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SEMIHYPERGROUPS CHARACTERIZED BY MEANS OF THEIR FERMATEAN FUZZY BI-HYPERIDEALS

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Abstract. *The concept of Fermatean fuzzy sets, was introduced by Senapati and Yager in 2019, and is direct extension of the concepts of fuzzy sets, intuitionistic fuzzy sets and Pythagorean fuzzy sets. In this article, the concept of Fermatean fuzzy sets was applied to studying the semihypergroups. The notions of Fermatean fuzzy subsemihypergroups, Fermatean fuzzy (resp., left, right) hyperideals and Fermatean fuzzy (resp., generalized) bi-hyperideals of semihypergroups are introduced and some their properties are investigated. Finally, some characterizations of regular semihypergroups by the concepts of Fermatean fuzzy (resp., left, right) hyperideals and Fermatean fuzzy (resp., generalized) bi-hyperideals of semihypergroups are investigated.*
Keywords: Fermatean fuzzy hyperideal, Fermatean fuzzy bi-hyperideal, Fermatean fuzzy generalized bi-hyperideal, Regular semihypergroup

1. **Introduction.** Zadeh [29] defined the concept of fuzzy subsets or fuzzy sets as a function from a nonempty set X to the unit interval $[0, 1]$. This concept is a mathematical extension of classical sets in mathematics. Rosenfeld [21] coined the term fuzzy sets to characterize the concept of fuzzy subgroups of groups, which inspired a variety of algebraic structures. Then, the notion of fuzzy sets was investigated in semigroups by Kuroki [10]. In addition, several researchers have debated the extensions of fuzzy sets with applications in algebraic structures (see, e.g., [1, 3, 27]). Atanassov [2] proposed the notion of intuitionistic fuzzy sets as a generalization of the concept of fuzzy sets. In fact, the degree of membership of an element in a given set is determined by the fuzzy sets, while the intuitionistic fuzzy sets provide both membership and non-membership degrees. In 2013, Yager [28] introduced the concept of Pythagorean fuzzy sets as the sum of the squares of membership and that non-membership relates to the unit interval $[0, 1]$. Subsequently, Senapati and Yager [23] initiated the concept of Fermatean fuzzy sets in 2019, which is the cube sum of its membership and non-membership degrees belonging to $[0, 1]$. This concept generalizes the concepts of fuzzy sets, intuitionistic fuzzy sets and Pythagorean fuzzy sets. Fermatean fuzzy set theory has been studied in other algebraic structures by many mathematicians (see, e.g., [8, 9, 18, 26]).

Marty [11] presented algebraic hyperstructures in 1934 as an extension of a classical algebraic structure. Many mathematicians researched this idea in the decades that followed, and it is still studied nowadays (see, e.g., [12, 16, 20]). Regularities of algebraic structures are interesting and essential properties to examine. In 2015, Pibaljommee and Davvaz [19] characterized the completely regular and the strongly regular ordered semihypergroups

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by fuzzy bi-hyperideals. After that the regular LA-semihypergroups were characterized by means of their $(\in_\Gamma, \in_\Gamma \vee q_\triangle)$-cubic hyperideals by Gulistan et al. [5]. In 2021, Shabir et al. [24] provided the characterizations of regular and intra-regular semihypergroups by the properties of their $(\in, \in \vee q)$-bipolar fuzzy hyperideals. In the same year, Sanpan et al. [22] characterized some regularities in ordered Γ-semihypergroups by using interval-valued Q-fuzzy right Γ-hyperideals and interval-valued Q-fuzzy left Γ-hyperideals. It was known the semihypergroup as a generalization of a semigroup. Nakkhasen [13] characterized in regular and intra-regular semigroups by the properties of picture fuzzy left (resp., right) ideals, picture fuzzy quasi-ideals and picture fuzzy (resp., generalized) bi-ideals in 2021. Recently, the classes of weakly regular, regular and intra-regular LA-semihyperrings were characterized in terms of their hyperideals by Nakkhasen, see [14, 15].

In this paper, the focus is on studying the concept of Fermatean fuzzy sets in the semihypergroups, which are the generalizations of semigroups and ordered semigroups. Moreover, some basic concepts of semihypergroups are presented in Section 2. Then, in Section 3, we define the notions of Fermatean fuzzy subsemihypergroups, Fermatean fuzzy (resp., left, right) hyperideals and Fermatean fuzzy (resp., generalized) bi-hyperideals of semihypergroups and consider some of its properties. Section 4 shows that Fermatean fuzzy bi-hyperideals and Fermatean fuzzy generalized bi-hyperideals coincide in regular semihypergroups. Moreover, characterizations of regular semihypergroups by using the concepts of Fermatean fuzzy (resp., left, right) hyperideals and Fermatean fuzzy (resp., generalized) bi-hyperideals of semihypergroups are obtained. Finally, Section 5 concludes the paper.

2. **Preliminaries.** In this section, we recall some definitions and properties which will be used throughout this paper.

A *hypergroupoid* $(H, \cdot)$ is a nonempty set H together with a mapping $\cdot : H \times H \to \mathcal{P}^*(H)$, where $\mathcal{P}^*(H)$ denotes the set of all nonempty sets of H (see [11]). The image of the pair (x, y) is denoted by $x \cdot y$. If $A, B \in \mathcal{P}^*(H)$ and $x \in H$, then we denote $A \cdot B = \bigcup_{a \in A, b \in B} a \cdot b$, $A \cdot x = A \cdot \{x\}$ and $x \cdot B = \{x\} \cdot B$. A hypergroupoid $(S, \cdot)$ is called a *semihypergroup* if for every $x, y, z \in S$, $(x \cdot y) \cdot z = x \cdot (y \cdot z)$, that is, $\bigcup_{a \in x \cdot y} a \cdot z = \bigcup_{b \in y \cdot z} x \cdot b$ (see [4]). Throughout this paper, we write a semihypergroup S instead of a semihypergroup $(S, \cdot)$, AB instead of $A \cdot B$ for every nonempty sets A and B of a semihypergroup S and xy instead of $x \cdot y$ for each element x and y of a semihypergroup S.

Next, we recall the concepts of many types of hyperideals in semihypergroups (see, [6, 25]). Let S be a semihypergroup and A be any nonempty subset of S. Then,

(i) A is called a *subsemihypergroup* of S if $AA \subseteq A$;
(ii) A is called a *left hyperideal* of S if $SA \subseteq A$;
(iii) A is called a *right hyperideal* of S if $AS \subseteq A$;
(iv) A is called a *hyperideal* of S if it is both a left and a right hyperideal of S;
(v) A is called a *bi-hyperideal* of S if $AA \subseteq A$ and $ASA \subseteq A$;
(vi) A is called a *generalized bi-hyperideal* of S if $ASA \subseteq A$.

Let A be a nonempty subset of a semihypergroup S. We denote by $\langle A \rangle_B$ the bi-hyperideal of S generated by A. If $A = \{a\}$, then we write $\langle a \rangle_B$ instead of $\langle \{a\} \rangle_B$.

Lemma 2.1 ([17]). *Let A be a nonempty subset of a semihypergroup S and $a \in S$. Then,*
(i) $\langle A \rangle_B = A \cup AA \cup ASA$;
(ii) $\langle a \rangle_B = a \cup aa \cup aSa$.

Now, we review the concepts of fuzzy sets [29], intuitionistic fuzzy sets [2], Pythagorean fuzzy sets [28] and Fermatean fuzzy sets [23] as follows.

Definition 2.1 ([29]). *Let X be a nonempty set. A fuzzy set (briefly, FS) μ of X is a mapping $\mu : X \to [0,1]$. One more meaning, a fuzzy set $\mathcal{A}$ in the set X is defined by the form*

$$\mathcal{A} := \{\langle x, \mu_{\mathcal{A}}(x)\rangle \mid x \in X\},$$

where $\mu_{\mathcal{A}} : X \to [0,1]$ denotes the degree of membership of each $x \in X$ in the set $\mathcal{A}$.

For any two fuzzy sets μ and λ of a nonempty set X, $\mu \cap \lambda$ and $\mu \cup \lambda$ are fuzzy sets of X defined by $(\mu \cap \lambda)(x) = \min\{\mu(x), \lambda(x)\}$ and $(\mu \cup \lambda)(x) = \max\{\mu(x), \lambda(x)\}$ for all $x \in X$, respectively.

Definition 2.2 ([2]). *An intuitionistic fuzzy set (briefly, IFS) $\mathcal{A}$ in a nonempty set X is defined as the form*

$$\mathcal{A} := \{\langle x, \mu_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(x)\rangle \mid x \in X\},$$

where $\mu_{\mathcal{A}} : X \to [0,1]$ and $\lambda_{\mathcal{A}} : X \to [0,1]$ represent the degree of membership and non-membership of each $x \in X$ to the set $\mathcal{A}$, respectively, and also $0 \leq \mu_{\mathcal{A}}(x) + \lambda_{\mathcal{A}}(x) \leq 1$, for all $x \in X$.

Definition 2.3 ([28]). *A Pythagorean fuzzy set (briefly, PFS) $\mathcal{A}$ on a nonempty set X is defined as the form*

$$\mathcal{A} := \{\langle x, \mu_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(x)\rangle \mid x \in X\},$$

where the functions $\mu_{\mathcal{A}} : X \to [0,1]$ and $\lambda_{\mathcal{A}} : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ to $\mathcal{A}$, respectively, and for every $x \in X$, $0 \leq (\mu_{\mathcal{A}}(x))^2 + (\lambda_{\mathcal{A}}(x))^2 \leq 1$.

Definition 2.4 ([23]). *Let X be a universal set. A Fermatean fuzzy set (briefly, FFS) $\mathcal{A}$ on X is to be a structure:*

$$\mathcal{A} := \{\langle x, \mu_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(x)\rangle \mid x \in X\},$$

where $\mu_{\mathcal{A}} : X \to [0,1]$ and $\lambda_{\mathcal{A}} : X \to [0,1]$ represent the degree of membership and the degree of non-membership of the element $x \in X$ to the set $\mathcal{A}$, respectively, with condition that $0 \leq (\mu_{\mathcal{A}}(x))^3 + (\lambda_{\mathcal{A}}(x))^3 \leq 1$, for all $x \in X$.

This concept generalizes the concepts of FSs, IFSs and PFSs. For the sake of simplicity, we use the symbol $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ for the FFS $\mathcal{A} := \{\langle x, \mu_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(x)\rangle \mid x \in X\}$.

Let $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ and $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$ be any two FFSs on a universe set X. Then, we denote

(i) $\mathcal{A} \subseteq \mathcal{B}$ iff $\mu_{\mathcal{A}}(x) \leq \mu_{\mathcal{B}}(x)$ and $\lambda_{\mathcal{A}}(x) \geq \lambda_{\mathcal{B}}(x)$, for all $x \in X$,
(ii) $\mathcal{A} \cap \mathcal{B} = \{\langle x, (\mu_{\mathcal{A}} \cap \mu_{\mathcal{B}})(x), (\lambda_{\mathcal{A}} \cup \lambda_{\mathcal{B}})(x)\rangle \mid x \in X\}$,
(iii) $\mathcal{A} \cup \mathcal{B} = \{\langle x, (\mu_{\mathcal{A}} \cup \mu_{\mathcal{B}})(x), (\lambda_{\mathcal{A}} \cap \lambda_{\mathcal{B}})(x)\rangle \mid x \in X\}$.

Let S be a semihypergroup. We denote the collection of FFSs on S by $\mathcal{FFS}(S)$ with $\mathcal{S} = \{\langle x, 1, 0\rangle \mid x \in S\}$ and $\emptyset = \{\langle x, 0, 1\rangle \mid x \in S\}$. Let A be any subset of S. The *Fermatean characteristic function* of A is defined by $\mathcal{C}_A = \{\langle x, \mu_{\mathcal{C}_A}(x), \lambda_{\mathcal{C}_A}(x)\rangle \mid x \in S\}$, where

$$\mu_{\mathcal{C}_A}(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise,} \end{cases} \quad \text{and} \quad \lambda_{\mathcal{C}_A}(x) = \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{otherwise.} \end{cases}$$

We observe that if $A = S$ (resp., $A = \emptyset$), then $\mathcal{C}_A = \mathcal{S}$ (resp., $\mathcal{C}_A = \emptyset$).

Let $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ and $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$ be FFSs on a semihypergroup S. The *Fermatean fuzzy product* of $\mathcal{A}$ and $\mathcal{B}$ is defined by

$$\mathcal{A} \circ \mathcal{B} := \{\langle x, \mu_{\mathcal{A}\circ\mathcal{B}}(x), \lambda_{\mathcal{A}\circ\mathcal{B}}(x)\rangle \mid x \in S\},$$

where

$$\mu_{\mathcal{A}\circ\mathcal{B}}(x) = \begin{cases} \sup\limits_{x\in yz} [\min\{\mu_{\mathcal{A}}(y), \mu_{\mathcal{B}}(z)\}] & \text{if } \exists\, y,z \in S \text{ such that } x \in yz, \\ 0 & \text{otherwise,} \end{cases}$$

$$\lambda_{\mathcal{A}\circ\mathcal{B}}(x) = \begin{cases} \inf\limits_{x\in yz} [\max\{\lambda_{\mathcal{A}}(y), \lambda_{\mathcal{B}}(z)\}] & \text{if } \exists\, y,z \in S \text{ such that } x \in yz, \\ 1 & \text{otherwise.} \end{cases}$$

The following lemmas can be proved straightforward.

Lemma 2.2. *Let $\mathcal{C}_A = (\mu_{\mathcal{C}_A}, \lambda_{\mathcal{C}_A})$ and $\mathcal{C}_B = (\mu_{\mathcal{C}_B}, \lambda_{\mathcal{C}_B})$ be FFSs on a semihypergroup S, where A and B are any nonempty subsets of S. Then the following conditions hold:*
(i) $\mathcal{C}_{A\cap B} = \mathcal{C}_A \cap \mathcal{C}_B$;
(ii) $\mathcal{C}_{AB} = \mathcal{C}_A \circ \mathcal{C}_B$.

Lemma 2.3. *Let A and B be any nonempty subsets of a semihypergroup S and μ be a fuzzy set of S. If $A \subseteq B$, then $\inf_{x\in A}\mu(x) \geq \inf_{x\in B}\mu(x)$ and $\sup_{x\in A}\mu(x) \leq \sup_{x\in B}\mu(x)$.*

Lemma 2.4. *Let $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$, $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$, $\mathcal{C} = (\mu_{\mathcal{C}}, \lambda_{\mathcal{C}})$ and $\mathcal{D} = (\mu_{\mathcal{D}}, \lambda_{\mathcal{D}})$ be FFSs on a semihypergroup S. If $\mathcal{A} \subseteq \mathcal{B}$ and $\mathcal{C} \subseteq \mathcal{D}$, then $\mathcal{A}\circ\mathcal{C} \subseteq \mathcal{B}\circ\mathcal{D}$.*

3. **Fermatean Fuzzy Hyperideals in Semihypergroups.** In this section, we define the notions of Fermatean fuzzy subsemihypergroups, Fermatean fuzzy (resp., left, right) hyperideals and Fermatean fuzzy (resp., generalized) bi-hyperideals of semihypergroups. Then, these concepts are characterized by the Fermatean fuzzy products. Also, we survey the connections between different types of hyperideals and their Fermatean characteristic functions in semihypergroups.

Definition 3.1. *Let S be a semihypergroup. A FFS $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ on S is called:*
(i) a Fermatean fuzzy subsemihypergroup (briefly, FFSub) of S if for every $x,y \in S$, $\inf_{z\in xy}\mu_{\mathcal{A}}(z) \geq \min\{\mu_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(y)\}$ *and* $\sup_{z\in xy}\lambda_{\mathcal{A}}(z) \leq \max\{\lambda_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(y)\}$;
(ii) a Fermatean fuzzy left hyperideal (briefly, FFL) of S if for every $x,y \in S$, $\inf_{z\in xy}\mu_{\mathcal{A}}(z) \geq \mu_{\mathcal{A}}(y)$ *and* $\sup_{z\in xy}\lambda_{\mathcal{A}}(z) \leq \lambda_{\mathcal{A}}(y)$;
(iii) a Fermatean fuzzy right hyperideal (briefly, FFR) of S if for every $x,y \in S$, $\inf_{z\in xy}\mu_{\mathcal{A}}(z) \geq \mu_{\mathcal{A}}(x)$ *and* $\sup_{z\in xy}\lambda_{\mathcal{A}}(z) \leq \lambda_{\mathcal{A}}(x)$;
(iv) a Fermatean fuzzy hyperideal (briefly, FFH) of S if it is both a FFL and a FFR of S.

Definition 3.2. *A FFSub $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ of a semihypergroup S is called a Fermatean fuzzy bi-hyperideal (briefly, FFB) of S if for every $w,x,y \in S$:*
(i) $\inf_{z\in xwy}\mu_{\mathcal{A}}(z) \geq \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\}$;
(ii) $\sup_{z\in xwy}\lambda_{\mathcal{A}}(z) \leq \max\{\lambda_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(y)\}$.

Definition 3.3. *A FFS $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ of a semihypergroup S is called a Fermatean fuzzy generalized bi-hyperideal (briefly, FFGB) of S if for every $w,x,y \in S$:*
(i) $\inf_{z\in xwy}\mu_{\mathcal{A}}(z) \geq \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\}$;
(ii) $\sup_{z\in xwy}\lambda_{\mathcal{A}}(z) \leq \max\{\lambda_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(y)\}$.

Let S be a semihypergroup. We note that every FFL (resp., FFR) of S is a FFB of S, while every FFB of S is a FFGB of S. In contrast, a FFB of S may not be a FFL and a FFR of S, and a FFGB need not be a FFB of S. This can be shown by the following examples.

Example 3.1. *Let $S = \{a, b, c, d\}$. Define the hyperoperation $\cdot$ on S as follows:*

$\cdot$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a\}$	$\{a, b\}$
d	$\{a\}$	$\{a\}$	$\{a, b\}$	$\{a, c\}$

Then, $(S, \cdot)$ is a semihypergroup. Next, we define the FFS $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ on S as follows:

$$\mu_{\mathcal{A}}(a) = 0.9,\ \mu_{\mathcal{A}}(b) = 0.5,\ \mu_{\mathcal{A}}(c) = 0.8,\ \mu_{\mathcal{A}}(d) = 0.4,$$
$$\lambda_{\mathcal{A}}(a) = 0.5,\ \lambda_{\mathcal{A}}(b) = 0.8,\ \lambda_{\mathcal{A}}(c) = 0.7,\ \lambda_{\mathcal{A}}(d) = 0.9.$$

It turns out that $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is a FFB of S, but it is not a FFL of S, since

$$\inf_{z \in dc} \mu_{\mathcal{A}}(z) < \mu_{\mathcal{A}}(c) \text{ and } \sup_{z \in dc} \lambda_{\mathcal{A}}(z) > \lambda_{\mathcal{A}}(c).$$

Moreover, $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is not a FFR of S, because

$$\inf_{z \in cd} \mu_{\mathcal{A}}(z) < \mu_{\mathcal{A}}(c) \text{ and } \sup_{z \in cd} \lambda_{\mathcal{A}}(z) > \lambda_{\mathcal{A}}(c).$$

Example 3.2. *Let $S = \{a, b, c, d\}$. Define the hyperoperation $\cdot$ on S by the following table:*

$\cdot$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a, b\}$	$\{a\}$
d	$\{a\}$	$\{a\}$	$\{a, b\}$	$\{a, b\}$

Then, $(S, \cdot)$ is a semihypergroup. We define a FFS $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ on S by

$$\mu_{\mathcal{A}}(a) = 0.8,\ \mu_{\mathcal{A}}(b) = 0.3,\ \mu_{\mathcal{A}}(c) = 0.7,\ \mu_{\mathcal{A}}(d) = 0.5,$$
$$\lambda_{\mathcal{A}}(a) = 0.4,\ \lambda_{\mathcal{A}}(b) = 0.9,\ \lambda_{\mathcal{A}}(c) = 0.8,\ \lambda_{\mathcal{A}}(d) = 0.6.$$

It is not difficult to calculate that $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is a FFGB of S, but it is not a FFSub of S, because

$$\inf_{z \in cc} \mu_{\mathcal{A}}(z) < \min\{\mu_{\mathcal{A}}(c), \mu_{\mathcal{A}}(c)\} \text{ and } \sup_{z \in cc} \lambda_{\mathcal{A}}(z) > \max\{\lambda_{\mathcal{A}}(c), \lambda_{\mathcal{A}}(c)\}.$$

This implies that $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ is not a FFB of S.

Theorem 3.1. *Let $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ be a FFS on a semihypergroup S. Then,*

(i) $\mathcal{A}$ is a FFSub if and only if $\mathcal{A} \circ \mathcal{A} \subseteq \mathcal{A}$;
(ii) $\mathcal{A}$ is a FFL if and only if $\mathcal{S} \circ \mathcal{A} \subseteq \mathcal{A}$;
(iii) $\mathcal{A}$ is a FFR if and only if $\mathcal{A} \circ \mathcal{S} \subseteq \mathcal{A}$;
(iv) $\mathcal{A}$ is a FFGB if and only if $\mathcal{A} \circ \mathcal{S} \circ \mathcal{A} \subseteq \mathcal{A}$;
(v) $\mathcal{A}$ is a FFB if and only if $\mathcal{A} \circ \mathcal{A} \subseteq \mathcal{A}$ and $\mathcal{A} \circ \mathcal{S} \circ \mathcal{A} \subseteq \mathcal{A}$.

Proof: (i) Assume that $\mathcal{A}$ is a FFSub of S. Let $a \in S$. Clearly, $\mathcal{A} \circ \mathcal{A} \subseteq \mathcal{A}$ for all $x, y \in S$ such that $a \notin xy$. Suppose that there exist $b, c \in S$ such that $a \in bc$. Thus,

$$\begin{aligned} \mu_{\mathcal{A} \circ \mathcal{A}}(a) &= \sup_{a \in bc} [\min\{\mu_{\mathcal{A}}(b), \mu_{\mathcal{A}}(c)\}] \\ &\leq \sup_{a \in bc} \left[\inf_{a \in bc} \mu_{\mathcal{A}}(a) \right] \\ &\leq \mu_{\mathcal{A}}(a), \end{aligned}$$

$$\begin{aligned}\lambda_{\mathcal{A}\circ\mathcal{A}}(a) &= \inf_{a\in bc}[\max\{\lambda_{\mathcal{A}}(b), \lambda_{\mathcal{A}}(c)\}]\\ &\geq \inf_{a\in bc}\left[\sup_{a\in bc}\lambda_{\mathcal{A}}(a)\right]\\ &\geq \lambda_{\mathcal{A}}(a).\end{aligned}$$

Hence, $\mathcal{A}\circ\mathcal{A}\subseteq\mathcal{A}$. Conversely, let $x, y \in S$. Then, we have

$$\begin{aligned}\inf_{z\in xy}\mu_{\mathcal{A}}(z) &\geq \inf_{z\in xy}\mu_{\mathcal{A}\circ\mathcal{A}}(z)\\ &= \inf_{z\in xy}\left[\sup_{z\in xy}[\min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\}]\right]\\ &\geq \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\},\end{aligned}$$

$$\begin{aligned}\sup_{z\in xy}\lambda_{\mathcal{A}}(z) &\leq \sup_{z\in xy}\lambda_{\mathcal{A}\circ\mathcal{A}}(z)\\ &= \sup_{z\in xy}\left[\inf_{z\in xy}[\max\{\lambda_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(y)\}]\right]\\ &\leq \max\{\lambda_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(y)\}.\end{aligned}$$

Therefore, $\mathcal{A}$ is a FFSub of S.

(ii) Assume that $\mathcal{A}$ is a FFL of S. Let $a \in S$. If there do not exist any $x, y \in S$ such that $a \in xy$, then $\mathcal{S}\circ\mathcal{A}\subseteq\mathcal{A}$. Suppose that $a \in bc$ for some $b, c \in S$. Thus, we have

$$\begin{aligned}\mu_{\mathcal{S}\circ\mathcal{A}}(a) &= \sup_{a\in bc}[\min\{\mu_{\mathcal{S}}(b), \mu_{\mathcal{A}}(c)\}]\\ &= \sup_{a\in bc}[\mu_{\mathcal{A}}(c)]\\ &\leq \sup_{a\in bc}\left[\inf_{a\in bc}\mu_{\mathcal{A}}(a)\right]\\ &\leq \mu_{\mathcal{A}}(a),\end{aligned}$$

$$\begin{aligned}\lambda_{\mathcal{S}\circ\mathcal{A}}(a) &= \inf_{a\in bc}[\max\{\lambda_{\mathcal{S}}(b), \lambda_{\mathcal{A}}(c)\}]\\ &= \inf_{a\in bc}[\lambda_{\mathcal{A}}(c)]\\ &\geq \inf_{a\in bc}\left[\sup_{a\in bc}\lambda_{\mathcal{A}}(a)\right]\\ &\geq \lambda_{\mathcal{A}}(a).\end{aligned}$$

Hence, $\mathcal{S}\circ\mathcal{A}\subseteq\mathcal{A}$. Conversely, let $x, y \in S$. Then, we have

$$\begin{aligned}\inf_{z\in xy}\mu_{\mathcal{A}}(z) &\geq \inf_{z\in xy}\mu_{\mathcal{S}\circ\mathcal{A}}(z)\\ &= \inf_{z\in xy}\left[\sup_{z\in xy}[\min\{\mu_{\mathcal{S}}(x), \mu_{\mathcal{A}}(y)\}]\right]\\ &\geq \min\{\mu_{\mathcal{S}}(x), \mu_{\mathcal{A}}(y)\}\\ &= \mu_{\mathcal{A}}(y),\end{aligned}$$

$$\begin{aligned}\sup_{z\in xy}\lambda_{\mathcal{A}}(z) &\leq \sup_{z\in xy}\lambda_{\mathcal{S}\circ\mathcal{A}}(z)\\ &= \sup_{z\in xy}\left[\inf_{z\in xy}[\max\{\lambda_{\mathcal{S}}(x), \lambda_{\mathcal{A}}(y)\}]\right]\end{aligned}$$

$$\leq \max\{\lambda_{\mathcal{S}}(x), \lambda_{\mathcal{A}}(y)\}$$
$$= \lambda_{\mathcal{A}}(y).$$

This shows that, $\mathcal{A}$ is a FFL of S.

(iii) The proof is similar to the proof of (ii).

(iv) Assume that $\mathcal{A}$ is a FFGB of S. Let $a \in S$. Obviously, if $a \notin xyz$ for all $x, y, z \in S$, then $\mathcal{A} \circ \mathcal{S} \circ \mathcal{A} \subseteq \mathcal{A}$. Suppose that there exist $b, c, d \in S$ such that $a \in bcd$. Thus, we have

$$\begin{aligned}
\mu_{\mathcal{A}\circ\mathcal{S}\circ\mathcal{A}}(a) &= \sup_{a\in bcd}[\min\{\mu_{\mathcal{A}}(b), \mu_{\mathcal{S}}(c), \mu_{\mathcal{A}}(d)\}] \\
&= \sup_{a\in bcd}[\min\{\mu_{\mathcal{A}}(b), \mu_{\mathcal{A}}(d)\}] \\
&\leq \sup_{a\in bcd}\left[\inf_{a\in bcd}\mu_{\mathcal{A}}(a)\right] \\
&\leq \mu_{\mathcal{A}}(a), \\
\lambda_{\mathcal{A}\circ\mathcal{S}\circ\mathcal{A}}(a) &= \inf_{a\in bcd}[\max\{\lambda_{\mathcal{A}}(b), \lambda_{\mathcal{S}}(c), \lambda_{\mathcal{A}}(d)\}] \\
&= \inf_{a\in bcd}[\max\{\lambda_{\mathcal{A}}(b), \lambda_{\mathcal{A}}(d)\}] \\
&\geq \inf_{a\in bcd}\left[\sup_{a\in bcd}\lambda_{\mathcal{A}}(a)\right] \\
&\geq \lambda_{\mathcal{A}}(a).
\end{aligned}$$

This means that $\mathcal{A} \circ \mathcal{S} \circ \mathcal{A} \subseteq \mathcal{A}$. Conversely, let $w, x, y \in S$. Then, we have

$$\begin{aligned}
\inf_{z\in xwy}\mu_{\mathcal{A}}(z) &\geq \inf_{z\in xwy}\mu_{\mathcal{A}\circ\mathcal{S}\circ\mathcal{A}}(z) \\
&= \inf_{z\in xwy}\left[\sup_{z\in xwy}[\min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{S}}(w), \mu_{\mathcal{A}}(y)\}]\right] \\
&= \inf_{z\in xwy}\left[\sup_{z\in xwy}[\min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\}]\right] \\
&\geq \min\{\mu_{\mathcal{A}}(x), \mu_{\mathcal{A}}(y)\}, \\
\sup_{z\in xwy}\lambda_{\mathcal{A}}(z) &\leq \sup_{z\in xwy}\lambda_{\mathcal{A}\circ\mathcal{S}\circ\mathcal{A}}(z) \\
&= \sup_{z\in xwy}\left[\inf_{z\in xwy}[\max\{\lambda_{\mathcal{A}}(x), \lambda_{\mathcal{S}}(w), \lambda_{\mathcal{A}}(y)\}]\right] \\
&= \sup_{z\in xwy}\left[\inf_{z\in xwy}[\max\{\lambda_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(y)\}]\right] \\
&\leq \max\{\lambda_{\mathcal{A}}(x), \lambda_{\mathcal{A}}(y)\}.
\end{aligned}$$

Therefore, $\mathcal{A}$ is a FFGB of S.

(v) It follows by (i) and (iv). □

Theorem 3.2. *Let A be a nonempty subset of a semihypergroup S. Then, the following statements hold:*

(i) A is a subsemihypergroup of S if and only if $\mathcal{C}_A = (\mu_{\mathcal{C}_A}, \lambda_{\mathcal{C}_A})$ is a FFSub of S;
(ii) A is a left hyperideal of S if and only if $\mathcal{C}_A = (\mu_{\mathcal{C}_A}, \lambda_{\mathcal{C}_A})$ is a FFL of S;
(iii) A is a right hyperideal of S if and only if $\mathcal{C}_A = (\mu_{\mathcal{C}_A}, \lambda_{\mathcal{C}_A})$ is a FFR of S;
(iv) A is a hyperideal of S if and only if $\mathcal{C}_A = (\mu_{\mathcal{C}_A}, \lambda_{\mathcal{C}_A})$ is a FFH of S;
(v) A is a generalized bi-hyperideal of S if and only if $\mathcal{C}_A = (\mu_{\mathcal{C}_A}, \lambda_{\mathcal{C}_A})$ is a FFGB of S;

(vi) A is a bi-hyperideal of S if and only if $\mathcal{C}_A = (\mu_{\mathcal{C}_A}, \lambda_{\mathcal{C}_A})$ is a FFB of S.

Proof: (i) Assume that A is a subsemihypergroup of S. Suppose that $\inf\limits_{z\in ab}\mu_{\mathcal{C}_A}(z) < \min\{\mu_{\mathcal{C}_A}(a), \mu_{\mathcal{C}_A}(b)\}$ for some $a, b \in S$. Then, $\inf_{z\in ab}\mu_{\mathcal{C}_A}(z) = 0$ and $\min\{\mu_{\mathcal{C}_A}(a), \mu_{\mathcal{C}_A}(b)\} = 1$. Thus, $\mu_{\mathcal{C}_A}(a) = 1$ and $\mu_{\mathcal{C}_A}(b) = 1$. It follows that $a, b \in A$. Also, $ab \subseteq A$. This means that for every $z \in ab$, we have that $\mu_{\mathcal{C}_A}(z) = 1$. This implies that $\inf_{z\in ab}\mu_{\mathcal{C}_A}(z) = 1$. This is a contradiction. Hence,

$$\inf_{z\in xy} \mu_{\mathcal{C}_A}(z) \geq \min\{\mu_{\mathcal{C}_A}(x), \mu_{\mathcal{C}_A}(y)\},$$

for all $x, y \in S$. If $\sup_{z\in ab} \lambda_{\mathcal{C}_A}(z) > \max\{\lambda_{\mathcal{C}_A}(a), \lambda_{\mathcal{C}_A}(b)\}$ for some $a, b \in S$. It turns out that $\sup_{z\in ab} \lambda_{\mathcal{C}_A}(z) = 1$ and $\max\{\lambda_{\mathcal{C}_A}(a), \lambda_{\mathcal{C}_A}(b)\} = 0$. Also, $\lambda_{\mathcal{C}_A}(a) = 0$ and $\lambda_{\mathcal{C}_A}(b) = 0$. Then, $a, b \in A$, and so $ab \subseteq A$. This implies that for every $z \in ab$, we get that $\lambda_{\mathcal{C}_A}(z) = 0$. Thus, $\sup_{z\in ab} \lambda_{\mathcal{C}_A}(z) = 0$, which is a contradiction. Hence,

$$\sup_{z\in xy} \lambda_{\mathcal{C}_A}(z) \leq \max\{\lambda_{\mathcal{C}_A}(x), \lambda_{\mathcal{C}_A}(y)\},$$

for all $x, y \in S$. This shows that $\mathcal{C}_A = (\mu_{\mathcal{C}_A}, \lambda_{\mathcal{C}_A})$ is a FFSub of S.

Conversely, assume that $\mathcal{C}_A = (\mu_{\mathcal{C}_A}, \lambda_{\mathcal{C}_A})$ is a FFSub of S. Let $x, y \in A$. Then, $\inf_{z\in xy} \mu_{\mathcal{C}_A}(z) \geq \min\{\mu_{\mathcal{C}_A}(x), \mu_{\mathcal{C}_A}(y)\} = 1$. That is, $\mu_{\mathcal{C}_A}(z) = 1$ for all $z \in xy$. This means that for each $z \in xy$, we have that $z \in A$. Hence, $xy \subseteq A$. Therefore, A is a subsemihypergroup of S.

The other conditions can be proved in a similar way. □

4. **Characterizing Regular Semihypergroups.** In this section, we present that Fermatean fuzzy generalized bi-hyperideals and Fermatean fuzzy bi-hyperideals in regular semihypergroups coincide. Then, we provide some characterizations of regular semihypergroups in terms of Fermatean fuzzy (resp., left, right) hyperideals and Fermatean fuzzy (resp., generalized) bi-hyperideals of semihypergroups.

A semihypergroup S is called *regular* (see [7]) if for each element $a \in S$, there exists element $x \in S$ such that $a \in axa$. This is equivalent to saying that $a \in aSa$, for every $a \in S$ or $A \subseteq ASA$, for every $A \subseteq S$.

Theorem 4.1. *Let S be a regular semihypergroup. Every FFGB of S is a FFB of S.*

Proof: Let $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ be a FFGB of S, and let $a, b \in S$. Then, there exists $x \in S$ such that $b \in bxb$. It follows that $ab \subseteq abxb$. By Lemma 2.3, we have

$$\inf_{z\in ab} \mu_{\mathcal{A}}(z) \geq \inf_{z\in abxb} \mu_{\mathcal{A}}(z) \geq \min\{\mu_{\mathcal{A}}(a), \mu_{\mathcal{A}}(b)\},$$
$$\sup_{z\in ab} \lambda_{\mathcal{A}}(z) \leq \sup_{z\in abxb} \lambda_{\mathcal{A}}(z) \leq \max\{\lambda_{\mathcal{A}}(a), \lambda_{\mathcal{A}}(b)\}.$$

Hence, $\mathcal{A}$ is a FFSub of S. Therefore, $\mathcal{A}$ is a FFB of S. □

Lemma 4.1 ([17]). *Let S be a semihypergroup. Then, S is regular if and only if $R \cap L = RL$, for every left hyperideal L and every right hyperideal R of S.*

Theorem 4.2. *Let S be a semihypergroup. Then, S is regular if and only if $\mathcal{R} \cap \mathcal{L} = \mathcal{R} \circ \mathcal{L}$, for every FFL $\mathcal{L} = (\mu_{\mathcal{L}}, \lambda_{\mathcal{L}})$ and every FFR $\mathcal{R} = (\mu_{\mathcal{R}}, \lambda_{\mathcal{R}})$ of S.*

Proof: Assume that S is regular. Let $\mathcal{L} = (\mu_{\mathcal{L}}, \lambda_{\mathcal{L}})$ and $\mathcal{R} = (\mu_{\mathcal{R}}, \lambda_{\mathcal{R}})$ be a FFL and a FFR of S, respectively. Then, $\mathcal{R} \circ \mathcal{L} \subseteq \mathcal{S} \circ \mathcal{L} \subseteq \mathcal{L}$ and $\mathcal{R} \circ \mathcal{L} \subseteq \mathcal{R} \circ \mathcal{S} \subseteq \mathcal{R}$. Thus, $\mathcal{R} \circ \mathcal{L} \subseteq \mathcal{R} \cap \mathcal{L}$. On the other hand, let $a \in S$. So, there exists $x \in S$ such that $a \in axa$. It follows that

$$\mu_{\mathcal{R}\circ\mathcal{L}}(a) = \sup_{a\in pq}[\min\{\mu_{\mathcal{R}}(p), \mu_{\mathcal{L}}(q)\}]$$

$$\begin{aligned}
&\geq \min\left\{\inf_{p\in ax}\mu_{\mathcal{R}}(p), \mu_{\mathcal{L}}(a)\right\}\\
&\geq \min\{\mu_{\mathcal{R}}(a), \mu_{\mathcal{L}}(a)\}\\
&= \mu_{\mathcal{R}\cap\mathcal{L}}(a),
\end{aligned}$$

$$\begin{aligned}
\lambda_{\mathcal{R}\circ\mathcal{L}}(a) &= \inf_{a\in pq}[\max\{\lambda_{\mathcal{R}}(p), \lambda_{\mathcal{L}}(q)\}]\\
&\leq \max\left\{\sup_{p\in ax}\lambda_{\mathcal{R}}(p), \lambda_{\mathcal{L}}(a)\right\}\\
&\leq \max\{\lambda_{\mathcal{R}}(a), \lambda_{\mathcal{L}}(a)\}\\
&= \lambda_{\mathcal{R}\cup\mathcal{L}}(a).
\end{aligned}$$

This shows that $\mathcal{R}\cap\mathcal{L} \subseteq \mathcal{R}\circ\mathcal{L}$. Hence, $\mathcal{R}\cap\mathcal{L} = \mathcal{R}\circ\mathcal{L}$.

Conversely, let L and R be any left hyperideal and right hyperideal of S, respectively. Clearly, $RL \subseteq R\cap L$. Next, let $x \in R\cap L$. By Theorem 3.2, $\mathcal{C}_L = (\mu_{\mathcal{C}_L}, \lambda_{\mathcal{C}_L})$ and $\mathcal{C}_R = (\mu_{\mathcal{C}_R}, \lambda_{\mathcal{C}_R})$ are a FFL and a FFR of S, respectively. Then, by the given assumption and Lemma 2.2, we have that $\mathcal{C}_{RL} = \mathcal{C}_R \circ \mathcal{C}_L = \mathcal{C}_R \cap \mathcal{C}_L$. Thus,

$$\begin{aligned}
\mu_{\mathcal{C}_{RL}}(x) &= \mu_{\mathcal{C}_R\cap\mathcal{C}_L}(x)\\
&= \min\{\mu_{\mathcal{C}_R}(x), \mu_{\mathcal{C}_L}(x)\}\\
&= 1.
\end{aligned}$$

We obtain that $x \in RL$. Hence, $R\cap L \subseteq RL$. It turns out that $R\cap L = RL$. By Lemma 4.1, we conclude that S is regular. □

Lemma 4.2. *Let S be a semihypergroup. Then, S is regular if and only if $B = BSB$, for every bi-hyperideal B of S.*

Proof: Assume that S is regular. Let B be any bi-hyperideal of S. Then, $BSB \subseteq B$. Next, let $a \in B$. Thus, there exists $x \in S$ such that $a \in axa$. So, $a \in axa \subseteq BSB$. Hence, $B \subseteq BSB$. It follows that $B = BSB$. Conversely, let $a \in S$. By assumption and Lemma 2.1, we have

$$a \in \langle a\rangle_B = \langle a\rangle_B S\langle a\rangle_B = (a\cup aa\cup aSa)S(a\cup aa\cup aSa) \subseteq aSa.$$

Therefore, S is regular. □

Theorem 4.3. *Let S be a semihypergroup. Then, S is regular if and only if $\mathcal{B} = \mathcal{B}\circ\mathcal{S}\circ\mathcal{B}$, for every FFB $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$ of S.*

Proof: Assume that S is regular. Let $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$ be a FFB of S. By Theorem 3.1, $\mathcal{B}\circ\mathcal{S}\circ\mathcal{B} \subseteq \mathcal{B}$. Otherwise, let $a \in S$. Then, there exists $x \in S$ such that $a \in axa$. Thus, we have

$$\begin{aligned}
\mu_{\mathcal{B}\circ\mathcal{S}\circ\mathcal{B}}(a) &= \sup_{a\in bcd}[\min\{\mu_{\mathcal{B}}(b), \mu_{\mathcal{S}}(c), \mu_{\mathcal{B}}(d)\}]\\
&\geq \min\{\mu_{\mathcal{B}}(a), \mu_{\mathcal{S}}(x), \mu_{\mathcal{B}}(a)\}\\
&= \min\{\mu_{\mathcal{B}}(a), \mu_{\mathcal{B}}(a)\}\\
&= \mu_{\mathcal{B}}(a),
\end{aligned}$$

$$\begin{aligned}
\lambda_{\mathcal{B}\circ\mathcal{S}\circ\mathcal{B}}(a) &= \inf_{a\in bcd}[\max\{\lambda_{\mathcal{B}}(b), \lambda_{\mathcal{S}}(c), \lambda_{\mathcal{B}}(d)\}]\\
&\leq \max\{\lambda_{\mathcal{B}}(a), \lambda_{\mathcal{S}}(x), \lambda_{\mathcal{B}}(a)\}\\
&= \max\{\lambda_{\mathcal{B}}(a), \lambda_{\mathcal{B}}(a)\}
\end{aligned}$$

$$= \lambda_{\mathcal{B}}(a).$$

This means that $\mathcal{B} \subseteq \mathcal{B} \circ \mathcal{S} \circ \mathcal{B}$. Therefore, $\mathcal{B} = \mathcal{B} \circ \mathcal{S} \circ \mathcal{B}$.

Conversely, let B be a bi-hyperideal of S. Then, $BSB \subseteq B$. Let $a \in B$. By Theorem 3.2, $\mathcal{C}_B = (\mu_{\mathcal{C}_B}, \lambda_{\mathcal{C}_B})$ is a FFB of S. By the given assumption, $\mathcal{C}_B = \mathcal{C}_B \circ \mathcal{S} \circ \mathcal{C}_B$. Then, by Lemma 2.2, we have

$$\mu_{\mathcal{C}_{BSB}}(a) = \mu_{\mathcal{C}_B \circ \mathcal{S} \circ \mathcal{C}_B}(a) = \mu_{\mathcal{C}_B}(a) = 1.$$

It turns out that $a \in BSB$. Hence, $B \subseteq BSB$. We obtain that $B = BSB$. By Lemma 4.2, we obtain that S is regular. □

The following theorem follows from Theorem 4.1 and Theorem 4.3.

Theorem 4.4. *Let S be a semihypergroup. Then, S is regular if and only if $\mathcal{G} = \mathcal{G} \circ \mathcal{S} \circ \mathcal{G}$, for every FFGB $\mathcal{G} = (\mu_{\mathcal{G}}, \lambda_{\mathcal{G}})$ of S.*

Theorem 4.5. *Let S be a semihypergroup. Then, S is regular if and only if $\mathcal{B} \cap \mathcal{A} = \mathcal{B} \circ \mathcal{A} \circ \mathcal{B}$, for every FFH $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ and every FFB $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$ of S.*

Proof: Assume that S is regular. Let $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ and $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$ be a FFH and a FFB of S, respectively. By Lemma 2.4 and Theorem 3.2, we obtain that $\mathcal{B} \circ \mathcal{A} \circ \mathcal{B} \subseteq \mathcal{B} \circ \mathcal{S} \circ \mathcal{B} \subseteq \mathcal{B}$ and $\mathcal{B} \circ \mathcal{A} \circ \mathcal{B} \subseteq (\mathcal{S} \circ \mathcal{A}) \circ \mathcal{S} \subseteq \mathcal{A} \circ \mathcal{S} \subseteq \mathcal{A}$. That is, $\mathcal{B} \circ \mathcal{A} \circ \mathcal{B} \subseteq \mathcal{B} \cap \mathcal{A}$. On the other hand, let $a \in S$. Then, there exists $x \in S$ such that $a \in axa$. Also, $a \in axaxa$. Thus, we have

$$\begin{aligned}
\mu_{\mathcal{B} \circ \mathcal{A} \circ \mathcal{B}}(a) &= \sup_{a \in bcd} [\min\{\mu_{\mathcal{B}}(b), \mu_{\mathcal{A}}(c), \mu_{\mathcal{B}}(d)\}] \\
&\geq \min\left\{\mu_{\mathcal{B}}(a), \inf_{c \in xax} \mu_{\mathcal{A}}(c), \mu_{\mathcal{B}}(a)\right\} \\
&\geq \min\left\{\mu_{\mathcal{B}}(a), \inf_{y \in ax} \mu_{\mathcal{A}}(y), \mu_{\mathcal{B}}(a)\right\} \\
&\geq \min\{\mu_{\mathcal{B}}(a), \mu_{\mathcal{A}}(a), \mu_{\mathcal{B}}(a)\} \\
&= \min\{\mu_{\mathcal{B}}(a), \mu_{\mathcal{A}}(a)\} \\
&= \mu_{\mathcal{B} \cap \mathcal{A}}(a), \\
\lambda_{\mathcal{B} \circ \mathcal{A} \circ \mathcal{B}}(a) &= \inf_{a \in bcd} [\max\{\lambda_{\mathcal{B}}(b), \lambda_{\mathcal{A}}(c), \lambda_{\mathcal{B}}(d)\}] \\
&\leq \max\left\{\lambda_{\mathcal{B}}(a), \sup_{c \in xax} \lambda_{\mathcal{A}}(c), \lambda_{\mathcal{B}}(a)\right\} \\
&\leq \max\left\{\lambda_{\mathcal{B}}(a), \sup_{y \in ax} \lambda_{\mathcal{A}}(y), \lambda_{\mathcal{B}}(a)\right\} \\
&\leq \max\{\lambda_{\mathcal{B}}(a), \lambda_{\mathcal{A}}(a), \lambda_{\mathcal{B}}(a)\} \\
&= \max\{\lambda_{\mathcal{B}}(a), \lambda_{\mathcal{A}}(a)\} \\
&= \lambda_{\mathcal{B} \cup \mathcal{A}}(a).
\end{aligned}$$

Hence, $\mathcal{B} \cap \mathcal{A} \subseteq \mathcal{B} \circ \mathcal{A} \circ \mathcal{B}$. Therefore, $\mathcal{B} \cap \mathcal{A} = \mathcal{B} \circ \mathcal{A} \circ \mathcal{B}$.

Conversely, let $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$ be a FFB of S. Since S itself is a FFH of S and by the hypothesis, we have that $\mathcal{B} = \mathcal{B} \cap \mathcal{S} = \mathcal{B} \circ \mathcal{S} \circ \mathcal{B}$. By Theorem 4.3, S is regular. □

The following result can be obtained by Theorem 4.1 and Theorem 4.5.

Theorem 4.6. *Let S be a semihypergroup. Then, S is regular if and only if $\mathcal{G} \cap \mathcal{A} = \mathcal{G} \circ \mathcal{A} \circ \mathcal{G}$, for every FFH $\mathcal{A} = (\mu_{\mathcal{A}}, \lambda_{\mathcal{A}})$ and every FFGB $\mathcal{G} = (\mu_{\mathcal{G}}, \lambda_{\mathcal{G}})$ of S.*

Theorem 4.7. *Let S be a semihypergroup. Then, the following statements are equivalent:*

(i) S is regular;
(ii) $\mathcal{G} \cap \mathcal{L} \subseteq \mathcal{G} \circ \mathcal{L}$, for every FFGB $\mathcal{G} = (\mu_{\mathcal{G}}, \lambda_{\mathcal{G}})$ and every FFL $\mathcal{L} = (\mu_{\mathcal{L}}, \lambda_{\mathcal{L}})$ of S;
(iii) $\mathcal{B} \cap \mathcal{L} \subseteq \mathcal{B} \circ \mathcal{L}$, for every FFB $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$ and every FFL $\mathcal{L} = (\mu_{\mathcal{L}}, \lambda_{\mathcal{L}})$ of S.

Proof: (i) ⇒ (ii). Let $\mathcal{G} = (\mu_{\mathcal{G}}, \lambda_{\mathcal{G}})$ and $\mathcal{L} = (\mu_{\mathcal{L}}, \lambda_{\mathcal{L}})$ be a FFGB and a FFL of S, respectively. Let $a \in S$. Since S is regular, there exists $x \in S$ such that $a \in axa$. Then, we have

$$\begin{aligned}\mu_{\mathcal{G}\circ\mathcal{L}}(a) &= \sup_{a\in pq}[\min\{\mu_{\mathcal{G}}(p), \mu_{\mathcal{L}}(q)\}] \\ &\geq \min\left\{\mu_{\mathcal{G}}(a), \inf_{q\in xa} \mu_{\mathcal{L}}(q)\right\} \\ &\geq \min\{\mu_{\mathcal{G}}(a), \mu_{\mathcal{L}}(a)\} \\ &= \mu_{\mathcal{G}\cap\mathcal{L}}(a),\end{aligned}$$

$$\begin{aligned}\lambda_{\mathcal{G}\circ\mathcal{L}}(a) &= \inf_{a\in pq}[\max\{\lambda_{\mathcal{G}}(p), \lambda_{\mathcal{L}}(q)\}] \\ &\leq \max\left\{\lambda_{\mathcal{G}}(a), \sup_{q\in xa} \lambda_{\mathcal{L}}(q)\right\} \\ &\leq \max\{\lambda_{\mathcal{G}}(a), \lambda_{\mathcal{L}}(a)\} \\ &= \lambda_{\mathcal{G}\cup\mathcal{L}}(a).\end{aligned}$$

Hence, $\mathcal{G} \cap \mathcal{L} \subseteq \mathcal{G} \circ \mathcal{L}$.

(ii) ⇒ (iii). Since every FFB $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$ of S is a FFGB of S, we obtain that (iii) holds.

(iii) ⇒ (i). Let $\mathcal{L} = (\mu_{\mathcal{L}}, \lambda_{\mathcal{L}})$ and $\mathcal{R} = (\mu_{\mathcal{R}}, \lambda_{\mathcal{R}})$ be a FFL and a FFR of S, respectively. It follows that $\mathcal{R}$ is also a FFB of S. By the given assumption, we have that $\mathcal{R}\cap\mathcal{L} \subseteq \mathcal{R}\circ\mathcal{L}$. On the other hand, we get that $\mathcal{R} \circ \mathcal{L} \subseteq \mathcal{R} \cap \mathcal{L}$. Hence, $\mathcal{R} \cap \mathcal{L} = \mathcal{R} \circ \mathcal{L}$. By Theorem 4.2, S is regular. □

The following theorem can be proved similar to Theorem 4.7.

Theorem 4.8. *Let S be a semihypergroup. Then, the following statements are equivalent:*
(i) S is regular;
(ii) $\mathcal{R} \cap \mathcal{G} \subseteq \mathcal{R} \circ \mathcal{G}$, for every FFGB $\mathcal{G} = (\mu_{\mathcal{G}}, \lambda_{\mathcal{G}})$ and every FFR $\mathcal{R} = (\mu_{\mathcal{R}}, \lambda_{\mathcal{R}})$ of S;
(iii) $\mathcal{R} \cap \mathcal{B} \subseteq \mathcal{R} \circ \mathcal{B}$, for every FFB $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$ and every FFR $\mathcal{R} = (\mu_{\mathcal{R}}, \lambda_{\mathcal{R}})$ of S.

Theorem 4.9. *Let S be a semihypergroup. Then, the following conditions are equivalent:*
(i) S is regular;
(ii) $\mathcal{R} \cap \mathcal{G} \cap \mathcal{L} \subseteq \mathcal{R} \circ \mathcal{G} \circ \mathcal{L}$, for every FFGB $\mathcal{G} = (\mu_{\mathcal{G}}, \lambda_{\mathcal{G}})$, every FFL $\mathcal{L} = (\mu_{\mathcal{L}}, \lambda_{\mathcal{L}})$ and every FFR $\mathcal{R} = (\mu_{\mathcal{R}}, \lambda_{\mathcal{R}})$ of S;
(iii) $\mathcal{R} \cap \mathcal{B} \cap \mathcal{L} \subseteq \mathcal{R} \circ \mathcal{B} \circ \mathcal{L}$, for every FFB $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$, every FFL $\mathcal{L} = (\mu_{\mathcal{L}}, \lambda_{\mathcal{L}})$ and every FFR $\mathcal{R} = (\mu_{\mathcal{R}}, \lambda_{\mathcal{R}})$ of S.

Proof: (i) ⇒ (ii). Assume that S is regular. Let $\mathcal{G} = (\mu_{\mathcal{G}}, \lambda_{\mathcal{G}})$, $\mathcal{L} = (\mu_{\mathcal{L}}, \lambda_{\mathcal{L}})$ and $\mathcal{R} = (\mu_{\mathcal{R}}, \lambda_{\mathcal{R}})$ be a FFGB, a FFL and a FFR of S, respectively. Let $a \in S$. Then, there exists $x \in S$ such that $a \in axa$. Also, $a \in (ax)(axa)(xa)$. Thus, we have

$$\begin{aligned}\mu_{\mathcal{R}\circ\mathcal{G}\circ\mathcal{L}}(a) &= \sup_{a\in wyz}[\min\{\mu_{\mathcal{R}}(w), \mu_{\mathcal{G}}(y), \mu_{\mathcal{L}}(z)\}] \\ &\geq \min\left\{\inf_{w\in ax} \mu_{\mathcal{R}}(w), \inf_{y\in axa} \mu_{\mathcal{G}}(y), \inf_{z\in xa} \mu_{\mathcal{L}}(z)\right\} \\ &\geq \min\{\mu_{\mathcal{R}}(a), \min\{\mu_{\mathcal{G}}(a), \mu_{\mathcal{G}}(a)\}, \mu_{\mathcal{L}}(a)\}\end{aligned}$$

$$
\begin{aligned}
&= \min\{\mu_{\mathcal{R}}(a), \mu_{\mathcal{G}}(a), \mu_{\mathcal{L}}(a)\} \\
&= \mu_{\mathcal{R}\cap\mathcal{G}\cap\mathcal{L}}(a), \\
\lambda_{\mathcal{R}\circ\mathcal{G}\circ\mathcal{L}}(a) &= \inf_{a\in wyz}[\max\{\lambda_{\mathcal{R}}(w), \lambda_{\mathcal{G}}(y), \lambda_{\mathcal{L}}(z)\}] \\
&\leq \max\left\{\sup_{w\in ax}\lambda_{\mathcal{R}}(w), \sup_{y\in axa}\lambda_{\mathcal{G}}(y), \sup_{z\in xa}\lambda_{\mathcal{L}}(z)\right\} \\
&\leq \max\{\lambda_{\mathcal{R}}(a), \max\{\lambda_{\mathcal{G}}(a), \lambda_{\mathcal{G}}(a)\}, \lambda_{\mathcal{L}}(a)\} \\
&= \max\{\lambda_{\mathcal{R}}(a), \lambda_{\mathcal{G}}(a), \lambda_{\mathcal{L}}(a)\} \\
&= \lambda_{\mathcal{R}\cup\mathcal{G}\cup\mathcal{L}}(a).
\end{aligned}
$$

It turns out that $\mathcal{R}\cap\mathcal{G}\cap\mathcal{L} \subseteq \mathcal{R}\circ\mathcal{G}\circ\mathcal{L}$.

(ii) $\Rightarrow$ (iii). Since every FFB $\mathcal{B} = (\mu_{\mathcal{B}}, \lambda_{\mathcal{B}})$ of S is also a FFGB of S, this implies that (iii) holds.

(iii) $\Rightarrow$ (i). Let $\mathcal{L} = (\mu_{\mathcal{L}}, \lambda_{\mathcal{L}})$ and $\mathcal{R} = (\mu_{\mathcal{R}}, \lambda_{\mathcal{R}})$ be a FFL and a FFR of S, respectively. Since $\mathcal{S}$ itself is a FFB of S and by the hypothesis, we have that $\mathcal{R}\cap\mathcal{L} = \mathcal{R}\cap\mathcal{S}\cap\mathcal{L} \subseteq \mathcal{R}\circ\mathcal{S}\circ\mathcal{L} \subseteq \mathcal{R}\circ\mathcal{L}$. Otherwise, we get that $\mathcal{R}\circ\mathcal{L} \subseteq \mathcal{R}\cap\mathcal{L}$. Hence, $\mathcal{R}\cap\mathcal{L} = \mathcal{R}\circ\mathcal{L}$. Therefore, S is regular by Theorem 4.2. □

5. **Conclusions.** The concepts of Fermatean fuzzy sets as generalizations of fuzzy sets, intuitionistic fuzzy sets and Pythagorean fuzzy sets are presented. In this paper, we applied the notion of Fermatean fuzzy sets to describing the concepts of Fermatean fuzzy subsemihypergroups, Fermatean fuzzy (resp., left, right) hyperideals and Fermatean fuzzy (resp., generalized) bi-hyperideals of semihypergroups. Then, some characterizations of various types of Fermatean fuzzy hyperideals of semihypergroups by the Fermatean fuzzy products were studied and discussed the relationships between many types of hyperideals and their Fermatean characteristic functions in semihypergroups. In a regular semihypergroup, it was shown that Fermatean fuzzy generalized bi-hyperideals and Fermatean fuzzy bi-hyperideals coincide. Finally, we characterized regular semihypergroups using the concepts of Fermatean (resp., left, right) hyperideals and Fermatean fuzzy (resp., generalized) bi-hyperideals of semihypergroups. Future work, will be able to characterize many classes of regularities in semihypergroups and other algebraic structures by the properties of Fermatean fuzzy sets.

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