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LOW-SENSITIVITY CONTROL FOR MINIMUM PHASE PLANTS USING DOUBLE FEEDBACK CONTROL

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Abstract. *We consider a design method of a control system by using double feedback control strucutre with low sensitivity and robust stability. Several researches are conducted on the robust stabilization problem. According to some researches, for the large uncertainty, it is well-known to be difficult to design a control system with robust stability and low sensitivity, in the meaning of disturbance attenuation and so on. In order to design the control system with robust stability and low sensitivity, Yu et al. proposed a new control structure called the double feedback control structure. The double feedback control structure can bring a robust stability and low sensitivity characteristic to the control system. Yu et al. clarified a robust stability condition of the control system by using double feedback control structure, which is called the double feedback control system, for systems with certain class of uncertainty. This robust stability condition implies that low sensitivity control guarantees robust stability of the double feedback control system. However, Yu et al. do not show the complete proof of a robust stability condition of the double feedback control system. In this paper, we show the complete proof of a robust stability condition of the double feedback control system. In addition, we propose a design method and a design procedure of the double feedback control system.*

Keywords: Minimum phase system, Low-sensitivity control, Sensitivity control, Robust stability, Uncertainty, Plant having varying number of unstable poles

1. **Introduction.** An uncertainty, which is an error between the plant and the nominal plant, often makes a control system unstable. In order to keep the control system being stable for the plant with uncertainty, several studies are conducted on the robust stabilization problem. Doyle and Stein built the basic solution to this problem [1]. They show the necessary and sufficient conditions for the multiplicative uncertainty and the additive uncertainty. Chen and Desoer derived complete proof of the solution in [2]. Kimura considered the robust stabilizability problem for single-input and single-output systems [8]. Vidyasagar and Kimura expanded the result by Kimura [8] for multiple-input and multiple-output systems [9].

According to [1, 2, 3], in order to keep stability for the large uncertainty, the complementary sensitivity function must be small value. However, by making the complimentary sensitivity function of the control system small, the control system is brought lower performance in the meaning of disturbance attenuation property and so on. In order to produce the control system with high disturbance attenuation property, we must make the sensitivity function of the control system small. Since the sum of the sensitivity function and the

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complementary sensitivity function is equal to 1, we cannot obtain either low-sensitivity or high robust stability characteristics.

Low-sensitivity does not always make the system unstable. Maeda and Vidya-sagar considered this problem as an infinite gain margin problem [10, 11]. Nogami et al. clarified the condition that high-gain controller does not make the control system unstable and also proposed a design method [12]. Doyle et al. considered this low-sensitivity control problem from another view point; there exists a class of uncertainty that has the property low-sensitivity making the system robustly stable [14]. Therefore, if the uncertainty is discribed in [14], we can construct low-sensitivity characteristics with robust stability. In this meaning, the uncertainty in [14] is suitable for high-performance robust control system design. However, this method cannot apply for plants having varying number of unstable poles. There exist plants such that the number of unstable poles changes. For example, the number of right half plane poles of a large flexible spacecraft changes when the configuration of the spacecraft is changed [16]. Yamada considered this problem and clarified the robust stability conditions using phase information [20]. The result of [20] is suitable to design the control system with robust stability and low-sensitivity characteristics for plants having varying number of unstable poles. However, even if the control system satisfies the result in [20], the control system does not always have a high performance on low-sensitivity characteristics of the control system.

The performance of the low-sensitivity characteristics of the control system is related to control structure. In order to design the control system with low-sensitivity characteristics, it is important to consider a control structure. The internal model control structure [21] is known as an effective control structure for low-sensitivity control. However, the internal model control structure cannot apply for unstable systems. Zhou and Ren overcame this problem and proposed a new control structure named generalized internal model control (GIMC) [22]. The GIMC structure is a reconfigureable control structure for fault-tolerant control. Campos-Delgado and Zhou adopted the GIMC structure to the fault tolerant control problem in multi-input/multi-output system [23]. Campos-Delgado et al. provided a design procedure of the control system by using GIMC structure with fault tolerance by using robust control theory [24]. The fault tolerant control by using the GIMC structure is applied to several applications: magnetic suspention systems [25]; heating, ventilation, and air-conditioning system [27]; automotive electric power steering system [28]. In addition, Okajima et al. proposed a model error compensator control structure [29, 30, 31]. Model error compensator control structure is a control structure that the output trajectory of the nominal plant can be made close to that of the model. The model error compensator control structure is applied to several control such as an indoor platoon driving system of welfare personal vehicles [32].

Yu et al. expanded the result in [20] and proposed the double feedback control system [40]. According to [40], the double feedback control system has a structure ensuring the robust stability, reducing the influence of the uncertainty to the output and providing the low-sensitivity characteristics. This control system is expected to apply for a low sensitivity control of the plant with varying number of unstable pole such as large flexible spacecraft, and multi-agent-system [41]. Yu et al. also clarified the robust stability condition of double feedback control system. However, Yu et al. do not show complete proof of the robust stability condition of the double feedback control system. In addition, the design procedure of the double feedback control system is not clarified. If the design procedure of the double feedback control system can be clarified, we can construct a high performance control system, systematically.

In this paper, we show complete proof of the robust stability condition of the double feedback control system for single-input/single-output time-invariant minimum phase plants having varying number of unstable poles. In addition, we propose a design method for double feedback control systems, which reduces the influence of the uncertainty to the output. This paper is organized as follows. In Section 2, we explain the preliminary results of two-degree-of-freedom control systems and the problem considered in this paper. In Section 3, the proof of robust stability condition of the double feedback control systems is clarified. In Section 4, a design method for double feedback control system with robust stability and low-sensitivity characteristics is presented. In Section 5, a design procedure of double freedom control system with robust stability and low-sensitivity characteristics is shown. In Section 6, we show a numerical example to illustrate the effectiveness of the proposed method. Section 7 gives concluding remarks.

Notation

R	the set of real numbers.
C	the set of complex numbers.
$R(s)$	the set of real relational functions with s.
RH_∞	the set of stable proper real relational functions.
$\|\cdot\|_\infty$	H_∞ norm.
$\Re\{\cdot\}$	real part of $\{\cdot\}$.

2. **Preliminary Results and Problem Formulation.** In this section, we explain preliminarily two-degree-of freedom control systems and the problem considered in this paper.

Consider the two-degree-of-freedom control system in Figure 1. Here, $G(s) \in R(s)$ is a strictly proper plant. $G(s)$ is assumed to be of minimum phase, that is, $G(s)$ has no zero in the closed right half plane. $F_1(s) \in RH_\infty$ and $C_1(s) \in R(s)$ are controllers. $r(s) \in R(s)$ is the reference input, $y(s) \in R(s)$ is the output and

$$e(s) = F_1(s)r(s) - y(s). \tag{1}$$

The nominal plant of $G(s)$ is denoted by $F_0(s) \in RH_\infty$. $F_0(s)$ is also assumed to be strictly proper and of minimum phase. Using $F_0(s)$, $G(s)$ is written by the form in

$$G(s) = F_0(s)\left(1 + \Delta(s)\right), \tag{2}$$

where $\Delta(s)$ is an uncertainty.

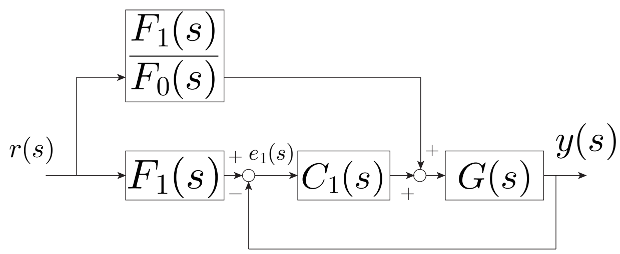

Figure 1. The two-degree-of-freedom control system

The transfer functions from $r(s)$ to $y(s)$ and $r(s)$ to $e(s)$ in Figure 1 are written by

$$y(s) = F_1(s)\left(1 + H_1(s)\right) r = G_1(s)r \tag{3}$$

and

$$e(s) = -F_1(s)H_1(s)r(s), \tag{4}$$

respectively, where

$$H_1(s) = \frac{S_1(s)\frac{\Delta(s)}{1+\Delta(s)}}{1 - S_1(s)\frac{\Delta(s)}{1+\Delta(s)}} \tag{5}$$

and

$$S_1(s) = \frac{1}{1 + C_1(s)F_0(s)}. \tag{6}$$

From (4) and (5), in order to reduce the influence of $\Delta(s)$ to $y(s)$, we must make $S_1(s)$ small. $S_1(s)$ is called a sensitivity function of the control system in Figure 1.

According to [20], it is well-known that the control system with low-sensitivity characteristics sometimes becomes unstable due to the uncertainty $\Delta(s)$. However, the control system with low-sensitivity characteristics is not always unstable for the uncertainty. In [20], Yamada clarified the necessary and sufficient condition that the control system with low-sensitivity characteristics is robustly stable for a class of uncertainty defined as follows.

Definition 2.1. *$G(s)$ is called the elementary of the set Ω if the following expressions hold.*

- *$\Delta(s)$ satisfies*

$$\left|\frac{\Delta(j\omega)}{1 + \Delta(j\omega)}\right| < |W(j\omega)| \ (\omega \in R), \tag{7}$$

where $W(s) \in R(s)$ satisfies

$$\lim_{\omega \to \infty} |W(j\omega)| < 1. \tag{8}$$

- *Both of $G(s)$ and $F_0(s)$ are of minimum phase.*

From [20], Definition 2.1 and the assumption that $G(s)$ and $F_0(s)$ are strictly proper, we have the following theorem.

Theorem 2.1. [20] *If $G(s)$ and $F_0(s)$ hold (8), then the relative degree of $F_0(s)$ is equal to that of $G(s)$.*

Theorem 2.1 gives a condition of the nominal plant that the relative degree of the nominal plant satisfying (8) is equal to that of the plant.

According to [20, 40], the robust stability condition of the two-degree-of-freedom control system in Figure 1 is summarized as following theorem.

Theorem 2.2. [20, 40] *Assume that $C_1(s)$ stabilizes $F_0(s) \in RH_\infty$ and $F_1(s)/F_0(s) \in RH_\infty$.*

The control system in Figure 1 is robustly stable for $G(s) \in \Omega$ if and only if

$$\|S_1(s)W(s)\|_\infty \leq 1. \tag{9}$$

Theorem 2.2 is summarized as follows.

- In order to design the control system with robust stability and low sensitivity, the controller $C_1(s)$ needs to minimize $\|S_1(s)W(s)\|_\infty$, at worst $C_1(s)$ must satisfy

$$\|S_1(s)W(s)\|_\infty < 1.$$

- The control system in Figure 1 satisfying Theorem 2.2 is robustly stable for the plant included in the set Ω. Since there has no expression of a number of poles in the closed right half plane in Definition 2.1, the control system in Figure 1 satisfying Theorem 2.2 can be made robustly stable for the plant with varying number of pole in the closed right half plane.

Yu et al. expanded the above discribed two-degree-of-freedom control system and proposed the double feedback control system [40]. In addition, the robust stability condition of the double feedback control system is clarified, but they do not show complete proof

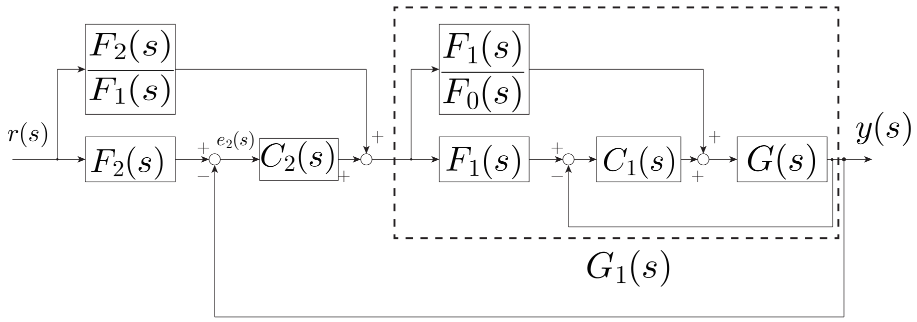

Figure 2. The double feedback control system

of the robust stability condition. The problem considered in this paper is to give complete proof of the robust stability condition of the double feedback control system [40] shown in Figure 2 for the plants having varying number of unstable poles and propose a design method for the double feedback control systems, which reduces the influence of the uncertainty to the output rather than the two-degree-of-freedom control system in Figure 1. The control system shown in Figure 2 is called the double feedback control system, since the control system shown in Figure 2 has a structure including the system $G_1(s)$, which is the transfer function from $r(s)$ to $y(s)$ of the two-degree-of-freedom control system in Figure 1 [40]. Here, $F_2(s) \in RH_\infty$ and $C_2(s) \in R(s)$ are controllers and

$$e_2(s) = F_2(s)r(s) - y(s). \tag{10}$$

The transfer functions from $r(s)$ to $y(s)$ and from $r(s)$ to $e_2(s)$ of the double feedback control system in Figure 2 are given by

$$y(s) = F_2(s)\left(1 + H_2(s)\right)r(s) \tag{11}$$

and

$$e_2(s) = F_2(s)r(s) - F_2(s)(1 + H_2(s))r(s) = -F_2(s)H_2(s)r(s), \tag{12}$$

where

$$H_2(s) = \frac{S_1(s)S_2(s)\frac{\Delta(s)}{1+\Delta(s)}}{1 - S_1(s)S_2(s)\frac{\Delta(s)}{1+\Delta(s)}} \tag{13}$$

and

$$S_2(s) = \frac{1}{1 + F_2(s)C_2(s)}. \tag{14}$$

From (12) and (13), in order to reduce the influence of $\Delta(s)$ to $y(s)$, we must make $S(s) = S_1(s)S_2(s)$ small, and $S(s)$ is called the sensitivity function of the double feedback control system in Figure 2. The problem in this paper is to show the robust stability condition of the double feedback control system in Figure 2 that a low sensitivity control guarantees robust stability, and provide a design method of the double feedback control system with robust stability and low sensitivity.

3. **Robust Stability Condition of the Double Feedback Control System.**

In this section, we give complete proof of robust stability condition of the double feedback control system in Figure 2 for $G(s) \in \Omega$.

According to [40], the robust stability condition of the double feedback control system in Figure 2 is summarized as follows.

Theorem 3.1. [40] *Assume that $C_1(s)$ stabilizes $F_0(s) \in RH_\infty$, $C_2(s)$ stabilizes $F_1(s) \in RH_\infty$, and $F_2(s)/F_1(s) \in RH_\infty$. The control system in Figure 2 is stable for $G(s) \in \Omega$ if and only if*

$$\|S_1(s)S_2(s)W(s)\|_\infty \leq 1. \tag{15}$$

The proof of Theorem 3.1 requires following lemma.

Lemma 3.1. [20] *It is assumed that $F_0(s)$ has no zero in the closed right half plane and the p_m-th number of the pole in the closed right half plane and $G(s)$ has no zero in the closed right half plane and the p-th number of the pole in the closed right half plane. Then the Nyquist plot of $1 + \Delta(j\omega)$ for $-\infty \leq \omega \leq \infty$ encircles the origin $(0,0)$ $p - p_m$ times in the counter-clockwise direction.*

Using Lemma 3.1, we will prove Theorem 3.1.

Proof: The characteristic polynomial of the double feedback control system in Figure 2 is given by

$$1 + \left\{\frac{F_1(s)}{F_0(s)}C_2(s)(1 + C_1(s)F_0(s)) + C_1(s)\right\}G(s). \tag{16}$$

Since $F_1(s) \in RH_\infty$ and $F_0(s)$ is of minimum phase, if the Nyquist plot of the characteristic polynomial in (16) for $-\infty \leq \omega \leq \infty$ encircles the origin $(0,0)$ $p + p_{c1} + p_{c2}$ times in the counter-clockwise direction, then the control system in Figure 2 is stable. The characteristic polynomial in (16) is rewritten by

$$\begin{aligned}&1 + \left\{\frac{F_1(s)}{F_0(s)}C_2(s)(1 + C_1(s)F_0(s)) + C_1(s)\right\}G(s)\\&= (1 + F_1(s)C_2(s))(1 + F_0(s)C_1(s))(1 + \Delta(s))\left(1 - S_2(s)S_1(s)\frac{\Delta(s)}{1 + \Delta(s)}\right)\end{aligned} \tag{17}$$

From the assumption that $C_1(s)$ stabilizes $F_0(s)$, the Nyquist plot of $1 + C_1(s)F_0(s)$ for $-\infty \leq \omega \leq \infty$ encircles the origin $(0,0)$ $p_m + p_{c1}$ times in the counter-clockwise direction, where p_m is the number of unstable poles of $F_0(s)$ and p_{c1} is that of $C(s)$. Since $C_2(s)$ stabilizes $F_1(s) \in RH_\infty$, the Nyquist plot of $1 + C_2(s)F_1(s)$ for $-\infty \leq \omega \leq \infty$ encircles the origin $(0,0)$ p_{c2} times in the counter-clockwise direction, where p_{c2} is the number of unstable poles of $C_2(s)$. From Lemma 3.1, the Nyquist plot of $1 + \Delta(s)$ for $-\infty \leq \omega \leq \infty$ encircles the origin $(0,0)$ $p - p_m$ times in the counter-clockwise direction. Thus, the necessary and sufficient condition that the double feedback control system in Figure 2 is stable for $G(s) \in \Omega$ is that the Nyquist plot of $1 - S_1(s)S_2(s)\Delta(s)/(1 + \Delta(s))$ does not encircle the origin $(0,0)$ any times.

The remaining problem is to prove the necessary and sufficient condition that $1 - S_1(s)S_2(s)\Delta(s)/(1 + \Delta(s))$ does not encircle the origin any times is equivalent to (15). We adopt the same procedure in [13] to prove this.

Sufficiency is shown, that is, we show that if $\|S_1(s)S_2(s)W(s)\|_\infty < 1$, then the Nyquist plot of $1 - S_1(s)S_2(s)\Delta(s)/(1 + \Delta(s))$ encircles the origin no time. It is clear that the Nyquist plot of $1 - S_1(s)S_2(s)\Delta(s)/(1 + \Delta(s))$ encircles the origin no time for any $\Delta(s)$. The sufficiency has been proved.

Next necessity is shown, that is, we show that if $\|S_1(s)S_2(s)W(s)\|_\infty > 1$, then there exists $\Delta(s) \in \Omega$ to make the Nyquist plot of $1 - S_1(s)S_2(s)\Delta(s)/(1 + \Delta(s))$ encircle the origin $(0,0)$. From the assumption of $\|S_1(s)S_2(s)W(s)\|_\infty \geq 1$, their exists ω and $\epsilon > 0$ satisfying

$$|S_1(j\omega)S_2(j\omega)W(j\omega)| = 1 + \epsilon. \tag{18}$$

If we set

$$\frac{\Delta(j\omega)}{1+\Delta(j\omega)} = \frac{|W(j\omega)|}{1+\epsilon} < |W(j\omega)|, \tag{19}$$

then we have

$$1 - S_1(j\omega)S_2(j\omega)\frac{\Delta(j\omega)}{1+\Delta(j\omega)} = 0. \tag{20}$$

This implies that the Nyquist plot of $1 - S_1(s)S_2(s)\Delta(s)/(1+\Delta(s))$ passes through the origin. Therefore, for this $\Delta(s) \in \Omega$, the system in Figure 2 is unstable. Thus, the necessity has been proved.

We have thus proved Theorem 3.1. □

Theorem 3.1 is summarized as follows.

- In order to design the control system with robust stability and low sensitivity, the controllers $C_1(s)$, $F_1(s)$ and $C_2(s)$ need to minimize $\|S_1(s)S_2(s)W(s)\|_\infty$, at worst $C_1(s)$ must satisfy

$$\|S_1(s)S_2(s)W(s)\|_\infty < 1.$$

- It is not only related with $C_1(s)$ but also $C_2(s)$ to minimize $\|S_1(s)S_2(s)W(s)\|$.
- The control system in Figure 1 to satisfy Theorem 2.1 is robustly stable for the plant included in the set Ω.

4. A Design Method for Double Feedback Control.

In this section, we propose a design method for double feedback control system in Figure 2 that reduces the influence of the uncertainty $\Delta(s)$ to the output $y(s)$ rather than two-degree-of-freedom control system in Figure 1.

In order to compare the influence of the uncertainty $\Delta(s)$ to the output $y(s)$ in Figure 2 and that in Figure 1, we set $F_1(s) = F_2(s)$. $F_1(s) \in RH_\infty$ and $F_2(s) \in RH_\infty$ are designed to satisfy $F_1(s)/F_0(s) \in RH_\infty$. The controller $C_1(s)$ is designed to satisfy

$$\|S_1(s)W(s)\|_\infty \le 1. \tag{21}$$

The problem to obtain the controller $C_1(s)$ satisfying (21) is equivalent to the following H_∞ control problem. In order to obtain the controller $C_1(s)$ satisfying (21), we consider the control system in Figure 3. $P(s)$ is selected such that the transfer function from w to z in Figure 3 is equal to $S_1(s)W(s)$. The state space description of $P(s)$ is

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t) \\ z(t) = C_1x(t) + D_{11}w(t) + D_{12}u(t) \\ y(t) = C_2x(t) + D_{21}w(t) + D_{22}u(t) \end{cases}, \tag{22}$$

where $A \in R^{n\times n}$, $B_1 \in R^n$, $B_2 \in R^n$, $C_1 \in R^{1\times n}$, $C_2 \in R^{1\times n}$, $D_{11} \in R$, $D_{12} \in R$, $D_{21} \in R$, $D_{22} \in R$. The controller $C_1(s)$ can be obtained by using LMI, and so on [17, 18].

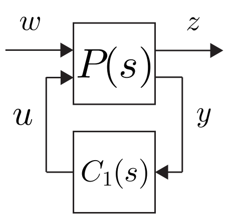

FIGURE 3. The block diagram of H_∞ control problem

The controller $C_2(s)$ satisfying (15) can be designed as follows. From (14) and the result in [21], since $F_1(s) \in RH_\infty$, the parameterization of all stabilizing controllers $C_2(s)$ for $F_1(s)$ is given by

$$C_2(s) = \frac{Q_2(s)}{1 - Q_2(s)F_1(s)}, \tag{23}$$

where $Q_2(s) \in RH_\infty$ is any function. According to [40], in order to make $|e_2(j\omega)|$ smaller than $|e_1(j\omega)|$,

$$\left|\frac{e_2(j\omega)}{e_1(j\omega)}\right| = \left|\frac{H_2(j\omega)}{H_1(j\omega)}\right| \leq 1 \tag{24}$$

is needed. From (6), (14) and (24), $H_1(s)/H_2(s)$ is rewritten as

$$\frac{H_1(s)}{H_2(s)} = \frac{S_2(s)\left(1 - S_2(s)\frac{\Delta(s)}{1+\Delta(s)}\right)}{1 - S_2(s)S_1(s)\frac{\Delta(s)}{1+\Delta(s)}} = 1 - K(s), \tag{25}$$

where

$$K(s) = \frac{1 - S_2(s)}{1 - S_2(s)S_1(s)\frac{\Delta(s)}{1+\Delta(s)}}. \tag{26}$$

From (24), (25) and (26), if we design $C_2(s)$ satisfying

$$|1 - K(j\omega)| \simeq 0 \tag{27}$$

in wide frequency range, then we can make $|e_2(j\omega)| < |e_1(j\omega)|$. Satisfying (27) is equivalent to making $|S_2(j\omega)|$ small. From (14) and (23), $S_2(s)$ is rewritten as

$$S_2(s) = 1 - Q_2(s)F_1(s). \tag{28}$$

When $F_1(s)$ is designed to be of minimum phase, in order to satisfy $|S_2(j\omega)| \simeq 0$ in the wide frequency range, $Q_2(s)$ is settled by

$$Q_2(s) = \frac{1}{F_1(s)}\frac{1}{(1 + \tau_{Q_2}s)^{\alpha_{Q_2}}}, \tag{29}$$

where $\tau_{Q_2} > 0$ is an arbitrary real number and α_{Q_2} is an arbitrary positive integer that makes $Q_2(s)$ proper. τ_{Q_2} and α_{Q_2} are designed to satisfy

$$|S_2(j\omega)| = \left|1 - \frac{1}{(1 + \tau_{Q_2}j\omega)^{\alpha_{Q_2}}}\right| \leq \frac{1}{|S_1(j\omega)W(j\omega)|} \quad (\forall\omega \in R). \tag{30}$$

5. **Design Procedure.** In this section, a design procedure of the double feedback control system in Figure 2 is presented.

A design procedure of the double feedback control system with robust stability and low-sensitivity characteristics satisfying Theorem 3.1 is summarized as follows.

Step 1. In order for the output y to follow the step reference input r, $F_1(s) \in RH_\infty$ and $F_2(s)$ are settled by

$$F_1(s) = F_2(s) = \frac{1}{(1 + \tau_f s)^{\alpha_f}}, \tag{31}$$

where τ_f is a positive real number and α_f is a positive integer to make $F_1(s)/F_0(s)$ proper.

Step 2. Obtain $C_1(s)$ satisfying (21) by solving a linear matrix inequality based on the H_∞ problem.

Step 3. $Q_2(s)$ in (29) is designed satisfying (30). Using $Q_2(s)$, $C_2(s)$ is designed as (23).

6. **Numerical Example.** In this section, we show a numerical example to illustrate the features of the proposed design method for the double feedback control system in Figure 2. The numerical example in this paper illustrates that the double feedback control system by using the design procedure in Section 5 has robust stability and low sensitivity characteristics. In addition, the influence of $\Delta(s)$ to $y(s)$ double feedback control system by using the design procedure in Section 5 is smaller than that of the two-degree-of-freedom control system satisfying Theorem 3.1.

Consider the problem to design the two-degree-of-freedom control system in Figure 1 and the double feedback control system in Figure 2 with the robust stability and low-sensitivity characteristics for the plant $G(s)$ in (2), where

$$F_0(s) = \frac{s+3}{s^2-s-6}, \tag{32}$$

$\Delta(s)$ is any function satisfying (7) and

$$W(s) = \frac{0.95(s+200)}{s+2}. \tag{33}$$

In order to compare the response of the output y in Figure 1 and that of the output in Figure 2, $F_1(s) \in RH_\infty$ and $F_2(s)$ are set as

$$F_1(s) = F_2(s) = \frac{1}{1+0.1s}. \tag{34}$$

$C_1(s)$ satisfying (21) is obtained by using LMI and given by

$$C_1(s) = \frac{753700s^2 + 6654000s + 10290000}{s^3 + 24290s^2 + 126300s + 155300}. \tag{35}$$

The gain plot of $S_1(s)$ and $1/W(s)$ is shown in Figure 4. Here, the solid line shows the gain plot of $S_1(s)$ and the dashed line shows that of $1/W(s)$. Figure 4 shows that $C_1(s)$ in (35) satisfies (21).

$Q_2(s)$ satisfying (30) is settled by (29) as

$$Q_2(s) = \frac{1}{F_1(s)}\frac{1}{1+0.01s} = \frac{10s+100}{s+100}. \tag{36}$$

From (23), we have

$$C_2(s) = \frac{10s+100}{s}. \tag{37}$$

The gain plot of $S_2(s)$ and that of $1/(S_1(s)W(s))$ are shown in Figure 5. Here, the solid line shows the gain plot of $S_2(s)$ and the dashed line shows that of $1/(S_1(s)W(s))$. Figure 5 shows that $C_2(s)$ satisfies the robust stability condition in (15).

When $G(s)$ is given by

$$G(s) = \frac{s^2+3s+2}{s^3-s^2-14s+24}, \tag{38}$$

the gain plot of $\Delta(s)/(1+\Delta(s))$ and that of $W(s)$ are shown in Figure 6. Here, the solid line shows the gain plot of $\Delta(s)/(1+\Delta(s))$ and the dashed line shows that of $W(s)$. Figure 6 shows that $G(s) \in \Omega$.

Next we show that $|H_2(j\omega)|$ is smaller than $|H_1(j\omega)|$ in the wide frequency. Figure 7 shows the gain plot of $H_1(s)$ and that of $H_2(s)$. Here the dashed line shows the gain plot of $H_1(s)$ and the solid line shows that of $H_2(s)$. From Figure 7, since $|H_2(j\omega)|$ is smaller than $|H_1(j\omega)|$ in wide frequency range, we find that we can design the double feedback control system in Figure 2 with robust stability and low-sensitivity characteristics.

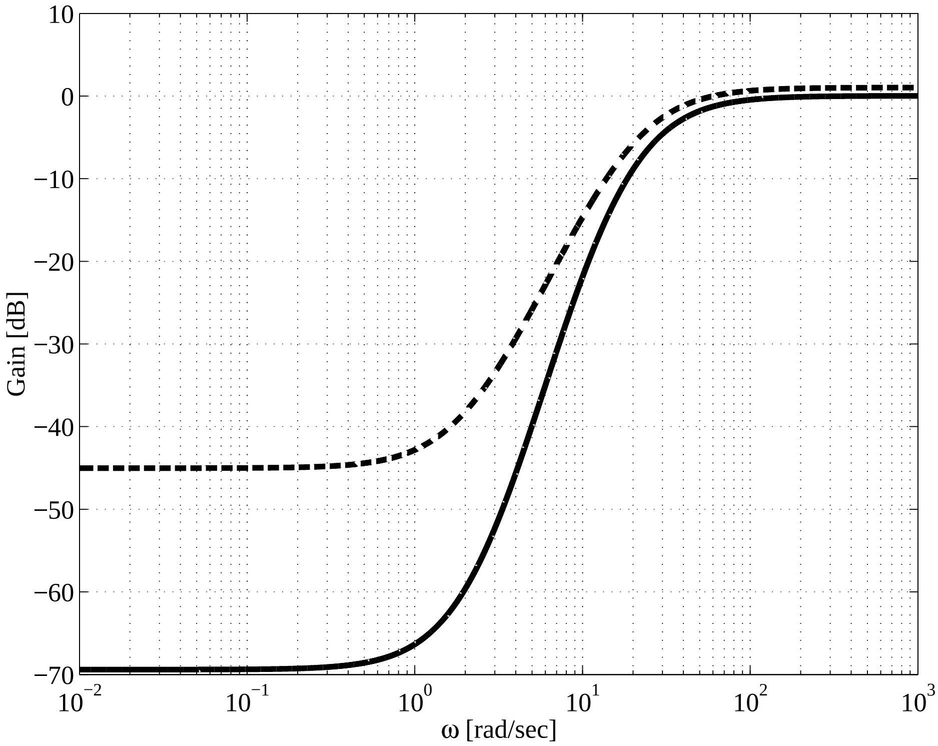

FIGURE 4. The gain plot of $S_1(s)$ and $1/W(s)$

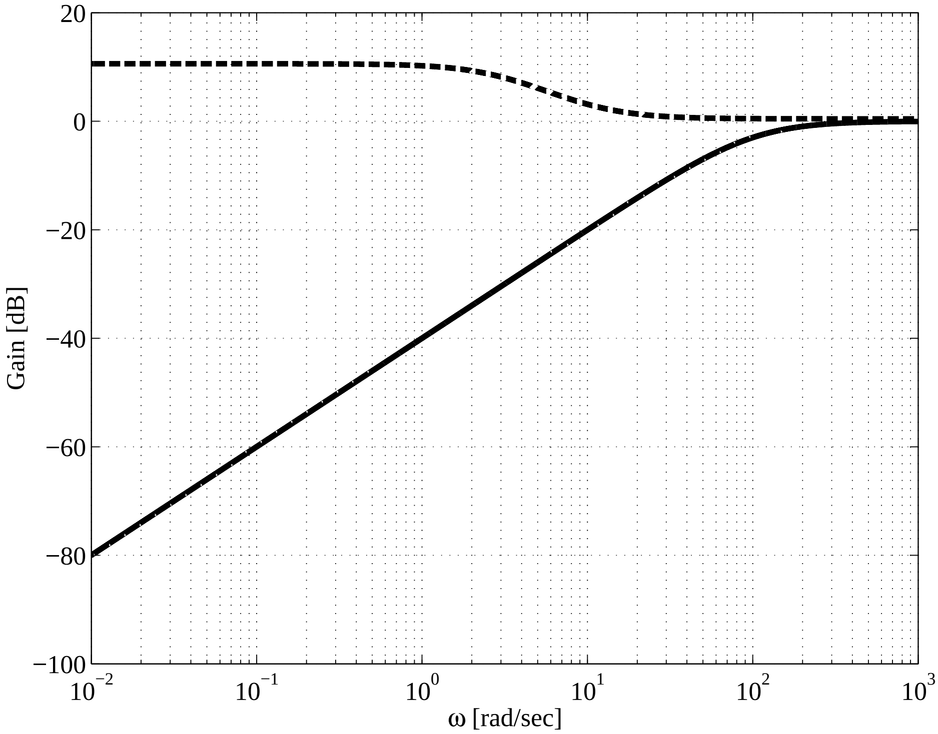

FIGURE 5. The gain plot of $S_2(s)$ and $1/(S_1(s)W(s))$

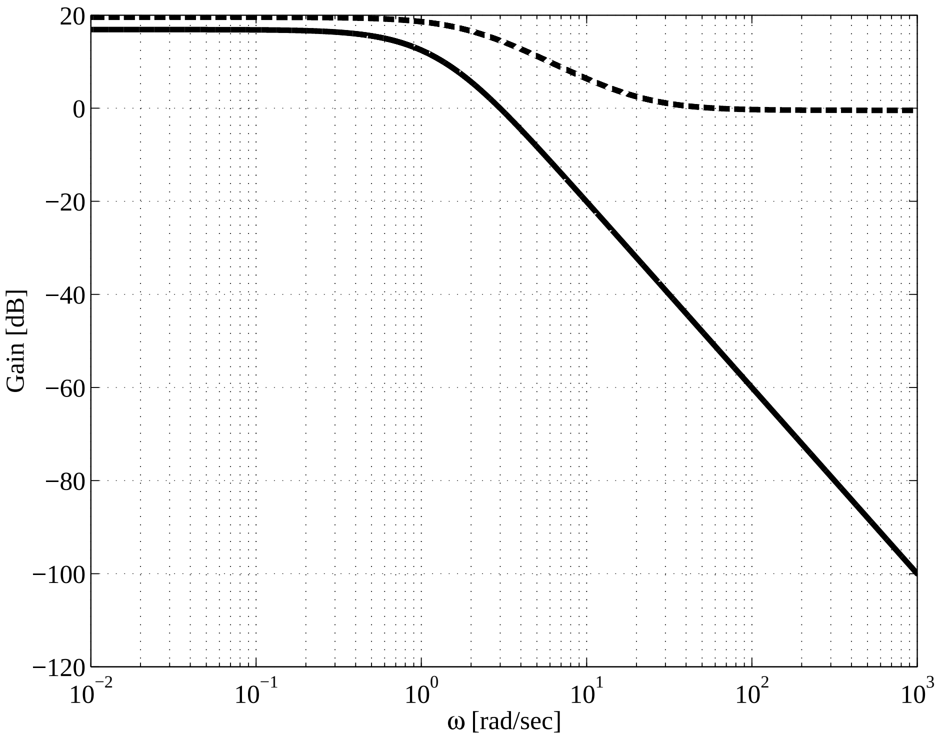

FIGURE 6. The gain plot of $\Delta(s)/(1+\Delta(s))$ and $W(s)$

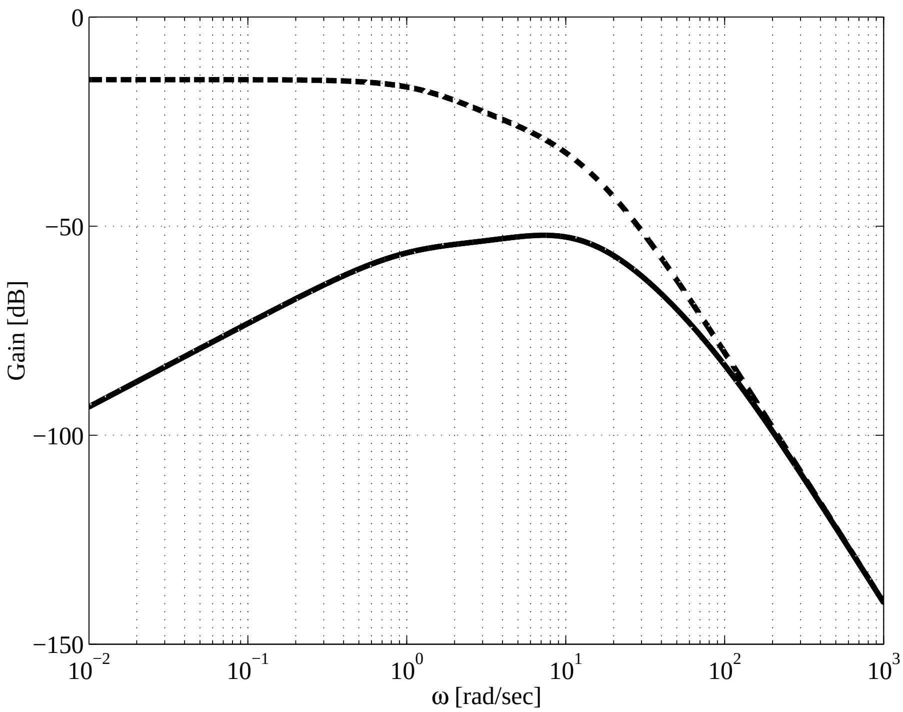

FIGURE 7. The gain plot of $H_1(s)$ and $H_2(s)$

In order to compare the influence of the uncertainty $\Delta(s)$ to the output $y(s)$ of the two-degree-of-freedom control system in Figure 1 and that of the double feedback control system in Figure 2, we show responses of the output $y(s)$ for the step reference input

$$r(t) = 0.5 + 0.5 \sin t \quad (t \geq 0). \tag{39}$$

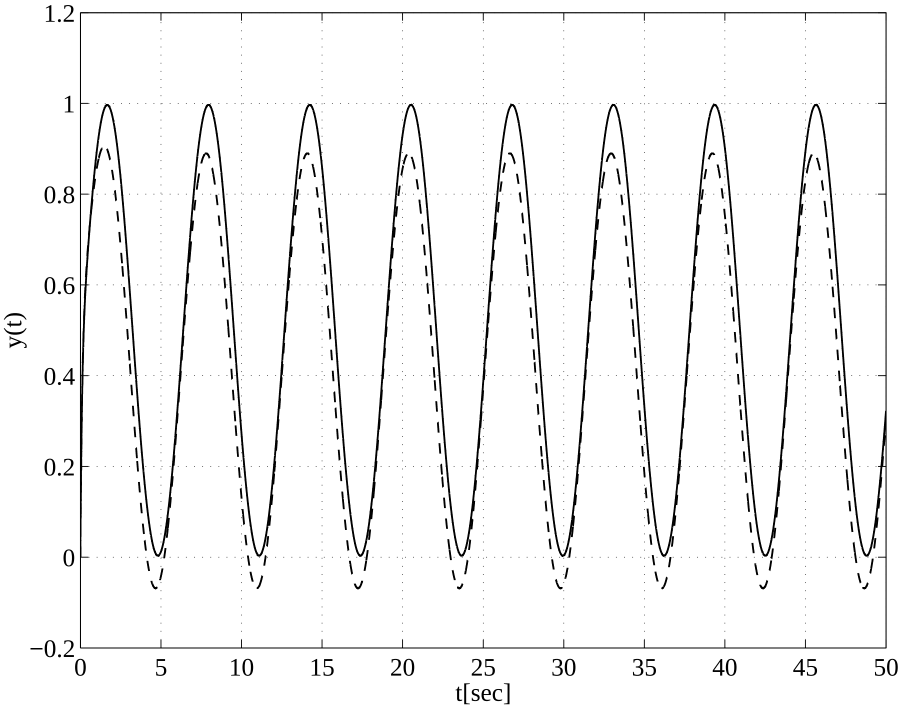

FIGURE 8. The response of the two-degree-of-freedom control system in Figure 1 for $G(s)$, $F_0(s)$ and the double feedback control system in Figure 2 for $G(s)$ for the reference input $r(t)$ in (39)

The responses of the output $y(t)$ of the two-degree-of-freedom control system in Figure 1 and the double feedback control system in Figure 2 are shown in Figure 8. Here, the dotted line shows the step response of the two-degree-of-freedom control system in Figure 1 for $G(s)$ and the solid line shows the step response of the double feedback control system in Figure 2 for $G(s)$. Figure 8 shows that for the step reference input $r(t)$ in (39), the two-degree-of-freedom control system in Figure 1 and the double feedback control system are stable.

The errors $e_1(s)$ in (4) and $e_2(s)$ in (12) are shown in Figure 9. Here, the dotted line shows the step response of $e_1(t)$ and the solid line shows the step response of $e_2(t)$. Figure 9 shows that for $G(s) \in \Omega$, the error between the reference input and the output of the double feedback control system in Figure 2 is smaller than that of the two-degree-of-freedom control system in Figure 1. In addition, from Figure 9, the controller designed by the design procedure in Section 5 makes the double feedback control system in Figure 2 robustly stable.

In this way, it is shown that using the proposed design method of the double feedback control system in Figure 2, we can design the double feedback control system in Figure 2 with robust stability and low-sensitivity characteristics.

7. **Conclusion.** In this paper, we have shown complete proof of the robust stability condition of the double feedback control system. In addition, a design method and a design procedure of the double feedback control system are described. Numerical example is illustrated to show the effectiveness of the proposed method. However, we do not consider a design method of the control system by using double feedback control with robust stability and low sensitivity for SISO non-minimum phase system with varying number of unstable pole. It is well-known that it is difficult to design a control system with low

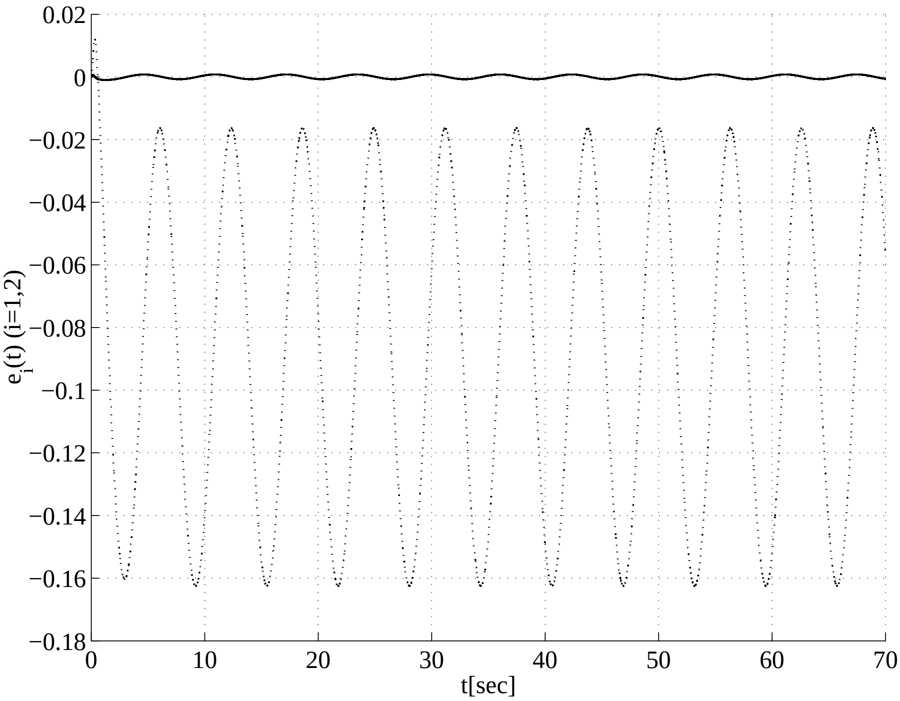

FIGURE 9. The response of $e_1(t)$ in (4) and $e_2(t)$ in (12) for the step reference input $r(t)$ in (39)

sensitivity for a non-minimum phase system. This design method will be clarified in another papers as future works.

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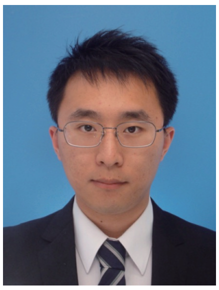

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