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THE PARAMETERIZATION OF ALL DISTURBANCE OBSERVERS FOR PERIODIC INPUT AND OUTPUT DISTURBANCES

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Abstract. *In this paper, we examine the parameterization of all linear functional disturbance observers for periodic input and output disturbances. The disturbance observers have been used to estimate the disturbance in the plant. Several papers on design methods for disturbance observers have been published. Recently, the parameterization of all disturbance observers and all linear functional disturbance observers for plants with any input and output disturbance were clarified. If parameterizations of all such observers for plants with any input and output disturbance are used, there is a possibility that we could design a control system to attenuate input and output disturbances effectively. However, no paper examines the parameterization of all linear functional disturbance observers for plants with any input and output disturbances. In this paper, we propose the parameterization of all linear functional disturbance observers for periodic input and output disturbances. We presented the proposed method could attenuate periodic disturbances effectively without using repetitive controllers. Besides, a design method and a design procedure of a linear functional disturbance observer were presented. Finally, we explained the features of the proposed design method and they were illustrated through numerical examples.*

Keywords: Disturbances, Disturbance observer, Periodic input and output disturbance, Parameterization, Linear functional disturbance observer

1. **Introduction.** In this paper, we examine the parameterization of all linear functional disturbance observers for periodic input and output disturbances. A disturbance observer is used to estimate the disturbances in the factory plant [1, 2, 3, 4, 5, 6]. Several papers on design methods for disturbance observers have been published [4, 5, 6, 7, 8]. Currently, the applications of disturbance observers have been used in many control systems such as a motion-control field [3, 5, 9]. A disturbance observer is used in motion control to cancel the disturbance or to make the closed-loop system robustly stable [1, 2, 7, 8, 9, 10, 11, 12, 13]. Typically disturbance observers include disturbance signal generators and an observer. Disturbances that are normally considered step disturbances are estimated by

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the observer. Since the disturbance observer is simple to understand the structure, it is used in many cases [1, 2, 8, 9, 10, 11, 12, 13].

Mita et al. pointed out that disturbance observers are not the only alternative design of complete controllers [7]. That is, a control system with a disturbance observer does not guarantee robust stability. Extended H_∞ control in [7] has therefore been proposed as an effective motion control method that cancels disturbances. This implies that using the method in [7], a control system with a disturbance observer could be designed to guarantee robust stability. From another point of view, Kobayashi et al. considered an observer design method for obtaining phase compensation based on disturbance observers [8]. Compared to using a phase compensator, the control system in [8] is simple and easy to design. In this way, a robustness analysis of the control system that has observed disturbances has been considered.

Another important control problem is the parameterization problem which is the problem of finding all stable controllers for the plant [14, 15, 16, 17, 18, 19, 20, 21]. If the parameterization of all disturbance observers for any disturbances could be obtained, we could express results from previous studies of disturbance observers in a uniform manner. In addition, disturbance observers for any disturbances could be designed systematically. From this point of view, Yamada et al. examined parameterizations of all disturbance observers and all linear functional disturbance observers for plants with any input-output disturbances [22, 23, 24, 25, 26]. Ando et al. examined parameterizations of all disturbance observers and all linear functional disturbance observers for plants with any input and output disturbances [27]. However, methods in [22, 23, 24, 25, 26, 27] could not apply for periodic disturbance. Phukapak et al. overcame this problem and examined the parameterizations of all disturbance observers for periodic output disturbances [28]. In addition, the previous study [29] explained the parameterization of all disturbance observers for periodic input disturbances. However, no paper examines the parameterization of all disturbance observers for periodic input and output disturbances. Methods in [22, 23, 24, 25, 26, 27] can estimate disturbances with finite number of frequency component but cannot estimate disturbances with infinite number of frequency component. In addition, when we control practical systems, many disturbances appear as periodic disturbances, such as robot arms, heat-flow experiments, multi-axis manipulators, noises and vibrations [3, 26, 30, 31]. Therefore, it is important for the control system to attenuate periodic disturbances. In addition, parameterization and disturbance observers are taken into account for periodic input and output disturbances in this paper. Based on a control system to attenuate periodic disturbances, hence, this research would contribute to the performance improvement of the parameterization of all disturbance observers in order to periodic input and output disturbances. For these reasons, the purpose of this study is to propose the parameterization of all disturbance observers for periodic input and output disturbances.

In this paper, we propose the parameterization for disturbance observers for periodic input and output disturbances. First, the necessary structure and characteristics of disturbance observers and the linear functional disturbance observer for periodic input and output disturbances are introduced. Next, to attenuate periodic input and output disturbances effectively, a design method for the control system using these parameterizations of all disturbance observers and of all linear functional disturbance observers for periodic input and output disturbance is proposed. In addition, control characteristics of the control system using these parameterizations are clarified. A design procedure is also given. Finally, we offer numerical examples to illustrate the features of the proposed design method. This paper is organized as follows. In Section 2, we formulate the problem considered in this paper. In Section 3, we clarify the conditions to estimate the periodic input and output disturbances. In Section 4, we propose the parameterization of all disturbance observers for periodic input and output disturbances. In Section 5, we define the parameterization of all linear functional disturbance observers for periodic input and output disturbances. In Section 6, we show a design method for the linear functional disturbance observer. In Section 7, we present a procedure for linear functional disturbance observers for periodic input and output disturbances. In Section 8, we provide numerical examples to illustrate the features of the proposed method. Section 9 gives concluding remarks. Therefore, this research could provide a different disturbance observers parameterization and this could attenuate periodic disturbances effectively without using repetitive controllers.

Notations

R	the set of real numbers.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
$\mathcal{U}$	the unimodular procession in RH_∞. That is, $P(s) \in \mathcal{U}$ means that $P(s) \in RH_\infty$ and $P^{-1}(s) \in RH_\infty$.
$\bar{\sigma}(\{\cdot\})$	largest singular value of $\{\cdot\}$.
diag $(a_1, \ldots, a_n)$	an $n \times n$ diagonal matrix with a_i as its i-th diagonal element.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	the state space description $C(sI - A)^{-1}B + D$.
$\mathcal{L}\{\cdot\}$	the Laplace transformation of $\{\cdot\}$.

2. Problem Formulation.

Consider the plant described by

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + B(u(t) + d_1(t)) \\ y(t) = Cx(t) + d_2(t) \end{array} \right. , \tag{1}$$

where $A \in R^{n\times n}$, $B \in R^{n\times p}$ and $C \in R^{m\times n}$, $x \in R^n$ is the state variable, $u \in R^p$ is the control input, $y \in R^m$ is the output, $d_1(t) \in R^m$ and $d_2(t) \in R^m$ are periodic disturbances with period $T > 0$ satisfying

$$d_1(t + T) = d_1(t) \quad (\forall t \geq 0) \tag{2}$$

and

$$d_2(t + T) = d_2(t) \quad (\forall t \geq 0)\,. \tag{3}$$

It is assumed that (A, B) is stabilizable and (C, A) is detectable, A has no eigenvalue on the imaginary axis and $u(t)$ and $y(t)$ are available, but $d_1(t)$ and $d_2(t)$ are unavailable. The transfer function from $u(s)$ to $y(s)$ in (1) is denoted by

$$y(s) = G(s)u(s) + G(s)d_1(s) + d_2(s), \tag{4}$$

where

$$G(s) = C(sI - A)^{-1}B \in R^{m\times p}(s). \tag{5}$$

When the disturbances $d_1(t)$ and $d_2(t)$ are unavailable, a disturbance estimator called the disturbance observer is frequently used. The disturbance observer estimates the disturbance of the periodic input disturbance $d_1(t)$ and periodic output disturbance $d_2(t)$ by using an available measurement. Since the available measurements of the plant in (1) are $u(t)$ and $y(t)$, that is, $d_1(s)$ and $d_2(s)$ are estimated by the form in

$$\tilde{d}(s) = F_1(s)e^{-sT}y(s) + F_2(s)e^{-sT}u(s), \tag{6}$$

where $F_1(s) \in RH_\infty^{m\times m}$, $F_2(s) \in RH_\infty^{m\times p}$, $u(s) = \mathcal{L}\{u(t)\}$, $y(s) = \mathcal{L}\{y(t)\}$, $\tilde{d}(s) = \mathcal{L}\left\{\tilde{d}(t)\right\}$ and $\tilde{d}(t) \in R^m$. The structure of disturbance observer $\tilde{d}(s)$ in (6) is shown in

Figure 1. In the following, we call the system $\tilde{d}(s)$ in (6) a disturbance observer for periodic input and output disturbances, if $e(t)$ written by

$$e(t) = \mathcal{L}^{-1}\{G(s)d_1(s)\} + d_2(t) - \tilde{d}(t) \tag{7}$$

satisfies

$$\lim_{t\to\infty} e(t) = 0 \tag{8}$$

for any initial state $x(0)$, control input $u(t)$ and periodic disturbances $d_1(t)$ and $d_2(t)$. We denote $e(s)$ the Laplace transformation of $e(t)$.

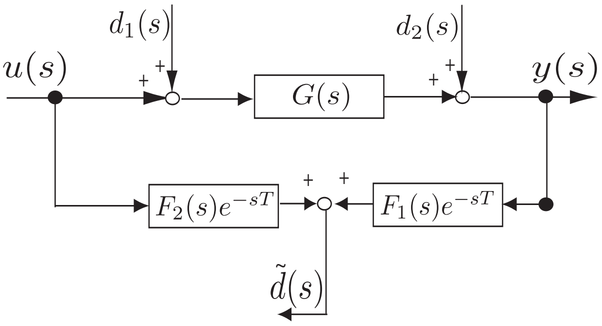

Figure 1. Structure of a disturbance observer

The problem considered in this paper is to obtain the parameterization of all disturbance observers $\tilde{d}(s)$ in (6) for the periodic input and output disturbances.

3. **Condition to Estimate the Periodic Input and Output Disturbances.** In this section, we clarify the condition of $\tilde{d}(s)$ in (6) to satisfy (8).

The condition of $\tilde{d}(s)$ in (6) satisfying (8) is summarized in the following theorem.

Theorem 3.1. *$\tilde{d}(s)$ in (6) works as a disturbance observer for periodic input and output disturbances if and only if*

$$F_1(s)N(s) + F_2(s)D(s) = 0 \tag{9}$$

and

$$\left(1 - e^{-s_iT}\right) e(s_i) = 0 \ \ \forall s_i, \tag{10}$$

where

$$s_i = j\omega_i \tag{11}$$

and

$$\omega_i = \frac{2\pi i}{T} \ \ (i = 0, 1, \ldots) \tag{12}$$

and j is the imaginary unit.

Proof: First the necessity is shown, that is, if $\tilde{d}(s)$ in (6) satisfies (8), then (9) and (10) are satisfied. $\tilde{d}(s)$ in (6) is rewritten by

$$\begin{aligned}\tilde{d}(s) = {} & \left(F_1(s)e^{-sT}N(s) + F_2(s)e^{-sT}D(s)\right)\xi(s) + F_1(s)G(s)e^{-sT}d_1(s) \\ & + F_1(s)e^{-sT}d_2(s),\end{aligned} \tag{13}$$

where $\xi(s)$ is the pseudo-state variable satisfying

$$u(s) = D(s)\xi(s), \tag{14}$$

$N(s) \in RH_\infty^{m\times p}$ and $D(s) \in RH_\infty^{m\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = N(s)D^{-1}(s). \tag{15}$$

$\xi(s)$ in (13) is factorized as

$$\xi(s) = \tilde{\xi}(s) + \bar{\xi}(s) = \frac{1}{1-e^{-sT}}\tilde{\xi}(s) + \bar{\xi}(s), \tag{16}$$

where $\tilde{\xi}(s)$ is denoted by

$$\tilde{\xi}(s) = \int_0^T e^{-sT}\xi(\tau)d\tau, \tag{17}$$

$\tilde{\xi}(s)/\left(1-e^{-sT}\right)$ means the periodic signal with period T and $\bar{\xi}(s)$ includes all other signals. From (2) and (3), $d_1(s)$ and $d_2(s)$ in (13) are written by

$$d_1(s) = \frac{1}{1-e^{-sT}}\hat{d}_1(s) \tag{18}$$

and

$$d_2(s) = \frac{1}{1-e^{-sT}}\hat{d}_2(s), \tag{19}$$

where

$$\hat{d}_i(s) = \int_0^T e^{-sT}d_i(\tau)d\tau \quad (i=1,2). \tag{20}$$

From (13), (18) and (19), $e(t)$ in (7) is given by

$$\begin{aligned} e(s) &= G(s)d_1(s) + d_2(s) - \tilde{d}(s) \\ &= \left(I - F_1(s)e^{-sT}\right)G(s)\frac{1}{1-e^{-sT}}\hat{d}_1(s) + \left(I - F_1(s)e^{-sT}\right)\frac{1}{1-e^{-sT}}\hat{d}_2(s) \\ &\quad - (F_1(s)N(s) + F_2(s)D(s))\frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s) \\ &\quad - (F_1(s)N(s) + F_2(s)D(s))e^{-sT}\bar{\xi}(s). \end{aligned} \tag{21}$$

From the assumption that $e(t)$ satisfies (8) for any $\bar{\xi}(s)$,

$$(F_1(s)N(s) + F_2(s)D(s))e^{-sT}\bar{\xi}(s) = 0 \tag{22}$$

is satisfied for any $\bar{\xi}(s)$. That is, we have (9). Substitution of (9) to (21) gives

$$e(s) = \left(I - F_1(s)e^{-sT}\right)G(s)\frac{1}{1-e^{-sT}}\hat{d}_1(s) + \left(I - F_1(s)e^{-sT}\right)\frac{1}{1-e^{-sT}}\hat{d}_2(s). \tag{23}$$

From the assumption that $e(t)$ satisfies (8) and internal model principle [32, 33], (10) is satisfied. We have thus proved the necessity.

Next the sufficiency is shown. That is, if (9) and (10) are satisfied, then $e(t)$ in (7) satisfies (8). From (9), $e(t)$ in (7) is written by

$$\begin{aligned} e(s) &= \left(I - F_1(s)e^{-sT}\right)G(s)\frac{1}{1-e^{-sT}}\hat{d}_1(s) + \left(I - F_1(s)e^{-sT}\right)\frac{1}{1-e^{-sT}}\hat{d}_2(s) \\ &\quad - (F_1(s)N(s) + F_2(s)D(s))\frac{e^{-sT}}{1-e^{-sT}}\tilde{\xi}(s). \end{aligned} \tag{24}$$

From (9) and (10), $e(t)$ in (7) satisfies (8). Thus, the sufficiency is shown.

We have thus proved Theorem 3.1.

Note that from Theorem 3.1, (9) is a condition of disturbance observers for any state variable. In addition, (10) is a condition to estimate any periodic signals. Therefore, this is the most important condition to estimate the periodic disturbances and the mentioned condition could solve the problem.

In this section, we obtained the conditions to estimate the periodic input and output disturbances. In the next section, using the result of Theorem 3.1, we clarify the parameterization of all disturbance observers for periodic disturbances.

4. Parameterization of All Disturbance Observers for Periodic Disturbances.

In this section, we propose the parameterization of all disturbance observer $\tilde{d}(s)$ in (6) for periodic input and output disturbances.

The parameterization is summarized in the following theorem.

Theorem 4.1. *The system $\tilde{d}(s)$ in (6) is the disturbance observer for periodic input and output disturbance if and only if $F_1(s)$ and $F_2(s)$ are written by*

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s) \tag{25}$$

and

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s) \in RH_\infty^{m\times p}, \tag{26}$$

where $\tilde{D}(s)\in RH_\infty^{m\times m}$ and $\tilde{N}(s)\in RH_\infty^{m\times p}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \tilde{D}^{-1}(s)\tilde{N}(s), \tag{27}$$

respectively. $Q(s) \in RH_\infty$ is any function satisfying

$$\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i) = I \ \ \forall s_i \ (i = 0, \ldots). \tag{28}$$

Proof of Theorem 4.1 requires the following lemma.

Lemma 4.1. [17] *Assume that $A(s) \in RH_\infty^{m\times n}$, $B(s) \in RH_\infty^{q\times p}$, $C(s) \in RH_\infty^{m\times p}$ and*

$$\operatorname{rank}\begin{bmatrix} A^T(s) & B^T(s) \end{bmatrix} = \gamma \tag{29}$$

are satisfied. There exist $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ satisfying

$$X(s)A(s) + Y(s)B(s) = C(s) \tag{30}$$

if and only if there exists $U(s) \in \mathcal{U}$ satisfying

$$\begin{bmatrix} A(s) \\ B(s) \\ C(s) \end{bmatrix} = U(s)\begin{bmatrix} A(s) \\ B(s) \\ 0 \end{bmatrix}. \tag{31}$$

When $X_0(s) \in RH_\infty$ and $Y_0(s) \in RH_\infty$ are solution to (30), then all solutions to (30) are given by

$$\begin{bmatrix} X(s) & Y(s) \end{bmatrix} = \begin{bmatrix} X_0(s) & Y_0(s) \end{bmatrix} + Q(s)\begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix}, \tag{32}$$

where $W_1(s)$ and $W_2(s)$ satisfy

$$W_1(s)A(s) + W_2(s)B(s) = 0 \tag{33}$$

and

$$\operatorname{rank}\begin{bmatrix} W_1(s) & W_2(s) \end{bmatrix} = n + q - \gamma \tag{34}$$

and $Q(s) \in RH_\infty^{p\times(n+q-\gamma)}$ is any function.

Using Theorem 3.1 and Lemma 4.1, Theorem 4.1 is proved.

Proof: From Theorem 3.1, $\tilde{d}(s)$ works as a disturbance observer for periodic disturbance if and only if $F_1(s) \in RH_\infty^{m\times m}$ and $F_2(s) \in RH_\infty^{m\times p}$ satisfy (9). From Lemma 4.1, all solution of $F_1(s)$ and $F_2(s)$ to satisfy (9) are given by (25) and (26), respectively, since

$$\tilde{D}(s)N(s) - \tilde{N}(s)D(s) = 0, \tag{35}$$

and Lemma 4.1, where $\tilde{D}(s) \in RH_\infty^{p\times p}$ and $\tilde{N}(s) \in RH_\infty^{p\times m}$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \tilde{D}(s)^{-1}(s)\tilde{N}(s). \tag{36}$$

The rest is to prove $\tilde{d}(s)$ in (6) works as a periodic input and output disturbance observer if and only if $Q(s)$ in (25) and (26) satisfies (28). From Theorem 3.1, $\tilde{d}(s)$ in (6) works as a periodic input and output disturbance observer if and only if $e(s)$ satisfies (10). The necessity is shown. That is if $\tilde{d}(s)$ in (6) works as a periodic input and output disturbance observer, then $Q(s)$ in (25) and (26) satisfies (28). From (25) and (26), $e(s)$ is written by

$$e(s) = \left\{I - F_1(s)e^{-sT}\right\} G(s)\frac{1}{1-e^{-sT}}\hat{d}_1(s) + \left\{I - F_1(s)e^{-sT}\right\} \frac{1}{1-e^{-sT}}\hat{d}_2(s). \tag{37}$$

This equation yields

$$\begin{aligned}\left(1-e^{-s_iT}\right)e(s_i) &= \left\{I - \left(\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i)\right)\right\} G(s)\tilde{d}_1(s_i)\\ &\quad + \left\{I - \left(\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i)\right)\right\} \tilde{d}_2(s_i)\\ &= 0.\end{aligned} \tag{38}$$

We have (28). Thus, we proved the necessity.

Next, the sufficiency is shown. That is, we show that if $Q(s)$ in (25) and (26) satisfies (28), then (10) is satisfied. $e(s)$ is written by (37). Substituting (28) to (37), it is obvious that (10) is satisfied. In this way, the sufficiency has been proved.

From the above discussion, we have thus proved Theorem 4.1. Note that from Theorem 4.1, when $G(s)$ is stable, if $Q(s)$ is settled by

$$Q(s) = \tilde{D}^{-1}(s) - I, \tag{39}$$

then $Q(s)$ in (39) satisfies (28). However, when $G(s)$ is unstable, it is difficult to set $Q(s)$ satisfying (28). For the unstable plant $G(s)$, a disturbance observer for periodic input and output disturbances is often used to attenuate disturbances effectively in [36], even if the system $\tilde{d}(s)$ in (6) satisfying (10) could not be designed. This means that in order to attenuate periodic disturbances, it is enough to estimate $(I - F(s))G(s)\tilde{d}_1(s) + (I - F(s))\tilde{d}_2(s)$, where $F(s) \in RH_\infty$ is any function. From this point of view, in the next section, when $G(s)$ is unstable, we define a linear functional disturbance observer for periodic input and output disturbance observer and clarify the parameterization of disturbance observer for periodic input and output observers for periodic input and output disturbances.

5. **Parameterization of All Linear Functional Disturbance Observers for Periodic Input and Output Disturbances.** In this section, we define a linear functional disturbance observer and present the parameterization of all linear functional disturbance observers for periodic input and output disturbances.

We call $\tilde{d}(s)$ in (6) the linear functional disturbance observer for periodic input and output disturbances if $\tilde{d}(s)$ written by

$$\left(1-e^{-s_iT}\right)e(s_i) = F(s_i)G(s_i)\hat{d}_1(s_i) + F(s_i)\hat{d}_2(s_i) \tag{40}$$

is satisfied, where $F(s) \in RH_\infty$ is any function satisfying

$$\bar{\sigma}\left\{F(s_i)\right\} \simeq 0 \;\; \forall s_i \; (i = 1, \ldots, n_{\max}) \tag{41}$$

and $n_{\max}$ is the maximum frequency satisfying (41). Since the available measurements of the plant $G(s)$ in (1) are $u(t)$ and $y(t)$ and the input disturbance $d_1(t)$ satisfies (2) and the output disturbance $d_2(t)$ satisfies (3), the periodic disturbance $d(t)$ is estimated by the form in (6), where $F_1(s) \in RH_\infty^{m\times m}$ and $F_2(s) \in RH_\infty^{m\times p}$.

The parameterization of the linear functional disturbance observer for periodic input and output disturbance is summarized as follows.

Theorem 5.1. *The system $\tilde{d}(s)$ in (6) is the linear functional disturbance observer for periodic input and output disturbance if and only if $F_1(s)$, $F_2(s)$ and $F(s)$ are described by*

$$F_1(s) = \tilde{D}(s) + Q(s)\tilde{D}(s), \tag{42}$$

$$F_2(s) = -\tilde{N}(s) - Q(s)\tilde{N}(s), \tag{43}$$

and

$$F(s) = I - F_1(s) = I - \left(\tilde{D}(s) + Q(s)\tilde{D}(s)\right), \tag{44}$$

respectively, where

$$\bar{\sigma}(I - F_1(s_i)) = \bar{\sigma}\left\{I - \left(\tilde{D}(s_i) + Q(s_i)\tilde{D}(s_i)\right)\right\} \simeq 0 \;\; \forall s_i \; (i = 1, \ldots, n_{\max}), \tag{45}$$

respectively.

Proof: First, the necessity is shown. That is, we show that if the system $\tilde{d}(s)$ in (6) is a linear functional disturbance observer for a periodic input and output disturbances, then (42), (43), (44) and (45) are satisfied. From (6), (13), (14), (15), (16) and (17), for the system $\tilde{d}(s)$ in (6), $e(s)$ is written as (21). From the assumption that $e(s)$ satisfies (8) for any $\bar{\xi}(s)$, (23) holds for any $\bar{\xi}(s)$. That is, we have (10). From (35) and Lemma 4.1, all solutions of $F_1(s)$ and $F_2(s)$ to satisfy (10) are given by (42) and (43), respectively. Substitution of (10) to (23) gives (24). From (24) the assumption that $e(s)$ satisfies (40), we have (44) and (45). In this way, the necessity has been proved.

Next, the sufficiency is shown. That is, we show that if (42), (43), (44) and (45) are satisfied, then $\tilde{d}(s)$ is a linear functional disturbance observer. Since $e(s)$ is written by (21), substituting (42), (43), (44) and (45) to (21), it is obvious that (40) is satisfied. In this way, the sufficiency has been proved.

From the above, we have thus proved Theorem 5.1.

6. **Design Method for Linear Functional Disturbance Observers.**

In this section, we show a design method for a linear functional disturbance observer $\tilde{d}(s)$.

In order to design the linear functional disturbance observer $\tilde{d}(s)$ for periodic input and output disturbances, $Q(s)$ in (42) and (43) needs to satisfy (45).

When $G(s)$ is unstable, $Q(s)$ is set as

$$Q(s) = \hat{Q}(s)\left(I - \tilde{D}(s)\right)\tilde{D}_o^{-1}(s), \tag{46}$$

where $\tilde{D}_o(s) \in RH_\infty^{m\times m}$ is an outer function of $\tilde{D}(s)$ satisfying

$$\tilde{D}(s) = \tilde{D}_o(s)\tilde{D}_i(s), \tag{47}$$

$\tilde{D}_i(s) \in RH_\infty^{m\times m}$ is a co-inner function of $\tilde{D}(s)$ satisfying $\tilde{D}_i(0) = I$ and $\tilde{D}_i(s)\tilde{D}_i(-s)^T = I$, $\hat{Q}(s) \in RH_\infty^{m\times m}$ is any function satisfying

$$\bar{\sigma}\left\{I - \hat{Q}(s_i)\tilde{D}_i(s_i)\right\} \simeq 0 \;\; \forall s_i \; (i = 1, \ldots, n_{\max}). \tag{48}$$

7. **Procedure for Linear Functional Disturbance Observers for Periodic Input and Output Disturbances.** In this section, we show a design procedure for linear functional disturbance observers for periodic input and output disturbances satisfying Theorem 5.1.

A design procedure is summarized as follows.

Procedure

1) Obtain coprime factors $\tilde{N}(s) \in RH_\infty^{m\times p}$ and $\tilde{D}(s) \in RH_\infty^{m\times m}$ of $G(s) \in R(s)^{m\times p}$ satisfying (27). The parameterization of all linear functional disturbance observers is given by (6), where $F_1(s)$, $F_2(s)$ and $F(s)$ are written by (42), (43) and (44), respectively.
2) The maximum frequency range $n_{\max}$ in (45) to estimate the periodic disturbance $d(s)$ is settled.
3) Factorize $\tilde{D}(s)$ as (47) satisfying $\tilde{D}_i(0) = I$.
4) Settle $Q(s) \in RH_\infty^{m\times m}$ satisfying (45). In order to satisfy (45), $Q(s) \in RH_\infty^{m\times m}$ is set according to (46). Here $\hat{Q}(s)$ is a low-pass filter satisfying $\hat{Q}(0) = I$, as
$$\hat{Q}(s) = \mathrm{diag}\left\{\frac{k_1}{(1+s\tau_1)^{\alpha_1}}, \ldots, \frac{k_m}{(1+s\tau_m)^{\alpha_m}}\right\}, \tag{49}$$
α_i $(i = 1, 2, \ldots, m)$ is an arbitrary positive integer and k_i $(i = 1, 2, \ldots, m)$ satisfying τ_i $(i = 1, 2, \ldots, m)$
$$\sigma\left\{I - \hat{Q}(s_i)\tilde{D}_i(s_i)\right\} \simeq 0 \ \ \forall s_i \ (i = 1, 2, \ldots, m) \tag{50}$$
are real numbers.
5) Substituting $Q(s)$ for (42), (43) and (44), $F_1(s)$, $F_2(s)$ and $F(s)$ are obtained. Then we could design disturbance observer $\tilde{d}(s)$ for periodic input and output disturbances as (6).

8. **Numerical Example.** In this section, we show numerical examples to illustrate the effectiveness of the proposed parameterizations.

Firstly, we show that the proposed design method of the disturbance observer for the stable plant in this paper could estimate the periodic disturbance more effectively than the other design method of disturbance observers. To compare the effectiveness of the proposed design method in this paper, we show a result that the disturbance observer designed by using a design method of [4] and a proposed method in this paper estimates the periodic disturbance for a Single-Input/Single-Output stable plant. Next, we show that the linear functional disturbance observer for the periodic disturbances designed by using the proposed design method in this paper could estimate the periodic disturbances for Single-Input/Single-Output unstable plant.

8.1. **Numerical example 1. A numerical example of disturbance observers for step disturbance by using a design method in [4] for the stable plant.** Consider the problem to estimate the periodic disturbance by designing a disturbance observer using a design method in [4] for stable plant $G(s)$ given as
$$G(s) = \frac{25s+1}{s^2+11s+10}. \tag{51}$$
The period T of the periodic disturbance $d(t)$ is
$$T = \pi. \tag{52}$$

The disturbance observer is denoted as

$$\tilde{d}(s) = Q(s)G(s)^{-1}y(s) + Q(s)u(s), \tag{53}$$

where $Q(s)$ in (53) is the filter satisfying $\lim_{s\to 0} Q(s) = 1$. $Q(s)$ in (53) is settled by

$$Q(s) = \frac{1}{(1+0.1s)^2}. \tag{54}$$

When the control input $u(t)$, periodic input disturbance $d_1(t)$ and periodic output disturbance $d_2(t)$ are given by

$$u(t) = 0, \tag{55}$$

$$d_1(t) = \sum_{i=1}^{3} \sin(it) \tag{56}$$

and

$$d_2(t) = \sum_{i=1}^{3} \sin(it), \tag{57}$$

respectively, the response curves of disturbance estimations are got by using the disturbance observers for the step disturbance. The response curves of disturbance estimations are shown in Figure 2. Here, the dotted line shows the periodic disturbances of $Gd_1(t) + d_2(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 2 shows that the disturbance observer $\tilde{d}(s)$ in (53) for step disturbance could not estimate $\tilde{d}(t)$ effectively.

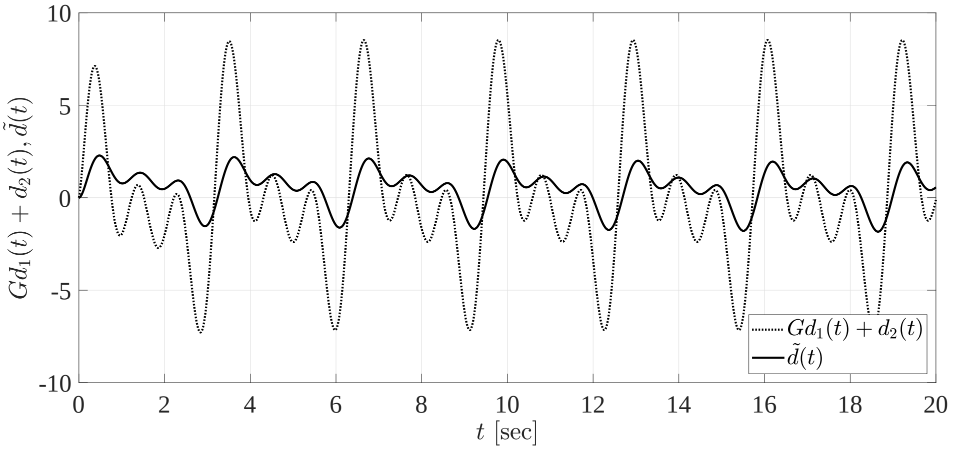

FIGURE 2. Response curves of the disturbance estimation by using a design method of [4]

8.2. **Numerical example 2. A numerical example of disturbance observers for step disturbance by using a proposed method for the stable plant.** Consider the problem to obtain the parameterization of all disturbance observers for stable plant $G(s)$ written by

$$G(s) = \frac{25s+1}{s^2+11s+10}. \tag{58}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{59}$$

Coprime factorization of $G(s)$ in (58) satisfying (27) is given by

$$\tilde{N}(s) = G(s) = \frac{25s+1}{s^2+11s+10} \tag{60}$$

and

$$\tilde{D}(s) = 1. \tag{61}$$

From Theorem 4.1, the parameterization of all disturbance observers $\tilde{d}(s)$ for stable plant $G(s)$ in (58) is given by (6), where

$$F_1(s) = 1 + Q(s), \tag{62}$$

$$F_2(s) = -\frac{25s+1}{s^2+11s+10} - Q(s)\frac{25s+1}{s^2+11s+10} \tag{63}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, we design a disturbance observer $\tilde{d}(s)$ for the periodic input and output disturbances, that is, $Q(s)$ is settled satisfying (28). In order to satisfy (28), $Q(s)$ is settled by (39).

When the control input $u(t)$, periodic input disturbance $d_1(t)$ and periodic output disturbance $d_2(t)$ are given by

$$u(t) = 0, \tag{64}$$

$$d_1(t) = \sum_{i=1}^{3} \sin(it) \tag{65}$$

and

$$d_2(t) = \sum_{i=1}^{3} \sin(it), \tag{66}$$

respectively, the response curves of disturbance estimation are got by using a proposed method. The response curves of disturbance estimations are shown in Figure 3. Here, the dotted line shows the periodic disturbances of $Gd_1(t) + d_2(t)$ and the solid line shows the disturbance observer of $\tilde{d}(t)$. Figure 3 shows that disturbance observer $\tilde{d}(s)$ in (6) could estimate $\tilde{d}(t)$ effectively.

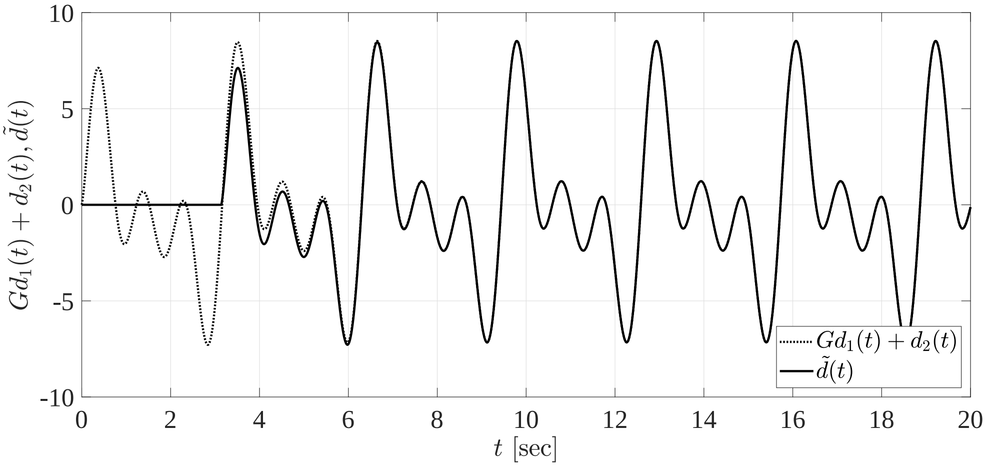

Figure 3. Response curves of the disturbance estimation

In this way, it is shown that using the obtained parameterization of all disturbance observers for periodic input and output disturbances, we could easily design a disturbance observer for periodic disturbances.

8.3. **Numerical example 3. A numerical example for disturbance observers.** Consider the problem to obtain the parameterization of all disturbance observers for stable plant $G(s)$ written by

$$G(s) = \frac{25s+1}{s^2+11s+10}. \tag{67}$$

The period T of the periodic disturbance $d(t)$ is

$$T = \pi. \tag{68}$$

Coprime factorization of $G(s)$ in (67) satisfying (27) is given by

$$\tilde{N}(s) = G(s) = \frac{25s+1}{s^2+11s+10} \tag{69}$$

and

$$\tilde{D}(s) = 1. \tag{70}$$

From Theorem 4.1, the parameterization of all disturbance observers $\tilde{d}(s)$ for stable plant $G(s)$ in (67) is given by (6), where

$$F_1(s) = 1 + Q(s), \tag{71}$$

$$F_2(s) = -\frac{25s+1}{s^2+11s+10} - Q(s)\frac{25s+1}{s^2+11s+10} \tag{72}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, we design a disturbance observer $\tilde{d}(s)$ for the periodic input and output disturbances, that is, $Q(s)$ is settled satisfying (28). In order to satisfy (28), $Q(s)$ is settled by (39).

When the control input $u(t)$, periodic input disturbance $d_1(t)$ and periodic output disturbance $d_2(t)$ are given by

$$u(t) = 0, \tag{73}$$

$$d_1(t) = \sum_{i=1}^{3} \sin(it) \tag{74}$$

and

$$d_2(t) = \sum_{i=1}^{3} \sin(it), \tag{75}$$

respectively, the response of the error $e(t)$ in (7) is shown in Figure 4. Here, the solid line shows the response of $e(t)$. Figure 4 shows that disturbance observer $\tilde{d}(s)$ in (6) for periodic input and output disturbances could estimate $\mathcal{L}^{-1}\{G(s)d_1(s)\} + d_2(t) - \tilde{d}(t)$ effectively.

In this way, it is shown that using the obtained parameterization of all disturbance observers for periodic input and output disturbances, we could easily design the disturbance observer for periodic input and output disturbances.

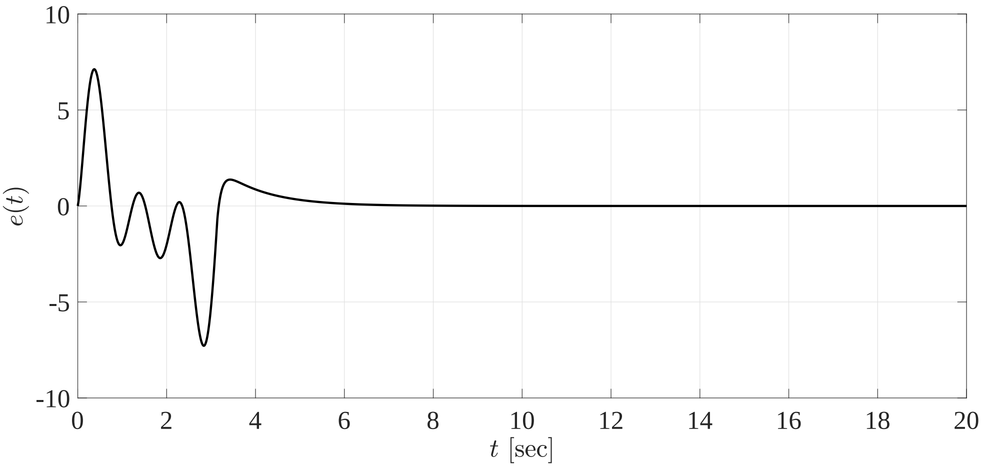

FIGURE 4. The response of the error $e(t)$ in (7)

8.4. **Numerical example 4. A numerical example for linear functional disturbance observers.** Consider the problem to obtain the parameterization of all linear functional disturbance observers for periodic input and output disturbances for unstable plant $G(s)$ described by

$$G(s) = \frac{s+1}{s^2 - 94s - 600}. \tag{76}$$

The period T of the periodic disturbances is

$$T = \pi. \tag{77}$$

A pair of coprime factors $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ of $G(s)$ in (76) satisfying (27) is given by

$$\tilde{N}(s) = \frac{-2s-2}{s^2+1006s+6000} \tag{78}$$

and

$$\tilde{D}(s) = \frac{-2s+200}{s+1000}, \tag{79}$$

respectively. From Theorem 5.1, the parameterization of all linear functional disturbance observers $\tilde{d}(s)$ is given by (6), where

$$F_1(s) = \frac{-2s+200}{s+1000} + Q(s)\frac{-2s+200}{s+1000}, \tag{80}$$

$$F_2(s) = \frac{2s+2}{s^2+1006s+6000} + Q(s)\frac{2s+2}{s^2+1006s+6000}, \tag{81}$$

$$F(s) = 1 - \frac{-2s+200}{s+1000} - Q(s)\frac{-2s+200}{s+1000} \tag{82}$$

and $Q(s) \in RH_\infty$ is any function.

Next using obtained parameterization, we design a linear functional disturbance observer $\tilde{d}(s)$ for the periodic input and output disturbances by using the procedure described in Section 7, that is, $Q(s)$ is settled satisfying (28). The maximum frequency range $n_{\max}$ in (45) to estimate the periodic disturbance $d(s)$, is settled by

$$n_{\max} = 3. \tag{83}$$

$\tilde{D}(s)$ in (79) is factorized as (47), where

$$\tilde{D}_o(s) = \frac{2s+200}{s+1000}, \tag{84}$$

and

$$\tilde{D}_i(s) = \frac{-s+100}{s+100}. \tag{85}$$

In order to satisfy (45), $\hat{Q}(s)$ is settled by

$$\hat{Q}(s) = 1. \tag{86}$$

In order to confirm that $\hat{Q}(s)$ in (86) satisfies (45), we show the gain plot of $1-\hat{Q}(s)\tilde{D}_i(s)$ in Figure 5. Figure 5 shows $\hat{Q}(s)$ in (86) satisfies (45). $Q(s)$ is set by (46) and written by

$$Q(s) = \frac{1.5s+400}{s+100}. \tag{87}$$

From (42), (43) and (44), we have $F_1(s)$, $F_2(s)$ and $F(s)$ are designed as

$$F_1(s) = \frac{-5s+500}{s+1000}, \tag{88}$$

$$F_2(s) = \frac{5s^2+1005s+1000}{s^3+1106s^2+106600s+600000} \tag{89}$$

and

$$F(s) = \frac{6s+500}{s+1000}. \tag{90}$$

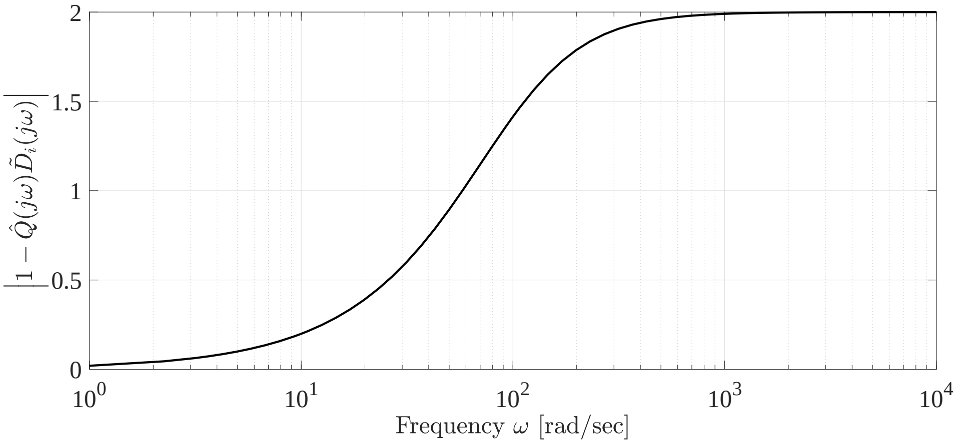

FIGURE 5. The gain plot of $1-\hat{Q}(s)\tilde{D}_i(s)$

When the control input $u(t)$, periodic input disturbance $d_1(t)$ and periodic output disturbance $d_2(t)$ are given by

$$u(t) = 0, \tag{91}$$

$$d_1(t) = \sum_{i=1}^{3} \sin(it) \tag{92}$$

and

$$d_2(t) = \sum_{i=1}^{3} \sin(it) \tag{93}$$

the response of the error $e(t)$ in (7) is shown in Figure 6. Here, the solid line shows the response of $e(t)$. Figure 6 shows that linear functional disturbance observer $\tilde{d}(s)$ in (6) for periodic input and output disturbances could estimate $\mathcal{L}^{-1}\{G(s)d_1(s)\} + d_2(t) - \tilde{d}(t)$ effectively.

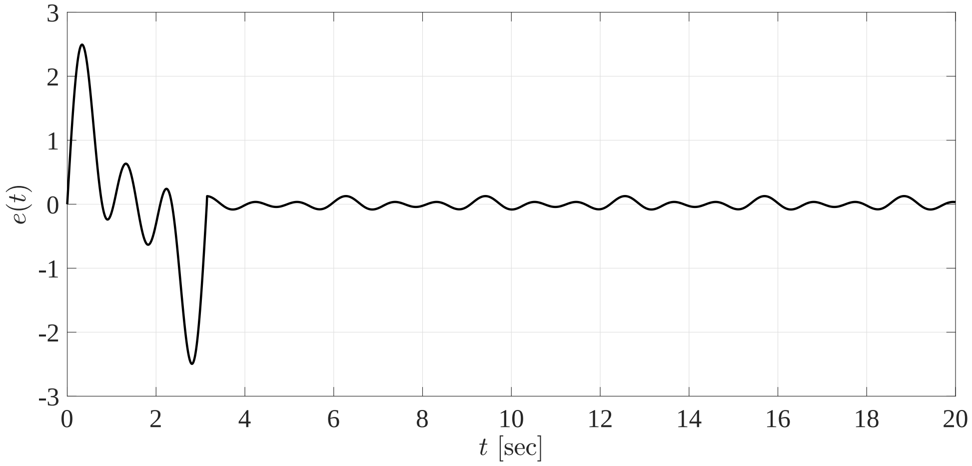

FIGURE 6. The response of the error $e(t)$ in (7)

We have thus shown that using the parameterization of all linear functional disturbance observers for periodic input and output disturbances, we could easily design a linear functional disturbance observer for periodic input and output disturbances.

9. **Conclusions.** In this paper, we have proposed parameterizations of all disturbance observers and of all linear functional disturbance observers for periodic input and output disturbances. We have shown that the proposed method could attenuate periodic disturbances effectively without using repetitive controllers. A design method and a design procedure of linear functional disturbance observer are presented. Finally, we have shown features of the proposed design method that were illustrated through numerical examples. Using obtained parameterizations, a design method of control systems will be discussed in another article.

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