International Journal of Innovative Computing, Information and Control
Volume **19**, Number **1**, February **2023**
ICIC International ©2023 ISSN 1349-4198
pp. 33–46

DIRECT DATA DRIVEN MODEL REFERENCE CONTROL THROUGH PRACTICAL IDENTIFICATION ANALYSIS

Jingwei Huang[1], Jianhong Wang[1], Ricardo Ambrocio Ramirez-Mendoza[2] and Yanfang Wang[3]

[1]School of Electronic Engineering and Automation
Jiangxi University of Science and Technology
No. 86, Hongqi Road, Ganzhou 341000, P. R. China
{ 9120110047; 9120180002 }@jxust.edu.cn

[2]School of Engineering and Science
Tecnologico de Monterrey
Monterrey, Ave. Eugenio Garza Sada 2501, Monterrey, N. L. 64849, Mexico
ricardo.ramirez@tec.mx

[3]School of Mechanical and Electronic Engineering
Jiangxi College of Applied Technology
No. 9, Wenfeng Road, Rongjiang New District, Ganzhou 341000, P. R. China
617420310@qq.com

Received May 2022; revised September 2022

Abstract. *For one commonly used closed loop system with an unknown feedback controller, the constructed cost function, used to identify the unknown plant and noise filter, is modified to its simplified form, then being convenient for the latter model reference control. Given an expected matching function, the problems of model matching and noise suppression are also considered simultaneously to get a constrained optimization problem. To alleviate the dependency on unknown plant and noise filter for model reference control, the direct data driven model reference control is proposed to generate the optimal feedback controller, and its recursive generation is also given. The merit of this direct data driven model reference control is to apply the measured data to designing the unknown feedback controller, while satisfying the model matching and noise suppression. After comparing the detailed expressions of the optimal feedback controller for model reference control and this new direct data direct model reference control respectively, we see they are the same with each other, and our given recursive expression is more convenient for practical engineers.*
Keywords: Model reference control, Direct data driven, Practical identification, Small gain theorem, Stability analysis

1. **Introduction.** The main mission of advanced control theory is to design one practical controller in open loop or closed loop structure, i.e., forward controller and feedback controller, so that this designed controller can drive the output of a plant to track an expected set point or to satisfy a given target. Two categories exist for controller design, i.e., model based approach and direct data driven approach. Consider the first model based approach, a mathematical model of the considered plant is required for the next controller design. It tells that no mathematical model means no controller. Constructing the corresponding mathematical model for the unknown plant is very necessary for this first type of model based approach, and it is also the most difficult step, as it needs some knowledge of other subjects, such as probability theory, linear and nonlinear system theory. This

DOI: 10.24507/ijicic.19.01.33

modeling process corresponds to model identification or system identification, which is adopted to obtain the mathematical model exploiting measured data from experiment on the considered open loop or closed loop system. The whole steps for system identification include four main steps, i.e., model structure selection, optimal input design, parameter estimation and model validation. These above four steps are implemented iteratively until one satisfying model is obtained, so system identification is the first step or premise for the next controller design, i.e., the idea of identification for control.

Due to the application of data driven approach widely in control field, and the similar point between data driven approach and system identification, we call their combination as identification for control, i.e., system identification for direct data driven control. In [1], the feasible parameter set is replaced by one confidence interval, as this confidence interval can accurately describe the actual probability that the future predictor will fall into the constructed confidence interval. The problem about how to construct this confidence interval is solved by a linear approximation/programming approach [2], which can identify the unknown parameter only for linear regression model. According to the obtained feasible parameter set or confidence interval, the midpoint or center can be deemed as the final parameter estimation; further a unified framework for solving the center of the confidence interval is modified to satisfy the robustness [3]. There are two kinds of descriptions on external noise, one is probabilistic description, the other is deterministic description, corresponding to the unknown but bounded noise here [4]. For the probabilistic description on external noise, the noise is always assumed to be one white noise, and its probabilistic density function is known in advance. On the contrary for deterministic description on external noise, the only information about noise is bound, so this deterministic description can relax the strict assumption on probabilistic description. In reality or practice, bounded noise is more common than white noise. Within the deterministic description on external noise, set membership identification is adjusted to design controllers with two degrees of freedom [5]. Set membership control is applied to designing feedback control in a closed loop system with nonlinear system in [6], where the considered system is identified by set membership identification, and [7] takes the derivative of the above cost function with respect to control input to achieve one optimal input.

Here this paper considers this direct data driven control through combining model reference and practical identification analysis [8]. More specifically, the main essence of our considered direct data driven control is to reformulate the controller design problem as one system identification problem, and then our contribution about practical identification analysis is applied [9]. On the other hand, model reference is also introduced in direct data driven control to get our studied direct data driven model reference control strategy. Generally, this paper makes use of practical identification, direct data driven control and model reference control, so that one more practical control strategy is convenient for some engineers [10]. To verify this, our considered control strategy is applied to flight simulator. Consider that practical identification analysis for direct data driven control, the original cost function, used to identify the unknown controller parameters, is modified to its another improved or simplified form, which is convenient for the latter controller design [11]. Before proposing this direct data driven model reference control, the classical model reference control is reviewed, and the optimal feedback control is derived. Furthermore, in the framework of direct data driven model reference control, the measured data are applied to designing the unknown controller directly, and their comparisons are also given.

2. **System Structure Description.**

A commonly used closed loop system structure in theory research or practical engineering is plotted in the following Figure 1, where in Figure 1, $y(t)$ is the closed loop output, $u(t)$ is the input signal for the plant $G_0(z)$, $r(t)$ is

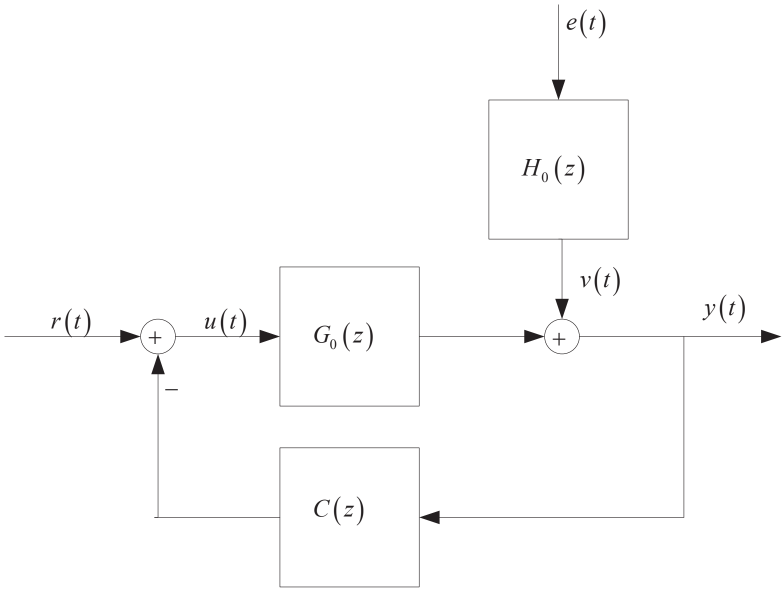


FIGURE 1. Closed loop system structure

the external excitation input signal, $e(t)$ is the external noise. For simplicity of discussion, external noise $e(t)$ is assumed to be a zero mean white Gaussain noise with variance λ_0, and external excitation input $r(t)$ and external noise $e(t)$ are mutually uncorrelated. $v(t)$ is a colored noise through passing that white noise $e(t)$ with a filter $H_0(z)$. z is the time delay operator. $\{G_0(z), H_0(z)\}$ denotes the true plant and true noise filter respectively, which are unknown. $C(z)$ is one feedback controller, whatever linear or nonlinear. From the theoretical perspective, the mission of system identification or data driven estimation is to apply measured data $\{r(t), u(t), y(t)\}_{t=1}^{N}$ to identifying those pair $\{G_0(z), H_0(z)\}$, where N is the number of measured data. On the other hand, direct data driven control proposes to use measured data $\{r(t), u(t), y(t)\}_{t=1}^{N}$ to design that feedback controller $C(z)$ without any knowledge about plant and filter $\{G_0(z), H_0(z)\}$.

From Figure 1, we have

$$v(t) = H_0(z)e(t) \tag{1}$$

Using the basic modern control theory and some mathematical equation computations, we get

$$y(t) = G_0(z)u(t) + H_0(z)e(t); \quad u(t) = r(t) - C(z)y(t) \tag{2}$$

Expanding each term in Equation (2), we have

$$\begin{aligned} y(t) &= \frac{G_0(z)}{1 + G_0(z)C(z)}r(t) + \frac{H_0(z)}{1 + G_0(z)C(z)}e(t) \\ u(t) &= \frac{1}{1 + G_0(z)C(z)}r(t) - \frac{C(z)H_0(z)}{1 + G_0(z)C(z)}e(t) \end{aligned} \tag{3}$$

Design a sensitivity function to reduce the computational complexity, i.e.,

$$S_0(z) = \frac{1}{1 + G_0(z)C(z)} \tag{4}$$

Based on above defined sensitivity function $S_0(z)$, we rewrite Equation (3) as that

$$\begin{aligned} y(t) &= G_0(z)S_0(z)r(t) + H_0(z)S_0(z)e(t) \\ u(t) &= S_0(z)r(t) - C(z)H_0(z)S_0(z)e(t) \end{aligned} \tag{5}$$

The main task of data driven estimation or system identification is to identify an unknown parameter vector in Equation (2) through some statistical methods.

3. **Practical Identification Analysis.** To give our new contributions about system identification and good understanding about direct data driven control, firstly some preliminaries about closed loop system are given as follows.

3.1. **Preliminaries about identification.** After parameterizing the unknown plant and noise filter $\{G_0(z), H_0(z)\}$ in Equation (2), then the parameterized closed loop system is that

$$\begin{aligned} y(t,\theta) &= \frac{G(z,\theta)}{1+G(z,\theta)C(z)}r(t) + \frac{H(z,\theta)}{1+G(z,\theta)C(z)}e(t) \\ u(t,\theta) &= \frac{1}{1+G(z,\theta)C(z)}r(t) - \frac{C(z)H(z,\theta)}{1+G(z,\theta)C(z)}e(t) \end{aligned} \tag{6}$$

where in Equation (6), θ is an unknown parameter vector, which is needed to identify. The goal of this section is to estimate the unknown parameter vector $\hat{\theta}_N$ based on the collected input and output data sequence $Z^N = \{y(t), u(t)\}_{t=1}^N$, where N is the number of total observed data. From Equation (6), one step ahead prediction is constructed to express an interesting output prediction $\hat{y}(t,\theta)$, i.e.,

$$\hat{y}(t,\theta) = \frac{G(z,\theta)}{H(z,\theta)}r(t) + \frac{H(z,\theta) - 1 - G(z,\theta)C(z)}{H(z,\theta)}y(t) \tag{7}$$

Combining Equation (3) and output prediction (7), the error function $\varepsilon(t,\theta)$ is yielded as

$$\varepsilon(t,\theta) = y(t) - \hat{y}(t,\theta) = \frac{1+G(z,\theta)C(z)}{H(z,\theta)}\left[y(t) - \frac{G(z,\theta)}{1+G(z,\theta)C(z)}r(t)\right] \tag{8}$$

Through using above defined error function, we have the following numerical optimization problem to estimate that unknown parameter vector θ, i.e.,

$$\hat{\theta}_N = \arg\min_\theta V_N\left(\theta, Z^N\right) = \arg\min_\theta \frac{1}{N}\sum_{t=1}^{N} \varepsilon^2(t,\theta) \tag{9}$$

Through using the smooth property with respect to the parameterized prediction error $\varepsilon(t,\theta)$, then the cost function in above optimization process can be reduced to one finding root process. As one step ahead prediction error or residual $\varepsilon(t,\theta)$ is a smooth function of parameter vector θ for every time instant t, the parameter estimator is equivalently given as a root of the following function:

$$D(\theta) = \partial_\theta \sum_{t=1}^{N} \frac{1}{2}\varepsilon^2(t,\theta) = -\sum_{t=1}^{N}[\partial_\theta \hat{y}(t,\theta)](y(t) - \hat{y}(t,\theta)) \tag{10}$$

It means that

$$\hat{\theta}_N \in sol[D(\theta) = 0] \tag{11}$$

where notation ∂_θ means partial derivative operation about θ, and after some tedious mathematical operations, $\partial_\theta \hat{y}(t,\theta)$ is expanded as that

$$\begin{aligned} \partial_\theta \hat{y}(t,\theta) = & \frac{\frac{\partial G(z,\theta)}{\partial\theta}H(z,\theta) - G(z,\theta)\frac{\partial H(z,\theta)}{\partial\theta}}{H^2(z,\theta)}r(t) - \frac{\frac{\partial G(z,\theta)}{\partial\theta}C(z)H(z,\theta)}{H^2(z,\theta)}y(t) \\ & - \frac{(1+G(z,\theta)C(z))\frac{\partial H(z,\theta)}{\partial\theta}}{H^2(z,\theta)}y(t) \end{aligned} \tag{12}$$

3.2. **Practical identification analysis.** Observing Figure 1 again, excitation input $\{r(t)\}$ is a quasi stationary reference signal and its spectrum is $\phi_r(w)$. Moreover by using the condition of that independent and identically distributed white noise with variance λ_0, that external white noise $e(t)$ has power spectrum λ_0, then the colored noise $v(t)$ has power spectrum, i.e.,

$$H_0\left(e^{jw}\right) H_0^*\left(e^{jw}\right) \lambda_0 = \left|H_0\left(e^{jw}\right)\right|^2 \lambda_0$$

where $*$ means complex conjugate.

From Equation (5), we see the uncorrelated property holds between excitation input $r(t)$ and white noise $e(t)$, and then we obtain some spectrum relations directly, such as

$$\phi_u(w) = |S_0|^2\phi_r(w) + |C|^2|S_0|^2|H_0|^2\lambda_0 = \phi_u^r(w) + \phi_u^e(w) \tag{13}$$

where $\phi_u^r(w)$ and $\phi_u^e(w)$ are two parts for the excitation input spectrum.

Similarly, we have the following relation about the output spectrum:

$$\phi_y(w) = |G_0|^2|S_0|^2\phi_r(w) + |H_0|^2|S_0|^2\lambda_0 \tag{14}$$

Similarly, we get another form for the cross spectrum:

$$\phi_{yu}(w) = G_0|S_0|^2\phi_r(w) - C|H_0|^2|S_0|^2\lambda_0; \quad \phi_{ue}(w) = -CH_0S_0\lambda_0 \tag{15}$$

Observing above three equations simultaneously, it holds that

$$\lambda_0\phi_u(w) - |\phi_{ue}(w)|^2 = \lambda_0|S_0|^2\phi_r(w) = \lambda_0\phi_u^r(w) \tag{16}$$

where in above derivation process, for simplified notation, e^{jw} is neglected.

The reason why we introduce the above spectral relations concerns on estimating the transfer function $G_0(z)$ with its spectral analysis estimation. More specifically, spectral analysis estimation $\hat{G}\left(e^{jw}\right)$ is got as follows:

$$\begin{aligned} \hat{G}\left(e^{jw}\right) &= \frac{G_0\left(e^{jw}\right)\phi_r(w) - C\left(e^{jw}\right)\left|H_0\left(e^{jw}\right)\right|^2\lambda_0}{\phi_r(w) + \left|C\left(e^{jw}\right)\right|^2\left|H_0\left(e^{jw}\right)\right|^2\lambda_0} \\ &= \frac{G_0\left(e^{jw}\right)\phi_r(w) - C\left(e^{jw}\right)\phi_v(w)}{\phi_r(w) + \left|C\left(e^{jw}\right)\right|^2\phi_v(w)} \end{aligned} \tag{17}$$

Taking the limit operation on both sides of Equation (17), when N is sufficiently large, that spectral analysis estimation $\hat{G}\left(e^{jw}\right)$ approaches to the following limit:

$$G_0\left(e^{jw}\right)\phi_r(w) - C\left(e^{jw}\right)\phi_v(w) \to \phi_{yu}(w); \quad \phi_r(w) + \left|C\left(e^{jw}\right)\right|^2\phi_v(w) \to \phi_u(w)$$

Then it means that

$$\frac{G_0\left(e^{jw}\right)\phi_r(w) - C\left(e^{jw}\right)\phi_v(w)}{\phi_r(w) + \left|C\left(e^{jw}\right)\right|^2\phi_v(w)}$$

Consider that cost function (9), assume the true parameter is in the considered parameter set, it means one true parameter vector θ_0 such that

$$G(z,\theta_0) = G_0(z), \quad H(z,\theta_0) = H_0(z)$$

As the error function (8) is not dependent of the unknown parameter explicitly, so to find its detailed relation with the unknown parameter, we need to rewrite it in another form, i.e.,

$$\varepsilon(t,\theta) = H^{-1}(z,\theta)[y(t) - G(z,\theta)u(t)] \tag{18}$$

Combining two equations (3) and (18), we can easily obtain the following detailed expression as

$$y(t) - G(z,\theta)u(t) = \frac{G_0(z) - G(z,\theta)}{1 + G_0(z)C(z)}r(t) + \frac{H_0(z)(1 - G(z,\theta)C(z))}{1 + G_0(z)C(z)}e(t) \tag{19}$$

Substituting (19) into the prediction error (18), we get

$$\varepsilon(t,\theta)=\frac{G_0(z)-G(z,\theta)}{H(z,\theta)}S_0(z)r(t)+\frac{H_0(z)}{H(z,\theta)}\frac{1-G(z,\theta)C(z)}{1+G_0(z)C(z)}e(t) \tag{20}$$

From above our derivations, another form for that cost function is rewritten as follows:

$$\begin{aligned}V_N(\theta) &= \frac{1}{N}\sum_{t=1}^{N}\varepsilon^2(t,\theta)=\int\frac{\left|G\left(e^{jw},\theta\right)-G_0\left(e^{jw},\theta\right)\right|^2}{\left|H\left(e^{jw},\theta\right)\right|^2}\frac{1}{\left|1+G_0\left(e^{jw}\right)C\left(e^{jw}\right)\right|^2}\phi_r(w)dw\\&+\int\left|\frac{1+G\left(e^{jw},\theta\right)C\left(e^{jw}\right)}{1+G_0\left(e^{jw}\right)C\left(e^{jw}\right)}\right|^2\frac{\left|H_0\left(e^{jw}\right)\right|^2}{|H(e^{jw},\theta)|^2}\lambda_0 dw\end{aligned} \tag{21}$$

It means that we can identify the unknown parameter estimations through minimizing the following improved cost function. Apply Equation (13) into above Equation (21) to get that

$$\begin{aligned}V_N(\theta) &= \int\frac{\left|G\left(e^{jw},\theta\right)-G_0\left(e^{jw},\theta\right)\right|^2}{\left|H\left(e^{jw},\theta\right)\right|^2}\phi_u^r(w)dw\\&+\int\left|\frac{1+G\left(e^{jw},\theta\right)C\left(e^{jw}\right)}{1+G_0\left(e^{jw}\right)C\left(e^{jw}\right)}\right|^2\frac{1}{\left|H\left(e^{jw},\theta\right)\right|^2}\phi_v(w)dw\end{aligned} \tag{22}$$

Above Equation (22) tells that if the number N of observed data is sufficiently large, the while minimum process for the improved cost function $V_N(\theta)$ is guaranteed to be the global minimum, i.e., the following relation holds that

$$G\left(e^{jw},\theta\right)\to G_0\left(e^{jw},\theta_0\right);\quad \theta\to\theta_0$$

The above obtained result is one simplified form of the classical result.

4. **Direct Data Driven Model Reference Control.** Section 3 concerns around the problem of system identification or data driven estimation, i.e., identify that unknown parameter vector θ as its parameter estimation $\hat{\theta}_N$. Then the unknown plant $G_0(z)$ and noise filter $H_0(z)$ are replaced by their identified values $\left\{G\left(z,\hat{\theta}_N\right),H\left(z,\hat{\theta}_N\right)\right\}$, so this pair of identified values $\left\{G\left(z,\hat{\theta}_N\right),H\left(z,\hat{\theta}_N\right)\right\}$ is applied for the next controller design. The above description corresponds to the explanation about model based control. From Section 4, we start to propose our named direct data driven model reference control. For clarity of presentation, firstly we review the concept of model reference control and do some improvement to get one approximate optimal feedback controller.

4.1. **Model reference control.** Rewrite that parameterized output again

$$y(t,\theta)=\frac{G(z,\theta)}{1+G(z,\theta)C(z)}r(t)+\frac{H(z,\theta)}{1+G(z,\theta)C(z)}e(t) \tag{23}$$

where the first transfer function is called as closed loop transfer function from input signal $r(t)$ to output signal $y(t)$, similarly the second transfer function corresponds to closed loop transfer function from external noise $e(t)$ to output signal $y(t)$. And that parameter vector θ can be replaced by its parameter estimation $\hat{\theta}_N$, while using above practical identification analysis.

The physical principle of model reference control is explained as follows. When given one expected or designed transfer function $M(z)$, the problem is turned to design that feedback controller $C(z)$, such that the first transfer function approaches to the designed transfer function $M(z)$. The structure of model reference control is plotted in Figure 2.

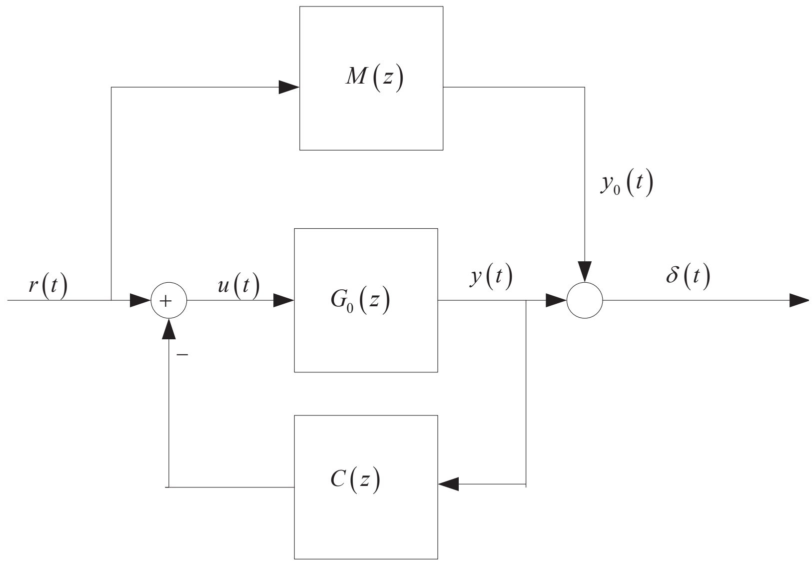


FIGURE 2. Structure of model reference control

From Figure 2, some relations are obtained:

$$y_0(t) = M(z)r(t); \quad y(t,\theta) = \frac{G(z,\theta)}{1+G(z,\theta)C(z)}r(t) + \frac{H(z,\theta)}{1+G(z,\theta)C(z)}e(t) \tag{24}$$

so the goal of model reference control is to design the feedback controller $C(z)$ to satisfy that

$$\frac{G(z,\theta)}{1+G(z,\theta)C(z)} \to \frac{G\left(z,\hat{\theta}_N\right)}{1+G\left(z,\hat{\theta}_N\right)C(z)} \to M(z) \tag{25}$$

Except condition (25), the other condition is imposed as that

$$\frac{H(z,\theta)}{1+G(z,\theta)C(z)} \to 0 \tag{26}$$

Then combine above two conditions to get

$$\frac{G(z)}{1+G(z)C(z)} \to M(z); \quad \frac{H(z)}{1+G(z)C(z)} \to 0 \tag{27}$$

where for the sake of brevity, parameter vector θ is abbreviated.

Define the output error function as

$$\delta(t) = y_0(t) - y(t) = \left[\frac{G(z)}{1+G(z)C(z)} - M(z)\right] r(t) \tag{28}$$

Then our extended model reference control is turned to the following optimization problem with one equity constraint, i.e.,

$$J_1(C(z)) = \left\| \frac{G(z)}{1+G(z)C(z)} - M(z) \right\|_2^2; \quad \text{subject to } \frac{H(z)}{1+G(z)C(z)} = 0 \tag{29}$$

where $\|.\|$ is the common Euclidean norm for above constrained optimization problem.

To solve above constrained optimization problem (29), we always change it as one unconstrained optimization problem by introducing one discount factor $\lambda \in [0,1]$, i.e.,

$$J_2(C(z)) = \left\| \frac{G(z)}{1+G(z)C(z)} - M(z) \right\|_2^2 + \lambda \left\| \frac{H(z)}{1+G(z)C(z)} \right\|_2^2 \tag{30}$$

Discount factor $\lambda \in [0, 1]$ can trade off the model matching and the effect from the external noise, using the equivalent property for each norm, then that common Euclidean norm $\|.\|_2^2$ can be replaced by L2 norm, such as

$$J_2(C(z)) = \left[\frac{G(z)}{1+G(z)C(z)} - M(z)\right]^2 + \lambda\left[\frac{H(z)}{1+G(z)C(z)}\right]^2 \tag{31}$$

where in optimization problem, $M(z)$ is the expected matching function, $\{G(z), H(z)\}$ can be replaced by their estimations $\left\{G\left(z, \hat{\theta}_N\right), H\left(z, \hat{\theta}_N\right)\right\}$. $\lambda \in [0, 1]$ is a discount factor, so only that feedback controller $C(z)$ is unknown.

4.2. **Optimal feedback controller.** The direct way to yield the optimal feedback controller is to differentiate that cost function with respect to controller, i.e.,

$$\frac{\partial J_2(C(z))}{\partial C(z)} = 2\left[\frac{G(z)}{1+G(z)C(z)} - M(z)\right]\frac{G^2(z)}{[1+G(z)C(z)]^2} + 2\lambda\frac{H(z)G(z)}{[1+G(z)C(z)]^2} = 0 \tag{32}$$

It means that

$$\left[\frac{G(z)}{1+G(z)C(z)} - M(z)\right]G(z) + \lambda H(z) = 0; \quad \frac{G(z)}{1+G(z)C(z)} - M(z) = -\frac{\lambda H(z)}{G(z)}$$

$$\frac{G(z)}{1+G(z)C(z)} = M(z) - \frac{\lambda H(z)}{G(z)} = \frac{M(z)G(z) - \lambda H(z)}{G(z)}$$

$$1+G(z)C(z) = \frac{G^2(z)}{M(z)G(z) - \lambda H(z)}; \quad G(z)C(z) = \frac{G^2(z) - M(z)G(z) + \lambda H(z)}{M(z)G(z) - \lambda H(z)} \tag{33}$$

Then optimal feedback controller is yielded as that

$$C(z) = \frac{G^2(z) - M(z)G(z) + \lambda H(z)}{M(z)G^2(z) - \lambda H(z)G(z)} \tag{34}$$

or

$$C(z) = \frac{G^2\left(z, \hat{\theta}_N\right) - M(z)G\left(z, \hat{\theta}_N\right) + \lambda H\left(z, \hat{\theta}_N\right)}{M(z)G^2\left(z, \hat{\theta}_N\right) - \lambda H\left(z, \hat{\theta}_N\right)G\left(z, \hat{\theta}_N\right)} \tag{35}$$

The difference between Equations (34) and (35) is that $\{G(z), H(z)\}$ are replaced by their estimations $\left\{G\left(z, \hat{\theta}_N\right), H\left(z, \hat{\theta}_N\right)\right\}$. Observing Equation (34), if no $H(z)$ exists, then $C(z)$ is reduced to

$$C(z) = \frac{G(z) - M(z)}{M(z)G(z)} \tag{36}$$

Equation (36) is very simple in practical analysis, due to its simplicity to satisfy the model matching. During our derivation process for that optimal feedback controller $C(z)$, matching function $M(z)$ and noise suppression for $e(t)$ are all considered. When more emphasis is on the effect from the external noise, then that discount factor λ may be chosen approximately, for example, let $\lambda = 1$. On the contrary, if we only consider the main task about tracking the reference or matching function $M(z)$, i.e., $\frac{G(z)}{1+G(z)C(z)} \to M(z)$, and do not care the effect from external noise, so we can let $\lambda = 0$. For this special case, the optimal feedback controller $C(z)$ is reduced to its simplified form, i.e., $C(z) = \frac{G(z)-M(z)}{M(z)G(z)}$.

4.3. **Data driven scheme.** From Equations (34), (35) and (36), we see within the framework of model reference control, optimal feedback controller $C(z)$ is dependent of plant and filter $\{G(z), H(z)\}$. If the identification accuracy for $\{G(z), H(z)\}$ is unsatisfied, then it will destroy the latter control performance. To weaken this dependency on $\{G(z), H(z)\}$, data driven scheme is introduced with model reference control to yield one new novel direct data driven model reference control strategy. Specifically in our information age, lots of data are collected very easily. As some important factors are contained in these data, our work is to abstract the important information.

In Figure 2, output error function $\delta(t)$ is defined as $\delta(t) = y_0(t) - y(t)$. Similarly, can we construct the corresponding input error function, and use this input error function to design the optimal feedback controller? More specifically, given the desired matching function $M(z)$, we have $y(t) = M(z)r(t)$. Use the inverse property of this desired matching function $M(z)$ to get

$$r(t) = M^{-1}(z)y(t) \tag{37}$$

Then input signal becomes

$$u(t) = r(t) - C(z)y(t) = M^{-1}(z)y(t) - C(z)y(t) = \left[M^{-1}(z) - C(z)\right] y(t) \tag{38}$$

Define the input error function as

$$\gamma(t) = u(t) - \left[M^{-1}(z) - C(z)\right] y(t) \tag{39}$$

where $\{u(t), y(t)\}_{t=1}^{N}$ are known, as they are collected through one kind of sensor. From Equation (39), a sequence of observed data $\{u(t), y(t)\}_{t=1}^{N}$ are collected, and then that unknown feedback controller $C(z)$ is yielded from the following optimization problem, i.e.,

$$\arg\min_{C(z)} \frac{1}{N} \sum_{t=1}^{N} \gamma^2(t) = \arg\min_{C(z)} \frac{1}{N} \sum_{t=1}^{N} \left[u(t) - \left[M^{-1}(z) - C(z)\right] y(t)\right]^2 \tag{40}$$

The detailed description about above process can be referred to our previous work. By differentiating with respect to $C(z)$ and by setting the derivative equal to zero, it holds that

$$\frac{\left[\partial \frac{1}{N} \sum_{t=1}^{N} \gamma^2(t)\right]}{\partial C(z)} = \frac{2}{N} \sum_{t=1}^{N} \left[u(t) - \left[M^{-1}(z) - C(z)\right] y(t)\right] y^T(t) = 0 \tag{41}$$

and

$$\sum_{t=1}^{N} u(t)y^T(t) = \sum_{t=1}^{N} M^{-1}(z)y(t)y^T(t) - \sum_{t=1}^{N} C(z)y(t)y^T(t) \tag{42}$$

The following spectral relations are used during our later mathematical derivations.

$$y(t) = G(z)u(t);\ \phi_{uy}(w) = G(z)\phi_u(w);\ \phi_y(w) = G(z)G^*(z)\phi_u(w) = |G(z)|^2\phi_u(w) \tag{43}$$

Substituting these priori spectral relations in Equation (42), we have

$$C(z)\phi_y(w) = M^{-1}(z)\phi_y(w) - \phi_{uy}(w) \tag{44}$$

i.e.,

$$C(z) = M^{-1}(z) - \frac{\phi_{uy}(w)}{\phi_y(w)} \tag{45}$$

Then we have

$$C(z) = M^{-1}(z) - \frac{G(z)\phi_u(w)}{|G(z)|^2\phi_u(w)} = \frac{1}{M(z)} - \frac{1}{G(z)} = \frac{G(z) - M(z)}{M(z)G(z)} \tag{46}$$

Based on the following relation,

$$\sum_{t=1}^{N} y(t)y^T(t)C(z) = -\sum_{t=1}^{N} u(t)y^T(t) + \sum_{t=1}^{N} M^{-1}(z)y(t)y^T(t) \tag{47}$$

it holds that

$$C(z) = \left[\frac{1}{N}\sum_{t=1}^{N} y(t)y^T(t)\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N} M^{-1}(z)y(t)y^T(t)\right] - \left[\frac{1}{N}\sum_{t=1}^{N} y(t)y^T(t)\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N} u(t)y^T(t)\right] \quad (48)$$

i.e.,

$$C(z) = \frac{1}{M(z)} - \left[\frac{1}{N}\sum_{t=1}^{N} y(t)y^T(t)\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N} u(t)y^T(t)\right] \quad (49)$$

As $M(z)$ is given in priori in Equation (49), for notational simplicity, define $C_1(N)$ as

$$C_1(N) = \left[\frac{1}{N}\sum_{t=1}^{N} y(t)y^T(t)\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N} u(t)y^T(t)\right] \quad (50)$$

so we have

$$C(z) = \frac{1}{M(z)} - C_1(N) \quad (51)$$

If more measured data are collected, the updated information is imposed on term $C_1(N)$, for example,

$$C_1(N+1) = \left[\frac{1}{N}\sum_{t=1}^{N} y(t)y^T(t) + \frac{1}{N}y(N+1)y^T(N+1)\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N} u(t)y^T(t) + \frac{1}{N}u(N+1)y^T(N+1)\right] \quad (52)$$

Let

$$P^{-1}(k) = \sum_{t=1}^{k} y(t)y^T(t); \quad P^{-1}(k-1) = \sum_{t=1}^{k-1} y(t)y^T(t)$$

so

$$P^{-1}(k) = \sum_{t=1}^{k-1} y(t)y^T(t) + y(k)y^T(k) = P^{-1}(k-1) + y(k)y^T(k)$$

and

$$C_1(k-1) = P(k-1)\sum_{t=1}^{k-1} u(t)y^T(t); \quad P^{-1}(k-1)C_1(k-1) = \sum_{t=1}^{k-1} u(t)y^T(t)$$

continuing to derive that

$$C_1(k) = P(k)\sum_{t=1}^{k} u(t)y^T(t) = P(k)\left[\sum_{t=1}^{k-1} u(t)y^T(t) + u(k)y(k)\right] = C_1(k-1) + P(k)[u(k) - y(k)C_1(k-1)]y^T(k) \quad (53)$$

After substituting the recursive expression about term $C_1(N)$ in Equation (51), the recursive form for optimal feedback controller $C(z)$ is yielded, for example,

$$[C(z)]_k = \frac{1}{M(z)} - C_1(k) = \frac{1}{M(z)} - C_1(k-1) - P(k)[u(k) - y(k)C_1(k-1)]y^T(k) \quad (54)$$

where $[C(z)]_k$ means the optimal feedback controller for k pairs of measured data.

5. **Simulation Examples.** Apply our derived practical identification analysis for a single input and single output system controlled by a model reference controller. A true data generating system considered here is given as

$$G_0(z) = \frac{0.25z^{-1} + 0.12z^{-2}}{1 - 1.6z^{-1} + 0.8z^{-2} - 0.64z^{-3} + 0.65z^{-4}}; \quad H_0(z) = \frac{1 + 0.2z^{-1}}{1 + 0.5z^{-1}} \tag{55}$$

Its parameterized form is given as

$$G(z, \theta) = \frac{a_5 z^{-1} + a_6 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}; \quad H_0(z) = \frac{1 + b_2 z^{-1}}{1 + b_1 z^{-1}} \tag{56}$$

A Gaussian white noise signal $e(t)$ with variance $\lambda_0 = 0.5$ is added through the noise filter $H_0(z)$. The sampled times $T_s = 1$ second, the true parameter vector θ_0 is that

$$\theta_0 = \begin{bmatrix} -1.6 & 0.8 & -0.64 & 0.65 & 0.25 & 0.12 & 0.5 & 0.2 \end{bmatrix}^T \tag{57}$$

The data generating system is operated in closed loop system with a model reference control based on our commissioning model (G_{init}, H_{init}), where θ_{init} is chosen as that

$$\theta_{init} = \begin{bmatrix} -1.7 & 0.7 & -0.4 & 0.8 & 0.15 & 0.1 & 0.4 & 0.1 \end{bmatrix}^T \tag{58}$$

All above parameters are chosen in such way that the obtained closed loop system must satisfy the bounded input and bounded output stability, which is proposed in model reference control. Model reference control is tuned so that we get sufficient performance for the commissioning model (G_{init}, H_{init}). Model reference control is set to have a prediction horizon of 100 and a control horizon of 100. The output variable $y(t)$ has a weight of 10, and the input variable $u(t)$ has a weight of 1. An excitation signal $r(t)$ has a bound, i.e., $-1 \leq r(t) \leq 1$.

The applied input signal is given in Figure 3, and we measure the output signal $y(t)$ by some measuring devices, where observed output signal is plotted in Figure 4. When using prediction error identification method to identify unknown parameters, if estimation error $\delta = \left\| \hat{\theta}(t) - \theta \right\| \Big/ \|\theta\|$ is less than one very small value 0.005, then terminate the recursive methods.

To verify the efficiency of the identified model $G\left(\hat{\theta}_N\right)$ by our derived results and make sure that this identified model can be used to replace the true model, we compare the

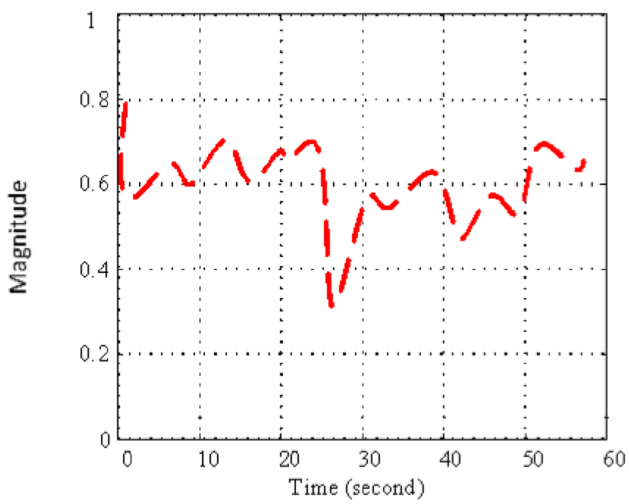


FIGURE 3. The applied input signal

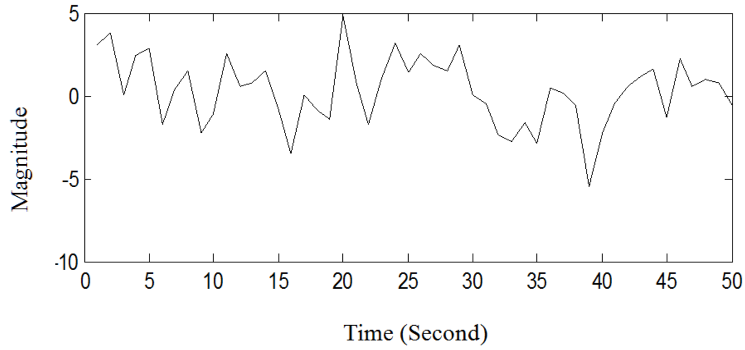


FIGURE 4. The observed output signal

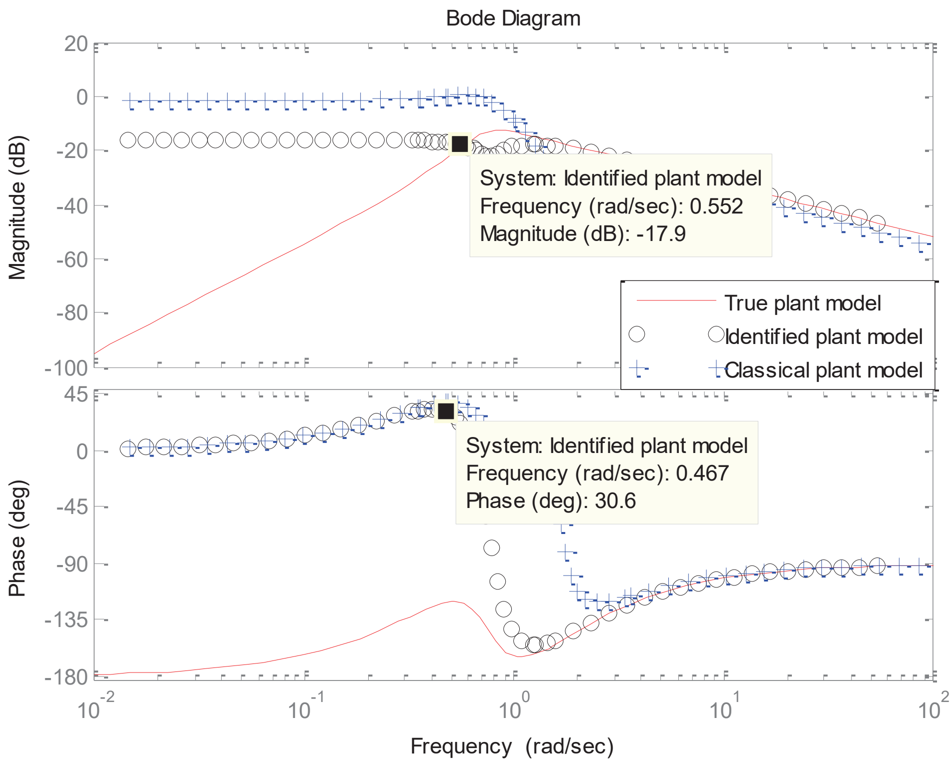


FIGURE 5. Bode plot for plant

Bode responses among true plant $G_0(q)$, our identified model $G\left(\hat{\theta}_N\right)$ and classical plant, respectively in Figure 5. From Figure 5, we see that these three Bode response curves coincide with each other, and our identified model $G\left(\hat{\theta}_N\right)$ can converge to its true plant quickly than classical identified plant. This means that our model error $\tilde{G}(q)$ will converge to zero with time increases very quickly.

6. **Conclusion.** In this paper, direct data driven model reference control is studied to design the unknown feedback controller through only applying the measured data. To easily understand our considered control strategy, practical identification analysis and model reference control are reviewed, and then our new contributions about them are described, such as simplified cost function, model matching and noise suppression in model reference control. The optimal feedback controllers for model reference control and our direct data driven model reference control are derived respectively to compare.

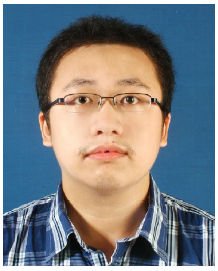
Although their final forms are the same with each other, but our devised recursive form is good for practical application. The problem of how to combine game theory and dynamic programming for direct data driven model reference control is our next ongoing work.

Acknowledgment. This work is partially supported by Jiangxi Provinical National Science Foundation (Nos. GJJ180484, GJJ214906).

REFERENCES

[1] D. Piga and S. Formentin, Direct data driven control of constrained systems, *IEEE Transactions on Control Systems Technology*, vol.25, no.2, pp.331-351, 2018.
[2] M. Tanaskovic, L. Fagiano and C. Novara, Data driven control on non-line systems: An on line direct approach, *Automatica*, vol.75, no.1, pp.1-10, 2017.
[3] M. Tanaskovic, L. Fagiano and V. Gligorovski, Adaptive model predictive control for linear time varying MIMO systems, *Automatica*, vol.105, no.2, pp.237-245, 2019.
[4] E. Terzi, L. Fagiano, M. Farina and R. Scattolini, Learning based predictive control for linear systems: A unitary approach, *Automatica*, vol.108, no.1, pp.2697-2705, 2019.
[5] A. Bemporad, M. Morari and V. Dua, The explicit linear quadratic regulator for constrained systems, *Automatica*, vol.38, no.3, pp.3-20, 2002.
[6] A. Bemporad, A. Casavola and E. Mosca, Control of constrained linear systems via predictive reference management, *IEEE Transactions on Automatic Control*, vol.42, no.2, pp.340-349, 1997.
[7] M. Gevers, Identification for control: From the early achievements to the revival of experiment design, *European Journal of Control*, vol.11, no.12, pp.1-10, 2017.
[8] P. Falugi and D. Q. Mayne, Getting robustness against unstructured uncertainty: A tube based MPC approach, *IEEE Transactions on Automatic Control*, vol.59, no.10, pp.1290-1295, 2014.
[9] U. Rosolia and F. Borrelli, Learning model predictive control for iterative tasks: A data driven control framework, *IEEE Transactions on Automatic Control*, vol.63, no.11, pp.1883-1896, 2018.
[10] E. Garone, S. Di Cairano and I. Kolmanovsky, Reference and command governors for systems with constraints: A survey on theory and applications, *Automatica*, vol.75, no.4, pp.306-328, 2017.
[11] M. Huang, Z. Hu and L. Wang, Optimal consensus control for heterogeneous nonlinear non-affine multi-agent systems with uncertain control directions, *ICIC Express Letters*, vol.16, no.2, pp.177-185, 2022.

Author Biography



Jingwei Huang received the master degree from Jiangxi University of Science and Technology, China, in 2017. He is currently a lecturer with Jiangxi University of Science and Technology, China. His research interests include machine learning and data driven control.

Jianhong Wang received the Ph.D. degree from Nanjing University of Aeronautics and Astronautics, China, in 2011. He is currently a professor with Jiangxi University of Science and Technology, China. His research interests include data driven control and adaptive control.

Ricardo Ambrocio Ramirez-Mendoza received a Ph.D. degree from INPG, France in 1997. He is now Professor of Mechatronics-Mechanics and Dean of Research, School of Engineering and Science at the Tecnologico de Monterrey. His main research interests include applications of advanced control to automotive systems. He is the (co-)author of 3 books, more than 80 papers in top journals and more than 200 international conference papers and more than 1,000 citations. He has worked as expert consulting for different industries and he is certified reviewer for regional development projects.

Yanfang Wang received the master degree from Jiangxi University of Science and Technology, China, in 2015. She is currently a lecturer and engineer with Jiangxi College of Applied Technology, China. Her research interests include deep learning and intelligent control.