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DISSIPATIVITY ANALYSIS OF STOCHASTIC LINEAR PARAMETER VARYING SYSTEMS

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Abstract. *This paper is concerned with the dissipativity analysis of linear parameter varying (LPV) system under stochastic perturbation. Two types of dissipativity are investigated: passivity and a more generalized form of dissipativity. With the use of parameter dependent Lyapunov functions, parameter dependent sufficient conditions are proposed and the corresponding controllers are designed to meet the requirements of dissipativity. Numerical examples are provided to illustrate the effectiveness of the proposed theories.*
Keywords: Linear parameter varying system, Dissipativity analysis, Stochastic systems, State feedback controller, Parameter dependent Lyapunov functions

1. Introduction.

Linear parameter varying (LPV) systems are those systems dependent on unknown but measurable time-varying parameters. It is an intuitive way to model plants with uncertain parameters and gains great popularity among researchers both in theoretical and practical fields. For example, it is used to describe the variations in temperature and mass flow in a reversible-SOC energy-storage plant [1], the bias terms are included into the electric vehicles dynamics and constitute an LPV system [2], and the changing operating point parameters of a grid-following inverter are treated as the scheduling variables in an LPV system [3].

Dissipativity is a system property which ensures the stability of the system. Dissipative systems are systems that can only dissipate but not generate energy, which means the stored energy in them would never be greater than that supplied from outer environment [5]. The concept of dissipativity is important not only in control theory, but also in physical systems such as circuits, and networks. It offers an efficient tool for stability analysis and control design for linear and nonlinear systems, such as switched systems [6], fuzzy systems [7], time-delayed systems [8], and fractional-order systems [9].

In practice, systems are facing stochastic perturbations, which results in stochastic systems. Considering the commonness of stochastic phenomenon and the popularity of LPV systems in practical engineering, investigation of LPV systems under stochastic perturbation would be of great practical meaning. This paper aims to carry out dissipativity analysis of LPV system facing stochastic perturbations; as far as authors know, there have no theoretical results in this area, which would contribute to theoretical innovation. The rest of this paper is organized as follows. The system description and preliminaries are put in Section 2. Section 3 presents our main results. Numerical examples are given in Section 4 and we conclude this paper in Section 5.

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2. **Problem Statement and Preliminaries.** In this paper, we consider the stochastic perturbed linear parameter varying system being established on the probability space, and describe it in Itô's form as follows:

$$(\Sigma)\colon dx(t) = [A(\rho(t))x(t) + B(\rho(t))u(t) + B_v(\rho(t))v(t)]dt + B_\omega(\rho(t))x(t)d\omega(t), \tag{1a}$$

$$y(t) = C(\rho(t))x(t) + D(\rho(t))v(t), \tag{1b}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input; $v(t) \in \mathbb{R}^q$ belonging to $\mathscr{L}_2[0,\infty)$, is either a disturbance input or a reference signal; $\omega(t)$ is a one-dimensional (1-D) Brownian motion satisfying $\mathbf{E}\{d\omega(t)\} = 0$ and $\mathbf{E}\{d\omega^2(t)\} = dt$; $\rho(t) \in \mathbb{R}^s$ is vector-valued parameter evolving continuously over time and its range is limited to a compact subset, and we will use ρ instead of $\rho(t)$ for simplicity in the rest of this paper. The value of ρ is unknown, but can be measured in real time. Matrix B is assumed to be of full column rank.

Thus, the autonomous system of (1) is formulated as

$$\left(\hat{\Sigma}\right)\colon dx(t) = [A(\rho)x(t) + B_v(\rho)v(t)]dt + B_\omega(\rho)x(t)d\omega(t), \tag{2a}$$

$$y(t) = C(\rho)x(t) + D(\rho)v(t). \tag{2b}$$

Definition 2.1. *The LPV stochastic system (2) is said to be passive if there exists a scalar $\gamma > 0$ such that*

$$2E\left\{\int_0^t v(t)^T y(t)dt\right\} \geq -\gamma E\left\{\int_0^t v(t)^T v(t)dt\right\} \tag{3}$$

for all $t > 0$ with zero initial conditions.

Definition 2.2. *([10] Supply Rate): The supply rate is a real valued function: $s(v,y)$: $V \times Y \to \mathbb{R}$, which is assumed to be locally Lebesgue integrable despite the input and the initial conditions, meaning that, for any $v \in V$ and $y \in Y$ and $t \geq 0$, it holds that $\int_0^t |s(v(t),y(t))|dt < +\infty$.*

Definition 2.3. *([10] Dissipative System): The LPV stochastic system (2) with supply rate $s(v,y)$ is considered as dissipative if there exists a nonnegative function $V(x)\colon X \to \mathbb{R}$, named as energy storage function, such that the following dissipation inequality holds:*

$$E\{V(x(t)) - V(x(0))\} \leq E\left\{\int_0^t s(v(t),y(t))dt\right\} \tag{4}$$

for all initial condition $x_0 \in X$, input $v \in V$ and $t \geq 0$.

Definition 2.4. *Given matrices $\Psi \in \mathbb{R}^{q\times q}$, $\Lambda \in \mathbb{R}^{p\times p}$, $\Phi \in \mathbb{R}^{q\times p}$, with Ψ being semi-negative symmetric and Λ being positive symmetric, the LPV stochastic system (2) is called (Ψ,Φ,Λ)-dissipative if for some real function $\gamma(\cdot)$ with $\gamma(0) = 0$, the following inequality holds:*

$$E\left\{\int_0^t \begin{bmatrix} y(t) \\ v(t) \end{bmatrix}^T \begin{bmatrix} \Psi & \Phi \\ * & \Lambda \end{bmatrix} \begin{bmatrix} y(t) \\ v(t) \end{bmatrix} dt\right\} + \gamma(x_0) \geq 0, \quad \forall t \geq 0. \tag{5}$$

Furthermore, if for some scalar $\delta > 0$, the following inequality holds:

$$E\left\{\int_0^t \begin{bmatrix} y(t) \\ v(t) \end{bmatrix}^T \begin{bmatrix} \Psi & \Phi \\ * & \Lambda \end{bmatrix} \begin{bmatrix} y(t) \\ v(t) \end{bmatrix} dt\right\} + \gamma(x_0) \geq \delta \int_0^t v^T(t)v(t)dt, \quad \forall t \geq 0, \tag{6}$$

then the LPV stochastic system (2) is called strictly (Ψ,Φ,Λ)-δ-dissipative.

3. Main Results.

3.1. Passivity analysis and synthesis.

Theorem 3.1. *If there exists a family of parameter dependent continuous differentiable symmetric positive matrices $P(\rho)$ and a scalar $\gamma > 0$, such that for all the parameter trajectories, it satisfies*

$$\Gamma(\rho) \triangleq \begin{bmatrix} \bar{\Gamma} & P(\rho)B_v(\rho) - C^T(\rho) \\ * & -D(\rho) - D^T(\rho) - \gamma I \end{bmatrix} < 0, \tag{7}$$

where $\bar{\Gamma} = \dot{P}(\rho) + \mathrm{sym}\{P(\rho)A(\rho)\} + {B_\omega}^T(\rho)P(\rho)B_\omega(\rho)$, then the autonomous system (2) is robustly passive in the sense of Definition 2.1.

Proof: Choose Lyapunov function candidate as $V(x, \rho) = x^T(t)P(\rho)x(t)$. According to Itô's formula, it follows that

$$dV(x, \rho) = LV(x, \rho)dt + 2x^T(t)P(\rho)B_\omega(\rho)x(t)d\omega(t), \tag{8}$$

where

$$\begin{aligned} LV(x, \rho) &= x^T(t)\dot{P}(\rho)x(t) + 2x^T(t)P(\rho)A(\rho)x(t) + 2x^T(t)P(\rho)B_v(\rho)v(t) \\ &\quad + x^T(t){B_\omega}^T(\rho)P(\rho)B_\omega(\rho)x(t) \\ &= x^T(t)\left[\dot{P}(\rho) + \mathrm{sym}\{P(\rho)A(\rho)\} + {B_\omega}^T(\rho)P(\rho)B_\omega(\rho)\right]x(t) \\ &\quad + 2x^T(t)P(\rho)B_v(\rho)v(t). \end{aligned} \tag{9}$$

Define a new function as

$$\Xi(x, \rho) \triangleq LV(x, \rho) - 2v(t)^T y(t) - \gamma v(t)^T v(t) \triangleq \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}^T \Gamma(\rho) \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}, \tag{10}$$

where $\Gamma(\rho)$ is defined in (7). By (7) we have $\Xi(x, \rho) < 0$, meaning that $LV(x, \rho) < 2v(t)^T y(t) + \gamma v(t)^T v(t)$, and then we obtain

$$dV(x, \rho) < \left(2v(t)^T y(t) + \gamma v(t)^T v(t)\right) dt + 2x^T(t)P(\rho)B_\omega(\rho)x(t)d\omega(t). \tag{11}$$

Integrating both sides of (11) from 0 to t, and taking expectations, we get

$$E\{V(x(t), \rho(t)) - V(x(0), \rho(0))\} < E\left\{\int_0^t 2v(t)^T y(t) + \gamma v(t)^T v(t)dt\right\}. \tag{12}$$

Note that $V(x(0), \rho(0)) = 0$ with the zero initial state; it follows that

$$E\left\{\int_0^t 2v(t)^T y(t) + \gamma v(t)^T v(t)dt\right\} > E\{V(x(t), \rho(t))\} \geq 0, \tag{13}$$

which is

$$2E\left\{\int_0^t v(t)^T y(t)dt\right\} \geq -\gamma E\left\{\int_0^t v(t)^T v(t)dt\right\}.$$

Proof completed.

Theorem 3.2. *If there exists a family of parameter dependent continuous differentiable symmetric positive matrices $R(\rho)$, matrices $F(\rho)$, and a scalar $\gamma > 0$, such that for all the parameter trajectories, the following inequality holds:*

$$\begin{bmatrix} X & B_v(\rho) - R(\rho)C^T(\rho) & R(\rho)B_\omega^T(\rho) \\ * & -D(\rho) - D^T(\rho) - \gamma I & 0 \\ * & * & -R(\rho) \end{bmatrix} < 0, \tag{14}$$

where $X = \mathrm{sym}\{A(\rho)R(\rho) + B(\rho)F(\rho)\} - \sum_{i=1}^{s} \tau_i \frac{\partial R(\rho)}{\rho_i}$, *then system (1) is passive with zero initial state under* $u(t) = K(\rho)x(t)$ *with* $K(\rho) = F(\rho)R^{-1}(\rho)$.

Proof: Considering system (1) with $u(t) = K(\rho)x(t)$, in which $K(\rho)$ is a parameter dependent state feedback controller, then the closed-loop system can be written as

$$dx(t) = (A(\rho) + B(\rho)K(\rho))x(t)dt + B_v(\rho)v(t)dt + B_\omega(\rho)x(t)d\omega(t). \tag{15}$$

Performing a congruence transformation to (7) with diag $(P^{-1}(\rho), I)$ and applying Schur complement, we obtain

$$\begin{bmatrix} \bar{X} & B_v(\rho) - P^{-1}(\rho)C^T(\rho) & P^{-1}(\rho){B_\omega}^T(\rho) \\ * & -D(\rho) - D^T(\rho) - \gamma I & 0 \\ * & * & -P^{-1}(\rho) \end{bmatrix} < 0, \tag{16}$$

where $\bar{X} = P^{-1}(\rho)\dot{P}(\rho)P^{-1}(\rho) + \mathrm{sym}\{(A(\rho) + B(\rho)K(\rho))P^{-1}(\rho)\}$ and

$$\dot{P} = \frac{dP}{d\rho} \cdot \frac{d\rho}{dt} = \sum_{i=1}^{s} \tau_i \frac{\partial R(\rho)}{\partial \rho_i}, \quad \tau_i = \frac{d\rho_i}{dt}. \tag{17}$$

Let matrix $R(\rho) = P^{-1}(\rho)$, and then we have

$$\frac{\partial R(\rho)}{\partial \rho_i} = -R(\rho)\frac{\partial P(\rho)}{\partial \rho_i}R(\rho). \tag{18}$$

Let $F(\rho) = K(\rho)R(\rho)$, and then we can obtain (14). Proof completed.

3.2. Dissipativity analysis and synthesis.

Theorem 3.3. *Given matrices* $\Psi \in \mathbb{R}^{q\times q}$, $\Lambda \in \mathbb{R}^{p\times p}$, $\Phi \in \mathbb{R}^{q\times p}$, *with* Ψ *being semi-negative symmetric and* Λ *being positive symmetric, and scalars* $\lambda > 0$ *and* $\delta > 0$, *if there exists a family of parameter dependent continuous differentiable symmetric positive matrices* $P(\rho)$, *such that for all the parameter trajectories, the following inequality holds:*

$$\mathrm{H}(\rho) \triangleq \begin{bmatrix} \mathrm{H}_{11}(\rho) & \mathrm{H}_{12}(\rho) \\ * & \mathrm{H}_{22}(\rho) \end{bmatrix} < 0, \tag{19}$$

where

$$\begin{aligned} \mathrm{H}_{11}(\rho) &\triangleq \dot{P}(\rho) + \mathrm{sym}\{P(\rho)A(\rho)\} + {B_\omega}^T(\rho)P(\rho)B_\omega(\rho) + \lambda P(\rho) - C^T(\rho)\Psi C(\rho), \\ \mathrm{H}_{12}(\rho) &\triangleq P(\rho)B_v(\rho) - C^T(\rho)\Psi D(\rho) - C^T(\rho)\Phi, \\ \mathrm{H}_{22}(\rho) &\triangleq -\Lambda + \delta I - D^T(\rho)\Psi D(\rho) - D^T(\rho)\Phi - \Phi^T D(\rho), \end{aligned}$$

then the autonomous LPV stochastic system (2) is (Ψ, Φ, Λ)-δ-*dissipative in the sense of Definition 2.4.*

Proof: Choosing Lyapunov function candidate as $V(x,\rho) = x^T(t)P(\rho)x(t)$, then we have (8) and (9). Define a new function as

$$\begin{aligned} \Pi(x,\rho) &\triangleq LV(x,\rho) + \lambda V(x,\rho) - y^T(t)\Psi y(t) - 2y(t)^T\Phi v(t) - v(t)^T(\Lambda - \delta I)v(t) \\ &\triangleq \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}^T \mathrm{H}(\rho) \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}, \end{aligned} \tag{20}$$

By (19) we know $\Pi(x,\rho) < 0$, meaning

$$\begin{aligned} dV(x,\rho) < {} & \Big(-\lambda V(x,\rho) + y^T(t)\Psi y(t) + 2y(t)^T\Phi v(t) + v(t)^T(\Lambda - \delta I)v(t)\Big)\, dt \\ & + 2x^T(t)P(\rho)B_\omega(\rho)x(t)d\omega(t). \end{aligned} \tag{21}$$

Note that

$$\begin{aligned} de^{\lambda t}V(x,\rho) &= \lambda e^{\lambda t}V(x,\rho)dt + e^{\lambda t}dV(x,\rho) \\ &< e^{\lambda t}\left(y^T(t)\Psi y(t) + 2y(t)^T\Phi v(t) + v(t)^T(\Lambda - \delta I)v(t)\right)dt \\ &\quad + 2e^{\lambda t}x^T(t)P(\rho)B_\omega(\rho)x(t)d\omega(t). \end{aligned} \tag{22}$$

Integrating both sides of (22) from 0 to t, and taking expectations, we get

$$\begin{aligned} E\{V(x(t),\rho(t))\} &< E\left\{\int_0^t y^T(t)\Psi y(t) + 2y(t)^T\Phi v(t) + v(t)^T(\Lambda - \delta I)v(t)dt\right\} \\ &\quad + e^{-\lambda t}V(x(0),\rho(0)) \\ &= E\left\{\int_0^t \begin{bmatrix} y(t) \\ v(t) \end{bmatrix}^T \begin{bmatrix} \Psi & \Phi \\ * & \Lambda \end{bmatrix} \begin{bmatrix} y(t) \\ v(t) \end{bmatrix} dt\right\} - \delta\int_0^t v^T(t)v(t)dt \\ &\quad + e^{-\lambda t}V(x(0),\rho(0)). \end{aligned} \tag{23}$$

Let $\gamma(x_0) = e^{-\lambda t}V(x(0),\rho(0))$, and we can obtain Inequality (6). Proof completed.

Theorem 3.4. *Given matrices $\Psi \in \mathbb{R}^{q\times q}$, $\Lambda \in \mathbb{R}^{p\times p}$, $\Phi \in \mathbb{R}^{q\times p}$, with Ψ being semi-negative symmetric and Λ being positive symmetric, and scalars $\lambda > 0$ and $\delta > 0$, if there exists a family of parameter dependent continuous differentiable symmetric positive matrices $R(\rho)$ and matrices $F(\rho)$, such that for all the parameter trajectories, the following inequality holds:*

$$\hat{\mathrm{H}}(\rho) \triangleq \begin{bmatrix} \hat{\mathrm{H}}_{11}(\rho) & \hat{\mathrm{H}}_{12}(\rho) & R(\rho){B_\omega}^T(\rho) & R(\rho)C^T(\rho)\Psi \\ * & \hat{\mathrm{H}}_{22}(\rho) & 0 & 0 \\ * & * & -R(\rho) & 0 \\ * & * & * & \Psi \end{bmatrix} < 0, \tag{24}$$

where

$$\begin{aligned} \hat{\mathrm{H}}_{11}(\rho) &\triangleq \mathrm{sym}\{A(\rho)R(\rho) + B(\rho)F(\rho)\} + \lambda R(\rho) - \sum_{i=1}^{s}\tau_i\frac{\partial R(\rho)}{\rho_i}, \\ \hat{\mathrm{H}}_{12}(\rho) &\triangleq B_v(\rho) - R(\rho)C^T(\rho)\Psi D(\rho) - R(\rho)C^T(\rho)\Phi, \\ \hat{\mathrm{H}}_{22}(\rho) &= \mathrm{H}_{22}(\rho) = -\Lambda + \delta I - D^T(\rho)\Psi D(\rho) - D^T(\rho)\Phi - \Phi^T D(\rho), \end{aligned}$$

then the LPV stochastic system (1) is (Ψ,Φ,Λ)-δ-dissipative in the sense of Definition 2.4 under $u(t) = K(\rho)x(t)$ with $K(\rho) = F(\rho)R^{-1}(\rho)$.

Proof: Considering $u(t) = K(\rho)x(t)$, substitute the $A(\rho)$ in (19) with $A(\rho)+B(\rho)K(\rho)$ according to Theorem 3.3, then we can obtain that the controlled system in (15) is (Ψ,Φ,Λ)-δ-dissipative if there exist parameter dependent positive definite matrices $P(\rho)$, such that for all parameter changing trajectories, it follows that

$$\begin{bmatrix} \bar{\mathrm{H}}_{11}(\rho) & P(\rho)B_\omega(\rho) - C^T(\rho)\Psi D(\rho) - C^T(\rho)\Phi \\ * & -\Lambda + \delta I - D^T(\rho)\Psi D(\rho) - D^T(\rho)\Phi - \Phi^T D(\rho) \end{bmatrix} < 0, \tag{25}$$

where

$$\begin{aligned} \bar{\mathrm{H}}_{11}(\rho) &= \dot{P}(\rho) + \mathrm{sym}\{P(\rho)[A(\rho) + B(\rho)K(\rho)]\} + {B_\omega}^T(\rho)P(\rho)B_\omega(\rho) \\ &\quad + \lambda P(\rho) - C^T(\rho)\Psi C(\rho). \end{aligned}$$

Performing a congruent transformation on (25) with diag $(P^{-1}(\rho), I)$, and defining $R(\rho) = P^{-1}(\rho)$, we obtain

$$\begin{bmatrix} \tilde{\mathrm{H}}_{11}(\rho) & B_\omega(\rho) - R(\rho)C^T(\rho)\Psi D(\rho) - R(\rho)C^T(\rho)\Phi \\ * & -\Lambda + \delta I - D^T(\rho)\Psi D(\rho) - D^T(\rho)\Phi - \Phi^T D(\rho) \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned} \tilde{\mathrm{H}}_{11}(\rho) =\ & \mathrm{sym}\{A(\rho)R(\rho) + B(\rho)F(\rho)\} + \lambda R(\rho) - \sum_{i=1}^{s} \tau_i \frac{\partial R(\rho)}{\rho_i} \\ & + R(\rho){B_\omega}^T(\rho)R^{-1}(\rho)B_\omega(\rho)R(\rho) - R(\rho)C^T(\rho)\Psi C(\rho)R(\rho). \end{aligned}$$

Letting $F(\rho) = K(\rho)R(\rho)$ and applying Schur complement to (26), we obtain (24). Proof completed.

4. **Numerical Examples.**

Consider stochastically perturbed LPV system (1) with the following matrices:

$$\begin{aligned} A &= \begin{bmatrix} -2.0 + 0.1\rho_1(t) & 1.2 \\ -1.8 & -1.0 + 0.2\rho_2(t) \end{bmatrix}, \ B = \begin{bmatrix} 1.5 + 0.1\rho_1(t) \\ 1.0 \end{bmatrix}, \\ B_\omega &= \begin{bmatrix} 1.4 + 0.1\rho_1(t) & 1.0 + 0.2\rho_1(t) \\ 0.7 & 0.9 \end{bmatrix}, \ B_v = \begin{bmatrix} 0.2 + 0.1\rho_1(t) \\ 0.1 + 0.1\rho_2(t) \end{bmatrix}, \\ C &= \begin{bmatrix} 1.2 + 0.2\rho_1(t) + 0.1\rho_2(t) & 0.8 \end{bmatrix}, \ D = 2.5 + 0.1\rho_1(t) + 0.1\rho_2(t), \end{aligned}$$

where $\rho_1(t) = \sin(t)$ and $\rho_2(t) = |\cos 5t|$ are time-varying parameters, satisfying $\rho_1(t) \in [-1, 1]$, $\rho_2(t) \in [0, 1]$, $\tau_1 \in [-1, 1]$, and $\tau_2 \in [-5, 5]$. As can be seen, the time-varying parameters can influence the system from less than one order or of the same order, which are chosen to test the robustness of the controller to be designed.

To carry out simulations, we use the following set of basis functions to approximate the parameter dependent matrices that appear in the stability conditions:

$$f_1(\rho) = 1, \ \ f_2(\rho) = \rho_1(t), \ \ f_3(\rho) = \rho_2(t).$$

Thus, we have

$$R(\rho) = \sum_{j=1}^{3} f_j(\rho)R_j, \quad F(\rho) = \sum_{j=1}^{3} f_j(\rho)F_j, \tag{27}$$

and then the problem becomes finding matrices R_j $(j = 1 \sim 3)$ and F_j $(j = 1 \sim 3)$, to satisfy (14) and (24).

4.1. **Passification.**

Add a disturbance signal $v(t) = 1/(1 + 0.5t^2)$ to the system. The task is to design a state feedback controller $K(\rho)$ to ensure the system (1) is passive with some index γ. By solving the conditions in (14) in Theorem 3.2, we obtain

$$\begin{aligned} R_1 &= \begin{bmatrix} 2.0141 & -1.8045 \\ -1.8045 & 2.6789 \end{bmatrix}, \ R_2 = \begin{bmatrix} -0.0300 & -0.0932 \\ -0.0932 & 0.1440 \end{bmatrix}, \\ R_3 &= \begin{bmatrix} -0.0044 & -0.0014 \\ -0.0014 & 0.0124 \end{bmatrix}, \ F_1 = \begin{bmatrix} 1.8683 & -4.2010 \end{bmatrix}, \\ F_2 &= \begin{bmatrix} -0.1829 & 0.0784 \end{bmatrix}, \ F_3 = \begin{bmatrix} 0.1021 & -0.1664 \end{bmatrix}, \end{aligned}$$

and the obtained controller is as follows:

$$\begin{aligned} K_1 &= \begin{bmatrix} -1.2040 & -2.3792 \end{bmatrix}, \ K_2 = \begin{bmatrix} 1.4635 & 1.4914 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} -18.3168 & -15.4934 \end{bmatrix}. \end{aligned}$$

The results are shown in Figure 1 and Figure 2. Figure 1 shows one set of state response under passification control, whereas Figure 2 shows ten sets of state response, from which

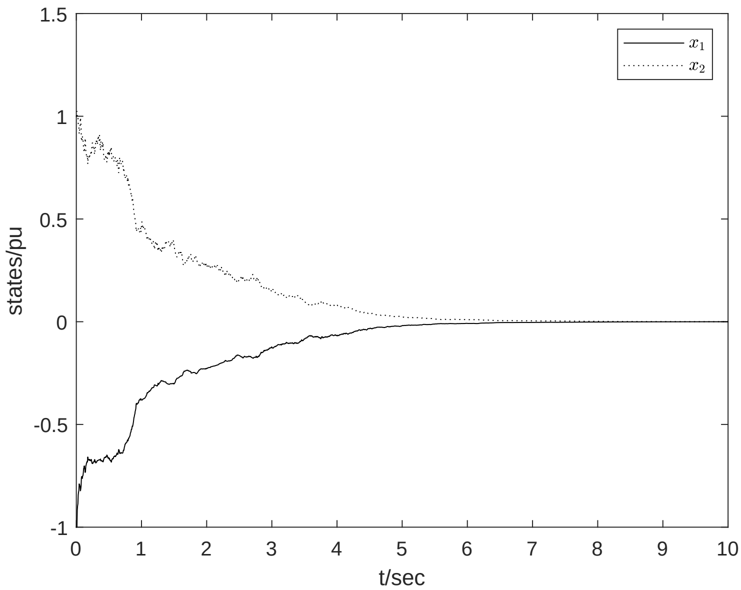

FIGURE 1. One set of state response under passification

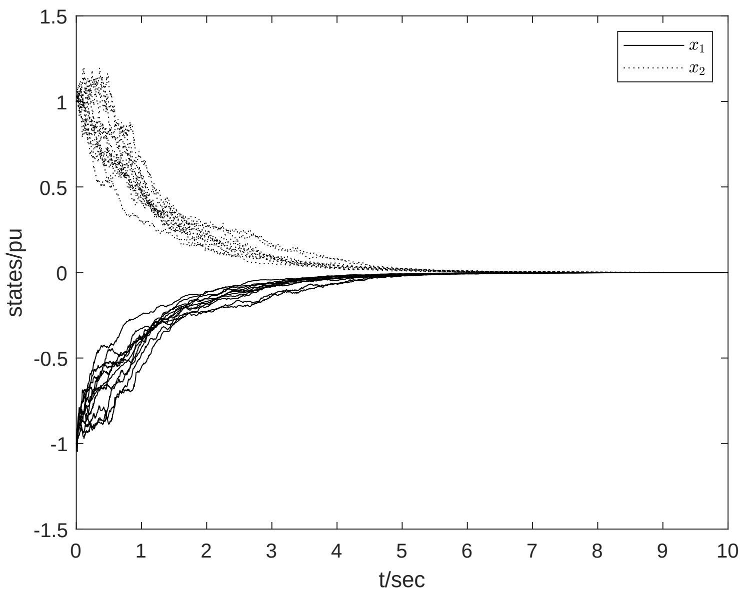

FIGURE 2. Ten sets of state response under passification

the stochastic phenomenon can be observed. As can be seen, with the designed controller, the system is stabilized in the face of stochastic perturbation, and the achieved γ is 1.3171.

4.2. **Dissipation.** Set $\Psi = -1.5$, $\Phi = 1.5$, $\Lambda = 1.8$, $\delta = 0.2$, $\lambda = 0.5$, and design a state feedback controller $K(\rho)$ to ensure the system (1) is dissipative with the disturbance $v(t) = 1/(1 + 0.5t^2)$. By solving the conditions in (24) in Theorem 3.4, we obtain

$$R_1 = \begin{bmatrix} 2.3370 & -2.7852 \\ -2.7852 & 4.5375 \end{bmatrix}, \quad R_2 = \begin{bmatrix} -0.9430 & 0.7264 \\ 0.7264 & -0.7148 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} -0.3426 & -0.0596 \\ -0.0596 & 0.0512 \end{bmatrix}, \quad F_1 = \begin{bmatrix} -0.8745 & -8.6499 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} -1.5818 & 1.9743 \end{bmatrix}, \quad F_3 = \begin{bmatrix} 0.0124 & 0.1261 \end{bmatrix},$$

and the obtained controller is as follows:

$$K_1 = \left[\begin{array}{cc} -9.8563 & -7.9562 \end{array} \right], \quad K_2 = \left[\begin{array}{cc} -2.0721 & -4.8676 \end{array} \right],$$
$$K_3 = \left[\begin{array}{cc} -0.3863 & 2.0132 \end{array} \right].$$

The results are shown in Figure 3 and Figure 4, which respectively show one set and ten sets of state response under dissipation control, and the stochastic phenomenon can be observed in Figure 4. As can be seen, with the designed controller, system (1) is stabilized while satisfying the dissipation requirements. As mentioned earlier that dissipativity includes passivity, if we set $\Psi = 0$, $\Phi = 1$, $\Lambda = 0$, $\delta = -\gamma$, then the dissipation problem reduced to the passification problem.

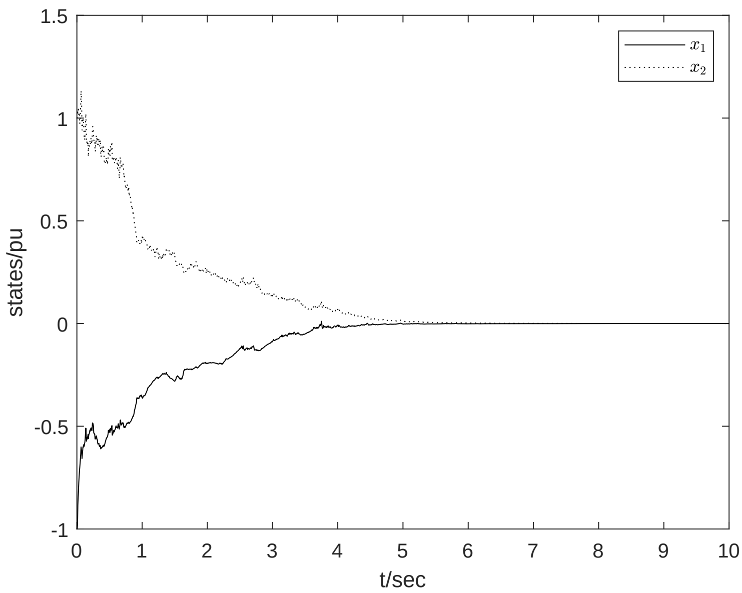

FIGURE 3. One set of state response under dissipation

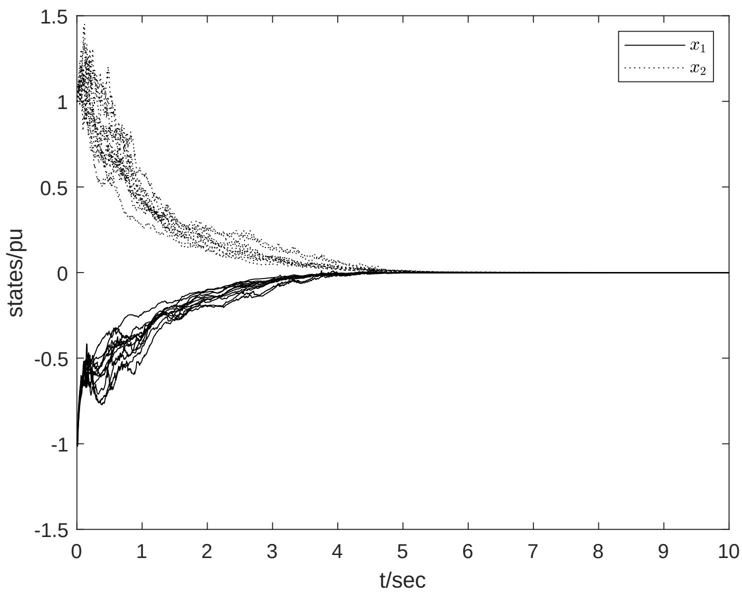

FIGURE 4. Ten sets of state response under dissipation

5. **Conclusions.** The problem of dissipativity analysis of linear parameter varying systems under stochastic perturbation has been addressed in this paper. Two types of dissipativity are analyzed, which are passivity and the more generalized (Ψ, Φ, Λ)-δ-dissipativity. Sufficient conditions for dissipativity have been given. Then, the controller synthesis is carried out. Numerical examples have been provided to illustrate the effectiveness of the proposed theories. Further directions for the work would take time delay and event-trigger mechanism into consideration.

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REFERENCES

[1] X. Xing, J. Lin, N. Brandon, A. Banerjee and Y. Song, Time-varying model predictive control of a reversible-SOC energy-storage plant based on the linear parameter-varying method, *IEEE Transactions on Sustainable Energy*, vol.11, no.3, pp.1589-1600, 2020.

[2] N. Vafamand, M. M. Arefi, M. H. Khooban, T. Dragičević and F. Blaabjerg, Nonlinear model predictive speed control of electric vehicles represented by linear parameter varying models with bias terms, *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol.7, no.3, pp.2081-2089, 2019.

[3] M. Z. Mansour, M. H. Ravanji, A. Karimi and B. Bahrani, Linear parameter-varying control of a power-synchronized grid-following inverter, *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol.10, no.2, pp.2547-2558, 2022.

[4] Y. Yang, J. Liu, B. Liu and Y. Shi, Mean-square exponential stability analysis and H_∞ performance of impulsive stochastic systems with time-varying uncertain parameters, *International Journal of Innovative Computing, Information and Control*, vol.17, no.1, pp.275-285, 2021.

[5] J. Zhao and D. J. Hill, Dissipativity theory for switched systems, *IEEE Transactions on Automatic Control*, vol.53, no.4, pp.941-953, 2008.

[6] Y. Tian and Z. Wang, Finite-time extended dissipative filtering for singular T-S fuzzy systems with nonhomogeneous Markov jumps, *IEEE Transactions on Cybernetics*, vol.52, no.6, pp.4574-4584, 2022.

[7] G. Ran, C. Li, H.-K. Lam, D. Li and C. Han, Event-based dissipative control of interval type-2 fuzzy Markov jump systems under sensor saturation and actuator nonlinearity, *IEEE Transactions on Fuzzy Systems*, vol.30, no.3, pp.714-727, 2022.

[8] Z. Feng, H. Zhang and H.-K. Lam, New results on dissipative control for a class of singular Takagi-Sugeno fuzzy systems with time delay, *IEEE Transactions on Fuzzy Systems*, vol.30, no.7, pp.2466-2475, 2022.

[9] X. Sun and X. Song, Dissipative analysis and event-triggered exponential synchronization for fractional-order complex-valued reaction-diffusion neural networks, *International Journal of Innovative Computing, Information and Control*, vol.18, no.5, pp.1519-1536, 2022.

[10] D. J. Hill and P. J. Moylan, Dissipative dynamical systems: Basic input-output and state properties, *Journal of the Franklin Institute*, vol.309, no.5, pp.327-357, 1980.

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