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SYNCHRONIZATION BETWEEN TWO NON-AUTONOMOUS CHAOTIC SYSTEMS VIA INTERMITTENT EVENT-TRIGGERED CONTROL

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Abstract. *An intermittent event-triggered control, as a combination of event-triggered control and periodically intermittent control, is proposed to realize the synchronization between two non-autonomous chaotic systems. Intermittent event-triggered control combines the advantages of event-triggered control and periodically intermittent control, which can reduce the number of control input updates and prolong the service life of control equipment. Under the intermittent event-triggered control, the system only needs to monitor, sample and transmit the synchronization error, and update the control input in the specific working time, while these operations are not needed in the rest time. Based on Lyapunov stability theory and linear matrix inequality technology, the sufficient conditions for synchronization between two non-autonomous chaotic systems are given. In particular, the event generator and controller are co-designed. Furthermore, it is proved that Zeno behavior will not occur in the control process. The conditions of event-triggered control and periodically intermittent control as special cases of intermittent event-triggered control in this paper are also discussed. Finally, two numerical examples are used to verify the theoretical results proposed in this paper.*

Keywords: Chaos synchronization, Event-triggered control, Periodically intermittent control, Lyapunov stability

1. **Introduction.**

Since the pioneering work of Pecora and Carroll [1], chaos synchronization has become a research hotspot in the field of nonlinear science because of its wealth applications in many fields of engineering and technology [2, 3, 4, 5, 6]. So far, many effective methods have been proposed to realize chaos synchronization, which can be roughly divided into "time-triggered control" and "event-triggered control". The time-triggered control methods not only have good control effect but also have the characteristics of predictability and easy implementation [7, 8, 9, 10]. As a discontinuous control method in time-triggered control, intermittent control divides each control period into two parts: "working time" for operative control and "rest time" for inoperative control, and changes the state of the system by adding a certain amount of control quantity to the system in the working time [11, 12, 13]. Compared with other continuous control methods, intermittent control is often used to realize chaos synchronization because it can reduce the amount of information transmission and prolong the service life of the control equipment. However, there is a problem in most chaotic synchronization schemes based on intermittent control that even when the system has achieved the desired control accuracy in the working time,

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the controller will still maintain the same frequency to control the system, resulting in a waste of resources.

The proposal of event-triggered mechanism in 1999 provides a new solution to this problem of resource waste [14, 15]. Under event-triggered control, the system only needs to update the control input when certain conditions are met, so as to reduce unnecessary signal transmission and save resources. At present, there have been many studies on using event-triggered control to achieve synchronization or consensus of systems [16, 17, 18, 19], and the co-design method of event generator and controller has also been proposed [20]. Compared with other discontinuous control methods, event-triggered control still has one drawback. The event generator and controller are still running although the controller does not need to continuously update the control input. Therefore, in order to further save communication resources as well as prolong the service life of the control equipment, it is meaningful to combine intermittent control with event-triggered control and propose a new control strategy. In [21], Hu and Cao proposed to introduce event-triggered mechanism into intermittent control, so that the controller only needs to work in a specific time, and no longer needs to continuously update the control input during the working time. And based on this intermittent event-triggered control method, the consensus of multi-agent systems is studied. After that, Hu and Cao proposed intermittent dynamic event-triggered control by improving the event-triggered condition on the basis of [21] to realize the consensus of nonlinear multi-agent systems [22]. In [23], Liu et al. proposed periodically intermittent event-triggered control by combining event-triggered control with periodically intermittent control, and studied the leader-following mean square consensus of stochastic multi-agent systems. In addition, the event-triggered condition is improved to reduce the number of triggering. Based on the event triggered mechanism, an adaptive linear sliding model control algorithm is employed to achieve spacecraft formation position coordination with bounded external disturbance and uncertain mass [24]. In [25], Chen et al. combined event-triggered control with intermittent control to study the exponential quasi-synchronization of coupled delayed memristive neural networks. The control cost in above references can be effectively reduced by reducing the number of control input updates. As far as we know, the problem about the synchronization of chaotic systems via intermittent event-triggered control is rarely studied. The intermittent event-triggered control is different from the periodically intermittent control and the event-triggered control. Is it a better control method between the event-triggered control and the periodically intermittent control? Therefore, an intermittent event-triggered method is designed by combining the periodically intermittent control and the event-triggered control in this paper. In addition, the synchronization results of drive and response systems are compared by three kinds of different control methods. The experimental results show that the proposed intermittent event-triggered method reduces the control cost and improves the control efficiency, and provides a more effective control method.

Inspired by the above literature, this paper studies the synchronization of non-autonomous chaotic systems under intermittent event-triggered control by combining the event-triggered control with the periodically intermittent control. In Theorem 3.1, the synchronization of drive and response systems is studied by the intermittent event-triggered control. The event-triggered control is designed when the intermittent controller is at a state of working, and it is at a state of rest during the rest period of the intermittent controller. However, if the event-triggered mechanism is triggered and updated infinitely in the control period, then it will show a terrible result of Zeno behavior. Therefore, in order to show there is no Zeno behavior in the proposed event-triggered control, it is strictly proved that the event-triggered mechanism is triggered and updated for a limited number of times in Theorem 3.2. Based on the experimental results, the intermittent event-triggered control can reduce the control cost by reducing the number of updates of the controller, and it provides a better control method than the periodically intermittent control or the event-triggered control. The main contributions are summarized as follows.

1) By combining event-triggered control with periodically intermittent control, an intermittent event-triggered control is proposed and applied to realizing the synchronization of two non-autonomous chaotic systems. This synchronization scheme can not only reduce communication consumption, but also prolong the service life of the control equipment.
2) Based on Lyapunov stability theory and linear matrix inequality technology, the sufficient conditions for the synchronization between two non-autonomous chaotic systems are given, and the event generator and controller are co-designed. Moreover, another theorem is given to show that Zeno behavior does not appear in the proposed intermittent event-triggered control.
3) The relationship between intermittent control, event control and intermittent event control is discussed, and the control cost, update rate and control time of three different controllers are compared via the simulation examples. The intermittent event-triggered control provides a good choice, which is helpful to design a better control method.

The rest of this paper is organized as follows. In Section 2, non-autonomous chaotic systems are modeled, and the synchronization scheme under intermittent event-triggered control is formulated. The main theoretical results are shown in Section 3. In Section 4, two numerical examples are given to verify the theoretical results. Finally, the conclusion is drawn in Section 5.

2. **Synchronization Scheme.**

The n-dimensional drive and response systems in the synchronization scheme are described as follows:

Drive system:

$$\dot{x}(t) = A(t)x(t) + f(x) + m(t), \tag{1}$$

Response system:

$$\dot{y}(t) = A(t)y(t) + f(y) + m(t) + u(t), \tag{2}$$

where $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^n$ and $y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}} \in R^n$ are the state vectors, $A(t) \in R^{n\times n}$ is a bounded matrix, $f(\cdot) \in R^n$ is a continuous nonlinear function, $m(t) \in R^n$ is the external excitation function, and $u(t)$ is the intermittent event-triggered controller to be designed.

Defining the synchronization error $e(t) = y(t) - x(t)$, one can obtain the synchronization error system

$$\dot{e}(t) = A(t)e(t) + f(y) - f(x) + u(t). \tag{3}$$

Under the periodically intermittent control mechanism, we divide the time intervals $t \in [0, +\infty)$ into working time intervals $t \in [mT, mT + \delta)$ and rest time intervals $t \in [mT + \delta, (m + 1)T)$, $m = 0, 1, 2, 3, \ldots$, where $T > 0$ is the control period and $0 < \delta \leqslant T$ is the control width. The controller works only during the working time intervals $t \in [mT, mT + \delta)$.

In addition, the event-triggered control mechanism is also considered to reduce the waste of resources caused by unnecessary control input updates. Under the intermittent event-triggered control, the event generator also works only during the working time intervals $t \in [mT, mT + \delta)$. If the i-th event-triggered instant in the m-th working time interval $t \in [mT, mT + \delta)$ is t_i^m, the next event-triggered instant t_{i+1}^m can be defined as

$$t_{i+1}^m = \min\left\{ t > t_i^m \left| z^{\mathrm{T}}(t)z(t) > \sigma e^{\mathrm{T}}(t)e(t) \right.\right\}, \tag{4}$$

where

$$z^{\mathrm{T}}(t)z(t) > \sigma e^{\mathrm{T}}(t)e(t) \tag{5}$$

is the corresponding event-triggered condition to be designed, $z(t) = e(t_i^m) - e(t)$, $\sigma > 0$ is the event-triggered parameter. When $t \in [mT, mT+\delta)$, the sequence of event-triggered instants determined by event-triggered condition (5) is denoted by $t_0^m, t_1^m, t_2^m, \ldots, t_i^m, t_{i+1}^m, \ldots$, $m = 0, 1, 2, 3, \ldots$. Illustration for time division under intermittent event-triggered control is shown in Figure 1. The $t_0^m, t_1^m, t_2^m, \ldots, t_i^m, t_{i+1}^m, \ldots \in [mT, mT+\delta)$ are the instants that the event-triggered condition $z^{\mathrm{T}}(t)z(t) > \sigma e^{\mathrm{T}}(t)e(t)$ happens, so the controller is updated and given by

$$u(t) = Ke(t_i^m),\ \ t \in [t_i^m, t_{i+1}^m)$$

where $i = 0, 1, 2, \ldots$, $m = 0, 1, 2, 3, \ldots$.

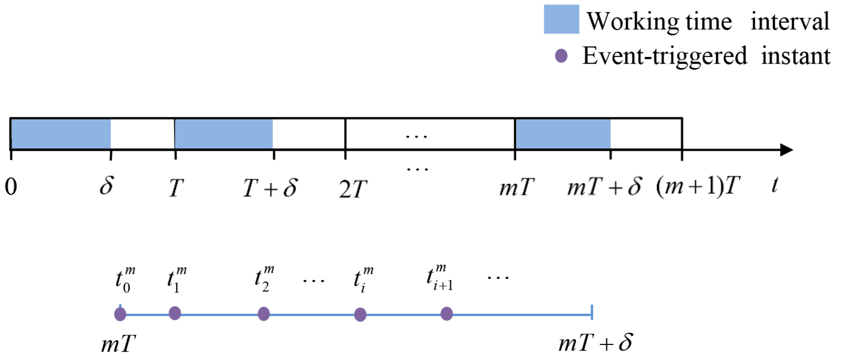

Figure 1. Illustration for working time division under intermittent event-triggered control

Thus, the intermittent event-triggered controller can be designed as follows:

$$u(t) = \begin{cases} Ke(t_i^m), & t \in [t_i^m, t_{i+1}^m) \cap [mT, mT+\delta) \\ 0, & t \in [mT+\delta, (m+1)T) \end{cases},$$
$$i = 0, 1, 2, \ldots, N_m,\ \ m = 0, 1, 2, 3\ldots, \tag{6}$$

where $K = \mathrm{diag}\{k_1, k_2, \ldots, k_n\} \in R^{n\times n}$ is the control gain, $\mathrm{diag}\{\cdots\}$ represents the diagonal matrix, $k_1, k_2, \ldots, k_n$ are constants, and N_m is the number of triggers. Furthermore, we suppose that the instant mT is the first event-triggered instant, i.e., $t_0^m = mT$ [21].

Under the intermittent event-triggered control, both the event generator and controller work intermittently in each control period. In the working time intervals $t \in [mT, mT+\delta)$, if $z^{\mathrm{T}}(t)z(t) > \sigma e^{\mathrm{T}}(t)e(t)$ holds at the current time t, an event will be triggered, i.e., $t_{i+1}^m = t$. At this time, the event generator will sample the synchronization error, i.e., $e(t_{i+1}^m) = e(t)$, and then the control input will be updated. Otherwise, the event generator will not perform sampling, and correspondingly, the zero-order holder will retain the synchronization error updated at the last event-triggered instant t_i^m until the next event-triggered instant t_{i+1}^m comes, so that the control input will not be updated. While in the rest time intervals $t \in [mT+\delta, (m+1)T)$, neither the event generator nor the controller works, the response system will operate without control. In order to illustrate the chaotic synchronization scheme under intermittent event-triggered control more clearly, the synchronization framework and synchronization algorithm of drive-response systems (1) and (2) based on intermittent event-triggered control are given in Figure 2 and Algorithm 1, respectively.

In Algorithm 1, the initial values of $x(0)$, $y(0)$, parameter σ, control period T and control gain K are firstly given, and then the event-triggered condition is calculated by

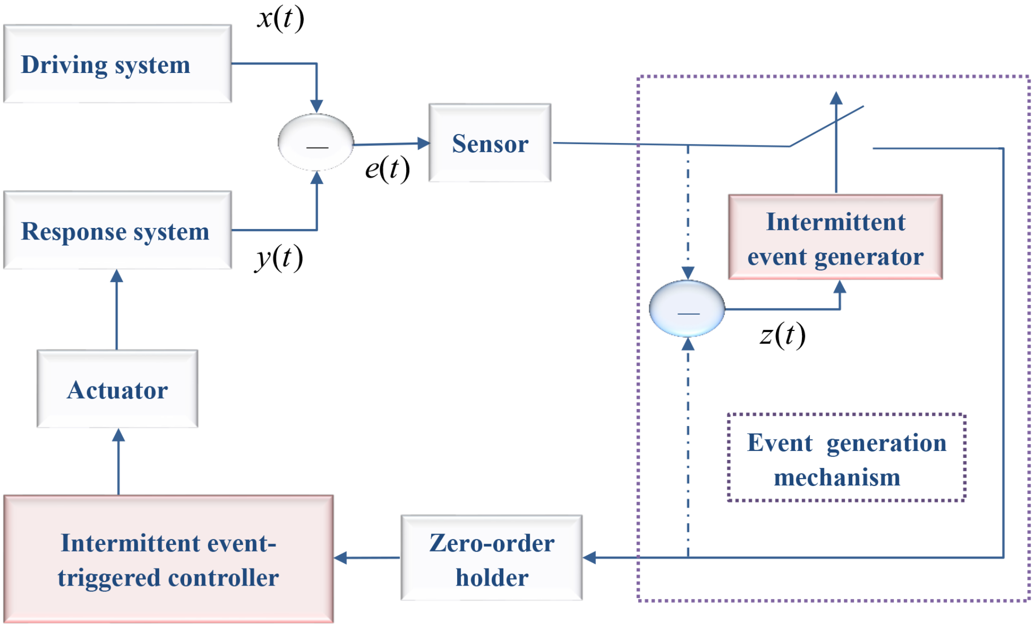

FIGURE 2. Synchronization framework of drive-response systems (1) and (2) based on intermittent event-triggered control

Algorithm 1: Synchronization algorithm of intermittent event-triggered control

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Input: Initialize all parameters, such as the initial states of the drive and response
       systems $x(0)$ and $y(0)$, control gain $K$, event-triggered parameter $\sigma$, control
       period $T$, and control width $\delta$. Update $e(0) = y(0) - x(0)$ $(t_0^m = mT)$.
for $t$ from $t_{\text{start}}$ to $t_{\text{end}}$ do
    if $t \in [mT, mT + \delta]$ then
        Update the states of drive-response systems (1) and (2);
        Calculate $e(t)$: $e(t) = y(t) - x(t)$;
        Calculate $z(t)$: $z(t) = e(t_i^m) - e(t)$;
        if $z^{\mathrm{T}}(t)z(t) > \sigma e^{\mathrm{T}}(t)e(t)$ then
            Record the time $t$ as the event-triggered instant $t_{i+1}^m$, and $z(t) = e(t_{i+1}^m)$
             $-e(t)$;
            Update the control input at time $t_{i+1}^m$, i.e., $u(t) = Ke(t_{i+1}^m)$;
        else
            Maintain the control input of the last time $t_i^m$, i.e., $u(t) = Ke(t_i^m)$;
        end
        Update the states of the response system with controller $u(t)$ at time $t$;
    end
    if $t \in [mT + \delta, (m + 1)T]$ then
        Update the states of the response system without controller at time $t$;
    end
end
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$z^{\mathrm{T}}(t)z(t)$ and $\sigma e^{\mathrm{T}}(t)e(t)$, where $e(t) = y(t) - x(t)$ and $z(t) = e(t_i^m) - e(t)$. The error $e(t_i^m)$ of $z(t) = e(t_i^m) - e(t)$ will be replaced by $e(t)$ when the event-triggered condition is satisfied $z^{\mathrm{T}}(t)z(t) > \sigma e^{\mathrm{T}}(t)e(t)$ at the time $t = t_{i+1}^m$, i.e., $z(t) = e(t_{i+1}^m) - e(t)$ when $t \geqslant t_{i+1}^m$, and then the controller is updated by

$$u(t) = Ke(t_{i+1}^m),\ t \in [t_{i+1}^m, t_{i+2}^m),$$

where t_{i+2}^m is the next time that the event-triggered condition is satisfied $z^{\mathrm{T}}(t)z(t) > \sigma e^{\mathrm{T}}(t)e(t)$, and $t_i^m, t_{i+1}^m, \ldots \in [mT, mT+\delta)$. Otherwise, the controller will remain unchanged, i.e.,

$$u(t) = Ke(t_i^m),\ t \in [t_i^m, t_{i+1}^m).$$

On the other hand, the controller is given by

$$u(t) = 0,\ t \in [\delta, (m+1)T) \subset [mT, (m+1)T],$$

and the event generator is at a state of rest when $t \in [\delta, (m+1)T)$.

For example, the event-triggered condition $z^{\mathrm{T}}(t)z(t)$ and the error $\sigma e^{\mathrm{T}}(t)e(t)$ in the first period $[0, 1]$ based on Example 4.1 are shown in Figure 3.

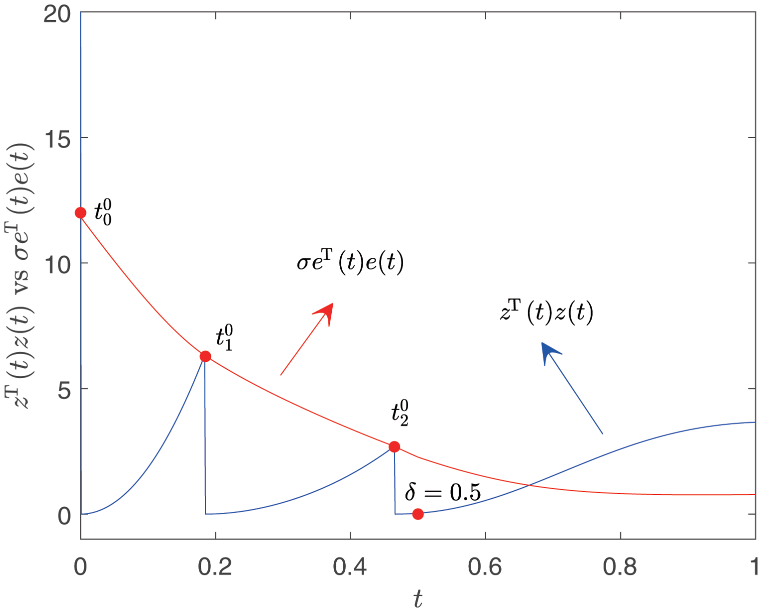


FIGURE 3. The condition $z^{\mathrm{T}}(t)z(t)$ and $\sigma e^{\mathrm{T}}(t)e(t)$ of the intermittent event-triggered control in the first period $[0, 1]$

To facilitate the subsequent discussion, the following Hypothesis 2.1, Lemma 2.1 and Lemma 2.2 are given, which will be needed in the proof of the main results.

Hypothesis 2.1. *The nonlinear function* $f(\cdot)$ *satisfies*

$$f(y) - f(x) = M(x, y)(y - x), \tag{7}$$

where $M(x, y)$ *is a bounded matrix. Such a matrix* $M(x, y)$ *does exist in many practical chaotic systems, such as Lorenz system, Chen system as well as the examples in Section 4. Moreover, due to the boundedness of* $A(t)$ *and* $M(x, y)$*, we can assume that there are positive definite matrices* A *and* M *such that*

$$A - A(t) \geqslant 0,\ M - M(x, y) \geqslant 0. \tag{8}$$

Lemma 2.1. [26] *If* X *and* Y *are real matrices with appropriate dimensions, then there exists* $\mu > 0$ *such that*

$$X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leqslant \mu X^{\mathrm{T}}X + \mu^{-1}Y^{\mathrm{T}}Y. \tag{9}$$

Lemma 2.2. [27] *The following linear matrix inequality*

$$\begin{pmatrix} N(x) & S(x) \\ S^{\mathrm{T}}(x) & R(x) \end{pmatrix} > 0$$

is equivalent to

$$R(x) > 0,\ N(x) - S(x)R^{-1}(x)S^{\mathrm{T}}(x) > 0,$$

where $N(x) = N^{\mathrm{T}}(x)$*,* $R(x) = R^{\mathrm{T}}(x)$ *and* $S(x)$ *depend affinely on* x*.*

From $z(t) = e(t_i^m) - e(t)$, we can rewrite Equation (6) as

$$u(t) = \begin{cases} K(z(t) + e(t)), & t \in [t_i^m, t_{i+1}^m) \cap [mT, mT + \delta) \\ 0, & t \in [mT + \delta, (m+1)T) \end{cases},$$
$$i = 0, 1, 2, \ldots, N_m,\ m = 0, 1, 2, 3, \ldots. \tag{10}$$

By substituting Equations (7) and (10) into Equation (3), the synchronization error system can be rewritten as follows:

$$\dot{e}(t) = \begin{cases} A(t)e(t) + M(x, y)e(t) + K(z(t) + e(t)), & t \in [t_i^m, t_{i+1}^m) \cap [mT, mT + \delta) \\ A(t)e(t) + M(x, y)e(t), & t \in [mT + \delta, (m+1)T) \end{cases},$$
$$i = 0, 1, 2, \ldots, N_m,\ m = 0, 1, 2, 3, \ldots. \tag{11}$$

Next, our goal is to find the appropriate event-triggered parameter σ, control gain K, control period T and control width δ such that the trajectories of drive and response systems satisfy

$$\lim_{t \to +\infty} \|y(t) - x(t)\| = \lim_{t \to +\infty} \|e(t)\| = 0, \tag{12}$$

where $\|\cdot\|$ denotes the Euclidean norm of vector.

Therefore, the synchronization of chaotic systems (1) and (2) in the sense of (12) is equivalent to asymptotical stability of the synchronization error system (11) at the origin $e = 0$. Based on the analyses of error system (11), the synchronization of drive-response systems (1) and (2) is discussed, and Lemma 2.1 and Hypothesis 2.1 are used to prove Theorem 3.1 in Section 3.

3. Main Results.

In this section, Lyapunov stability theory and linear matrix inequality technology are used to provide sufficient conditions for the error system of chaotic systems (1) and (2) to achieve asymptotical stability. The main results are given as follows.

Theorem 3.1. *If there exists a positive definite diagonal matrix* P*, positive scalars* λ_1*,* λ_2 *and* α *such that the following conditions hold:*

i) $PA + A^{\mathrm{T}}P + PM + M^{\mathrm{T}}P + PK + K^{\mathrm{T}}P + \alpha PKK^{\mathrm{T}}P + \alpha^{-1}\sigma I + \lambda_1 P \leqslant 0$*,*
ii) $PA + A^{\mathrm{T}}P + PM + M^{\mathrm{T}}P - \lambda_2 P \leqslant 0$*,*
iii) $\delta\lambda_1 - (T - \delta)\lambda_2 > 0$*,*

then the origin of synchronization error system (11) is asymptotically stable; moreover,

$$\|e(t)\| \leqslant \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\|e(0)\| \exp(-\gamma(t - \delta)),\ \forall t > 0,$$

where $\gamma = \frac{\delta\lambda_1 - (T-\delta)\lambda_2}{2T}$*,* $\lambda_{\min}(P)$ *and* $\lambda_{\max}(P)$ *represent the minimum and maximum eigenvalues of* P*, respectively.*

Proof: Choose the following Lyapunov function

$$V(e(t)) = e^{\mathrm{T}}(t)Pe(t).$$

Obviously, we have

$$\lambda_{\min}(P)\|e(t)\|^2 \leqslant V(e(t)) \leqslant \lambda_{\max}(P)\|e(t)\|^2. \tag{13}$$

When $t \in [t_i^m, t_{i+1}^m) \cap [mT, mT+\delta)$, $i = 0, 1, 2, \ldots, N_m$, $m = 0, 1, 2, \ldots$, calculating the derivative of $V(e(t))$ with respect to time t along the trajectory of the first subsystem of synchronization error system (11), we can get

$$\begin{aligned}
&\dot{V}(e(t)) \\
&= \dot{e}^{\mathrm{T}}(t)Pe(t) + e^{\mathrm{T}}(t)P\dot{e}(t) \\
&= e^{\mathrm{T}}(t)\big[P(A(t)+M(x,y)+K) + (A(t)+M(x,y)+K)^{\mathrm{T}}P\big]e(t) + 2e^{\mathrm{T}}(t)PKz(t).
\end{aligned}$$

According to Hypothesis 2.1 and Lemma 2.1, we can obtain

$$\begin{aligned}
&\dot{V}(e(t)) \\
&\leqslant e^{\mathrm{T}}(t)\big[P(A+M+K) + (A+M+K)^{\mathrm{T}}P\big]e(t) + 2e^{\mathrm{T}}(t)PKz(t) \\
&\leqslant e^{\mathrm{T}}(t)\big[P(A+M+K) + (A+M+K)^{\mathrm{T}}P\big]e(t) + \alpha e^{\mathrm{T}}(t)PKK^{\mathrm{T}}Pe(t) + \alpha^{-1}z^{\mathrm{T}}(t)z(t).
\end{aligned}$$

In addition, according to the event-triggered condition (5) and the discussion in Section 2, we can get that $z^{\mathrm{T}}(t)z(t) \leqslant \sigma e^{\mathrm{T}}(t)e(t)$ is always established when $t \in [mT, mT+\delta)$. Thus, the following inequality holds:

$$\begin{aligned}
&\dot{V}(e(t)) \\
&\leqslant e^{\mathrm{T}}(t)\big[P(A+M+K) + (A+M+K)^{\mathrm{T}}P\big]e(t) + \alpha e^{\mathrm{T}}(t)PKK^{\mathrm{T}}Pe(t) + \alpha^{-1}\sigma e^{\mathrm{T}}(t)e(t) \\
&= -\lambda_1 V(e(t)) + e^{\mathrm{T}}(t)\big[PA + A^{\mathrm{T}}P + PM + M^{\mathrm{T}}P + PK + K^{\mathrm{T}}P + \alpha PKK^{\mathrm{T}}P + \alpha^{-1}\sigma I \\
&\quad + \lambda_1 P\big]e(t).
\end{aligned}$$

Therefore, when the condition i) in Theorem 3.1 is satisfied, we have

$$\dot{V}(e(t)) \leqslant -\lambda_1 V(e(t)),\ t \in [mT, mT+\delta).$$

Thus,

$$V(e(t)) \leqslant V(e(mT))\exp(-\lambda_1(t-mT)),\ t \in [mT, mT+\delta). \tag{14}$$

Similarly, when $t \in [mT+\delta, (m+1)T)$,

$$\begin{aligned}
\dot{V}(e(t)) &= \dot{e}^{\mathrm{T}}(t)Pe(t) + e^{\mathrm{T}}(t)P\dot{e}(t) \\
&= e^{\mathrm{T}}(t)\big[P(A(t)+M(x,y)) + (A(t)+M(x,y))^{\mathrm{T}}P\big]e(t) \\
&\leqslant \lambda_2 V(e(t)) + e^{\mathrm{T}}(t)\big[PA + A^{\mathrm{T}}P + PM + M^{\mathrm{T}}P - \lambda_2 P\big]e(t).
\end{aligned}$$

And if the condition ii) in Theorem 3.1 is satisfied, then

$$\dot{V}(e(t)) \leqslant \lambda_2 V(e(t)),\ t \in [mT+\delta, (m+1)T).$$

Thus,

$$V(e(t)) \leqslant V(e(mT+\delta))\exp(\lambda_2(t-mT-\delta)),\ t \in [mT+\delta, (m+1)T). \tag{15}$$

And then according to the discussion in [26], it can be obtained that

$$V(e(t)) \leqslant V(e(0))\exp\left(-\frac{\delta\lambda_1 - (T-\delta)\lambda_2}{T}(t-\delta)\right),\ \forall t > 0. \tag{16}$$

Substituting Inequality (13) into Inequality (16) and sorting it out, one can obtain that

$$\|e(t)\| \leqslant \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\|e(0)\|\exp(-\gamma(t-\delta)),\ \forall t > 0, \tag{17}$$

where $\gamma = \frac{\delta\lambda_1 - (T-\delta)\lambda_2}{2T}$. When the condition iii) in Theorem 3.1 holds, i.e., $\gamma > 0$, we can obtain $\lim_{t\to+\infty}\|e(t)\| = 0$. Consequently, drive system (1) and response system (2) can achieve synchronization. Hence, the proof of Theorem 3.1 is completed. □

Remark 3.1. *The parameter γ represents the exponential convergence rate of the controlled system, which depends on the control period T and the control width δ, but also depends implicitly on the control gain K and the event-triggered parameter σ for the reason that λ_1 depends on K and σ.*

Remark 3.2. *In Figure 1, the rest time will be shortened to zero when $\delta \to T$, so the intermittent event-triggered control will become the traditional event-triggered control. At this time, the condition i) in Theorem 3.1 gives a sufficient condition for the asymptotical stability of the system (11) under the event-triggered control. A similar conclusion is found in* [24].

Remark 3.3. *In the traditional periodically intermittent control, the control input is continuously updated in the working time interval $t \in [mT, mT+\delta)$. Therefore, if $\sigma = 0$, the intermittent event-triggered control will become periodically intermittent control. At this time, Theorem 3.1 gives the sufficient conditions for the asymptotical stability of the system (11) under the periodically intermittent control.*

Remark 3.4. *According to Lemma 2.2, the condition i) of Theorem 3.1 is equivalent to the following linear matrix inequality* [26]*:*

$$\begin{pmatrix} (PA + A^{\mathrm{T}}P + PM + M^{\mathrm{T}}P + 2Q + \alpha^{-1}\sigma I + \lambda_1 P) & -Q \\ -Q & -\alpha^{-1}I \end{pmatrix} \leqslant 0, \tag{18}$$

where $Q = PK$, then K can be calculated by $K = P^{-1}Q$.

Obviously, if the event-triggered condition (5) is triggered and updated infinitely in the control period $[mT, mT+\delta)$, then it will invalidate the control method, which is called Zeno behavior. In order to avoid Zeno behavior in intervals $[mT, mT+\delta)$, $m = 0, 1, 2, \ldots$, the inter-event times $\{t_{i+1}^m - t_i^m\}$ are required to be strictly positive, $i = 0, 1, 2, \ldots, N_m$, $m = 0, 1, 2, \ldots$. According to [28], we will prove that there is a positive lower bound for the inter-event times in Theorem 3.1 when the event-triggered condition (5) and the control law (6) are satisfied. Therefore, Theorem 3.2 studies that the inter-event time is strictly positive for the proposed intermittent event-triggered control method, and it is given as follows.

Theorem 3.2. *Consider the synchronization error system (11) and the control law (6). When $t \in [mT, mT+\delta)$, $m = 0, 1, 2, \ldots$, there is a positive lower bound $\tau = \frac{\sqrt{\sigma}}{\rho(1+\sqrt{\sigma})} > 0$ for the inter-event times $\{t_{i+1}^m - t_i^m\}$ determined by the event-triggered condition (5), i.e., $t_{i+1}^m - t_i^m \geqslant \tau > 0$, $i = 0, 1, 2, \ldots, N_m$, $m = 0, 1, 2, \ldots$, where $\rho = \|A\| + \|M\| + \|K\|$.*

Proof: For each $[t_i^m, t_{i+1}^m)$,

$$\begin{aligned}
&\frac{d}{dt}\frac{\|z(t)\|}{\|e(t)\|} \\
&= \frac{d}{dt}\frac{(z^{\mathrm{T}}(t)z(t))^{\frac{1}{2}}}{(e^{\mathrm{T}}(t)e(t))^{\frac{1}{2}}} \\
&= \frac{\left(\left(z(t)^{\mathrm{T}}z(t)\right)^{-\frac{1}{2}}\left(z(t)^{\mathrm{T}}\dot{z}(t)\right)\left(e(t)^{\mathrm{T}}e(t)\right)^{\frac{1}{2}} - \left(e(t)^{\mathrm{T}}e(t)\right)^{-\frac{1}{2}}\left(e(t)^{\mathrm{T}}\dot{e}(t)\right)\left(z(t)^{\mathrm{T}}z(t)\right)^{\frac{1}{2}}\right)}{e(t)^{\mathrm{T}}e(t)} \\
&= -\frac{z(t)^{\mathrm{T}}\dot{e}(t)}{\|z(t)\|\|e(t)\|} - \frac{e(t)^{\mathrm{T}}\dot{e}(t)\cdot\|z(t)\|}{\|e(t)\|^2\|e(t)\|}
\end{aligned}$$

$$
\begin{aligned}
&\leqslant \frac{\|z(t)\|\|\dot{e}(t)\|}{\|z(t)\|\|e(t)\|} + \frac{\|e(t)\|\|\dot{e}(t)\|\|z(t)\|}{\|e(t)\|^3}\\
&= \left(1 + \frac{\|z(t)\|}{\|e(t)\|}\right)\frac{\|\dot{e}(t)\|}{\|e(t)\|}\\
&= \left(1 + \frac{\|z(t)\|}{\|e(t)\|}\right)\frac{(\|A(t)e(t) + M(x,y)e(t) + K(z(t)+e(t))\|)}{\|e(t)\|}\\
&\leqslant \left(1 + \frac{\|z(t)\|}{\|e(t)\|}\right)\left(\frac{((\|A\| + \|M\|)\|e(t)\| + \|K\|(\|z(t)\| + \|e(t)\|))}{\|e(t)\|}\right)\\
&\leqslant \rho\left(1 + \frac{\|z(t)\|}{\|e(t)\|}\right)^2,
\end{aligned}
$$

where $\rho = \|A\| + \|M\| + \|K\|$. The previous inequality becomes $\dot{y} \leqslant \rho(1+y)^2$ under a substitution $y = \frac{\|z\|}{\|e\|}$. Obviously, y satisfies $y(t) \leqslant \varphi(t, \varphi_0)$, where $\varphi(t, \varphi_0)$ is the solution of

$$
\dot{\varphi} = \rho(1+\varphi)^2 \tag{19}
$$

satisfying $\varphi(0, \varphi_0) = 0$. According to the event-triggered condition (5), the inter-event times are bounded by the time it takes for φ to evolve from 0 to $\sqrt{\sigma}$, i.e., inter-event times are bounded by the solution τ of $\varphi(\tau, 0) = \sqrt{\sigma}$. By solving Equation (19), we can obtain $\varphi(\tau, 0) = \tau\rho/(1-\tau\rho)$. Thus, $\sqrt{\sigma} = \tau\rho/(1-\tau\rho)$. Finally, through sorting we can get that $\tau = \frac{\sqrt{\sigma}}{\rho(1+\sqrt{\sigma})} > 0$. Theorem 3.2 is proved. □

4. Numerical Simulation.

In this section, two numerical examples will be used to verify the above theoretical results.

Example 4.1. *Consider the horizontal platform system as follows* [29]*:*

$$
\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -ax_2 - b\sin x_1 + c\cos x_1 \sin x_1 + f\cos\omega t. \end{cases} \tag{20}
$$

This system exhibits chaos behavior when $a = \frac{4}{3}$*,* $b = 3.77615$*,* $c = 0.0000046$*,* $f = \frac{34}{3}$ *and* $\omega = 1.8$*, as shown in Figure 4.*

Compared with system (1), we can get the following:

$$
A(t) = \begin{pmatrix} 0 & 1 \\ 0 & -a \end{pmatrix}, \quad m(t) = \begin{pmatrix} 0 \\ f\cos\omega t \end{pmatrix}, \quad f(x) = \begin{pmatrix} 0 \\ -b\sin x_1 + c\sin x_1 \cos x_1 \end{pmatrix}.
$$

Similarly, compared with system (2), it can be obtained that

$$
\dot{y}(t) = A(t)y(t) + f(y) + m(t) + u(t), \tag{21}
$$

where

$$
f(y) = \begin{pmatrix} 0 \\ -b\sin y_1 + c\sin y_1 \cos y_1 \end{pmatrix}.
$$

And from function $f(y)$, the matrix $M(x, y)$ mentioned in Hypothesis 2.1 equals

$$
M(x,y) = \begin{pmatrix} 0 & 0 \\ q(t) & 0 \end{pmatrix},
$$

where

$$
q(t) = \frac{(-b(\sin y_1 - \sin x_1) + c(\sin y_1 \cos y_1 - \sin x_1 \cos x_1))}{y_1 - x_1}.
$$

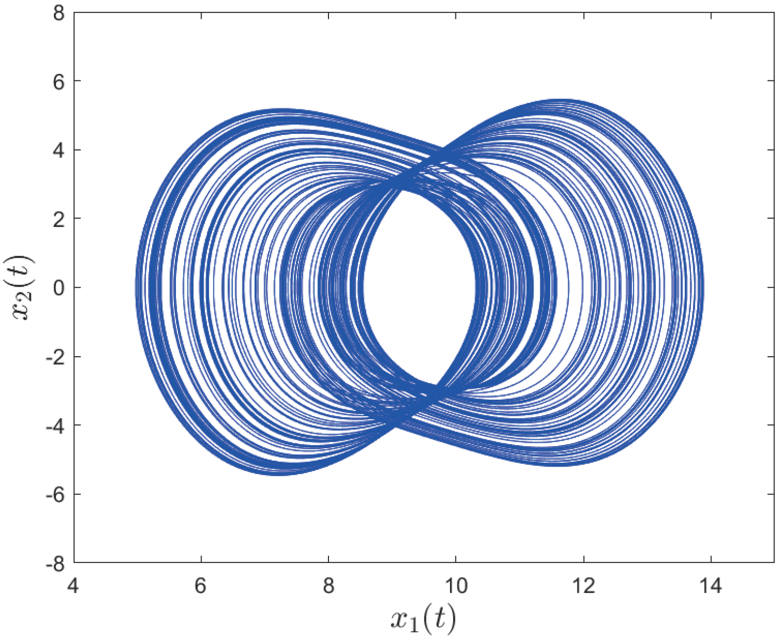


Figure 4. Chaotic attractor of the non-autonomous horizontal platform system (20)

According to the discussion in [29], the function $q(t)$ satisfies $|q(t)| < b + |c|$. Then we can find a positive definite matrix

$$M = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix}$$

satisfying $M - M(x, y) > 0$. In addition, from $a = \frac{4}{3}$ we can get that

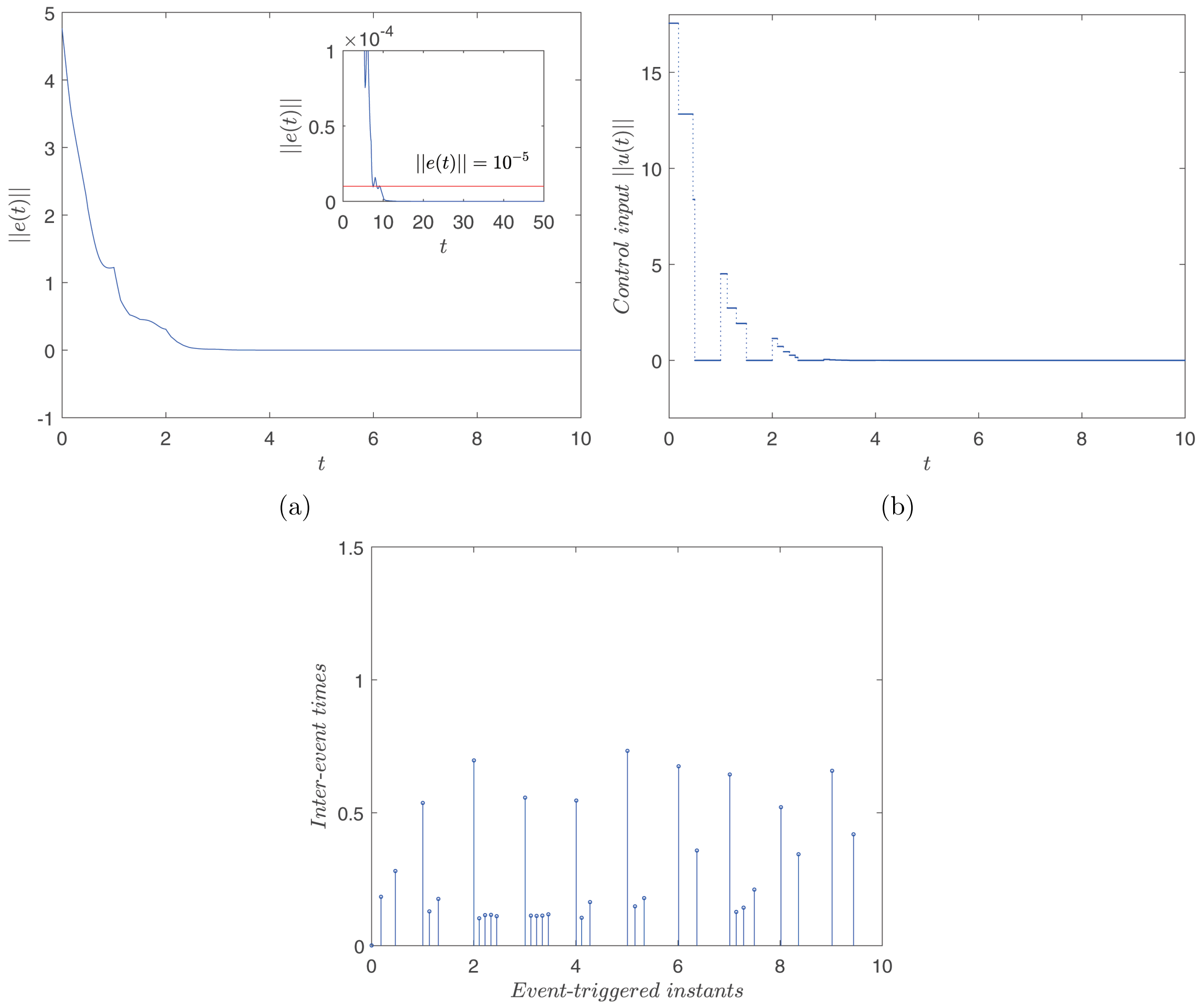
$$A = \begin{pmatrix} 0 & 1 \\ 0 & -\dfrac{4}{3} \end{pmatrix}.$$

Take $\alpha = 1$, $\lambda_1 = 1$, then substitute the values of A and M into the linear matrix inequality (18) and solve it. Finally, we can obtain $\sigma = 0.52$,

$$P = \begin{pmatrix} 0.16 & 0 \\ 0 & 1.03 \end{pmatrix}, \; K = \begin{pmatrix} -3.68 & 0 \\ 0 & -0.03 \end{pmatrix}.$$

If the event-triggered condition $z^{\mathrm{T}}(t)z(t) > 0.52e^{\mathrm{T}}(t)e(t)$ is satisfied, then the control input will be updated. As the $\lambda_2 = 0.34$ can be obtained by solving the condition ii) of Theorem 3.1, we can find the control period $T = 1$ and the control width $\delta = 0.5$ to ensure $\delta\lambda_1 - (T - \delta)\lambda_2 > 0$. If the step size $h = 0.001$, the total running time of the system $t = 50$ s, and the initial values of the drive-response systems (20) and (21) $(x_1(0), x_2(0)) = (-2, 1.9)$ and $(y_1(0), y_2(0)) = (0.6, -2.1)$, then the synchronization of drive-response systems (20) and (21) under intermittent event-triggered control is simulated by Matlab. The simulation results are given in Figure 5.

Figure 5(a) shows the synchronization error curve of the drive-response systems (20) and (21) under intermittent event-triggered control. The synchronization error curve tends to zero gradually in an alternating manner of decreasing and increasing, which is consistent with the working principle of the intermittent event-triggered control proposed in this paper. The synchronization error is almost equal to zero at about 17 seconds, which means that the drive-response systems (20) and (21) achieve synchronization.



(a)

(b)

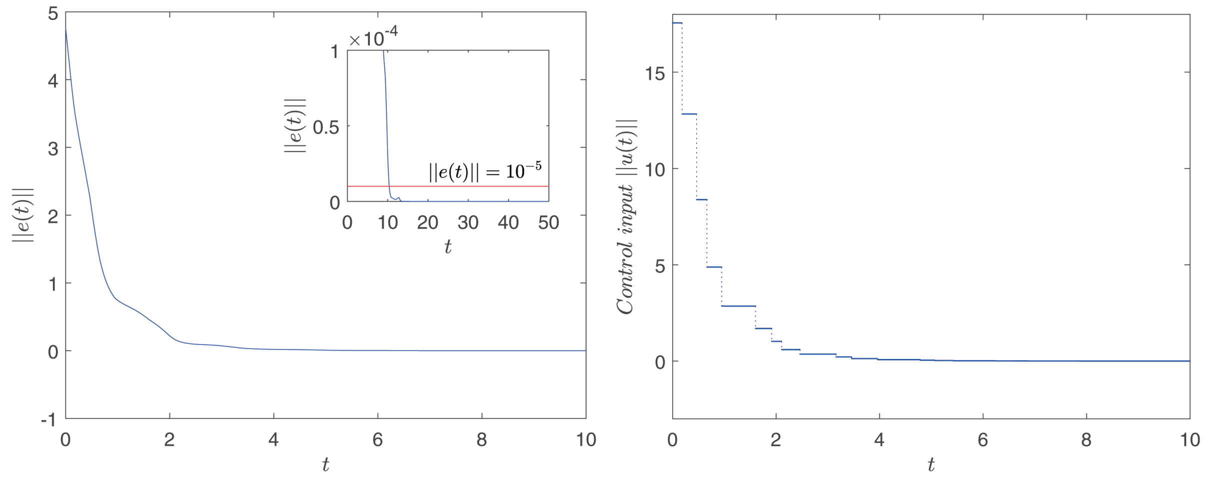
(c)

FIGURE 5. Simulation results of synchronization of drive-response systems (20) and (21) under intermittent event-triggered control with $k_1 = -3.68$, $k_2 = -0.03$, $\sigma = 0.52$, $T = 1$, $\delta = 0.5$: (a) Synchronization error curve; (b) control input; (c) event-triggered instants and inter-event times

Figure 5(b) reflects the update of the control input under the intermittent event-triggered control. The controller works intermittently rather than continuously, and it does not continuously update the control input during the working time. Moreover, with the realization of synchronization, the control input gradually converges to zero. The total number of control input updates is 98. Considering the function of control cost as $R = \int_0^{+\infty} \|u(t)\| dt$, through simulation, we can get that the control cost of this synchronization scheme is about 8872.24.

Figure 5(c) shows the event-triggered instants and inter-event times. The event-triggered control can only occur during the working time of each control period, and there is no Zeno behavior in the control process. The simulation results of synchronization of drive-response systems (20) and (21) are consistent with Theorems 3.1 and 3.2, which verifies that the synchronization scheme based on intermittent event-triggered control is feasible.

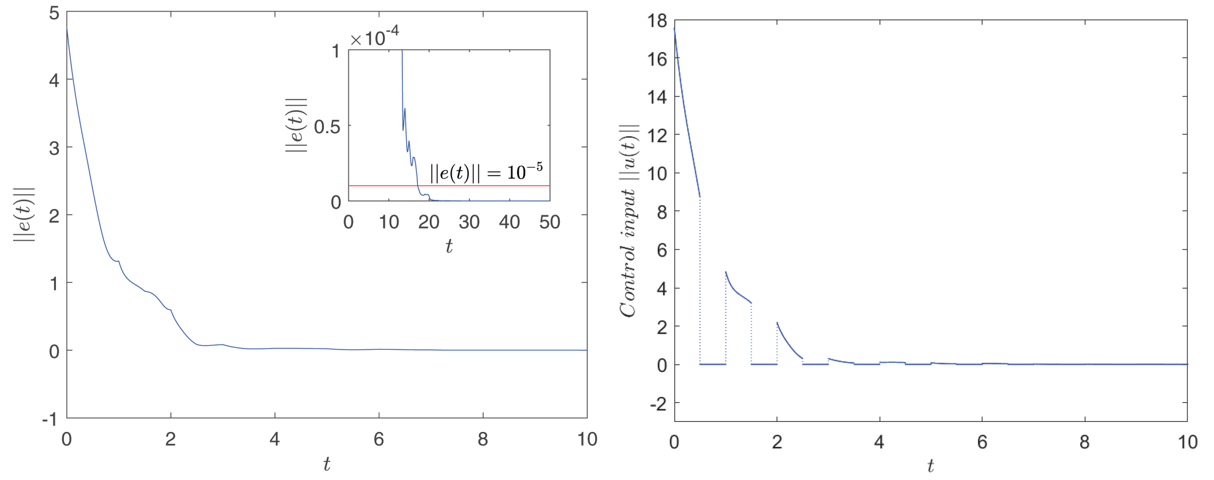
According to Remark 3.2, the intermittent event-triggered control will become the event-triggered control when $\delta = T$. Therefore, if the control width $\delta = T$ and the other conditions are unchanged, then the synchronization error curve of drive-response systems (20) and (21) and the update of the control input are obtained in Figure 6. The simulation results show that the synchronization scheme based on the event-triggered control is feasible, and the control input tends to zero as $t \to \infty$.



(a) (b)

FIGURE 6. Simulation results of synchronization of drive-response systems (20) and (21) under event-triggered control with $k_1 = -3.68$, $k_2 = -0.03$, $\sigma = 0.52$, $T = 1$, $\delta = 1$: (a) Synchronization error curve; (b) control input

By Remark 3.3, the intermittent event-triggered control will become the periodically intermittent control when $\sigma = 0$. If the event-triggered parameter $\sigma = 0$ and the other conditions are unchanged, then the corresponding synchronization results of drive-response systems (20) and (21) are shown in Figure 7. The drive-response systems (20) and (21) are synchronized by the periodically intermittent control, and the control input tends to zero during the control periods $[m, m + \delta)$ $(m = 0, 1, 2, \ldots)$ as $t \to \infty$.



(a) (b)

FIGURE 7. Simulation results of synchronization of drive-response systems (20) and (21) under periodically intermittent control with $k_1 = -3.68$, $k_2 = -0.03$, $\sigma = 0$, $T = 1$, $\delta = 0.5$: (a) Synchronization error curve; (b) control input

Furthermore, the simulation results with three different controllers are shown in Table 1. Compared with the method of event-triggered control, the intermittent event-triggered control cannot get better synchronization results, but the event generator and controller have a rest interval in each working period, which can prolong the service life of the control equipment and save resources to a certain extent. Similarly, compared with the periodically intermittent control, the intermittent event-triggered control requires an event generator to observe the synchronization error, but the number of control input updates is greatly reduced, which can reduce the cost of communication.

Table 1. Numerical results of synchronization of drive-response systems (20) and (21) under different control methods with $k_1 = -3.68$, $k_2 = -0.03$, $t = 50$ s

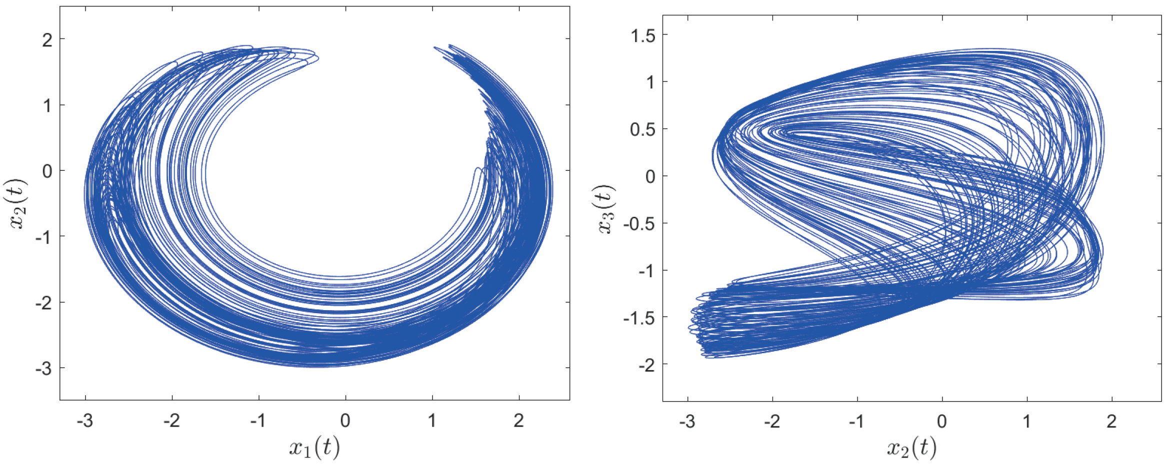
Control method	Control cost	The number of control input updates	Synchronization time
Intermittent event-triggered control	8872.24	98	17 s
Event-triggered control	13183.59	76	16 s
Periodically intermittent control	8962.57	25001	31 s

Therefore, the method of the intermittent event-triggered control can reduce the control cost and the number of control input updates, and provide a better choice than the methods of the periodically intermittent control and the event-triggered control. The experimental results of the synchronization of drive-response systems (20) and (21) show that the method of intermittent event-triggered control is feasible and effective.

Example 4.2. *Consider a non-autonomous gyrostat system as follows* [6]*:*

$$\begin{cases} \dot{x}_1 = -x_2x_3 - 0.5\big(1+6.5\cos t\big)x_2 + 0.4x_3 - 0.002\big(x_1 + x_1^3\big), \\ \dot{x}_2 = x_1x_3 + 0.5\big(1+6.5\cos t\big)x_1 - 0.4x_3 - 0.002\big(x_2 + x_2^3\big), \\ \dot{x}_3 = -0.2x_1 + 0.2x_2 - 0.2x_3 - 0.001\big(x_3 + x_3^3\big) + 1.625\sin t. \end{cases} \tag{22}$$

This system exhibits chaos behavior, as shown in Figure 8.



(a) (b)

Figure 8. Chaotic attractor of the non-autonomous gyrostat system (22)

Compared with system (1), it can be obtained that

$$A(t)=\begin{pmatrix} -0.002 & -0.5(1+6.5\cos t) & 0.4 \\ 0.5(1+6.5\cos t) & -0.002 & -0.4 \\ -0.2 & 0.2 & -0.201 \end{pmatrix},$$

$$f(x)=\begin{pmatrix} -x_2x_3-0.002x_1^3 \\ x_1x_3-0.002x_2^3 \\ -0.001x_3^3 \end{pmatrix},\ m(t)=\begin{pmatrix} 0 \\ 0 \\ 1.625\sin t \end{pmatrix}.$$

Similarly, the corresponding response system can be given by

$$\dot{y}(t)=A(t)y(t)+f(y)+m(t)+u(t), \tag{23}$$

where

$$f(y)=\begin{pmatrix} -y_2y_3-0.002y_1^3 \\ y_1y_3-0.002y_2^3 \\ -0.001y_3^3 \end{pmatrix}.$$

And from function $f(y)$, the matrix $M(x,y)$ mentioned in Hypothesis 2.1 equals

$$M(x,y)=\begin{pmatrix} -0.002\left(x_1^2+x_1y_1+y_1^2\right) & -y_3 & -x_2 \\ y_3 & -0.002\left(x_2^2+x_2y_2+y_2^2\right) & x_1 \\ 0 & 0 & -0.001\left(x_3^2+x_3y_3+y_3^2\right) \end{pmatrix}. \tag{24}$$

In addition, according to Sylvester criterion, matrices

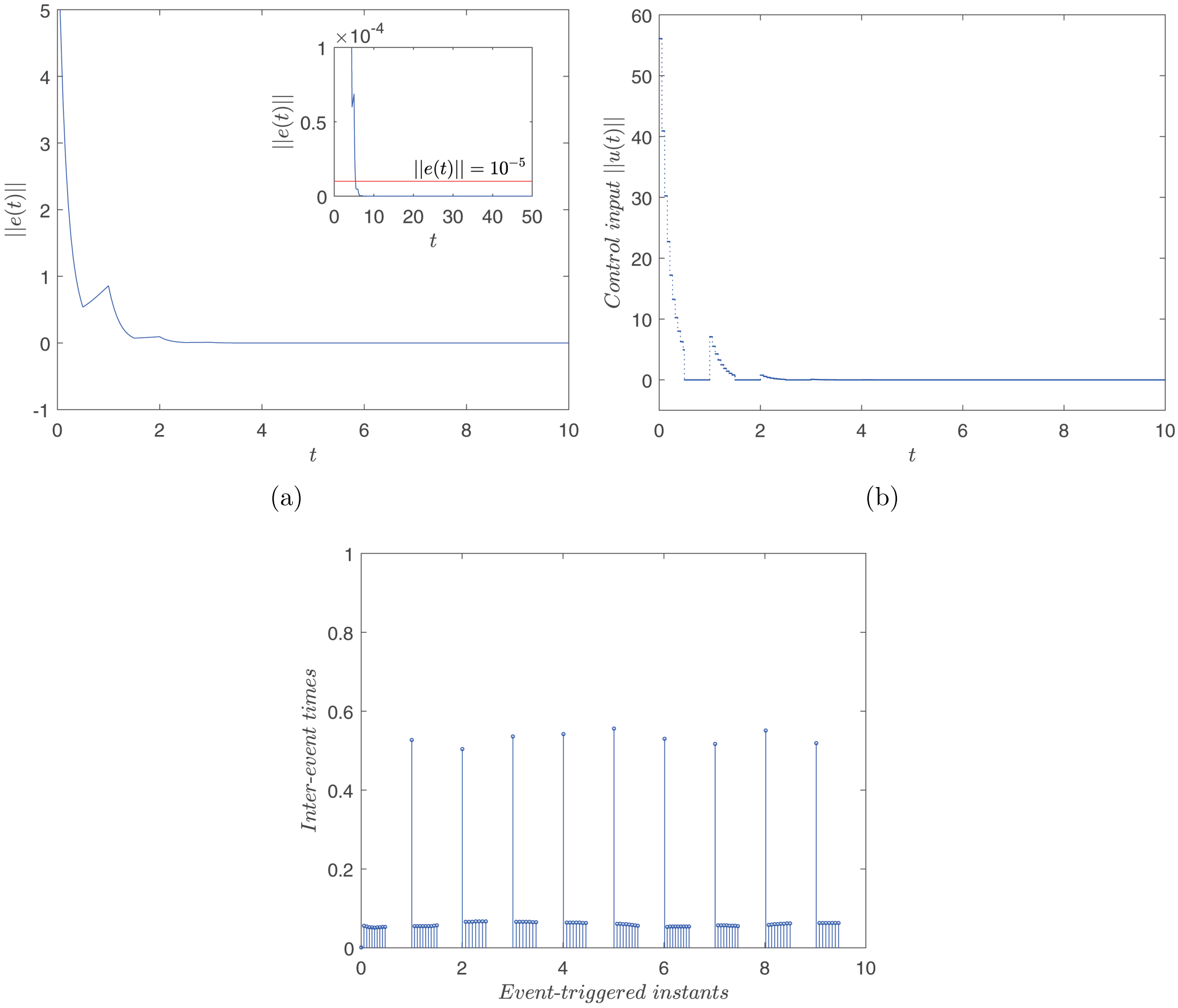
$$A=\begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix},\ M=\begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{pmatrix},$$

satisfy Inequality (8). If we take $\alpha=1$, $\lambda_1=3$, then substitute the values of A and M into the linear matrix inequality (18) and solve it. Finally, we can obtain $\sigma=0.17$,

$$P=\begin{pmatrix} 0.16 & 0 & 0 \\ 0 & 0.17 & 0 \\ 0 & 0 & 0.17 \end{pmatrix},\ K=\begin{pmatrix} -4.92 & 0 & 0 \\ 0 & -4.70 & 0 \\ 0 & 0 & -4.70 \end{pmatrix}.$$

And then $\lambda_2=1.94$ can be obtained by solving the condition ii) of Theorem 3.1. Therefore, we can find the control period $T=1$ and the control width $\delta=0.5$ to ensure $\delta\lambda_1-(T-\delta)\lambda_2>0$. The synchronization schemes under intermittent event-triggered control, event-triggered control and periodically intermittent control are verified by the simulation of Example 4.2 with the step size $h=0.001$, time $t=50$ s and the initial values $(x_1(0),x_2(0),x_3(0))=(2,-2,1.5)$, $(y_1(0),y_2(0),y_3(0))=(-0.6,1,-4)$. The simulation results are shown in Figures 9-11. It is shown that the three control methods can achieve the synchronization of drive-response systems (22) and (23).

In order to compare with the methods of event-triggered control and periodically intermittent control, the control cost and the number of control input updates are given in Table 2. In Example 4.2, the control cost of the event-triggered control and the periodically intermittent control is slightly lower than that of the intermittent event-triggered control, but the number of control input updates of the intermittent event-triggered control is much lower than the periodically intermittent control. Therefore, the methods of the intermittent event-triggered control and the event-triggered control are better than



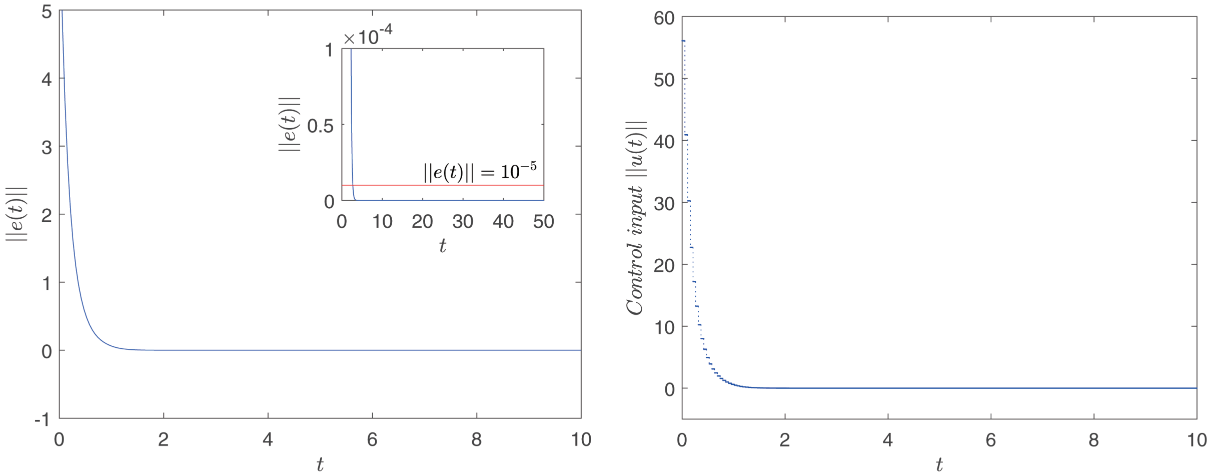
(a) (b)

(c)

FIGURE 9. Simulation results of synchronization of drive-response systems (22) and (23) under intermittent event-triggered control with $k_1 = -4.92$, $k_2 = -4.70$, $k_3 = -4.70$, $\sigma = 0.17$, $T = 1$, $\delta = 0.5$: (a) Synchronization error curve; (b) control input; (c) event-triggered instants and inter-event times

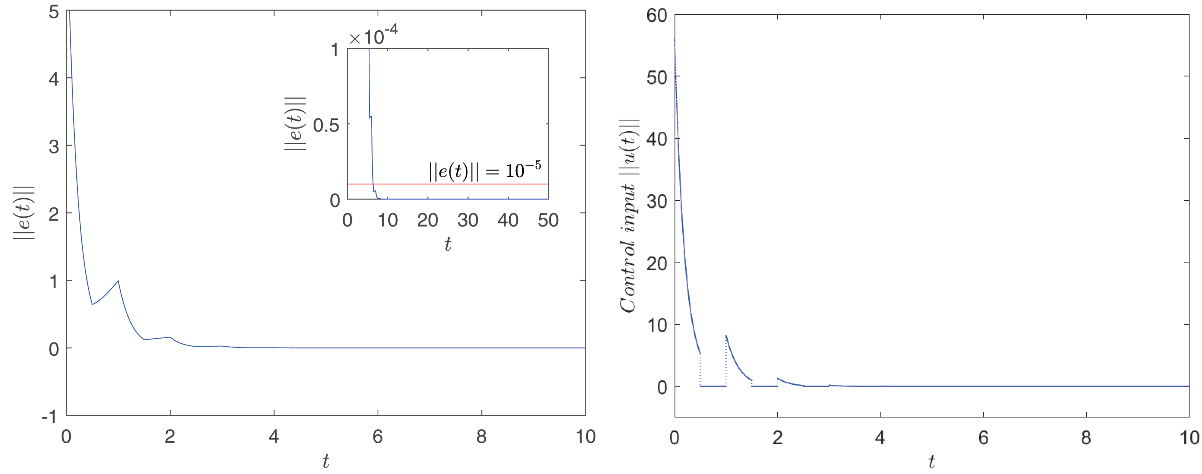
the periodically intermittent control for the synchronization of drive-response systems (22) and (23).

Remark 4.1. *In the case of a small external disturbance, the drive and response systems can also be synchronized by the intermittent event-triggered control. For example, if an external disturbance term* $\triangle = 0.001\sin(3t) * \cos(4t)$ *(or* $\triangle = 0.01\sin(3t) * \cos(4t)$*) is added to the variable* x_2 *of drive system (20), the synchronization error of drive-response systems (20) and (21) is shown in Figure 12. The synchronization error will be increased correspondingly with the increase of the external disturbance term* $\triangle$*. Similarly, if there is a small disturbance for the parameters, such as parameters* a*,* b*,* c*,* f*,* w*, then drive system (20) and response system (21) can also be synchronized. Furthermore, drive system (22) with a small external disturbance is discussed by the numerical simulations, and the experimental results show drive system (22) and response system (23) can also be synchronized.*



(a) (b)

Figure 10. Simulation results of synchronization of drive-response systems (22) and (23) under event-triggered control with $k_1 = -4.92$, $k_2 = -4.70$, $k_3 = -4.70$, $\sigma = 0.17$, $T = 1$, $\delta = 1$: (a) Synchronization error curve; (b) control input



(a) (b)

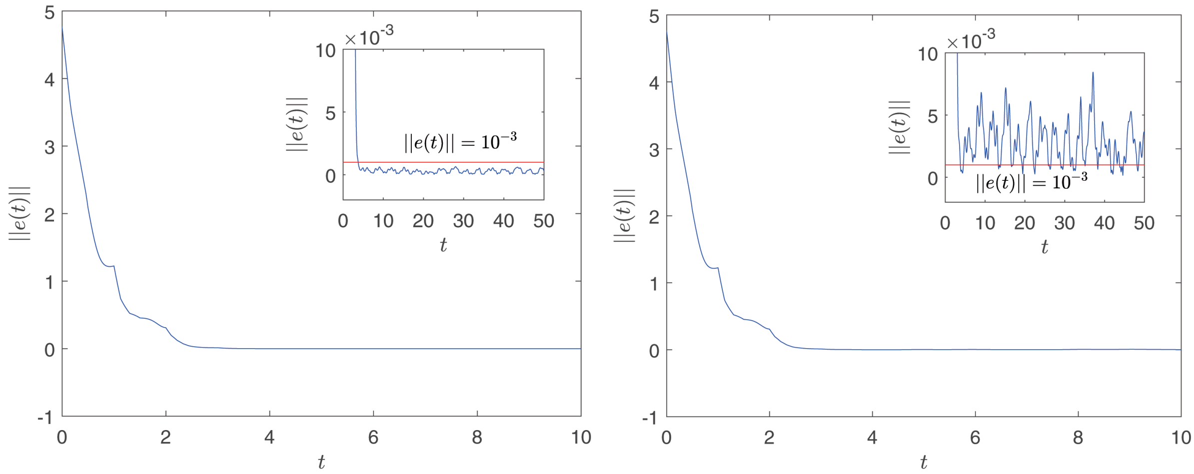
Figure 11. Simulation results of synchronization of drive-response systems (22) and (23) under periodically intermittent control with $k_1 = -4.92$, $k_2 = -4.70$, $k_3 = -4.70$, $\sigma = 0$, $T = 1$, $\delta = 0.5$: (a) Synchronization error curve; (b) control input

5. Conclusion.

Based on Lyapunov stability theory and linear matrix inequality technology, this paper has given sufficient conditions for the synchronizing two non-autonomous chaotic systems under intermittent event-triggered. The event generator and controller are co-designed, and Zeno behavior is excluded. Moreover, it is also discussed that under certain conditions, event-triggered control and periodically intermittent control are special cases of intermittent event-triggered control, and intermittent event-triggered control combines the advantages of both, which can not only reduce communication consumption, but also prolong the service life of control equipment. Finally, two numerical examples are used to verify the theoretical results. Nevertheless, it is worth noting that in the

Table 2. Numerical results of synchronization of drive-response systems (22) and (23) under different control methods with $k_1 = -4.92$, $k_2 = -4.70$, $k_3 = -4.70$, $t = 50$ s

Control method	Control cost	The number of control input updates	Synchronization time
Intermittent event-triggered control	12809.87	137	8 s
Event-triggered control	12229.11	126	4 s
Periodically intermittent control	12608.39	25001	10 s



(a) (b)

Figure 12. Simulation results of synchronization of drive-response systems (20) and (21) under intermittent event-triggered control with $k_1 = -3.68$, $k_2 = -0.03$, $\sigma = 0$, $T = 1$, $\delta = 0.5$: (a) Synchronization error curve with disturbance term $0.001\sin(3t)\cos(4t)$; (b) synchronization error curve with disturbance term $0.01\sin(3t)\cos(4t)$

actual synchronization process, it is difficult to avoid problems such as parameter mismatch or external disturbances, which is easy to lead to Zeno behavior. Therefore, the synchronization of chaotic systems with parameter mismatch or external disturbances via intermittent event-triggered control needs to be studied in the future.

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REFERENCES

[1] L. M. Pecora and T. L. Carroll, Synchronization in chaotic systems, *Physical Review Letters*, vol.64, no.8, pp.821-824, 1990.

[2] S. Boccaletti, J. Kurths, G. Osipov, D. Valladares and C. Zhou, The synchronization of chaotic systems, *Physics Reports*, vol.366, nos.1-2, pp.1-101, 2002.

[3] J. Yao, Z. H. Guan and D. J. Hill, Passivity-based control and synchronization of general complex dynamical networks, *Automatica*, vol.45, no.9, pp.2107-2113, 2009.

[4] M. H. Ma, J. P. Cai and H. Zhang, Quasi-synchronization of Lagrangian networks with parameter mismatches and communication delays via aperiodically intermittent pinning control, *Physica A: Statistical Mechanics and Its Applications*, vol.525, pp.1146-1160, 2019.

[5] V. K. Yadav, V. K. Shukla and S. Das, Exponential synchronization of fractional-order complex chaotic systems and its application, *Chaos, Solitons & Fractals*, vol.147, 110937, 2021.

[6] Z. M. Ge and T. N. Lin, Chaos, chaos control and synchronization of a gyrostat system, *Journal of Sound and Vibration*, vol.251, no.3, pp.519-542, 2002.

[7] X. R. Shi and Z. L. Wang, Adaptive synchronization of the energy resource systems with mismatched parameters via linear feedback control, *Nonlinear Dynamics*, vol.69, no.3, pp.993-997, 2012.

[8] J. P. Cai and M. H. Ma, Synchronization between two non-autonomous chaotic systems via intermittent control of sinusoidal state error feedback, *Optik*, vol.130, pp.455-463, 2017.

[9] M. H. Ma, H. Zhang, J. P. Cai and J. Zhou, Impulsive practical synchronization of n-dimensional nonautonomous systems with parameter mismatch, *Kybernetika*, vol.49, no.4, pp.539-553, 2013.

[10] M. Żochowski, Intermittent dynamical control, *Physica D: Nonlinear Phenomena*, vol.145, nos.3-4, pp.181-190, 2000.

[11] C. Hu, J. Yu, H. J. Jiang and Z. D. Teng, Exponential stabilization and synchronization of neural networks with time-varying delays via periodically intermittent control, *Nonlinearity*, vol.23, no.10, pp.2369-2391, 2010.

[12] J. Yu, C. Hu, H. J. Jiang and Z. D. Teng, Synchronization of nonlinear systems with delays via periodically nonlinear intermittent control, *Communications in Nonlinear Science and Numerical Simulation*, vol.17, no.7, pp.2978-2989, 2012.

[13] C. Xu, D. B. Tong, Q. Y. Chen, W. N. Zhou and Y. H. Xu, Exponential synchronization of chaotic systems with stochastic noise via periodically intermittent control, *International Journal of Robust and Nonlinear Control*, vol.30, no.7, pp.2611-2624, 2020.

[14] K. J. Aström and B. Bernhardsson, Comparison of periodic and event based sampling for first-order stochastic systems, *IFAC Proceedings Volumes*, vol.32, no.2, pp.5006-5011, 1999.

[15] K. J. Aström, Event based control, in *Analysis & Design of Nonlinear Control Systems*, Springer-Verlag Berlin Heidelberg, 2008.

[16] D. Yue, E. G. Tian and Q. L. Han, A delay system method for designing event-triggered controllers of networked control systems, *IEEE Trans. Automatic Control*, vol.58, no.2, pp.475-481, 2013.

[17] S. J. Liu and L. Zhou, Network synchronization and application of chaotic Lur'e systems based on event-triggered mechanism, *Nonlinear Dynamics*, vol.83, no.4, pp.2497-2507, 2016.

[18] W. Wei and J. Lv, Multi-agent event-triggered containment control with joint connected switching topology, *International Journal of Innovative Computing, Information and Control*, vol.18, no.3, pp.957-971, 2022.

[19] G. L. Zhao, C. C. Hua and X. P. Guan, A hybrid event-triggered approach to consensus of multiagent systems with disturbances, *IEEE Trans. Control of Network Systems*, vol.7, no.3, pp.1259-1271, 2020.

[20] D. Ma, J. X. Han, D. L. Zhang and Y. J. Liu, Co-design of event generator and dynamic output feedback controller for LTI systems, *Mathematical Problems in Engineering*, vol.2015, pp.1-7, 2015.

[21] A. H. Hu and J. D. Cao, Consensus of multi-agent systems via intermittent event-triggered control, *International Journal of Systems Science*, vol.48, no.2, pp.280-287, 2017.

[22] A. H. Hu, J. H. Park and M. F. Hu, Consensus of nonlinear multiagent systems with intermittent dynamic event-triggered protocols, *Nonlinear Dynamics*, vol.104, no.2, pp.1299-1313, 2021.

[23] X. Y. Liu, H. B. Fu and L. Liu, Leader-following mean square consensus of stochastic multi-agent systems via periodically intermittent event-triggered control, *Neural Processing Letters*, vol.53, no.1, pp.275-298, 2021.

[24] J. Wu, M. Liu and R. Fan, Relative position coordinated control of spacecraft formation based on event triggered transmission, *ICIC Express Letters*, vol.14, no.9, pp.873-881, 2020.

[25] J. J. Chen, B. S. Chen and Z. G. Zeng, Exponential quasi-synchronization of coupled delayed memristive neural networks via intermittent event-triggered control, *Neural Networks*, vol.141, pp.98-106, 2021.

[26] C. D. Li, G. Feng and X. F. Liao, Stabilization of nonlinear systems via periodically intermittent control, *IEEE Trans. Circuits and Systems II: Express Briefs*, vol.54, no.11, pp.1019-1023, 2007.

[27] S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.

[28] P. Tabuada, Event-triggered real-time scheduling of stabilizing control tasks, *IEEE Trans. Automatic Control*, vol.52, no.9, pp.1680-1685, 2007.

[29] X. F. Wu, J. P. Cai and M. H. Wang, Master-slave chaos synchronization criteria for the horizontal platform systems via linear state error feedback control, *Journal of Sound and Vibration*, vol.295, nos.1-2, pp.378-387, 2006.

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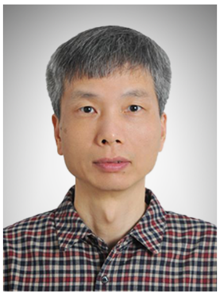

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