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STABILITY ANALYSIS AND CONTROL OF A CAR-FOLLOWING MODEL IN A CONNECTED AND AUTONOMOUS ENVIRONMENT

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Abstract. *In order to study the traffic jams and to understand the forming process of traffic congestion in traffic flow, the paper investigated the stability of a car-following model with considering the effect of front and rear vehicles on the basis of synchronization theory of complex network in a connected and autonomous environment. Through designing the appropriate controller and using the Lyapunov stability theory, the car-following model with considering the effect of front and rear vehicles is quickly stabilized and the stability condition of the model is obtained. In addition, based on the adaptive H_∞ synchronization theory for complex networks with external disturbances, the stability of the traffic flow is studied when the vehicles are subjected to the random external disturbance. Finally, the theoretical analysis is verified by numerical simulations, and the results show that the car-following model with effect of front and rear vehicles is rapidly stabilizing and congestion phenomenon is effectively alleviated under the controller designed.*
Keywords: Car-following model, Stability, Control, Complex network, Connected and autonomous environment

1. **Introduction.** In recent years, the traffic congestion has become more and more serious, which has caused many inconveniences to people's travel, and has also caused urban environmental pollution and frequent traffic accidents. Therefore, reducing and alleviating the traffic congestion is an urgent problem we need to solve at present. In addition, the existing modes of transportation and infrastructure also gradually fail to meet people's higher requirements for travel, and the timing and rationality of building these infrastructure are difficult to meet the increment speed of travel demand. Therefore, it causes traffic congestion, traffic accidents and traffic environmental pollution and other problems, and this series of problems has also become one of the many focal issues to be solved in the transportation industry.

Furthermore, the connected vehicle technology has gradually become a new direction for the development of intelligent transportation in recent years, and its wide application prospects will certainly overturn the existing traffic mode and provide new opportunities and challenges for the development of transportation. In the connected vehicle environment, the driver of the vehicle can obtain the information of his own vehicle in real time, and can also obtain the real-time information of other vehicles on the road. Based on this, the driver of the vehicle can predict the future traffic situation on the road, adjust the running state of the vehicle in time, and make the road traffic operation more smooth. Of course, the development of connected vehicle technology should still be based on the traditional traffic flow theory. Combined with the traditional traffic flow theory, use the

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interdisciplinary knowledge of mathematics, physics, communication engineering, information science, to establish a car-following model that can more accurately describe the basic characteristics of traffic flow, and reveal the fundamental characteristics of urban traffic congestion, so it can better serve the connected vehicle technology. Therefore, we must combine the traditional traffic flow theory and connected vehicle technology, establish suitable car-following mode in the connected vehicle environment, and use the idea of control theory to effectively organize traffic flow. This is of great significance to prevent and alleviate road traffic congestion and improve the transportation safety and efficiency of road network.

In recent decades, many scholars have studied the car-following model in connected vehicle environment [1-11], and the meaningful results have been achieved. For example, Qiu et al. studied the traffic flow mixed by manual and autonomous vehicles by using the numerical simulation method, and then some new conclusions are drawn [12]. Qin et al. considered the angle feedback of electronic throttle of multi-front vehicle, the CAV tracking model is constructed, and the stability criterion of the model is obtained by using the stability analysis method [13]. To achieve a good reflection for the dynamic characteristics of traffic flow, Yao et al. proposed a discrete model of dynamic heterogeneous traffic platoon in the Internet of Vehicles [14]. Wang et al. considered the concept of driving intent sharing, and then proposed a collision avoidance early-warning algorithm based on a complex V2V environment [15]. Wu et al. considered the influence of speed and acceleration of multiple preceding vehicles in V2V environment and then the longitudinal control model in connected autonomous environment was constructed [16]. Chang et al. analyzed 4 types of car-following characteristics and its spatial distribution probability in heterogeneous traffic flow and obtained the fundamental diagram model and the passenger car equivalents, and then the sensitivity analyses on penetration rate of connected vehicle and platoon size were carried out [17]. An et al. proposed a speed guidance decision method, and formulated three speed guidance strategies with considering the phase difference of signal lights and the traffic conditions for vehicles, and then an improved car-following model was established [18]. Jia et al. proposed a single-lane speed coordinated control strategy based on trajectory to reduce the adverse effects of stop-and-go vehicles in the bottleneck section of expressways [19]. Yao et al. constructed an analysis model of the mixed traffic flow stability of human-driven vehicles and intelligent connected vehicles considering time delay, and the influence of time delay on the stability of traffic flow is studied [20].

There are two basic kinds of stability for linear car-following model, which are asymptotic stability and local stability. The object of asymptotic stability discussion is the whole motorcade, which studies the influence of the disturbance of the first vehicle on the motorcade, that is, whether the following traffic flow can return to a steady state after a period of time. And the object of discussion on local stability is a pair of adjacent vehicles, which studies the influence of disturbance of leading vehicle on the following vehicles. The analysis of the stability of the nonlinear car-following model is more complex than the linear model, and the classical Lyapunov stability theory can be used to study the linear local stability and the nonlinear stability of nonlinear car-following model. At present, many scholars have studied the stability and control of nonlinear systems [21-23]. The synchronization of complex network refers to two or more dynamic systems whose properties are the same or similar under different initial conditions, and the evolution states of each system are gradually close to each other, and finally reach the same state through the interaction between the systems. In this paper, every vehicle driving in the driveway is regarded as a node of the network. If there is a communication relationship between vehicles, it constitutes the virtual edge of the network, that is, the communication relationship between vehicles can form the adjacent matrix of the network. The network composed of vehicles and communication relations between vehicles is realizing synchronization, namely, all vehicles in the lane are running in an orderly way with the ideal velocity and maintain the ideal headway distance. In fact, the synchronization problem of the network is transformed into the stability problem of the error system composed of the car-following model. The network composed of vehicles and communication relations between vehicles realized synchronization, namely, the traffic flow tends to be stable. Based on the above analysis, the stability of the car-following model with considering the effect of front and rear vehicles is studied by using the synchronization theory of complex networks in this study. And compared with the other methods, to study the stability of a car-following model by using the synchronization theory of complex network is more practical and accurate, for example, the adjacency matrix can accurately reflect the connection relation between vehicles, the controller designed is more practical, and the numerical simulation process is more accurate and easy to implement.

The remainder of this paper is organized as follows. The description of the car-following model with considering the effect of front and rear vehicles is given in Section 2. In Section 3, the stability of a car-following model is investigated in a connected and autonomous environment. The effect of random external disturbance on the stability of car-following model is studied in Section 4. In Section 5, the numerical simulations are carried out to show the validity of the theoretical analysis. Finally, Section 6 concludes the paper.

2. **A Car-Following Model with Effect of Front and Rear Vehicles.** By using the concept of "velocity-headway function", Newell proposed a new car-following model in 1961 [24]. Bando et al. further proposed a correction model in 1995 [25]. And we investigated the stability of a general nonlinear car-following model by using the synchronization theory of complex network in [26], but the model without considering the impact of rear vehicles. As we all know, the vehicles driving on the single lane are not only affected by the vehicle in front, but also may be urged by the nearest neighbor vehicles in the rear. In order to avoid collision, the current vehicle will also focus on the distance between the rear nearest neighboring vehicle and its own. Therefore, we study the car-following model with considering the effect of front and rear vehicles in this paper. In this model, the optimization velocity function depends on the headway distance between the front car and its own as well as the headway distance between the rear car and its own. Next, we give the differential equations of the car-following model with considering the effect of front and rear vehicles [27]:

$$\begin{cases} \dot{h}_i(t) = v_{i-1}(t) - v_i(t), \\ \dot{h}_{i+1}(t) = v_i(t) - v_{i+1}(t), \\ \dot{v}_i(t) = a\left[V(h_i(t), h_{i+1}(t)) - v_i(t)\right], \end{cases} \quad i = 1, 2, \ldots, N \tag{1}$$

where $a > 0$ is the sensitivity coefficient of driver, $v_i(t)$ is the velocity of vehicle i at time t, $v_{i-1}(t)$ is the velocity of vehicle $i-1$ at time t, $v_{i+1}(t)$ is the velocity of vehicle $i+1$ at time t, $h_i(t)$ is the headway distance between the $i-1$th and ith vehicles at time t, $h_{i+1}(t)$ is the headway distance between the ith and $i+1$th vehicles at time t, and $V(h_i(t), h_{i+1}(t))$ represents the modified optimized velocity function, which is defined as follows:

$$V(h_i(t), h_{i+1}(t)) = \tanh\left(\bar{h}_i(t) - h_d\right) + \tanh(h_d), \tag{2}$$

where $\bar{h}_i(t) = \alpha_1 h_i(t) + \alpha_2 h_{i+1}(t)$ is the average distance between two cars, α_1, α_2 are weighting coefficients of $h_i(t)$ and $h_{i+1}(t)$, respectively, $\alpha_1 + \alpha_2 = 1$, and h_d is the ideal headway distance.

Lemma 2.1. *$V(h_i(t))$ is a continuous, nonnegative and monotonic decreasing function. And there exists a constant $k > 0$, for $\forall h_i(t), h_j(t) \in R^n$, one has*

$$\big\|V(h_i(t)) - V(h_j(t))\big\| \le k\big\|h_i(t) - h_j(t)\big\|.$$

For the motion where N vehicles are running on a single lane without overtaking under an open boundary, we assume each vehicle as a node of a graph and then the coupling topology of the traffic flow system is described by a simple graph. Let $A = (a_{ij}) \in R^{N\times N}$ be an adjacency matrix of the relationship of communications among the vehicles, where $a_{ii} = -\sum_{j=1,i\neq j}^{N} a_{ij}$ and a_{ij} is defined as follows: if vehicle j is the front neighboring vehicle of vehicle i $(i \neq j)$, then $a_{ij} = a_{ji} = 1$; otherwise $a_{ij} = 0$. Consider that each vehicle i $(i = 2, 3, \dots, N)$ communicates with the front nearest neighboring vehicle $i - 1$, and it will also be urged by the rear nearest neighboring vehicle $i + 1$. Therefore, we can get the adjacency matrix A of the vehicles movement as follows:

$$A = \begin{pmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2 & 1 \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{pmatrix}.$$

Let the ideal velocity of the vehicle be $v_0(t)$, and the ideal headway distance be $h_0(t)$. Then the purpose of this study is to design a suitable controller by using synchronization control theory of complex networks, such that all vehicles are running in an orderly way with ideal velocity $v_0(t)$ and maintain the ideal headway distance $h_0(t)$.

3. **Stability Analysis of Car-Following Model Based on Complex Networks Synchronization.**

3.1. **Theory of synchronization of complex networks.**

Consider a general model of complex networks consisting of N nodes described as follows:

$$\dot{x}_i(t) = f(x_i(t)) + \varepsilon \sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + u_i(t), \quad i = 1, 2, \dots, N \tag{3}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^{\mathrm{T}} \in R^n$ denotes the state vector of the ith node, $f(\cdot) : R^n \to R^n$ represents the continuous nonlinear function of the network (3), $\Gamma \in R^{n\times n}$ represents the internal coupling function between two connected nodes, and the topology of complex networks is represented by the coupling matrix $A = (a_{ij}) \in R^{N\times N}$, where the entries a_{ij} are defined as follows: if there is a link from node j to i $(i \neq j)$, then let $a_{ij} > 0$ $(i \neq j)$; otherwise $a_{ij} = 0$ $(i \neq j,\ i, j = 1, 2, \dots, N)$, and $a_{ii} = -\sum_{j=1,i\neq j}^{N} a_{ij}$, and $u_i(t)$ is the controller of node i which needs to be designed according to the specific networks structure.

Definition 3.1. *Suppose that $x_i(t; t_0, X_0)$ $(i = 1, 2, \dots, N)$ is the solution of network (3), where $X_0 = \left(x_1^0, x_2^0, \dots, x_N^0\right)^{\mathrm{T}} \in R^{nN}$. And the mapping $f : \Omega \to R^n$ is continuous and differentiable with $\Omega \subseteq R^n$. If there exists a nonempty open subset $\Lambda \subseteq \Omega$ $(t \ge 0)$ with $x_i^0 \in \Lambda$, which makes $x_i(t; t_0, X_0) \in \Omega$ $(1 \le i \le N)$, and*

$$\lim_{t\to+\infty} \|x_i(t; t_0, X_0) - s(t, t_0, x_0)\| = 0, \quad (i = 1, 2, \dots, N) \tag{4}$$

then the network (3) has reached the synchronization, where $s(t, t_0, x_0)$ is the solution of $\dot{x}_i(t) = f(x_i(t))$ with $x_0 \subseteq \Omega$.

Let $e_i(t) = x_i(t) - s(t)$, $i = 1, 2, \ldots, N$ be the errors vector, the purpose of the controller $u_i(t)$ is to achieve network (3) synchronization, that is to say, $\lim_{t\to+\infty} \|e_i(t)\| = 0$ ($i = 1, 2, \ldots, N$), and then the error system can be described as follows:

$$\dot{e}_i(t) = f(x_i(t)) - f(s(t)) + \varepsilon \sum_{j=1}^{N} a_{ij}\Gamma(x_j(t) - s(t)) + u_i(t). \tag{5}$$

Assumption 3.1. *There exists a positive constant L, such that*

$$\|f(y(t)) - f(x(t))\| \le L\|y(t) - x(t)\|,$$

where $\forall x(t), y(t) \in R^n$.

Theorem 3.1. *Suppose that Assumption 3.1 holds, and the network (3) can realize synchronization under the following controller:*

$$\begin{cases} u_i(t) = -d_i e_i(t), \\ \dot{d}_i = k_i \|e_i(t)\|^2, \end{cases} \tag{6}$$

where k_i is any positive constant, and $i = 1, 2, \ldots, N$.

Proof: Choose the following Lyapunov candidate function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\frac{1}{k_i}\left(d_i - \bar{d}\right)^2, \tag{7}$$

where $\bar{d}$ is the larger enough positive constant which needs to be determined. Then we get the derivative of $V(t)$ along (5) as follows:

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\dot{e}_i(t) + \sum_{i=1}^{N}\frac{1}{k_i}\left(d_i - \bar{d}\right)\dot{d}_i \\ &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\left[f(x_i(t)) - f(s(t)) + \varepsilon\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) - d_i e_i(t)\right] + \sum_{i=1}^{N}\left(d_i - \bar{d}\right)\|e_i(t)\|^2 \\ &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\left[f(x_i(t)) - f(s(t)) + \varepsilon\sum_{j=1}^{N} a_{ij}\Gamma e_j(t) - \bar{d}e_i(t)\right]. \end{aligned}$$

Since

$$e_i^{\mathrm{T}}(t)\left[f(x_i(t)) - f(s(t))\right] \le \|e_i(t)\|\|f(x_i(t)) - f(s(t))\| \le L\|e_i(t)\|^2 = Le_i^{\mathrm{T}}(t)e_i(t),$$

we can get

$$\dot{V}(t) \le \sum_{i=1}^{N}\left(L - \bar{d}\right)\|e_i(t)\|^2 + \varepsilon\sum_{i=1}^{N}\sum_{j=1}^{N} e_i^{\mathrm{T}}(t)a_{ij}\Gamma e_j(t) = e(t)^{\mathrm{T}}Qe(t),$$

where $e(t) = (e_1(t), e_2(t), \ldots, e_N(t))^{\mathrm{T}} \in R^{nN}$, $Q = \left(P + P^{\mathrm{T}}\right)/2$, $P = (L - \bar{g})\, I_{nN} + \varepsilon(A \otimes \Gamma)$, I_{nN} is a unit $n \times N$ matrix. Apparently, there exists a larger enough $\bar{d} > 0$ which makes Q negative definite, that is, $\dot{V}(t) \le 0$. Here, the largest invariant set contained in set $E = \left\{\dot{V}(t) = 0\right\} = \{e_i(t) = 0, i = 1, 2, \ldots, N\}$ can be described as

$$M = \left\{(e, d) \in R^{nN} \times R^N : e = 0, \dot{d} = 0\right\},$$

where $d = (d_1, d_2, \ldots, d_N)^{\mathrm{T}}$. Based on the LaSalle's invariance principle, starting with arbitrary initial values, the trajectory asymptotically converges to the largest invariant

M which implies that $\lim_{t\to\infty}\|e_i(t)\| = 0$, $i = 1, 2, \ldots, N$. That is, the network (3) is synchronized.

3.2. **Stability analysis of car-following model.** Under the open boundary condition, we consider the situation of N vehicles running on the single lane without overtaking. Then take model (1) as the network's node, and the differential equations of the car-following model can be written as follows:

$$\begin{bmatrix} \dot{h}_i(t) \\ \dot{h}_{i+1}(t) \\ \dot{v}_i(t) \end{bmatrix} = \begin{bmatrix} v_{i-1}(t) - v_i(t) \\ v_i(t) - v_{i+1}(t) \\ a\left[\tanh\left(\bar{h}_i(t) - h_d\right) + \tanh(h_d) - v_i(t)\right] \end{bmatrix}. \tag{8}$$

Assume that $\Gamma = diag\{1, 1, \ldots, 1\}$, and the controller designed as follows:

$$\begin{cases} u_i(t) = -d_i e_i(t), \\ \dot{d}_i = k_i \|e_i(t)\|^2, \end{cases} \quad i = 1, 2, \ldots, N \tag{9}$$

where $e_i(t) = \left[e_i^1(t), e_i^2(t), e_i^3(t)\right]^{\mathrm{T}}$, $e_i^1(t) = h_i(t) - h_0(t)$, $e_i^2(t) = h_{i+1}(t) - h_0(t)$, $e_i^3(t) = v_i(t) - v_0(t)$, k_i is the positive constant.

On the basis of Equation (3), the dynamical equations of node i $(1 \le i \le N)$ can be described by

$$\begin{bmatrix} \dot{h}_i(t) \\ \dot{h}_{i+1}(t) \\ \dot{v}_i(t) \end{bmatrix} = \begin{bmatrix} v_{i-1}(t) - v_i(t) \\ v_i(t) - v_{i+1}(t) \\ a\left[\tanh\left(\bar{h}_i(t) - h_d\right) + \tanh(h_d) - v_i(t)\right] \end{bmatrix} + \begin{bmatrix} N_{i1}(t) \\ N_{i2}(t) \\ N_{i3}(t) \end{bmatrix} + u_i(t), \tag{10}$$

where

$$\begin{aligned} N_{i1}(t) &= \varepsilon\left[a_{i1}h_1(t) + a_{i2}h_2(t) + \cdots + a_{iN}h_N(t)\right], \\ N_{i2}(t) &= \varepsilon\left[a_{i1}h_2(t) + a_{i2}h_3(t) + \cdots + a_{iN}h_{N+1}(t)\right], \\ N_{i3}(t) &= \varepsilon\left[a_{i1}v_1(t) + a_{i2}v_2(t) + \cdots + a_{iN}v_N(t)\right]. \end{aligned}$$

Suppose that

$$\begin{aligned} &h'(t) = \left[h_1(t), h_2(t), \ldots, h_N(t)\right]^{\mathrm{T}}, \quad h''(t) = \left[h_2(t), h_3(t), \ldots, h_{N+1}(t)\right]^{\mathrm{T}}, \\ &v(t) = \left[v_1(t), v_2(t), \ldots, v_N(t)\right]^{\mathrm{T}}, \end{aligned}$$

and the model (10) can be converted to

$$\begin{bmatrix} \dot{h}'(t) \\ \dot{h}''(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} \Xi_1 v(t) + \varepsilon A_1 h'(t) \\ \Xi_2 v(t) + \varepsilon A_1 h''(t) \\ (\varepsilon A_1 - A_2)v(t) + A_2\left[\tanh\left(\bar{h}(t) - h_d\right) + \tanh(h_d)\right] \end{bmatrix} + \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix}, \tag{11}$$

where

$$A_1 = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix}, \quad A_2 = aI_N,$$

$$\begin{aligned} &\bar{h}(t) = \left[\bar{h}_1(t), \bar{h}_2(t), \ldots, \bar{h}_N(t)\right]^{\mathrm{T}}, \\ &u_1(t) = \left[-d_1(h_1(t) - h_0(t)), -d_2(h_2(t) - h_0(t)), \ldots, -d_N(h_N(t) - h_0(t))\right]^{\mathrm{T}}, \\ &u_2(t) = \left[-d_1(h_2(t) - h_0(t)), -d_2(h_3(t) - h_0(t)), \ldots, -d_N(h_{N+1}(t) - h_0(t))\right]^{\mathrm{T}}, \\ &u_3(t) = \left[-d_1(v_1(t) - v_0(t)), -d_2(v_2(t) - v_0(t)), \ldots, -d_N(v_N(t) - v_0(t))\right]^{\mathrm{T}}, \end{aligned}$$

$$\Xi_1 = \begin{bmatrix} -1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}, \quad \Xi_2 = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}.$$

And then we transformed model (11) to the following form:

$$\dot{x}(t) = A'x(t) + F(x(t)) + u(t), \tag{12}$$

where

$$x(t) = \begin{bmatrix} h'(t) \\ h''(t) \\ v(t) \end{bmatrix}, \quad A' = \begin{bmatrix} \varepsilon A_1 & 0 & \Xi_1 \\ 0 & \varepsilon A_1 & \Xi_2 \\ 0 & 0 & \varepsilon A_1 - A_2 \end{bmatrix}, \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix},$$

$$F(x(t)) = \begin{bmatrix} 0 \\ 0 \\ A_2 \left[\tanh\left(\bar{h}(t) - h_d\right) + \tanh(h_d)\right] \end{bmatrix}.$$

Owing to the fact that $s(t) = (h_0'(t), h_0''(t), v_0(t))^{\mathrm{T}}$ is the solution of $\dot{x}_i(t) = f(x_i(t))$, and we can get

$$F(x(t)) - F(s(t)) = \begin{bmatrix} 0 \\ 0 \\ A_2 \left[V\left(\bar{h}(t)\right)\right] - A_2 \left[V\left(\bar{h}_0(t)\right)\right] \end{bmatrix}, \tag{13}$$

we have

$$\|F(x(t)) - F(s(t))\| \leq \left\| \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_2 \end{bmatrix} \right\| \cdot \left\| \begin{bmatrix} 0 \\ 0 \\ V\left(\bar{h}(t)\right) - V\left(\bar{h}_0(t)\right) \end{bmatrix} \right\|. \tag{14}$$

Based on Lemma 2.1, there is a positive constant k, such that

$$\|V(h(t)) - V(h_0(t))\| \leq k\|h(t) - h_0(t)\|,$$

and we have

$$\|F(x(t)) - F(s(t))\| \leq \Theta' k\sqrt{\left(\bar{h}(t) - \bar{h}_0(t)\right)^2}$$
$$\leq \Theta' k\sqrt{\left[\alpha_1 \left(h'(t) - h_0'(t)\right) + \alpha_2 \left(h''(t) - h_0''(t)\right)\right]^2}$$

$$\Theta' k\sqrt{2\alpha_1^2 \left(h'(t) - h_0'(t)\right)^2 + 2\alpha_2^2 \left(h''(t) - h_0''(t)\right)^2}$$
$$\leq \sqrt{2}\Theta' k\sqrt{\left(h'(t) - h_0'(t)\right)^2 + \left(h''(t) - h_0''(t)\right)^2}$$
$$\leq \sqrt{2}\Theta' k\sqrt{\left(h'(t) - h_0'(t)\right)^2 + \left(h''(t) - h_0''(t)\right)^2 + \left(v(t) - v_0(t)\right)^2}$$
$$= \sqrt{2}\Theta' k \left\|x(t) - s(t)\right\|, \tag{15}$$

where $\Theta' = \left\| \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_2 \end{bmatrix} \right\|$.

Therefore, there exists a constant $L = \sqrt{2}\Theta' k > 0$, such that

$$\|F(x(t)) - F(s(t))\| \leq L \left\|x(t) - s(t)\right\|, \tag{16}$$

that is, Assumption 3.1 holds. Based on Theorem 3.1, the network of traffic flow formed by vehicles realized synchronization, namely, all vehicles are running in an orderly way with ideal velocity $v_0(t)$ and maintain the ideal headway distance $h_0(t)$.

4. Stability Analysis of Car-Following Model under External Disturbance.

4.1. Adaptive H_∞ synchronization for complex networks with external disturbance. Consider a general model of complex networks with external disturbances consisting of N nodes described as follows:

$$\dot{x}_i(t) = f(x_i(t)) + \varepsilon \sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + B\omega_i(t) + u_i(t), \quad i = 1, 2, \ldots, N \tag{17}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^{\mathrm{T}} \in R^n$ denotes the state vector of the ith node, $f(\cdot) : R^n \to R^n$ represents the continuous nonlinear vector-valued function of the network (17), $\Gamma \in R^{n\times n}$ represents the internal coupling function between two connected nodes, and the topology of complex networks (17) is represented by the coupling matrix $A = (a_{ij}) \in R^{N\times N}$, where the entries a_{ij} are defined as follows: if there is a link from node j to i ($i \neq j$), then let $a_{ij} > 0$ ($i \neq j$); otherwise, $a_{ij} = 0$ ($i \neq j$, $i, j = 1, 2, \ldots, N$), and $a_{ii} = -\sum_{j=1,i\neq j}^{N} a_{ij}$, B is a matrix with appropriate dimension, $\omega_i(t)$ is external disturbance, and $u_i(t)$ is the controller of node i which needs to be designed according to the specific networks structure.

Let $e_i(t) = x_i(t) - s(t)$, $i = 1, 2, \ldots, N$ be the errors vector, and then the error system can be described as follows:

$$\begin{aligned}\dot{e}_i(t) &= f(x_i(t)) + \varepsilon \sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + B\omega_i(t) + u_i(t) - f(s(t)) - \varepsilon \sum_{j=1}^{N} a_{ij}\Gamma s(t) \\ &= f(x_i(t)) - f(s(t)) + \varepsilon \sum_{j=1}^{N} a_{ij}\Gamma e_j(t) + B\omega_i(t) + u_i(t), \quad i = 1, 2, \ldots, N\end{aligned} \tag{18}$$

Definition 4.1. *For a given level $\gamma > 0$ and a symmetric positive definite matrix S, if*

a) when $\omega_i(t) = 0$, and the system (18) is asymptotically adaptive stable under the controller $u_i(t)$;

b) when $\omega_i(t) \neq 0$ and under zero initial conditions, one has

$$\int_0^\infty e_i^{\mathrm{T}}(t)Se_i(t)dt < \gamma^2 \int_0^\infty \omega_i^{\mathrm{T}}(t)\omega_i(t)dt, \tag{19}$$

then the network (17) is adaptive H_∞ synchronization.

Lemma 4.1. *For the given matrix $X, Y \in R^{n\times m}$, the following matrix inequality holds:*

$$X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leq X^{\mathrm{T}}AX + Y^{\mathrm{T}}A^{-1}Y, \tag{20}$$

where $A \in R^{n\times n}$ and $A^{\mathrm{T}} = A > 0$.

Theorem 4.1. *For $\gamma > 0$ and symmetric positive definite matrix S. Assume that Assumption 3.1 holds, and there exists positive definite matrix P and a sufficiently large positive constant $\tilde{d}$, such that*

$$\lambda_{\max}(P)L^{\mathrm{T}}L + P + \varepsilon a' \left(\Gamma^{\mathrm{T}}P + P\Gamma\right) - 2\tilde{d}P + \frac{1}{\gamma^2}PBB^{\mathrm{T}}P + S < 0, \tag{21}$$

and the network (17) can realize H_∞ synchronization under the following controller:

$$\begin{cases} u_i(t) = -d_i e_i(t), \\ \dot{d}_i = k_i e_i^{\mathrm{T}}(t) P e_i(t), \end{cases} \quad i = 1, 2, \ldots, N \tag{22}$$

where $a' = \max_{1 \le i \le N} \sum_{j=1}^{N} a_{ij}$ *and* $\lambda_{\max}(P)$ *is the maximum eigenvalue of matrix* P*,* k_i *is any positive constant.*

Proof: Choose the following Lyapunov candidate function:

$$V(t) = \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) P e_i(t) + \sum_{i=1}^{N} \frac{1}{k_i} \left(d_i - \tilde{d} \right)^2, \tag{23}$$

where $\tilde{d}$ is the larger enough positive constant which needs to be determined. Then we get the derivative of $V(t)$ along (18) as follows:

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^{N} \dot{e}_i^{\mathrm{T}}(t) P e_i(t) + \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) P \dot{e}_i(t) + 2 \sum_{i=1}^{N} \left(d_i - \tilde{d} \right) e_i^{\mathrm{T}}(t) P e_i(t) \\
&= \sum_{i=1}^{N} \left[(f(x_i(t)) - f(s(t)))^{\mathrm{T}} P e_i(t) + \varepsilon \sum_{j=1}^{N} e_i^{\mathrm{T}}(t) a_{ij} \Gamma^{\mathrm{T}} P e_j(t) + \omega_i^{\mathrm{T}}(t) B^{\mathrm{T}} P e_i(t) \right] \\
&\quad + \sum_{i=1}^{N} \left[e_i^{\mathrm{T}}(t) P (f(x_i(t)) - f(s(t))) + \varepsilon \sum_{j=1}^{N} e_i^{\mathrm{T}}(t) a_{ij} P \Gamma e_j(t) + e_i^{\mathrm{T}}(t) P B \omega_i(t) \right] \\
&\quad - 2\tilde{d} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t) P e_i(t) \\
&= \sum_{i=1}^{N} \left[(f(x_i(t)) - f(s(t)))^{\mathrm{T}} P e_i(t) + e_i^{\mathrm{T}}(t) P (f(x_i(t)) - f(s(t))) \right] \\
&\quad + \sum_{i=1}^{N} \left[\varepsilon \sum_{j=1}^{N} e_i^{\mathrm{T}}(t) a_{ij} \Gamma^{\mathrm{T}} P e_j(t) + \varepsilon \sum_{j=1}^{N} e_i^{\mathrm{T}}(t) a_{ij} P \Gamma e_j(t) - 2\tilde{d} e_i^{\mathrm{T}}(t) P e_i(t) \right] \\
&\quad + \sum_{i=1}^{N} \left[\omega_i^{\mathrm{T}}(t) B^{\mathrm{T}} P e_i(t) + e_i^{\mathrm{T}}(t) P B \omega_i(t) \right].
\end{aligned}$$

Based on Assumption 3.1, we have

$$\begin{aligned}
&\quad (f(x_i(t)) - f(s(t)))^{\mathrm{T}} P e_i(t) + e_i^{\mathrm{T}}(t) P (f(x_i(t)) - f(s(t))) \\
&= (f(x_i(t)) - f(s(t)))^{\mathrm{T}} P (f(x_i(t)) - f(s(t))) + e_i^{\mathrm{T}}(t) P e_i(t) \\
&\quad - \left[P^{\frac{1}{2}} (f(x_i(t)) - f(s(t))) - P^{\frac{1}{2}} e_i(t) \right]^{\mathrm{T}} \left[P^{\frac{1}{2}} (f(x_i(t)) - f(s(t))) - P^{\frac{1}{2}} e_i(t) \right] \\
&\le (f(x_i(t)) - f(s(t)))^{\mathrm{T}} P (f(x_i(t)) - f(s(t))) + e_i^{\mathrm{T}}(t) P e_i(t) \\
&\le \lambda_{\max}(P) e_i^{\mathrm{T}}(t) L^{\mathrm{T}} L e_i(t) + e_i^{\mathrm{T}}(t) P e_i(t).
\end{aligned}$$

And according to Lemma 4.1, we get

$$\omega_i^{\mathrm{T}}(t) B^{\mathrm{T}} P e_i(t) + e_i^{\mathrm{T}}(t) P B \omega_i(t) \le \frac{1}{\gamma^2} e_i^{\mathrm{T}}(t) P B B^{\mathrm{T}} P e_i(t) + \gamma^2 \omega_i^{\mathrm{T}}(t) \omega_i(t).$$

Thus, one has

$$\dot{V}(t) \le \sum_{i=1}^{N} \left[\varepsilon \sum_{j=1}^{N} a_{ij} e_i^{\mathrm{T}}(t) \Gamma^{\mathrm{T}} P e_j(t) + \varepsilon \sum_{j=1}^{N} a_{ij} e_i^{\mathrm{T}}(t) P \Gamma e_j(t) - 2\tilde{d} e_i^{\mathrm{T}}(t) P e_i(t) \right]$$

$$+\sum_{i=1}^{N}\left[\lambda_{\max}(P)e_i^{\mathrm{T}}(t)L^{\mathrm{T}}Le_i(t)+e_i^{\mathrm{T}}(t)Pe_i(t)\right]$$
$$+\sum_{i=1}^{N}\left[\frac{1}{\gamma^2}e_i^{\mathrm{T}}(t)PBB^{\mathrm{T}}Pe_i(t)+\gamma^2\omega_i^{\mathrm{T}}(t)\omega_i(t)\right].$$

Therefore, there is

$$\dot{V}(t)+\sum_{i=1}^{N}\left[e_i^{\mathrm{T}}(t)Se_i(t)-\gamma^2\omega_i^{\mathrm{T}}(t)\omega_i(t)\right]$$
$$\leq\sum_{i=1}^{N}\left[\varepsilon\sum_{j=1}^{N}a_{ij}e_i^{\mathrm{T}}(t)\Gamma^{\mathrm{T}}Pe_j(t)+\varepsilon\sum_{j=1}^{N}a_{ij}e_i^{\mathrm{T}}(t)P\Gamma e_j(t)-2\tilde{d}e_i^{\mathrm{T}}(t)Pe_i(t)\right]$$
$$+\sum_{i=1}^{N}\left[\lambda_{\max}(P)e_i^{\mathrm{T}}(t)L^{\mathrm{T}}Le_i(t)+e_i^{\mathrm{T}}(t)Pe_i(t)\right]$$
$$+\sum_{i=1}^{N}\frac{1}{\gamma^2}e_i^{\mathrm{T}}(t)PBB^{\mathrm{T}}Pe_i(t)+\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)Se_i(t)$$
$$\leq\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\left[\lambda_{\max}(P)L^{\mathrm{T}}L+P+\varepsilon a'\left(\Gamma^{\mathrm{T}}P+P\Gamma\right)-2\tilde{d}P+\frac{1}{\gamma^2}PBB^{\mathrm{T}}P+S\right]e_i(t).$$

If

$$\lambda_{\max}(P)L^{\mathrm{T}}L+P+\varepsilon a'\left(\Gamma^{\mathrm{T}}P+P\Gamma\right)-2\tilde{d}P+\frac{1}{\gamma^2}PBB^{\mathrm{T}}P+S<0,$$

then we have

$$\dot{V}(t)\leq\sum_{i=1}^{N}\left[\gamma^2\omega_i^{\mathrm{T}}(t)\omega_i(t)-e_i^{\mathrm{T}}(t)Se_i(t)\right],$$

and

$$\int_0^{\infty}e_i^{\mathrm{T}}(t)Se_i(t)dt<\gamma^2\int_0^{\infty}\omega_i^{\mathrm{T}}(t)\omega_i(t)dt,$$

and then the network (17) can realize H_∞ synchronization with level $\gamma>0$.

4.2. **Stability analysis of the model with external disturbance.** Under the open boundary condition, we also consider the situation of N vehicles are running on the single lane without overtaking. In the case of external disturbance, we take model (1) as the network's node. Suppose that $\Gamma=diag\{1,1,\dots,1\}$, and we design the following controller:

$$\begin{cases}u_i(t)=-d_ie_i(t),\\ \dot{d}_i=k_ie_i^{\mathrm{T}}(t)Pe_i(t),\end{cases}\quad i=1,2,\dots,N \tag{24}$$

where $e_i(t)=\left[e_i^1(t),e_i^2(t),e_i^3(t)\right]^{\mathrm{T}}$, $e_i^1(t)=h_i(t)-h_0(t)$, $e_i^2(t)=h_{i+1}(t)-h_0(t)$, $e_i^3(t)=v_i(t)-v_0(t)$, k_i is the positive constant.

On the basis of Equation (17), the dynamical equations of node i $(1\leq i\leq N)$ can be described by

$$\dot{x}_i(t)=\begin{pmatrix}v_{i-1}(t)-v_i(t)\\ v_i(t)-v_{i+1}(t)\\ a\left[\tanh(\alpha_1h_i(t)+\alpha_2h_{i+1}(t)-h_d)+\tanh(h_d)-v_i(t)\right]\end{pmatrix}+\begin{pmatrix}M_{i1}(t)\\ M_{i2}(t)\\ M_{i3}(t)\end{pmatrix}$$

$$+ B\omega_i(t) + u_i(t), \tag{25}$$

where

$$x_i(t) = [h_i(t), h_{i+1}(t), v_i(t)]^{\mathrm{T}}, \quad M_{i1}(t) = \varepsilon\left[a_{i1}h_1(t) + a_{i2}h_2(t) + \cdots + a_{iN}h_N(t)\right],$$
$$M_{i2}(t) = \varepsilon\left[a_{i1}h_2(t) + a_{i2}h_3(t) + \cdots + a_{iN}h_{N+1}(t)\right],$$
$$M_{i3}(t) = \varepsilon\left[a_{i1}v_1(t) + a_{i2}v_2(t) + \cdots + a_{iN}v_N(t)\right].$$

Based on Formula (16), Assumption 3.1 holds. Thus, according to Theorem 4.1, if there exist a sufficiently large positive constant $\tilde{d}$ and a positive definite matrix P, which makes $\lambda_{\max}(P)L^{\mathrm{T}}L + P + \varepsilon a'\left(\Gamma^{\mathrm{T}}P + P\Gamma\right) - 2\tilde{d}P + \frac{1}{\gamma^2}PBB^{\mathrm{T}}P + S < 0$, then the network of traffic flow formed by vehicles realized H_∞ synchronization, namely, all vehicles are running in an orderly way with ideal velocity $v_0(t)$ and maintain the ideal headway distance $h_0(t)$ under the random external disturbances.

5. **Numerical Simulations.** Under the open boundary condition, we consider the situation of 20 vehicles running on the single lane without overtaking. Let $\varepsilon = 0.6$, $a = 1.2$ s^{-1}, $h_d = 2$ m, $v_0(t) = 5.56$ m/s, $h_0(t) = 7.02$ m. Considering the influence of the front vehicle and the rear vehicle on the current vehicle speed, in the actual driving process of the driver, our sensitivity to the front car will be greater than the sensitivity to the rear vehicle, so we let $\alpha_1 = 0.8$, $\alpha_2 = 0.2$. In addition, consider that the shortest safe distance between vehicles is 7.02 m, and the ideal speed is 5.56 m/s. In order to ensure the driving safety at a high speed and the driver's driving behavior is more in line with

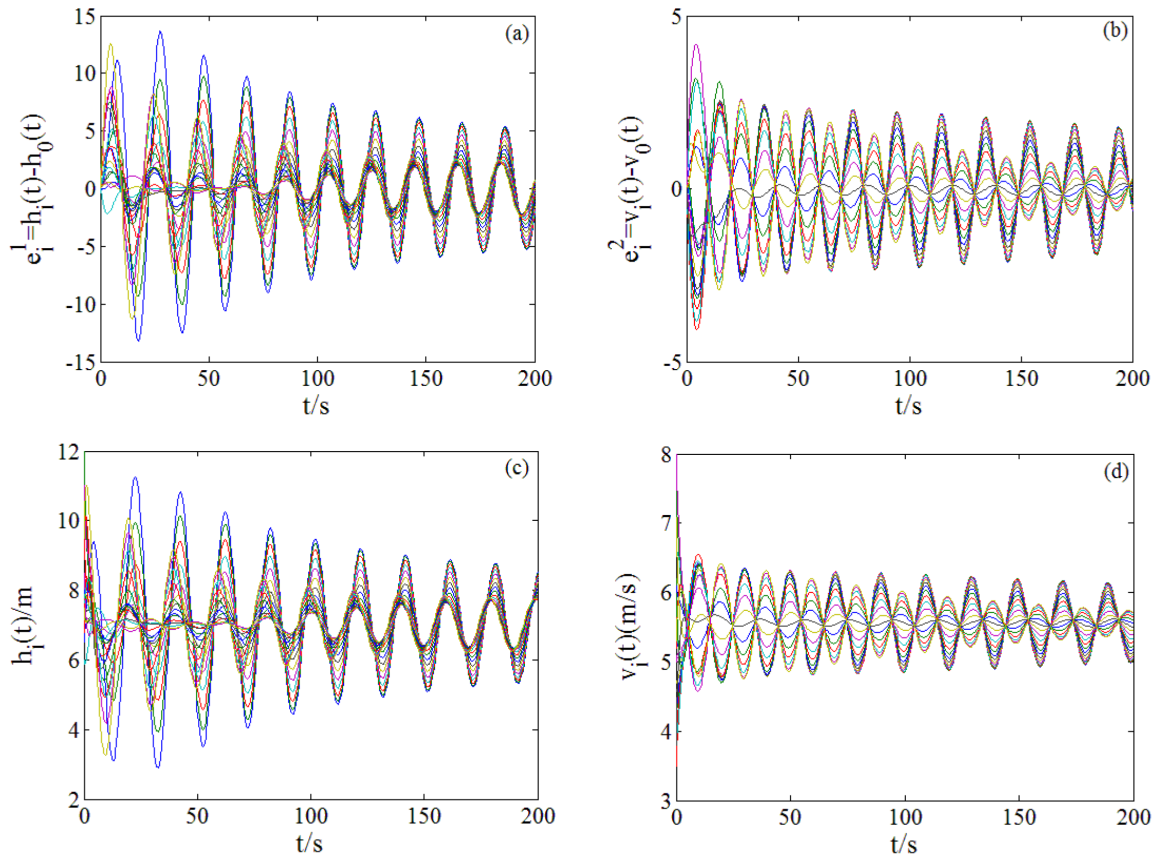

FIGURE 1. (a) Time evolution of error variables of $h_i(t)$; (b) time evolution of error variables of $v_i(t)$; (c) time evolution curves of $h_i(t)$; (d) time evolution curves of $v_i(t)$ without controller

the actual car-following behaviorthen we randomly selected the initial velocity $v_i(0)$ from $[3.5, 8]$, and the initial headway distance $h_i(0)$ from $[6.5, 12]$. And the initial velocity and the initial headway distance of the vehicle are not equal, so the study is more universal. First, we impose no controller on the traffic flow system and observe the evolution of the traffic flow over time through numerical simulations. The Matlab numerical simulation was performed and obtain the time evolution of error variables and time evolution curve of headway distance $h_i(t)$ and velocity $v_i(t)$ shown in Figure 1. We can see from Figure 1 that the traffic flow system is unstable without adding controllers, that is, there is a traffic jam.

Next, we added the controller to the traffic flow model and take the parameter $k_i = 0.5$, $i = 1, 2, \ldots, 20$. Through numerical simulation, we obtain the time evolution of error variables and time evolution curve of headway distance $h_i(t)$ and velocity $v_i(t)$ shown in Figure 2. From Figures 2(a) and 2(c), we can see that the difference between $h_i(t)$ $(i = 1, 2, \ldots, 19)$ and $h_0(t)$ tends to be 0 at 70 time units, and in the meantime $h_i(t)$ $(i = 1, 2, \ldots, 19)$ tends to $h_0(t) = 7.02$ m at 70 time units. As seen in Figures 2(b) and 2(d), the difference between $v_i(t)$ $(i = 1, 2, \ldots, 20)$ and $v_0(t)$ tends to be 0 at 70 time units, and meanwhile $v_i(t)$ $(i = 1, 2, \ldots, 20)$ tends to $v_0(t) = 5.56$ m/s at 70 time units. This indicates that the traffic flow system reaches the steady state in 70 time units when the controller is added; in other words, the controller designed in this paper can make the vehicle run smoothly and effectively suppresses and alleviates traffic congestion.

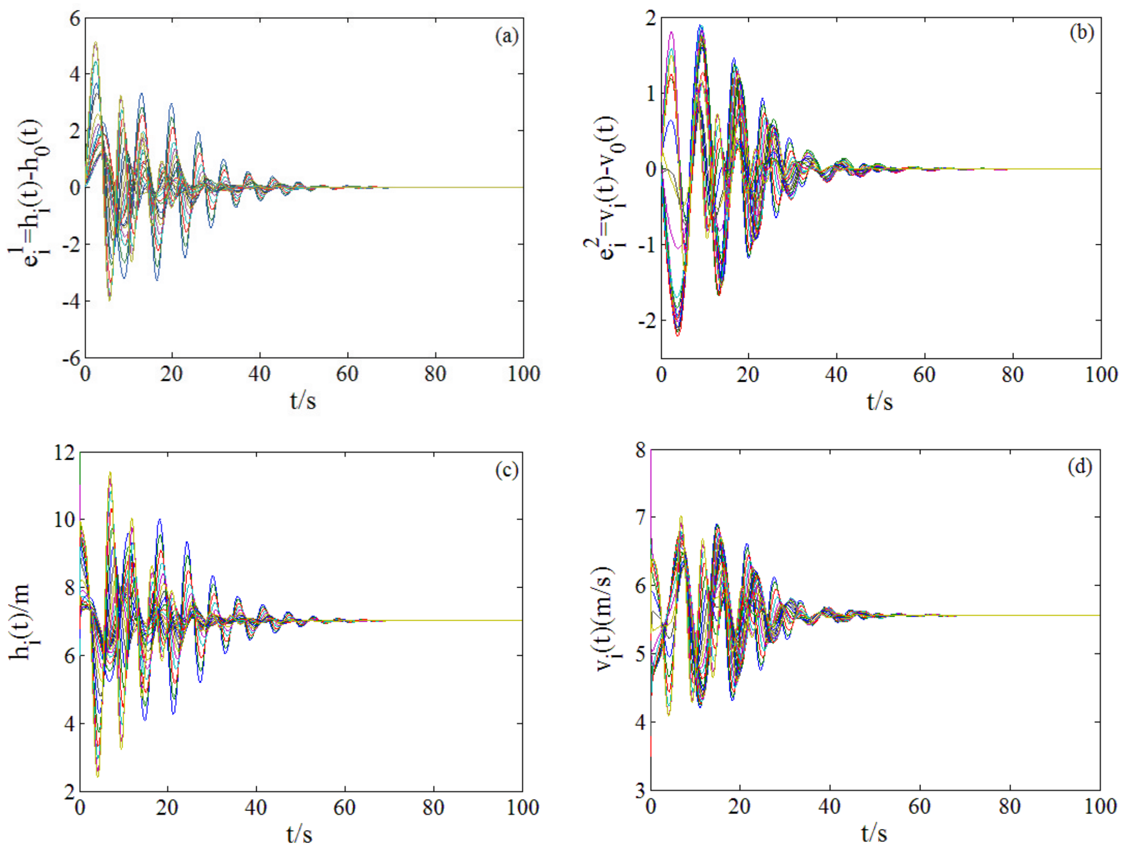

FIGURE 2. (a) Time evolution of error variables of $h_i(t)$; (b) time evolution of error variables of $v_i(t)$; (c) time evolution curves of $h_i(t)$; (d) time evolution curves of $v_i(t)$ when the controller is added

Finally, we discuss the stability of the car-following model under random external interference. Assume that vehicles 2, 8 and 14 are subject to random external interference as follows:

$$\omega_i(t) = \begin{cases} 1.5\sin(2t), & 90 \leq t \leq 100 \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

Moreover, we take $B = (1,1,1)^{\mathrm{T}}$, $\gamma = 0.5$, and $k_i = 0.5$, $i = 1, 2, \ldots, 20$. And then we get the time evolution of error variables and time evolution curve of headway distance $h_i(t)$ and velocity $v_i(t)$ as shown in Figure 3. As shown in Figure 3, even if the external

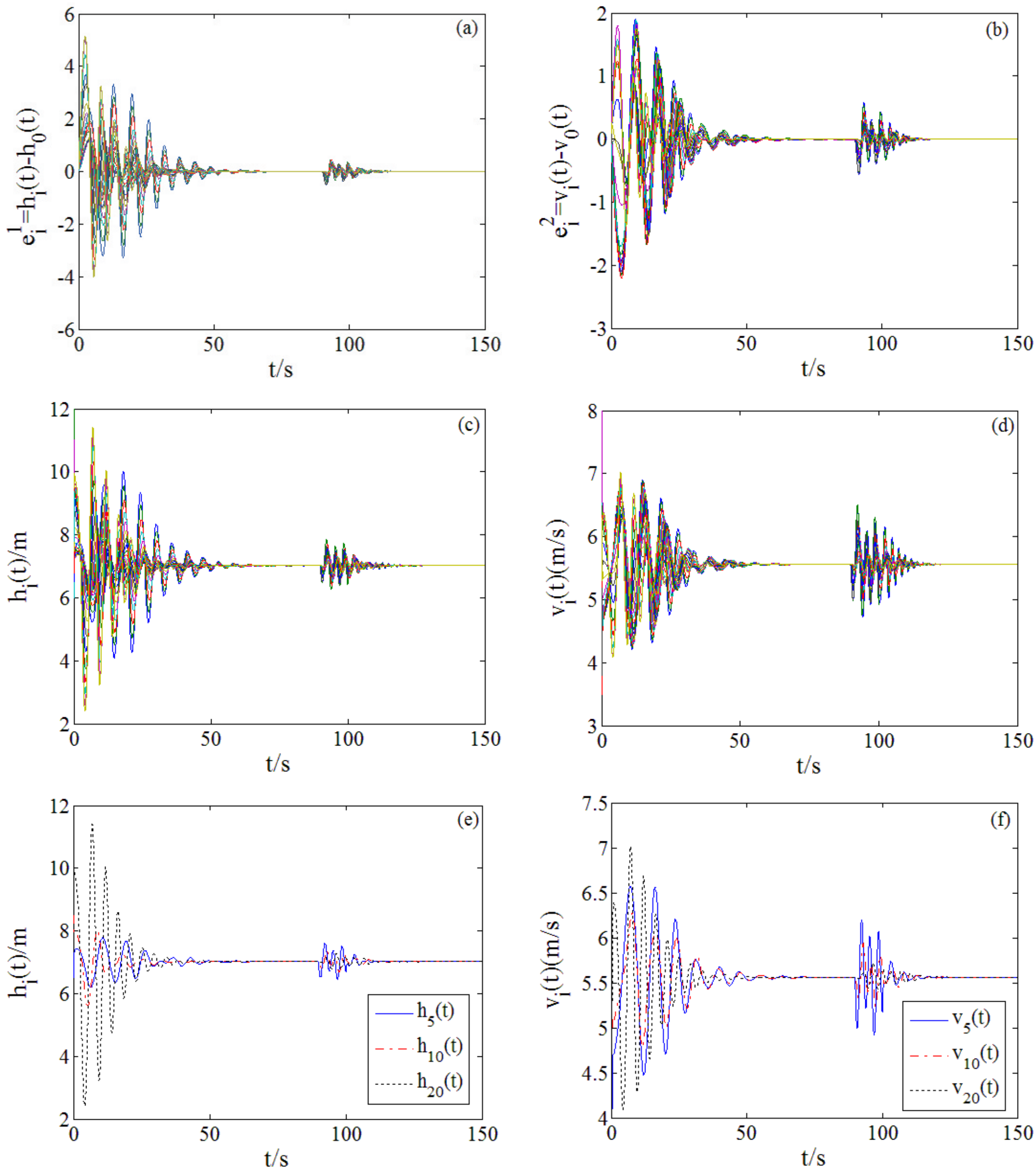

Figure 3. (a) Time evolution of error variables of $h_i(t)$; (b) time evolution of error variables of $v_i(t)$; (c) time evolution curves of $h_i(t)$; (d) time evolution curves of $v_i(t)$; (e) the headway distance variation curves between vehicle 5, vehicle 10 and vehicle 20 and front car, respectively; (f) instantaneous velocity of vehicle 5, vehicle 10 and vehicle 20 with external disturbance

interference applied to vehicles $i = 2, 8, 14$ has a clear effect on the vehicle behind them, the influence of external interference on the traffic flow system is effectively suppressed when the controller is added. And from Figures 3(a), 3(b), 3(c) and 3(d), we can see that the external interference fluctuates the velocity of the vehicles, but when the external interference disappeared and the controller is added, the velocity of all vehicles quickly tends to the preset velocity $v_0(t) = 5.56$ m/s, and all vehicles maintain the ideal headway distance $h_0(t) = 7.02$ m and running in an orderly way. Figure 3(e) shows the headway distance variation curves between vehicle 5, vehicle 10 and vehicle 20 and front car, respectively, and Figure 3(f) shows the instantaneous velocity of vehicle 5, vehicle 10 and vehicle 20 with external disturbance.

6. **Conclusions.** In this paper, the stability of a car-following model with effect of front and rear vehicles is studied by using the synchronization theory of complex networks in a connected and autonomous environment. Based on Lyapunov stability theory, an appropriate controller is designed to make the car-following model stable, and the stability conditions of the model are obtained. In addition, based on the H_∞ synchronization theory of complex network, the stability of the car-following model with external disturbances is obtained by designing a suitable controller under the condition that the model is subjected to random external disturbances. Numerical simulation results show that the traffic flow system rapidly stabilized through the controller designed in this paper, and the controller can effectively alleviate traffic congestion in a connected and autonomous environment. The research results of this paper can promote people's understanding of the formation process of traffic congestion and the influence of the effect of front and rear vehicles on traffic flow stability, and enrich the research results of traffic flow stability. The control algorithm designed in this paper can provide certain theoretical guidance for the design of intelligent driving system, and the conclusions can provide theoretical basis for preventing and controlling traffic congestion. Considering that the popularity of autonomous vehicles needs a certain process, there will be the human-driven vehicles and autonomous vehicles coexistence on the road in the future. Therefore, we will model the mixed traffic flow in the connected and autonomous environment and conduct theoretical and numerical analysis of its properties.

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