatic control engineering and M.E. degree from Xi'an Jiaotong University, China, in 1990 and 1993; the Ph.D. degree in electrical engineering from Keio University, Japan, 2000.

Prof. Sun is currently a full-time professor at Faculty of Environmental Engineering, The University of Kitakyushu, Japan. His research interests include optimization algorithms, system identification, adaptive signal processing, and applications in sensing systems and communication systems.

Jianing Liu received Master degree in information engineering from The University of Kitakyushu, Japan in 2020. Now she is a Ph.D. student in Graduate School of Environmental Engineering, The University of Kitakyushu. Her research interests include adaptive signal processing, machine learning and its applications.

Junbiao Zeng received Bachelor's degree from The University of Kitakyushu, Japan in 2021. Now he is a student in Graduate School of Environmental Engineering, The University of Kitakyushu. His research interests include system identification, time-varying systems, machine learning and their applications in signal processing systems.