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EXPONENTIAL STABILITY RESULTS FOR STOCHASTIC SEMI-LINEAR SYSTEMS WITH LÈVY NOISE

Ravi Ragul[1], Gunasekaran Sangeetha[2], Kalidass Mathiyalagan[1,*] and Huiyan Zhang[3]

[1]Department of Mathematics
Bharathiar University
Coimbatore-641 046, Tamilnadu, India
ragul500@gmail.com; *Corresponding author: kmathimath@buc.edu.in

[2]Department of Mathematics
KGISL Institute of Technology
Coimbatore-641 035, Tamilnadu, India
sgsangeethap@gmail.com

[3]National Research Base of Intelligent Manufacturing Service
Chongqing Technology and Business University
No. 19, Xuefu Avenue, Nan'an District, Chongqing 400067, P. R. China
huiyanzhang@ctbu.edu.cn

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Abstract. *In this paper, mean square exponential stability (MSES) for semi-linear partial differential equations (PDEs) involving Lèvy type noise is investigated. By constructing an appropriate Lyapunov function, a new set of sufficient conditions is established in terms of linear matrix inequalities (LMIs) ensuring that the given system with Neumann boundary conditions is MSES. The semilinear functions in the system are assumed to have sector bounds. The robust boundary feedback controller is designed to handle the noise and inappropriate behavior of the considered system. The boundary control gain is obtained by solving the derived results using the standard MATLAB software. Finally, two numerical examples are given to demonstrate effectiveness of the proposed results.*
Keywords: Parabolic system, Lèvy noise, Boundary control, Lyapunov stability, LMIs

1. **Introduction.** PDEs are widely used to describe the complex phenomenons existing in nature. Mainly, semi-linear reaction-diffusion PDEs are suitable model to determine the various real-life phenomenon such as population dynamics, and chemical reactions [4, 5]. Many researchers focused their attention on semi-linear reaction-diffusion equations due to their wide range of applications, see [6, 7, 8]. External disturbances, measurement error and lack of knowledge in specific parameters may contribute to random noise in dynamical systems [1, 2, 3]. To express such type of dynamical system, deterministic systems were extended to stochastic systems. Stochastic PDEs (SPDEs) help to describe the dynamics of chemical engineering, ecology, neurophysiology, statistical physics, biology and martial science [9, 10]. In recent years, SPDE is an active research area with many new results and developments, see [11, 12, 13] and references therein. One of the most fundamental concept in control theory is stability [14]. Lyapunov theory is a more familiar method in the field of stability analysis and found useful for nonlinear uncertain systems with bounded disturbances. Hung et al. [23] proposed a modified Takagi-Sugeno fuzzy model intelligent control design. In recent years, many stability results for SPDEs can be found

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in literature. For example, Luo and Zhang in [15] addressed the exponential stability results for PDEs with uncertainties and used the Lyapunov technique to achieve the necessary conditions. Ngoc [18] investigated the exponential stability analysis of stochastic functional differential equations.

On the other side, most of the stability results are presented for SPDEs in the literature dealt with Brownian motion. For instance, Pan et al. [16] studied mean square asymptotical stability of nonlinear stochastic reaction-diffusion systems with Brownian motion. Wu et al. [17] investigated MSES of nonlinear stochastic reaction-diffusion systems with Brownian motion. Wu and Zhang [34] proposed an MSES analysis for Takagi-Sugeno fuzzy stochastic nonlinear systems with Brownian motion. For instant, Brownian motion is a stochastic process with continuous paths; hence, it could not be applied to describing stochastic disturbances in real-world systems such as population dynamics, financial systems, neurobiology systems and genetic regulatory networks [19, 20]. These systems are complicated in nature, with discontinuous paths; as a result, the Brownian motion stochastic differential equation is falling to deal with these issues. In this connection, Lèvy noise is introduced to deal with the system's small and large fluctuations, as it combines Brownian motion and the Poisson process [21, 22, 24]. Lèvy process is one of the stochastic processes with fixed and independent increments. Stochastic results were analyzed for different kinds of systems in [25, 26]. Brzezniak et al. [27] have studied the concept of Lèvy noise to analyze the strong solutions of SPDE. Song et al. [28] addressed the robust stability analysis for stochastic systems with random jumps. The reaction-diffusion equations with Lèvy noise addressed for stochastic systems in [29] and sufficient conditions are established to analyze the exponential stability results. Recently, Li and Yang [30] investigated the results for continuous-time stochastic systems with Lèvy noise. Applications of Lèvy noise can be explained clearly through Chua's circuit in [31]. The effects of large fluctuations on reaction-diffusion equations were examined in [32].

The main objective of the proposed work is to derive the sufficient conditions for the considered reaction-diffusion equations to guarantee the MSES. We presented boundary control for semi-linear SPDEs with a Neumann boundary conditions driven by Lèvy noise. The main contributions of the present work are as the following.

- Boundary feedback control is proposed to guarantee MSES of the considered SPDE.
- Lèvy process which includes the Brownian motion and Poisson jump processes which are useful to match the demands of real situations such as random jumps or unexpected interferences.
- Semi-linear smooth function in the model is assumed to satisfy sector boundary conditions which makes the analysis more useful.
- By constructing suitable Lyapunov functional and with the help of Itö operator, a new set of sufficient conditions is derived in terms of LMIs for ensuring the MSES of the considered system.

Finally, two numerical examples are presented, and to demonstrate the effectiveness of the findings, an example using the Fisher equation with Lèvy noise is taken into consideration.

2. System Formulation and Preliminaries.

2.1. System description.

Consider the following stochastic semi-linear parabolic system

$$\frac{\partial\zeta(\vartheta,t)}{\partial t} = \frac{\partial^2\zeta(\vartheta,t)}{\partial x^2} + A\zeta(\vartheta,t) + f(t,\zeta(\vartheta,t)) + \sigma(t,\zeta(\vartheta,t))\frac{\partial\mathcal{W}(\vartheta,t)}{\partial t}$$

$$+\int_Z \phi(t,\zeta(\vartheta,t),z)\frac{\partial \tilde{N}(dt,dz)}{\partial t}, \tag{1}$$

$$\left.\frac{\partial\zeta(\vartheta,t)}{\partial\vartheta}\right|_{\vartheta=0}=0, \quad \left.\frac{\partial\zeta(\vartheta,t)}{\partial\vartheta}\right|_{\vartheta=1}=u(t), \quad \zeta(\vartheta,0)=\zeta_0(\vartheta)$$

where $t\geq 0$, $\vartheta\in(0,1)$ and $\zeta(\vartheta,t)\in\mathbb{R}^n$ are the time, space and system state respectively, A is a known constant matrix, $u(t)$ is the boundary control input given by $u(t)=K\zeta(1,t)$, where K is control gain to be designed. $\mathcal{W}(\vartheta,t)$ is a Brownian motion defined on a complete probability space $(\Sigma,\mathcal{F},P)$ adapted to a right continuous filtration $\mathcal{F}_{t\geq 0}$ and $\mathbb{E}\left(\frac{\partial\mathcal{W}(\vartheta,t)}{\partial t}\right)=0$. Denote by $N(dt,dz)$ the Poisson random measure with intensity measure $dtv(dz)$ where $z\in\mathcal{B}(Z)$. Then $\tilde{N}(dt,dz)=N(dt,dz)-dtv(dz)$ is the compensated Poisson measure. Assume that W and N are independent of each other and $\phi(\zeta(\vartheta,t),y)$ satisfy the inequality $\int_Z\int_0^1\phi^T(\zeta(\vartheta,t),z)\phi(\zeta(\vartheta,t),z)v(dz)<\infty$.

The assumptions listed below are important in getting our main results.

(A1) For $\forall y_1,y_2\in\mathbb{R}$, $y_1\neq y_2$, function $f(\cdot)$ satisfies

$$l^-\leq\frac{f(y_1)-f(y_2)}{y_1-y_2}\leq l^+, \tag{2}$$

where l^-, l^+ are known constant scalars and $f(0)=0$.

(A2) There exists a positive constant c, $\sigma(\zeta(\vartheta,t))$ satisfying the following condition

$$\operatorname{tr}\left(\sigma^T(\zeta(\vartheta,t))\sigma(\zeta(\vartheta,t))\right)\leq c\zeta^T(\vartheta,t)\zeta(\vartheta,t). \tag{3}$$

(A3) There exists a positive constant q, and the following inequality holds

$$\int_Z\left(\int_0^1\phi^T(\zeta(\vartheta,t),z)\phi(\zeta(\vartheta,t),z)d\vartheta\right)v(dz)\leq q\int_0^1\zeta^T(\vartheta,t)\zeta(\vartheta,t)d\vartheta. \tag{4}$$

2.2. **Preliminaries.** Here we introduce the basic definition and lemma, which are important in obtaining main results.

Definition 2.1. [17] *System* (1) *is said to be MSES if there exist positive constants $\beta>0$ and $\delta>0$ such that*

$$\mathbb{E}\|\zeta(\vartheta,t)\|^2\leq\beta e^{-\delta t}\mathbb{E}\left\|\zeta_0(\vartheta)\right\|^2, \quad t\geq 0, \quad \forall\zeta_0(\vartheta)\in\mathbb{L}_2(0,1),$$

for all $\zeta_0(\vartheta)\in L_2(0,1)$.

Lemma 2.1. [33] *Let $\zeta\in\mathbb{W}^{1,2}([0,l];\mathbb{R}^n)$ be a vector function with $\zeta(0)=0$ or $\zeta(l)=0$. Then, for matrix $R>0$, we have the following integral inequality:*

$$\int_0^l\zeta^T(s)R\zeta(s)ds\leq\frac{4l^2}{\pi^2}\int_0^l\left(\frac{d\zeta(s)}{ds}\right)^T R\left(\frac{d\zeta(s)}{ds}\right).$$

The following notations are used throughout this paper. $\mathbb{R}^n$ is n-dimensional Euclidean space. $\mathbb{Z}$ represents integer and $\mathcal{B}(Z)$ denotes Banach space. The superscript 'T' stands for matrix transposition. The symmetric elements are denoted by asterisk $(*)$. I stands for identity matrix. Let $\|\cdot\|$ denote $\mathbb{L}_2$ norm given by $\|\zeta(\vartheta,t)\|^2=\int_\Omega\zeta^T(\vartheta,t)\zeta(\vartheta,t)d\vartheta$. $\mathbb{W}_\Omega^{p,q}$ is the Sobolev space of absolutely continuous integrable functions defined over Ω with the norm $\|\zeta(\vartheta,t)\|_{\mathbb{W}_\Omega^{p,q}}=\left(\sum_{\|\alpha\|\leq p}\int_\Omega\left\|\frac{\partial\zeta(\vartheta,t)}{\partial\vartheta^\alpha}\right\|^q d\vartheta\right)^{(1/q)}$. '$\mathbb{E}$' denotes mathematical expectation. Next, the following Itö operator $\mathcal{L}$ is defined for later analysis (see [35])

$$\mathcal{L}V(\cdot)=\frac{\partial V}{\partial t}+\frac{\partial V}{\partial\wp}f(t,\zeta(\vartheta,t))+\frac{1}{2}\operatorname{tr}\left(\sigma^T(t,\zeta(\vartheta,t))\frac{\partial^2 V}{\partial y^2}\sigma(t,\zeta(\vartheta,t))\right)$$

$$+\frac{\partial V}{\partial \zeta}\sigma(t,\zeta(\vartheta,t))\frac{\partial \mathcal{W}(\vartheta,t)}{\partial t}+\int_Z\Big[V(t,\zeta(\vartheta,t)+\phi(t,\zeta(\vartheta,t),z))-V(t,\zeta(\vartheta,t))$$
$$-\frac{\partial V}{\partial \zeta}\phi(t,\zeta(\vartheta,t),z)\Big]\upsilon(d\zeta).$$

To handle the indeterministic part in stochastic systems, the above Itö type operator is used instead of regular differential operator to derive the main results.

3. **Mean Square Exponential Stability.** In this section, the boundary control design for semi-linear stochastic parabolic system is presented. The main objective of the work is to obtain sufficient conditions to find the suitable controller gain which ensures the MSES of the system (1).

Theorem 3.1. *Under the Assumptions (A1)-(A3), for given scalar $\delta > 0$, there exist symmetric matrix $P > 0$, scalar $\rho > 0$, and appreciate matrix $\mathcal{K}$ such that the following LMIs hold:*

$$P \leq \rho I,$$
$$\bar{\Xi}=\begin{bmatrix} \Xi_{11} & \frac{\pi^2}{2}P & P+\frac{1}{2}\epsilon(l^++l^-)I \\ * & \mathcal{K}+\mathcal{K}^T-\frac{\pi^2}{2}P & 0 \\ * & * & -\epsilon I \end{bmatrix}<0, \tag{5}$$

where $\Xi_{11}=PA+A^TP^T-\epsilon l^+l^-I+\rho cI+\rho qI-\frac{\pi^2}{2}P+\delta\left(P+P^T\right)$ and for any given initial condition, the decay rate satisfies

$$\mathbb{E}\left\|\zeta_0(\vartheta)\right\|^2 \leq \ \beta e^{-\delta t}\mathbb{E}\left\|\zeta_0(\vartheta)\right\|^2, \tag{6}$$

and then the system (1) *is MSES. Moreover, control gain is given by $K = P^{-1}\mathcal{K}$.*

Proof: Consider the Lyapunov functional

$$V(t,\zeta(\vartheta,t))=\int_0^1\zeta^T(\vartheta,t)P\zeta(\vartheta,t)d\vartheta.$$

In terms of the Itö formula, it is obtained as

$$\mathcal{L}V(\cdot)=\int_0^1\Bigg(2\zeta^T(\vartheta,t)P\left[\frac{\partial^2\zeta}{\partial x^2}+A\zeta(\vartheta,t)+f(t,\zeta(\vartheta,t))\right]$$
$$+\operatorname{tr}\left(\sigma^T(t,\zeta(\vartheta,t))P\sigma(t,\zeta(\vartheta,t))\right)\Bigg)d\vartheta+2\int_0^1\zeta^T(\vartheta,t)P\sigma(t,\zeta(\vartheta,t))\frac{\partial\mathcal{W}(\vartheta,t)}{\partial t}d\vartheta$$
$$+\int_Z\int_0^1\phi^T(t,\zeta(\vartheta,t),z)P\phi(t,\zeta(\vartheta,t),z)d\vartheta\upsilon(d\zeta).$$

Taking $\bar{\zeta}(\vartheta,t)=\zeta(\vartheta,t)-\zeta(1,t)$, obviously, we have $\bar{\zeta}(1,t)=0$, and $\frac{\partial\bar{\zeta}}{\partial\vartheta}=\frac{\partial\zeta}{\partial\vartheta}$.

By using integration by parts with the boundary conditions of system (1) and with the help of Lemma 2.1, we obtain

$$\int_0^1\zeta^T(\vartheta,t)P\frac{\partial^2\zeta}{\partial\vartheta^2}d\vartheta$$
$$=\zeta^T(1,t)PK\zeta(1,t)-\int_0^1\left(\frac{\partial\zeta}{\partial\vartheta}\right)^TP\left(\frac{\partial\zeta}{\partial\vartheta}\right)d\vartheta$$

$$
\begin{aligned}
&= \zeta^T(1,t)PK\zeta(1,t) - \int_0^1 \left(\frac{\partial \bar{\zeta}}{\partial \vartheta}\right)^T P \left(\frac{\partial \bar{\zeta}}{\partial \vartheta}\right) d\vartheta \\
&\le \zeta^T(1,t)PK\zeta(1,t) - \frac{\pi^2}{4}\int_0^1 \bar{\zeta}(\vartheta,t)P\bar{\zeta}(\vartheta,t)d\vartheta \\
&\le \zeta^T(1,t)PK\zeta(1,t) - \frac{\pi^2}{4}\int_0^1 [\zeta(\vartheta,t)-\zeta(1,t)]^T P[\zeta(\vartheta,t)-\zeta(1,t)]d\vartheta. \qquad (7)
\end{aligned}
$$

From assumptions (3) and (4), we get that

$$\operatorname{tr}\left(\sigma^T(t,\zeta(\vartheta,t))P\sigma(t,\zeta(\vartheta,t))\right) \le \rho c \zeta^T(\vartheta,t)\zeta(\vartheta,t) \qquad (8)$$

$$\int_Z \left(\int_0^1 \phi^T(t,\zeta(\vartheta,t),z)P\phi(t,\zeta(\vartheta,t),z)d\vartheta\right) v(dz) \le \rho q \int_0^1 \zeta^T(\vartheta,t)\zeta(\vartheta,t)d\vartheta. \qquad (9)$$

By compiling equation from (7) to (9), $\mathcal{L}V$ takes the form,

$$
\begin{aligned}
\mathcal{L}V(\cdot) \le& \int_0^1 \zeta^T(\vartheta,t)[2PA+\rho cI+\rho qI]\zeta(\vartheta,t)d\vartheta + 2\zeta^T(1,t)PK\zeta(1,t) \\
&- \frac{\pi^2}{2}\int_0^1 [\zeta(\vartheta,t)-\zeta(1,t)]^T P[\zeta(\vartheta,t)-\zeta(1,t)]d\vartheta \\
&+ \int_0^1 2\zeta^T(\vartheta,t)P\sigma(t,\zeta(\vartheta,t))\frac{\partial \mathcal{W}(\vartheta,t)}{\partial t}d\vartheta + \int_0^1 2\zeta^T(\vartheta,t)Pf(t,\zeta(\vartheta,t))d\vartheta.
\end{aligned}
$$

According to (2), the following inequalities are obtained

$$\frac{f(t,\zeta(\vartheta,t)) - l^+\zeta(\vartheta,t)}{\zeta(\vartheta,t)} \le 0, \quad \frac{f(t,\zeta(\vartheta,t)) - l^-\zeta(\vartheta,t)}{\zeta(\vartheta,t)} \ge 0, \qquad (10)$$

which implies there exists a $\epsilon > 0$, such that

$$\epsilon(f(\zeta(\vartheta,t)) - l^+\zeta(\vartheta,t))^T(f(\zeta(\vartheta,t)) - l^-\zeta(\vartheta,t)) \le 0. \qquad (11)$$

In view of (11), we have that

$$
\begin{aligned}
\mathcal{L}V(\cdot) \le& \int_0^1 \zeta^T(\vartheta,t)[2PA+\rho cI+\rho qI]\zeta(\vartheta,t)d\vartheta + 2\zeta^T(1,t)PK\zeta(1,t) \\
&- \frac{\pi^2}{2}\int_0^1 [\zeta(\vartheta,t)-\zeta(1,t)]^T P\,[\zeta(\vartheta,t)-\zeta(1,t)]\,d\vartheta \\
&+ \int_0^1 2\zeta^T(\vartheta,t)P\sigma(t,\zeta(\vartheta,t))\frac{\partial \mathcal{W}(\vartheta,t)}{\partial t}d\vartheta + \int_0^1 2\zeta^T(\vartheta,t)Pf(t,\zeta(\vartheta,t))d\vartheta \\
&- \epsilon\int_0^1 (f(t,\zeta(\vartheta,t)) - l^+\zeta(\vartheta,t))^T(f(t,\zeta(\vartheta,t)) - l^-\zeta(\vartheta,t))d\vartheta \\
\le& \int_0^1 \zeta^T(\vartheta,t)\left[2PA+\rho cI+\rho qI - \frac{\pi^2}{2}P - \epsilon l^+ l^-\right]\zeta(\vartheta,t)d\vartheta \\
&+ \zeta^T(1,t)\left(2PK - \frac{\pi^2}{2}P\right)\zeta(1,t) - \epsilon\int_0^1 f^T(t,\zeta(\vartheta,t))f(t,\zeta(\vartheta,t))d\vartheta \\
&+ \frac{\pi^2}{2}\int_0^1 2\zeta^T(\vartheta,t)P\zeta(1,t)d\vartheta + \int_0^1 2\zeta^T(\vartheta,t)P\sigma(t,\zeta(\vartheta,t))\frac{\partial \mathcal{W}(\vartheta,t)}{\partial t}d\vartheta \\
&+ \int_0^1 \zeta^T(\vartheta,t)(2P+\epsilon(l^+ + l^-))f(t,\zeta(\vartheta,t))d\vartheta.
\end{aligned}
$$

Then for decay rate δ and by setting $\zeta(\cdot) = \begin{bmatrix} \zeta^T(\vartheta,t) & \zeta^T(1,t) & f^T(\zeta(\vartheta,t)) \end{bmatrix}^T$, we have

$$\mathcal{L}V(\cdot) + 2\delta V(\cdot) \leq \int_0^1 \zeta^T(\cdot)\Xi\zeta(\cdot)d\vartheta + \int_0^1 2\zeta^T(\vartheta,t)P\sigma(t,\zeta(\vartheta,t))\frac{\partial\mathcal{W}(\vartheta,t)}{\partial t}d\vartheta,$$

where

$$\Xi = \begin{bmatrix} \Xi_{11} & \frac{\pi^2}{2}P & P + \frac{1}{2}\epsilon(l^+ + l^-)I \\ * & PK + K^TP^T - \frac{\pi^2}{2}P & 0 \\ * & * & -\epsilon I \end{bmatrix}.$$

By letting $\mathcal{K} = PK$, it is easy to reach LMI (5).

Taking expectations on both sides according to the properties of Itö integral, we arrive at

$$\mathbb{E}\left[\mathcal{L}V(t,\zeta(\vartheta,t)) + 2\delta V(\zeta(\vartheta,t))\right] \leq \mathbb{E}\left[\int_0^1 \zeta^T(\cdot)\Xi\zeta(\cdot)d\vartheta\right].$$

If LMI (5) holds, we get that

$$\mathbb{E}\left[\mathcal{L}V(\cdot) + 2\delta V(\cdot)\right] \leq 0.$$

Next by using comparison principle, for any arbitrary initial condition $\zeta(\vartheta,0) = \zeta_0(\vartheta)$, the following inequality holds

$$\begin{aligned} &\mathbb{E}[V(t,\zeta(\vartheta,t))] \leq e^{-2\delta t}\mathbb{E}[V(0,\zeta(\vartheta,0))], \\ \Rightarrow\ &\underline{\beta}\mathbb{E}\left\|\zeta(\vartheta,t)\right\|^2 \leq \bar{\beta}e^{-\delta t}\mathbb{E}\left\|\zeta_0(\vartheta)\right\|^2, \\ \Rightarrow\ &\mathbb{E}\left\|\zeta_0(\vartheta)\right\|^2 \leq \ \beta e^{-\delta t}\mathbb{E}\left\|\zeta_0(\vartheta)\right\|^2, \end{aligned} \tag{12}$$

where $\beta = \frac{\bar{\beta}}{\underline{\beta}}$, $\bar{\beta} = \lambda_{\max}(P)$, $\underline{\beta} = \lambda_{\min}(P)$. By Definition 2.1 system (1) is MSES. □

Remark 3.1. *It should be mentioned that Lèvy processes are stochastic processes with independent and stationary increments. Furthermore, Brownian motion, the Poisson process, stable and self-decomposable processes are all special examples of Lèvy processes. It is well-known that Brownian motion is a stochastic process that occurs continuously. Many practical systems, however, may be influenced by random jump type unexpected interference, such as the sharp fluctuations in the stock market impact of global financial crisis. In such instances, systems defined solely by Brownian motion are unable to meet the requirements of reality. Lèvy noise has been included into stochastic systems in order to develop more suitable results.*

4. **Numerical Example.**

Example 4.1. *This section presents a numerical example that demonstrates how the obtained results can be applied for a Fisher equation for the spatial spread of a favored gene in a population. Consider the stochastic PDEs with a Lèvy noise in the form of Fisher equation*

$$\begin{aligned} \frac{\partial\zeta(\vartheta,t)}{\partial t} = &\frac{\partial^2\zeta(\vartheta,t)}{\partial x^2} + A\zeta(\vartheta,t) + \zeta(\vartheta,t)(1-\zeta(\vartheta,t)) + \sigma(t,\zeta(\vartheta,t))\frac{\partial\mathcal{W}(\vartheta,t)}{\partial t} \\ &+ \int_Z \phi(t,\zeta(\vartheta,t),z)\frac{\partial\tilde{N}(dt,dz)}{\partial t}, \end{aligned} \tag{13}$$

with boundary and initial conditions

$$\begin{aligned} &\left.\frac{\partial\zeta(\vartheta,t)}{\partial\vartheta}\right|_{\vartheta=0} = 0, \quad \left.\frac{\partial\zeta(\vartheta,t)}{\partial\vartheta}\right|_{\vartheta=1} = K\zeta(1,t), \\ &\zeta(\vartheta,0) = \zeta_0(\vartheta), \end{aligned}$$

where

$$\mathcal{W}(\vartheta,t)=\begin{bmatrix}\mathcal{W}_1(\vartheta,t)\\ \mathcal{W}_2(\vartheta,t)\end{bmatrix},\quad \tilde{N}(dt,dz)=\begin{bmatrix}\tilde{N}_1(dt,dz)\\ \tilde{N}_2(dt,dz)\end{bmatrix},\quad \zeta(\vartheta,t)=\begin{bmatrix}\zeta_1(\vartheta,t)\\ \zeta_2(\vartheta,t)\end{bmatrix},$$

$$A=\begin{bmatrix}-0.1 & 0.15\\ 0.15 & -0.1\end{bmatrix},\quad \sigma(t,\zeta(\vartheta,t))=\begin{bmatrix}2.5\zeta^2(\vartheta,t) & 0\\ 0 & 1.5\zeta^2(\vartheta,t)\end{bmatrix},$$

$$\phi(t,\zeta(\vartheta,t),z)=\begin{bmatrix}12.5\zeta^3(\vartheta,t) & 0\\ 0 & 10.5\zeta^3(\vartheta,t)\end{bmatrix}.$$

It is easy to verify that the assumptions (2)-(4) *are held for* $l^+=0$, $l^-=-0.04$, $c=0.1$, $q=0.1$ *and* $\epsilon=2$.

Now, we solve the LMI (5) *and we can obtain the feasible solutions for the system under consideration. Also, the controller gain value is found as*

$$K=\begin{bmatrix}8.5743 & 0.5102\\ 0.5102 & 8.5743\end{bmatrix}.$$

Hence, by Theorem 3.1 *system* (13) *achieves mean square exponentially stable with decay rate* $\delta=1.6$. *Simulation results are presented to demonstrate the validity of the developed boundary controller. The state response of the system in the absence of the boundary controller is shown in Figure 1 and Figure 2. The effectiveness of the boundary controller is illustrated in Figure 3 and Figure 4, that is, with the help of the controller, the trajectories converge to equilibrium point. It is clear that the designed boundary control is very effective in controlling the states.*

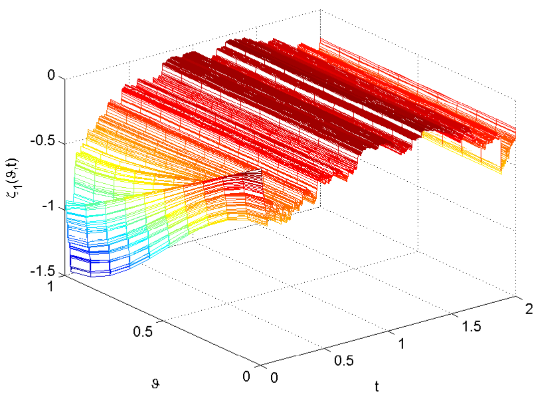

FIGURE 1. (color online) State $\zeta_1(\vartheta,t)$ without control

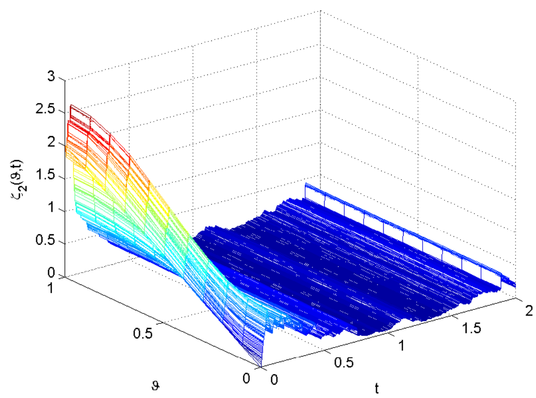

FIGURE 2. (color online) State $\zeta_2(\vartheta,t)$ without control

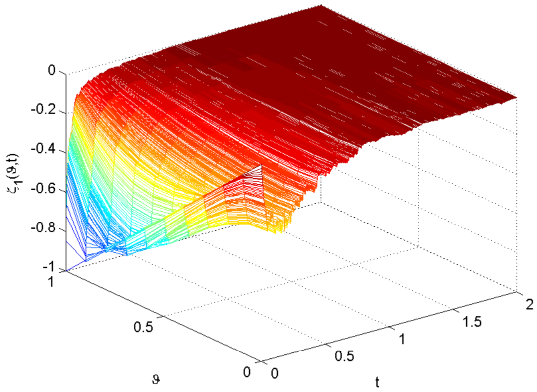

FIGURE 3. (color online) State $\zeta_1(\vartheta, t)$ with control

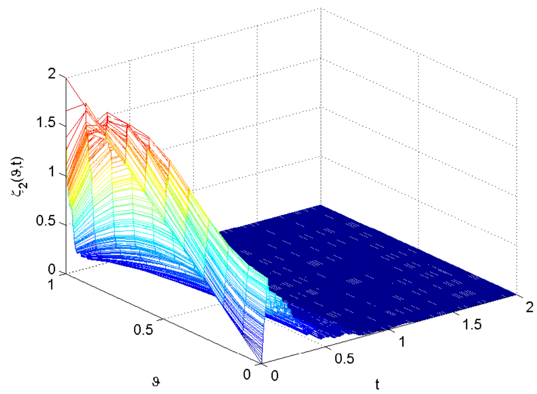

FIGURE 4. (color online) State $\zeta_2(\vartheta, t)$ with control

Example 4.2. *Consider following semi-linear SPDE*

$$\begin{aligned}\frac{\partial \zeta(\vartheta,t)}{\partial t} &= \left[\frac{\partial^2 \zeta(\vartheta,t)}{\partial x^2} + A\zeta(\vartheta,t) + f(t,\zeta(\vartheta,t))\right] + \sigma(t,\zeta(\vartheta,t))\frac{\partial \mathcal{W}(\vartheta,t)}{\partial t}\\ &\quad + \int_Z \phi(t,\zeta(\vartheta,t),z)\tilde{N}(dt,dz), \qquad (14)\end{aligned}$$

with boundary and initial conditions

$$\left.\frac{\partial \zeta(\vartheta,t)}{\partial \vartheta}\right|_{\vartheta=0} = 0, \qquad \left.\frac{\partial \zeta(\vartheta,t)}{\partial \vartheta}\right|_{\vartheta=1} = K\zeta(1,t),$$
$$\zeta(\vartheta,0) = \zeta_0(\vartheta),$$

where

$$f(\zeta(\vartheta,t)) = \begin{bmatrix} f_1(\zeta_1(\vartheta,t)) \\ f_2(\zeta_2(\vartheta,t)) \end{bmatrix}, \quad A = \begin{bmatrix} -0.5 & 0.01 \\ 0.1 & -0.5 \end{bmatrix},$$
$$f_j(\zeta(\vartheta,t)) = 0.01\cos(\zeta_j(\vartheta,t)), \quad j = 1,2.$$

$\sigma(\cdot)$ and $\phi(\cdot)$ are taken the same as in Example 4.1. It is easy to verify that the assumptions (2)-(4) are held for l^+, l^-, c, q, and ϵ, as follows: $l^+ = 0.01$, $l^- = 0.01$, $c = 0.1$, $q = 0.1$ and $\epsilon = 2$.

Now, we solve the LMI (5) and we can obtain the feasible solutions for the system under consideration. Also, the controller gain value is found as

$$K = \begin{bmatrix} 5.9641 & -0.5744 \\ -0.5744 & 5.9641 \end{bmatrix}.$$

Hence, by Theorem 3.1 *system* (14) *achieves mean square exponentially stable with decay rate* $\delta = 2.6$*. Simulation results are presented to demonstrate the validity of the developed boundary controller. The state response of the system in the absence of the boundary controller is shown in Figure 5 and Figure 6. The effectiveness of the boundary controller is illustrated in Figure 7 and Figure 8, that is, with the help of the controller, the trajectories converge to equilibrium point. It is clear that the designed boundary control is very effective in controlling the states.*

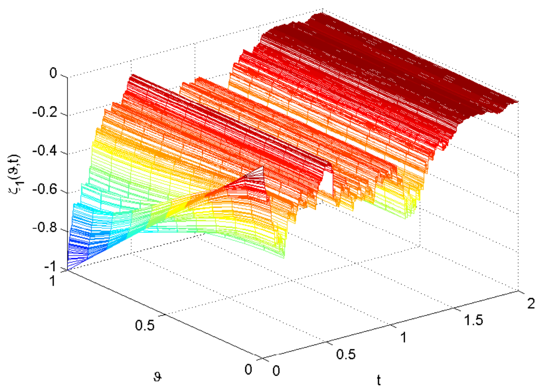

FIGURE 5. (color online) State $\zeta_1(\vartheta, t)$ without control

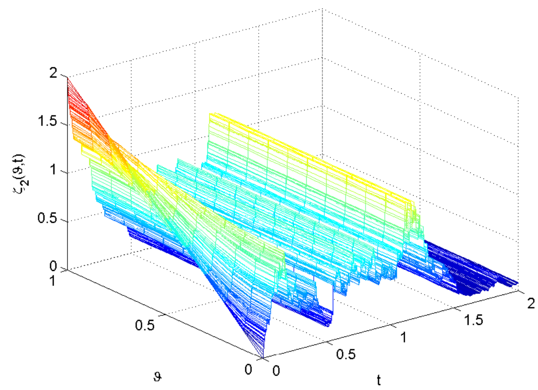

FIGURE 6. (color online) State $\zeta_2(\vartheta, t)$ without control

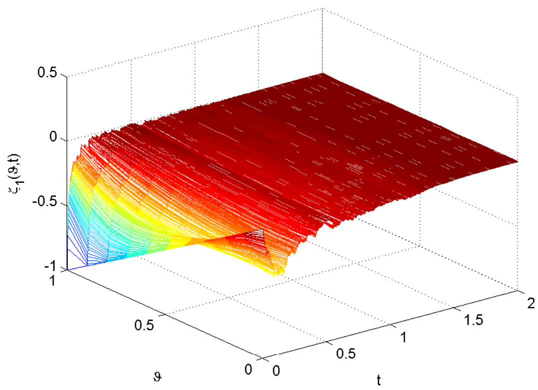

FIGURE 7. (color online) State $\zeta_1(\vartheta, t)$ with control

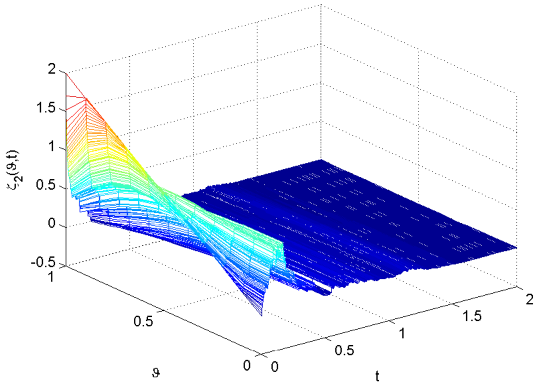

FIGURE 8. (color online) State $\zeta_2(\vartheta, t)$ with control

5. **Conclusion.** In this paper, the problem of mean-square exponential stability of semi-linear stochastic PDE driven by Lèvy noise using boundary feedback control is investigated. The system's mean-square exponential stability is guaranteed using Lyapunov theory and the LMI technique. Finally, numerical example validates the effectiveness of the proposed results. Further, due to the importance of fractional order dynamics, the proposed results will be extended to the time fractional SPDEs in future.

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Author Biography

Ravi Ragul received his B.Sc., M.Sc., and M.Phil. degrees in Mathematics from Bharathiar University, Coimbatore, India, in 2014, 2017 and 2019, respectively. He received his B.Ed. degree in Mathematics from Tamilnadu Teachers Education University, Chennai, India, in 2015. He is currently pursuing his Ph.D. degree in Bharathiar University, Coimbatore, India. His current research interests include stability, robust control for partial differential equations and time-delay systems.

Gunasekaran Sangeetha received the B.Sc. and M.Sc. degrees in Mathematics from Periyar and Bharathiar University, Coimbatore, in 2007 and 2009, respectively. She was awarded the M.Phil. and Ph.D. degrees in Mathematics from Bharathiar University, Coimbatore, in 2010 and 2021, respectively. Currently, she is working as an Assistant Professor in the Department of Mathematics, KGISL Institute of Technology, Coimbatore, India. To her credit, she has published 5 research papers in reputed journals. Her current research interests include control theory and its applications for different kinds of dynamical systems.

Kalidass Mathiyalagan received the B.Sc., M.Sc., and M.Phil. degrees in Mathematics from Bharathiar University, Coimbatore, India, in 2005, 2007 and 2008, respectively. He was awarded the Ph.D. degree in Mathematics from Anna University, Chennai, India, in 2012. Dr. Mathiyalagan was a Senior Engineering Research Fellow and CSIR Senior Research Fellow in the Department of Mathematics, Anna University of Technology, Coimbatore, India from June 2009 to December 2012. He was a Post-Doctoral Research Fellow in the Institute of Cyber-System and Control, Zhejiang University, China in 2013. He worked as Post-Doctoral Research Associate and Research Professor in Department of Electrical Engineering, Yeungnam University, Republic of Korea. Then he joined as Dr.D.S.Kothari postdoctoral fellow of University Grants Commission, India in Department of Mathematics, Bharathiar University, Coimbatore, India. Currently he is working as an Assistant Professor in the Department of Mathematics, Bharathiar University, Coimbatore, India. His current research interests include control theory and its applications for ordinary and partial differential equations, time delay systems. He serves as an Associate Editor in International Journal of Control, Automation and Systems. He is a recipient of the Highly Cited Researcher Award by Clarivate Analytics (formerly, Thomson Reuters) in 2019.

Huiyan Zhang received the M.Sc. degree in Control Engineering, and the Ph.D. degree in Control Theory and Control Engineering from Harbin Institute of Technology, Harbin, in Sep. 2014 and Apr. 2019, respectively. From Sep. 2015 to Sep. 2017, she was a Joint Training Ph.D. student with the School of Electrical and Electronic Engineering, The University of Adelaide. She is currently an Assistant Professor at Chongqing Technology and Business University. Her research interests include stochastic switched systems, event-triggered scheme, model reduction, balanced truncation, robust control and filtering design.