

ENHANCED ROBUST STABILITY FOR UNCERTAIN LUR'E SYSTEMS WITH TIME-VARYING DELAYS

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ABSTRACT. *This paper investigates the stability problem for nominal and uncertain Lur'e systems with time-varying delays. By applying the Lyapunov theory, the paper proposes an improved absolute stability criterion and a robustly absolute stability criterion, respectively. For one thing, an augmented Lyapunov-Krasovskii functional (LKF) with delay-dependent matrices and four single integral items is constructed. It can make full use of the system state variables and time delay information. For the other, to overcome the problem of nonlinear inequalities caused by the augmented LKF, the stability criteria can be expressed as a convex linear matrix inequality (LMI) via the new negative definite inequalities equivalence transformation lemma application. Finally, some case studies based on the uncertain Lur'e systems and Chua's circuit showed the effectiveness of the presented criteria.*

Keywords: Robust stability, Uncertain Lur'e system, Time-varying delays, Lyapunov method, LMI

1. Introduction. It is important that time delays can lead to system performance degradation and even instability, especially in networked control systems. At present, the stability and performance analysis of time-delayed systems have been extensively researched. The stability criteria of the time-delayed systems, as well as the controller design methods, can be derived by applying the Lyapunov stability theory. There are two main types of stability criteria: delay-dependent criteria [1, 2, 3] and delay-independent ones [4, 5]. It is obvious that time-varying delays are more general than time-invariant delays. Similarly, the delay-dependent stability criteria are less conservative than the delay-independent ones. Therefore, delay-dependent stability has become a popular research area and received fully adequate attention. The lower the conservativeness, the larger the upper bound of the delays. For real systems, a larger upper bound of time delays means a larger range of system stability. However, the stability criteria derived from the Lyapunov stability theory are a sufficient condition and are inevitably conservative. At present, there are three main concerns to reducing the conservativeness of stability criteria: construct Lyapunov-Krasovskii functional (LKF) to combine the information of state variables and time delays as much as possible; try to reduce the upper bound of the LKF derivative term; try to improve the degree of freedom of solving linear matrix inequality (LMI). In order to address the three aspects, many methods and techniques have been proposed. One is the construction of LKFs, such as augmented LKFs [6, 7], multiple-integral-based

LKFs [8, 9], and delay-decomposing LKFs [10, 11]. Another is to estimate the upper bound on the derivative of LKF, for example, Bessel-Legendre inequality [12], Jensen inequality [13], and relaxed integral inequality techniques [14]. And the third is to add some zero equations based on the Newton-Leibniz formula to increase the degrees of freedom of the LMI. It is worth noting that these three methods must be consistent; otherwise the final result is nonlinear matrix inequalities, which is difficult to solve. For example, in [15], LKF with double integral terms is constructed, and its time derivative contains an integral term, resulting in nonlinear matrix inequality. To deal with this nonlinear matrix inequality, three inequivalent LMIs are obtained via the improved integral inequality in the form of infinite series. Therefore, the stability criterion obtained is less conservative. Recently, to address the problem of nonlinear matrix inequalities in stability criteria, [16] proposed a new negative definite inequality equivalent transformation lemma. This lemma can equivalently convert the nonlinear matrix inequality in [15] into LMI without increasing conservativeness.

It is worth noting that most real systems are nonlinear. Lur'e systems are nonlinear systems consisting of feedback connections of linear dynamical systems, where the nonlinearities satisfy bounded constraints in certain sectors. Lur'e proposed the absolute stability of Lur'e systems [17]. Hereafter, more and more researchers have paid attention to the absolute stability of the time-delayed Lur'e systems [18, 19, 20, 21, 22]. [18, 19] considered the relationship between time delay and the upper bound difference of time delay, leaving out some information on time-varying delays, which can make the conclusion inevitably conservative to a certain extent. To address this shortcoming, [20, 21, 22] made use of free weight matrix theory and second-order Bessel-Legendre inequality to compensate for some neglected information about time-varying delays in previous literature. In practice, uncertainty exists in almost all systems due to inevitable approximations, data errors, system aging, or slow parameter changes. The study of robust stability of uncertain systems is significant due to the presence of uncertainty or time-varying parameters in the model. Therefore, the stability problem has received increasing attention, and the stability criteria for uncertain Lur'e systems with time-varying delays have been investigated. The basic idea is still based on the Lyapunov stability method. For the stability conditions of the Lur'e systems, the method of time-delay decomposition is adopted in [23, 24, 25], the method based on free matrix integral inequality is adopted in [26], and the new inequality technique is applied in [27, 28, 29], etc. Therefore, with the continuous update of LKF construction methods and inequality techniques, the conservativeness of stability criteria for uncertain Lur'e systems with time-varying delays can be further reduced. According to the above discussion, the new negative definite inequality equivalent transformation lemma proposed in [16] can further improve the construction of LKF to reduce the conservativeness of stability criteria for uncertain Lur'e systems with time-varying delays.

The principal purpose of this paper is to investigate robust stability for uncertain Lur'e systems with time-varying delays. By constructing an augmented LKF, an improved absolute stability criterion and a robustly absolute stability criterion are proposed, respectively. The primary achievements of this paper lie in the following three aspects.

- When constructing the LKFs, the delay-dependent Lyapunov matrices are introduced to the non-integral terms, which can describe the relationship between the time-varying delays and some state variables. Obviously, LKFs can contain more delay-related information than in some literature [22, 24, 27, 28, 29, 30], which may further reduce the conservativeness of the stability criteria.
- In addition, four additional integral components, $\int_{h_d}^s x(\alpha)d\alpha$, $\int_{h_1}^{h_d} x(\alpha)d\alpha$, $\int_{h_d}^t x(\alpha)d\alpha$ and $\int_{h_1}^s x(\alpha)d\alpha$, under two different subintervals $[0, h_t]$ and $[h_t, h]$ are augmented

to the single integral terms $\eta_3(s)$ and $\eta_4(s)$. It can better describe the coupling information between time-varying delays and state variables. Thus, this is another improvement that can obtain a larger stability margin than those in some literature [18, 19, 21, 22, 26, 31, 32, 33, 34].

- Using the novel negative definite inequality equivalent transformation lemma proposed in [16], the nonlinear matrix inequality containing h_t^2 is equivalently transformed into the corresponding LMI, which can avoid the conservativeness introduced in the transformation.

Notation: Throughout this paper, \mathbb{R}^n (respectively, $\mathbb{R}^{n \times m}$) refers to the n -dimensional Euclidean space (respectively, the set of $n \times m$ real matrices). The symmetric and symmetric positive definite matrices are denoted as \mathbb{S}^n and \mathbb{S}_+^n , respectively. For $P \in \mathbb{R}^{n \times n}$, P^{-1} and P^T stand for the inverse and transpose of the matrix P , respectively. $P > 0$ means that $P \in \mathbb{S}_+^n$. $\text{diag}\{\dots\}$ stands for the block-diagonal matrix. $\text{col}\{\cdot\}$ denotes a column vector. $\text{Sym}\{\Xi\} = \Xi + \Xi^T$. $\Delta_i = \begin{bmatrix} 0_{n \times (i-1)n} & I & 0_{n \times (11-i)n} \end{bmatrix}$.

2. Problem Statements. Consider a class of time-delayed Lur'e systems with parametric uncertainties described in the following state-space model

$$\begin{cases} \dot{x}(t) = (\Delta\mathcal{B}(t) + \mathcal{B})x(t) + (\Delta\mathcal{B}_d(t) + \mathcal{B}_d)x(h_d) + (\Delta\mathcal{D}(t) + \mathcal{D})\mathcal{Z}(t), \\ \mathcal{W}(t) = \mathcal{M}x(t) + \mathcal{N}x(h_d), \\ \mathcal{Z}(t) = -\gamma(t, \mathcal{W}(t)), \\ x(s) = \varpi(s), \quad s \in [-h, 0]. \end{cases} \tag{1}$$

Here, $x(t) \in \mathbb{R}^n$ is the state; let $t - h(t) = h_d$; $x(h_d)$ denotes delayed state variables with unknown time-varying delays, and the time-varying delay $h(t)$ is assumed to be bounded satisfying $0 \leq h(t) = h_t \leq h$, $|\dot{h}(t) = \dot{h}_t| \leq \mu < 1, \forall t \geq 0$, where h and μ are known constant values. $\mathcal{Z}(t) \in \mathbb{R}^m$ is the control input vector; $\mathcal{W}(t) \in \mathbb{R}^m$ is the system output vector. $\varpi(s)$ is an initial condition of $x(t)$ belonging to $[-h, 0]$. $\mathcal{B}, \mathcal{B}_d, \mathcal{D}, \mathcal{M}$, and \mathcal{N} are real known constant matrices with suitable dimensions.

In terms of structural uncertainties, the following form can be applied:

$$\begin{bmatrix} \Delta\mathcal{B}(t) & \Delta\mathcal{B}_d(t) & \Delta\mathcal{D}(t) \end{bmatrix} = \mathcal{A}\mathcal{F}(t) \begin{bmatrix} \mathcal{K}_{b1} & \mathcal{K}_{b2} & \mathcal{K}_d \end{bmatrix}, \tag{2}$$

where matrices $\mathcal{A}, \mathcal{K}_{b1}, \mathcal{K}_{b2}$, and \mathcal{K}_d are known, and unknown time-varying matrix $\mathcal{F}(t)$ satisfies $\mathcal{F}^T(t)\mathcal{F}(t) \leq I$.

$\gamma(t, \mathcal{W}(t)) = \text{col}\{\gamma_1(t, \mathcal{W}_1(t)), \gamma_2(t, \mathcal{W}_2(t)), \dots, \gamma_m(t, \mathcal{W}_m(t))\}$ is a memoryless, possibly time-varying nonlinear functional, piecewise continuous in t , globally Lipschitz in $\mathcal{W}(t)$. $\gamma(t, 0) = 0$ and nonlinear intervals satisfy the following conditions:

$$\gamma^T(t, \mathcal{W}(t)) [\gamma(t, \mathcal{W}(t)) - \mathcal{E}\mathcal{W}(t)] < 0, \tag{3}$$

or

$$[\gamma(t, \mathcal{W}(t)) - \mathcal{E}_1\mathcal{W}(t)]^T [\gamma(t, \mathcal{W}(t)) - \mathcal{E}_2\mathcal{W}(t)] < 0. \tag{4}$$

Remark 2.1. Let $\mathcal{E} = \mathcal{E}_1 - \mathcal{E}_2$. \mathcal{E}_1 and \mathcal{E}_2 in $\mathbb{R}^{m \times m}$ are constant real matrices. In other words, if it satisfies (3), the nonlinear constraint interval denotes $[0, \mathcal{E}]$ as shown in (a) of Figure 1. If $\gamma(t, \mathcal{W}(t))$ satisfies (4), the nonlinear constraint interval denotes $[\mathcal{E}_1, \mathcal{E}_2]$ as shown in (b) of Figure 1.

Definition 2.1. [32] (Robustly absolute stability) If the system is asymptotically stable for $\gamma(t, \mathcal{W}(t))$ satisfying (3) (or (4)) and all admissible uncertainties, the system (1) is said to be robustly absolutely stable in the sector $[0, \mathcal{E}]$ (or $[\mathcal{E}_1, \mathcal{E}_2]$).

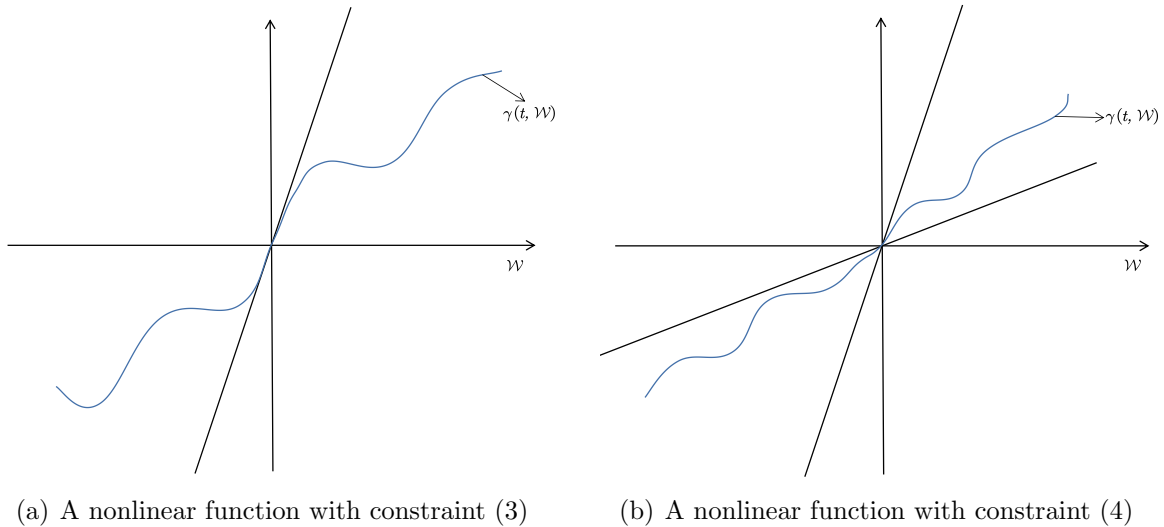


FIGURE 1. A nonlinear function with different sector constraints

This paper focuses on the robustly absolute stability of system (1) satisfying conditions (2)-(4) via Lyapunov stability theory. The following lemmas are necessary for deriving our main results.

Lemma 2.1. [12] For any $Q \in \mathbb{S}_+^n$ and $g : [m, n] \rightarrow \mathbb{R}^n$, the following integral inequality holds

$$\int_m^n \dot{g}^T(\theta) Q \dot{g}(\theta) d\theta \geq \frac{1}{n-m} \varrho^T \bar{Q} \varrho,$$

where $\bar{Q} = \text{diag}\{Q, 3Q, 5Q\}$, $\varrho = \text{col}\{\varrho_1, \varrho_2, \varrho_3\}$ with $\varrho_1 = g(n) - g(m)$, $\varrho_2 = g(n) + g(m) - \frac{2}{n-m} \int_m^n g(\theta) d\theta$, $\varrho_3 = \varrho_1 - \frac{6}{n-m} \int_m^n g(\theta) d\theta + \frac{12}{(n-m)^2} \int_m^n (n-\theta)g(\theta) d\theta$.

Lemma 2.2. [29] For given scalar $(0, 1]$, $\Lambda_1, \Lambda_2 \in \mathbb{S}_+^m$ and $\vartheta_1, \vartheta_2 \in \mathbb{R}^m$. If there exist $\varsigma_1, \varsigma_2 \in \mathbb{S}^m$ and $\Upsilon_1, \Upsilon_2 \in \mathbb{R}^{m \times m}$, then

$$\begin{bmatrix} \Lambda_1 - \varsigma_1 & \Upsilon_1 \\ * & \Lambda_1 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \Lambda_2 - \varsigma_2 & \Upsilon_2 \\ * & \Lambda_2 \end{bmatrix} \geq 0,$$

the following inequality holds

$$\begin{aligned} & \frac{1}{\alpha} \vartheta_1^T \Lambda_1 \vartheta_1 + \frac{1}{1-\alpha} \vartheta_2^T \Lambda_2 \vartheta_2 \\ & \geq 2\vartheta_1^T [\alpha \Upsilon_1 + (1-\alpha)\Upsilon_2] \vartheta_2 + \vartheta_1^T [\Lambda_1 + (1-\alpha)\varsigma_1] \vartheta_1 + \vartheta_2^T (\Lambda_2 + \alpha\varsigma_2) \vartheta_2. \end{aligned}$$

Lemma 2.3. [16] Let $P_2, P_1, P_0 \in \mathbb{S}^p$ and $\zeta \in \mathbb{R}^p$. Then the inequality

$$\zeta^T (h_t^2 P_2 + h_t P_1 + P_0) \zeta < 0$$

holds for all $h_t \in [0, h]$, if and only if there exist matrices $C = [(h/2)I \quad 0]$, $J = [(h/2)I \quad -I]$, $M \in \mathbb{S}_+^p$ and skew-symmetric matrix $N \in \mathbb{R}^{k \times k}$, then

$$\begin{bmatrix} P_0 & \frac{1}{2}P_1 \\ * & P_2 \end{bmatrix} - \begin{bmatrix} C \\ J \end{bmatrix}^T \begin{bmatrix} -M & N \\ * & M \end{bmatrix} \begin{bmatrix} C \\ J \end{bmatrix} < 0.$$

Lemma 2.4. [30] Given matrices M, N and $E = E^T$, the following inequality

$$E + M\Pi(\sigma)N + N^T\Pi^T(\sigma)M^T < 0$$

holds for any $\Pi(\sigma)$ satisfying $\Pi^T(\sigma)\Pi(\sigma) \leq I$, if and only if there exist a scalar $\varepsilon > 0$ such that $E + \varepsilon^{-1}MM^T + \varepsilon N^T N < 0$.

In the next section, we present our main results. A new LKF with delayed dependence matrices and four single integral terms is introduced. With the application of lemmas in this section, an absolute stability criterion of the system (5) and a robustly absolute stability criterion of system (1) are obtained.

3. Main Results. In the section, the time-delayed Lur'e system will derive some absolute and robustly absolute stability criteria. For brevity, we define the following notations.

$$\begin{aligned} h_{1d} &= 1 - \dot{h}_t, \quad h_d = t - h_t, \quad h_1 = t - h, \quad \eta_{0t} = \text{col} \{x(t), x(h_d), x(h_1)\}, \\ \eta_{1t} &= \text{col} \left\{ x(t), x(h_d), \int_{h_d}^t x(s) ds \right\}, \quad \eta_{2t} = \text{col} \left\{ x(h_d), x(h_1), \int_{h_1}^{h_d} x(s) ds \right\}, \\ \eta_3(s) &= \text{col} \left\{ \dot{x}(s), x(s), \eta_{0t}, \int_s^t x(\alpha) d\alpha, \int_{h_d}^s x(\alpha) d\alpha, \int_{h_1}^{h_d} x(\alpha) d\alpha \right\}, \\ \eta_4(s) &= \text{col} \left\{ \dot{x}(s), x(s), \eta_{0t}, \int_s^{h_d} x(\alpha) d\alpha, \int_{h_d}^t x(\alpha) d\alpha, \int_{h_1}^s x(\alpha) d\alpha \right\}, \\ \zeta_t &= \text{col} \{x(t), x(h_d), x(h_1), \rho_{1t}, \rho_{2t}, \rho_{3t}, \rho_{4t}, \dot{x}(t), \dot{x}(h_d), \dot{x}(h_1), \mathcal{Z}(t)\}, \\ \rho_{1t} &= \int_{h_1}^{h_d} \frac{x(s)}{h - h_t} ds, \quad \rho_{2t} = \int_{h_1}^{h_d} \frac{(h_d - s)x(s)}{(h - h_t)^2} ds, \quad \rho_{3t} = \int_{h_d}^t \frac{x(s)}{h_t} ds, \\ \rho_{4t} &= \int_{h_d}^t \frac{(t - s)x(s)}{h_t^2} ds. \end{aligned}$$

3.1. Absolute stability criterion. In this subsection, Loop-Transformation [12] is first applied to obtaining that the nominal form of system (1) in the sector $[\mathcal{E}_1, \mathcal{E}_2]$ without parametric uncertainty is identical to the following system (5) in the sector $[0, \mathcal{E}]$.

$$\begin{cases} \dot{x}(t) = \bar{\mathcal{B}}x(t) + \bar{\mathcal{B}}_d x(h_d) + \mathcal{D}\mathcal{Z}(t), \\ \mathcal{W}(t) = \mathcal{M}x(t) + \mathcal{N}x(h_d), \\ \mathcal{Z}(t) = -\gamma(t, \mathcal{W}(t)), \\ x(s) = \varpi(s), \quad s \in [-h, 0]. \end{cases} \tag{5}$$

Here, $\bar{\mathcal{B}} = \mathcal{B} - \mathcal{D}\mathcal{E}_1\mathcal{M}$ and $\bar{\mathcal{B}}_d = \mathcal{B}_d - \mathcal{D}\mathcal{E}_1\mathcal{N}$.

Theorem 3.1. *Let μ and h be given scalars. The system (5) satisfying conditions (2) and (3) is absolutely stable, if there exist matrices $S_{i1} \in \mathbb{S}^{3n}$, $S_{i2} \in \mathbb{S}_+^{3n}$, $\mathcal{X}_i \in \mathbb{S}^{3n}$, $\mathcal{Y}_i \in \mathbb{R}^{3n \times 3n}$, $Q_i \in \mathbb{S}_+^{8n}$, $U_j \in \mathbb{R}^{n \times n}$, $U_4 \in \mathbb{R}^{m \times n}$ and diagonal matrices $\mathcal{P}_i \in \mathbb{S}_+^n$, ($i = 1, 2$; $j = 1, 2, 3$), such that the following matrix inequalities hold for $\dot{h}_t \in \{-\mu, \mu\}$ and $\forall h_t \in [0, h]$.*

$$hS_{11} + S_{12} > 0, \quad hS_{21} + S_{22} > 0, \tag{6}$$

$$\begin{bmatrix} \bar{\mathcal{P}}_1 - \mathcal{X}_1 & \mathcal{Y}_1 \\ * & \bar{\mathcal{P}}_1 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \bar{\mathcal{P}}_2 - \mathcal{X}_2 & \mathcal{Y}_2 \\ * & \bar{\mathcal{P}}_2 \end{bmatrix} \geq 0, \tag{7}$$

$$h_t^2 \Psi_2(\dot{h}_t) + h_t \Psi_1(\dot{h}_t) + \Psi_0(\dot{h}_t) < 0, \tag{8}$$

where

$$\begin{aligned} \Psi_0(\dot{h}_t) &= \text{Sym} \{D_1^T S_{12} c_{11}\} + \dot{h}_t c_{11}^T S_{11} c_{11} + c_{31}^T Q_1 c_{31} - \dot{h}_t c_{21}^T S_{21} c_{21} \\ &\quad + \text{Sym} \{D_2^T (hS_{21} + S_{22}) c_{21} + c_{70}^T Q_1 D_3 + c_{80}^T Q_2 D_4\} \\ &\quad - h_{1d} c_{41}^T Q_1 c_{41} + h_{1d} c_{51}^T Q_2 c_{51} - c_{61}^T Q_2 c_{61} + h^2 \Delta_8^T \mathcal{P}_2 \Delta_8 \\ &\quad + h^2 h_{1d} \Delta_9^T (\mathcal{P}_1 - \mathcal{P}_2) \Delta_9 - \Gamma_1^T \bar{\mathcal{P}}_1 \Gamma_1 - \Gamma_2^T (\bar{\mathcal{P}}_2 + \mathcal{X}_2) \Gamma_2 \end{aligned}$$

$$\begin{aligned}
& - \text{Sym} \{ \Gamma_1^T \mathcal{Y}_1 \Gamma_2 + \Delta_{11}^T \mathcal{O} \Delta_{11} + \Delta_{11}^T \mathcal{O} \mathcal{E} [\mathcal{M} \Delta_1 + \mathcal{N} \Delta_2] - J_1^T \bar{U} P_0 \}, \\
\Psi_1 (\dot{h}_t) &= \text{Sym} \{ D_1^T S_{11} c_{11} + D_1^T S_{12} c_{12} + \dot{h}_t c_{12}^T S_{11} c_{11} + D_2^T (h S_{21} + S_{22} c_{22}) \\
& - D_2^T S_{21} c_{21} + c_{31}^T Q_1 c_{32} - h_{1d} c_{41}^T Q_1 c_{42} - \dot{h}_t c_{21}^T S_{21} c_{22} + c_{71}^T Q_1 D_3 \\
& + h_{1d} c_{51}^T Q_2 c_{52} - c_{61}^T Q_2 c_{62} - c_{81}^T Q_2 D_4 \} - h_{1d} h \Delta_9^T (\mathcal{P}_1 - \mathcal{P}_2) \Delta_9 \\
& - \frac{1}{h} \Gamma_1^T \mathcal{X}_1 \Gamma_1 + \frac{1}{h} \Gamma_2^T \mathcal{X}_2 \Gamma_2 + \text{Sym} \left\{ \frac{1}{h} \Gamma_1^T (\mathcal{Y}_1 - \mathcal{Y}_2) \Gamma_2 \right\}, \\
\Psi_2 (\dot{h}_t) &= \text{Sym} \{ D_1^T S_{11} c_{12} - D_2^T S_{21} c_{22} + D_3^T Q_1 c_{72} + D_4^T Q_2 c_{82} \} + c_{32}^T Q_1 c_{32} \\
& + \dot{h}_t c_{12}^T S_{11} c_{12} - h_{1d} c_{42}^T Q_1 c_{42} - \dot{h}_t c_{22}^T S_{21} c_{22} - c_{62}^T Q_2 c_{62} + h_{1d} c_{52}^T Q_2 c_{52}
\end{aligned}$$

with

$$\begin{aligned}
c_{11} &= \text{col} \{ \Delta_1, \Delta_2, \Delta_0 \}, \quad c_{12} = \text{col} \{ \Delta_0, \Delta_0, \Delta_6 \}, \\
c_{21} &= \text{col} \{ \Delta_2, \Delta_3, h \Delta_4 \}, \quad c_{22} = \text{col} \{ \Delta_0, \Delta_0, -\Delta_4 \}, \\
c_{31} &= \text{col} \{ \Delta_8, \Delta_1, \Delta_1, \Delta_2, \Delta_3, \Delta_0, \Delta_0, h \Delta_4 \}, \\
c_{32} &= \text{col} \{ \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_6, -\Delta_4 \}, \\
c_{41} &= \text{col} \{ \Delta_9, \Delta_2, \Delta_1, \Delta_2, \Delta_3, \Delta_0, \Delta_0, h \Delta_4 \}, \\
c_{42} &= \text{col} \{ \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_6, \Delta_0, -\Delta_4 \}, \\
c_{51} &= \text{col} \{ \Delta_9, \Delta_2, \Delta_1, \Delta_2, \Delta_3, \Delta_0, \Delta_0, h \Delta_4 \}, \\
c_{52} &= \text{col} \{ \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_6, -\Delta_4 \}, \\
c_{61} &= \text{col} \{ \Delta_{10}, \Delta_3, \Delta_1, \Delta_2, \Delta_3, h \Delta_4, \Delta_0, \Delta_0 \}, \\
c_{62} &= \text{col} \{ \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0, -\Delta_4, \Delta_6, \Delta_0 \}, \\
c_{70} &= \text{col} \{ \Delta_1 - \Delta_2, \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0 \}, \\
c_{71} &= \text{col} \{ \Delta_0, \Delta_6, \Delta_1, \Delta_2, \Delta_3, \Delta_0, \Delta_0, h \Delta_4 \}, \\
c_{72} &= \text{col} \{ \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_6 - \Delta_7, \Delta_7, -\Delta_4 \}, \\
c_{80} &= \text{col} \{ \Delta_2 - \Delta_3, h \Delta_4, h \Delta_1, h \Delta_2, h \Delta_3, h^2 (\Delta_4 - \Delta_5), \Delta_0, h^2 \Delta_5 \}, \\
c_{81} &= \text{col} \{ \Delta_0, \Delta_4, \Delta_1, \Delta_2, \Delta_3, 2h (\Delta_4 - \Delta_5), -h \Delta_6, 2h \Delta_5 \}, \\
c_{82} &= \text{col} \{ \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_0, \Delta_4 - \Delta_5, -\Delta_6, \Delta_5 \}, \\
\Gamma_1 &= \text{col} \{ \Delta_2 - \Delta_3, \Delta_2 + \Delta_3 - 2\Delta_4, \Delta_2 - \Delta_3 - 6\Delta_4 + 12\Delta_5 \}, \\
\Gamma_2 &= \text{col} \{ \Delta_1 - \Delta_2, \Delta_2 + \Delta_1 - 2\Delta_6, \Delta_1 - \Delta_2 - 6\Delta_6 + 12\Delta_7 \}, \\
D_1 &= \text{col} \{ \Delta_8, h_{1d} \Delta_9, \Delta_1 - h_{1d} \Delta_2 \}, \\
D_2 &= \text{col} \{ h_{1d} \Delta_9, \Delta_{10}, h_{1d} \Delta_2 - \Delta_3 \}, \\
D_3 &= \text{col} \{ \Delta_0, \Delta_0, \Delta_8, h_{1d} \Delta_9, \Delta_{10}, \Delta_1, -h_{1d} \Delta_2, h_{1d} \Delta_2 - \Delta_3 \}, \\
D_4 &= \text{col} \{ \Delta_0, \Delta_0, \Delta_8, h_{1d} \Delta_9, \Delta_{10}, h_{1d} \Delta_2, \Delta_1 - h_{1d} \Delta_2, -\Delta_3 \}.
\end{aligned}$$

Proof: The Lyapunov functional of the system (5) is constructed as below:

$$V(t) = \sum_{i=1}^3 V_i(t) \tag{9}$$

with

$$\begin{aligned}
V_1(t) &= \eta_{1t}^T S_{1t} \eta_{1t} + \eta_{2t}^T S_{2t} \eta_{2t}, \\
V_2(t) &= \int_{h_d}^t \eta_3^T(s) Q_1 \eta_3(s) ds + \int_{h_1}^{h_d} \eta_4^T(s) Q_2 \eta_4(s) ds,
\end{aligned}$$

$$V_3(t) = h \int_{h_1}^{h_d} (h - t + s) \dot{x}^T(s) \mathcal{P}_1 \dot{x}(s) ds + h \int_{h_d}^t (h - t + s) \dot{x}^T(s) \mathcal{P}_2 \dot{x}(s) ds.$$

S_{it} , ($i = 1, 2$) are affine functions in $h_t \in [0, h]$, $S_{1t} = h_t S_{11} + S_{12}$, $S_{2t} = (h - h_t) S_{21} + S_{22}$. By applying $S_{i2} > 0$, $h S_{i1} + S_{i2} > 0$, guarantee the LKF $V(t)$ positive definite. Now, taking the time-derivative of $V(t)$ along the solution of system (5) yields that

$$\dot{V}_1(t) = 2\dot{\eta}_{1t}^T S_{1t} \eta_{1t} + \eta_{1t}^T \dot{S}_{1t} \eta_{1t} + 2\dot{\eta}_{2t}^T S_{2t} \eta_{2t} + \eta_{2t}^T \dot{S}_{2t} \eta_{2t}, \tag{10}$$

$$\begin{aligned} \dot{V}_2(t) = & \eta_3^T(t) Q_1 \eta_3(t) - \eta_4^T(h_1) Q_2 \eta_4(h_1) - h_{1d} \eta_3^T(h_d) Q_1 \eta_3(h_d) + 2 \int_{h_d}^t \eta_3^T(s) ds Q_1 \frac{\partial}{\partial t} \eta_3(s) \\ & + h_{1d} \eta_4^T(h_d) Q_2 \eta_4(h_d) + 2 \int_{h_1}^{h_d} \eta_4^T(s) ds Q_2 \frac{\partial}{\partial t} \eta_4(s), \end{aligned} \tag{11}$$

$$\begin{aligned} \dot{V}_3(t) = & h^2 \dot{x}^T(t) \mathcal{P}_2 \dot{x}(t) - h \int_{h_1}^{h_d} \dot{x}^T(s) \mathcal{P}_1 \dot{x}(s) ds - h \int_{h_d}^t \dot{x}^T(s) \mathcal{P}_2 \dot{x}(s) ds \\ & + h_{1d} (h^2 - h_t h) \dot{x}^T(h_d) (\mathcal{P}_1 - \mathcal{P}_2) \dot{x}(h_d), \end{aligned} \tag{12}$$

where

$$\begin{aligned} \dot{\eta}_{1t}^T &= \zeta_t^T D_1^T, \quad \dot{\eta}_{2t}^T = \zeta_t^T D_2^T, \quad \eta_3(t) = (c_{31} + h_t c_{32}) \zeta_t, \\ \int_{h_d}^t \eta_3^T(s) ds Q_1 \frac{\partial}{\partial t} \eta_3(s) &= \zeta_t^T (c_{70} + h_t c_{71} + h_t^2 c_{72}) Q_1 D_3 \zeta_t, \\ \int_{h_1}^{h_d} \eta_4^T(s) ds Q_2 \frac{\partial}{\partial t} \eta_4(s) &= \zeta_t^T (c_{80} - h_t c_{81} + h_t^2 c_{82}) Q_2 D_4 \zeta_t, \\ \eta_3(h_d) &= (c_{41} + h_t c_{42}) \zeta_t, \quad \eta_4(h_d) = (c_{51} + h_t c_{52}) \zeta_t, \quad \eta_4(h_1) = (c_{61} + h_t c_{62}) \zeta_t. \end{aligned}$$

Let $\alpha_0 = \frac{h-h_t}{h}$, $\alpha_1 = \frac{1}{\alpha_0}$, $\alpha_2 = \frac{1}{1-\alpha_0}$. According to $\mathcal{P}_1 > 0$ and $\mathcal{P}_2 > 0$, we have the following integral inequalities from Lemmas 2.1 and 2.2

$$\begin{aligned} & h \left(\int_{h_1}^{h_d} \dot{x}^T(s) \mathcal{P}_1 \dot{x}(s) ds + \int_{h_d}^t \dot{x}^T(s) \mathcal{P}_2 \dot{x}(s) ds \right) \\ & \geq \alpha_1 \zeta_t^T \Gamma_1^T \bar{\mathcal{P}}_1 \Gamma_1 \zeta_t + \alpha_2 \zeta_t^T \Gamma_2^T \bar{\mathcal{P}}_2 \Gamma_2 \zeta_t \\ & \geq \zeta_t^T \left\{ \Gamma_1^T [\bar{\mathcal{P}}_1 + (1 - \alpha_0) \mathcal{X}_1] \Gamma_1 + 2\Gamma_1^T [\alpha_0 \mathcal{Y}_1 + (1 - \alpha_0) \mathcal{Y}_2] \Gamma_2 \right. \\ & \quad \left. + \Gamma_2^T [\bar{\mathcal{P}}_2 + \alpha_0 \mathcal{X}_2] \Gamma_2 \right\} \zeta_t. \end{aligned} \tag{13}$$

From (3), adding the positive definite diagonal matrix $\mathcal{O} \in \mathbb{R}^{m \times m}$, we can get

$$-2\mathcal{Z}^T(t) \mathcal{O} \mathcal{Z}(t) - 2\mathcal{Z}^T(t) \mathcal{O} \mathcal{E} [\mathcal{M}x(t) + \mathcal{N}x(h_d)] > 0. \tag{14}$$

For an appropriate matrix $\bar{U} = \text{col} \{U_1, U_2, U_3, U_4\}$, it is true that

$$2 \begin{bmatrix} x(t) \\ x(h_d) \\ \dot{x}(t) \\ \mathcal{Z}(t) \end{bmatrix}^T \bar{U} [\mathcal{B}x(t) + \mathcal{B}_d x(h_d) + \mathcal{D}\mathcal{Z}(t) - \dot{x}(t)] = 0. \tag{15}$$

Finally, from the above derivation (10)-(15), it can be summarized and simplified as follows

$$\dot{V}(t) \leq \zeta_t^T \left[h_t^2 \Psi_2 \left(\dot{h}_t \right) + h_t \Psi_1 \left(\dot{h}_t \right) + \Psi_0 \left(\dot{h}_t \right) \right] \zeta_t, \tag{16}$$

where $\Psi_0 \left(\dot{h}_t \right)$, $\Psi_1 \left(\dot{h}_t \right)$ and $\Psi_2 \left(\dot{h}_t \right)$ are given in Theorem 3.1, respectively.

Hereafter, if Inequalities (6)-(8) are satisfied, $\dot{V}(t) < 0$. Thus, the system (5) is absolutely stable for $\gamma(t, \mathcal{W}(t))$ satisfying (3) from Lyapunov stability theory. It can be known from Definition 2.1 that system (5) is absolutely stable in the sector $[0, \mathcal{E}]$. This completes the proof. \square

Remark 3.1. Recently, an improved stability criterion was proposed in [16], where the LKF was augmented by introducing some delay-dependent matrices and single integral terms. Inspired by this, when constructing the LKF, four additional integral components, $\int_{h_d}^s x(\alpha)d\alpha$, $\int_{h_1}^{h_d} x(\alpha)d\alpha$, $\int_{h_d}^{h_1} x(\alpha)d\alpha$ and $\int_{h_1}^s x(\alpha)d\alpha$, under two different subintervals $[0, h_t]$ and $[h_t, h]$ are augmented to single integral terms, $\eta_3(s)$ and $\eta_4(s)$. There is more coupling information that includes delayed state variables than [16]. It may further reduce the conservativeness of the stability criterion.

Remark 3.2. Obviously, the main Inequality (8) in Theorem 3.1 is the nonlinear matrix inequality, since Inequality (8) contains the product term of h_t^2 . At present, the common treatment method translates (8) into three sufficient restrictive constraints [15]. Moreover, such a solution method increases the constraint conditions of the inequality, resulting in the reduction of the degree of freedom of solving LMI. The novel negative definite inequality equivalent transformation lemma proposed in [16] is used to equivalently transform the nonlinear matrix inequality containing h_t^2 into the corresponding LMI, avoiding the conservativeness introduced in the transformation. Lemma 2.3 is used to convert Inequality (16) into LMI (8), which can be easily solved with the MATLAB LMI toolbox. Thus, Theorem 3.1 can be translated into the following form according to Lemma 2.3.

Corollary 3.1. Let μ and h be given scalars. The system (5) is absolutely stable if there exist matrices $S_{i1} \in \mathbb{S}^{3n}$, $S_{i2} \in \mathbb{S}_+^{3n}$, $\mathcal{X}_i \in \mathbb{S}^{3n}$, $\mathcal{Y}_i \in \mathbb{R}^{3n \times 3n}$, $Q_i \in \mathbb{S}_+^{8n}$, $\mathcal{P}_i \in \mathbb{S}_+^n$, $D_i \in \mathbb{S}_+^{11n}$, $U_j \in \mathbb{R}^{n \times n}$, $U_4 \in \mathbb{R}^{m \times n}$ and skew-symmetric matrices $G_i \in \mathbb{R}^{11n \times 11n}$, ($i = 1, 2; j = 1, 2, 3$), such that LMIs (6) and (7) and the following conditions hold for $\dot{h}_t \triangleq \mu_i \in \{-\mu, \mu\}$ and $\forall h_t \in [0, h]$.

$$\Pi_0 = \begin{bmatrix} \Psi_0(\mu_i) & \frac{1}{2}\Psi_1(\mu_i) \\ * & \Psi_2(\mu_i) \end{bmatrix} - \begin{bmatrix} C \\ J \end{bmatrix}^T \begin{bmatrix} -D_i & G_i \\ * & D_i \end{bmatrix} \begin{bmatrix} C \\ J \end{bmatrix} < 0, \tag{17}$$

where $C = [\frac{h}{2}I_p \quad 0_p]$ and $J = [\frac{h}{2}I_p \quad -I_p]$.

Proof: According to Lemma 2.3, the result is obviously true. \square

3.2. Robustly absolute stability criterion. In this section, the results gained before will be generalized to the problem of time-delayed Lur'e systems with parameter uncertainties. Moreover, the resulting robustly absolute stability criterion satisfies (4).

Theorem 3.2. Let μ and h be given scalars. The system (1) satisfying conditions (2) and (4) is robustly absolute stable, if there exist matrices $S_{i1} \in \mathbb{S}^{3n}$, $S_{i2} \in \mathbb{S}_+^{3n}$, $\mathcal{X}_i \in \mathbb{S}^{3n}$, $\mathcal{Y}_i \in \mathbb{R}^{3n \times 3n}$, $Q_i \in \mathbb{S}_+^{8n}$, $\mathcal{P}_i \in \mathbb{S}_+^n$, $D_i \in \mathbb{S}_+^{11n}$, $U_j \in \mathbb{R}^{n \times n}$, $U_4 \in \mathbb{R}^{m \times n}$, and skew-symmetric matrices $G_i \in \mathbb{R}^{11n \times 11n}$ for $i = 1, 2; j = 1, 2, 3$, and a scalar $\varepsilon > 0$, such that the LMIs (6) and (7) and the following conditions hold for $\dot{h}_t \triangleq \mu_i \in \{-\mu, \mu\}$ and $\forall h_t \in [0, h]$.

$$\begin{bmatrix} \Pi_0 & \Pi_1 & \varepsilon\Pi_2 \\ * & -\varepsilon I & 0 \\ * & 0 & -\varepsilon I \end{bmatrix} < 0, \tag{18}$$

where Π_0 can be found in Corollary 3.1, and $\Pi_1 = \begin{bmatrix} J_1^T \bar{U} D \\ 0 \end{bmatrix}$, $\Pi_2 = \begin{bmatrix} P \\ 0 \end{bmatrix}$.

Proof: We substitute $\bar{\mathcal{B}} + \mathcal{A}\mathcal{F}(t)\mathcal{K}_{b1}$, $\bar{\mathcal{B}}_d + \mathcal{A}\mathcal{F}(t)\mathcal{K}_{b2}$, and $\mathcal{D} + \mathcal{A}\mathcal{F}(t)\mathcal{K}_d$ for $\bar{\mathcal{B}}$, $\bar{\mathcal{B}}_d$, and \mathcal{D} of $\Psi_0(\dot{h}_t)$ in Inequality (17), respectively. Inequality (17) can be rewritten as the following:

$$\begin{bmatrix} \Psi_0(\dot{h}_t) & \frac{1}{2}\Psi_1(\dot{h}_t) \\ * & \Psi_2(\dot{h}_t) \end{bmatrix} - \begin{bmatrix} C \\ J \end{bmatrix}^T \begin{bmatrix} -D_i & G_i \\ * & D_i \end{bmatrix} \begin{bmatrix} C \\ J \end{bmatrix} + 2 \begin{bmatrix} J_1^T \bar{U} D \\ 0 \end{bmatrix} F(t) \begin{bmatrix} P^T \\ 0 \end{bmatrix} < 0,$$

where $P = \mathcal{K}_{b1}\Delta_1 + \mathcal{K}_d\Delta_2 + \mathcal{K}_{b2}\Delta_{11}$, $J_1 = \text{col}\{\Delta_1, \Delta_2, \Delta_8, \Delta_{11}\}$.

Because of $\mathcal{F}^T(t)\mathcal{F}(t) \leq I$, it follows from Lemma 2.4 that

$$\begin{bmatrix} \Psi_0(\dot{h}_t) & \frac{1}{2}\Psi_1(\dot{h}_t) \\ \frac{1}{2}\Psi_1(\dot{h}_t) & \Psi_2(\dot{h}_t) \end{bmatrix} - \begin{bmatrix} C \\ J \end{bmatrix}^T \begin{bmatrix} -D_i & G_i \\ * & D_i \end{bmatrix} \begin{bmatrix} C \\ J \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} J_1^T \bar{U} D \\ 0 \end{bmatrix} \begin{bmatrix} J_1^T \bar{U} D \\ 0 \end{bmatrix}^T + \varepsilon \begin{bmatrix} P \\ 0 \end{bmatrix} \begin{bmatrix} P \\ 0 \end{bmatrix}^T < 0. \tag{19}$$

Therefore, the LMI (18) holds, according to the Schur complement lemma, it is equivalent to (18). Thus, the system (1) is robustly absolute stable for $\gamma(t, \mathcal{W}(t))$ satisfying (4) from Lyapunov stability theory. It can be known from Definition 2.1 that system (1) is robustly absolute stable in the sector $[\mathcal{E}_1, \mathcal{E}_2]$. The proof is completed here. \square

Remark 3.3. *Inequality (3) is the nonlinear constraint of the Lur'e system. When $\mathcal{Z} = 0$, the system (1) without nonlinear terms can degenerate to a linear system. That is, our stability criteria are clearly true for linear systems as long as Inequality (14) obtained from the nonlinear constraint (3) is not considered in the derivation.*

4. Numerical Examples. This section provides two examples to demonstrate the advantages of the delay-dependent stability criteria obtained in the previous section. By solving the LMIs in Corollary 3.1 and Theorem 3.2 via the MATLAB LMI toolbox application, the maximum admissible delay upper bounds (MADUBs) for different μ are obtained. Numerical comparisons between our results and the recent ones in the literature are also provided. In addition, the complexity of the criteria is expressed as the number of variables (NoVs) to be determined to get the MADUBs.

Example 4.1. *We give the coefficients of the robust stability of an uncertain system (1) as follows:*

$$\begin{aligned} \mathcal{B} &= \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \mathcal{B}_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \mathcal{M} = [0.3 \quad 0.1], \\ \mathcal{N} &= [0.1 \quad 0.2], \mathcal{A} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \mathcal{D} = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}, \\ \mathcal{E}_1 &= 0.2, \mathcal{E}_2 = 0.5, \mathcal{K}_{b1} = \mathcal{K}_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathcal{K}_{b2} = 0. \end{aligned}$$

For various μ , Table 1 lists MADUBs h in Theorem 3.2. From Table 1, for the same method, MADUBs h increase with the increase of μ . By using our results and methods in [18, 19, 20, 21, 22], the system (1) is compared with various μ of MADUBs h . It can be seen from Table 1 that the MADUBs in Theorem 3.2 are larger than [18, 19, 20, 21, 22]. To verify the result ($h_t = 4.7528$), the simulation result is displayed in Figure 2, which demonstrates the state response of the system (1). According to the simulation result,

TABLE 1. MADUBs h for various μ (Example 4.1)

μ	[18]	[19]	[20]	[21]	[22]	Theorem 3.2
0	3.3057	3.3057	4.3332	4.3956	4.7527	4.7528
0.3	2.0787	2.2262	2.6873	2.9358	3.0951	3.1031
0.6	1.4195	1.7409	2.2021	2.4721	2.5775	2.5883
0.9	0.9228	1.4682	1.9897	2.2356	2.4081	2.4102
NoVs	10	24	105	539	495	1562

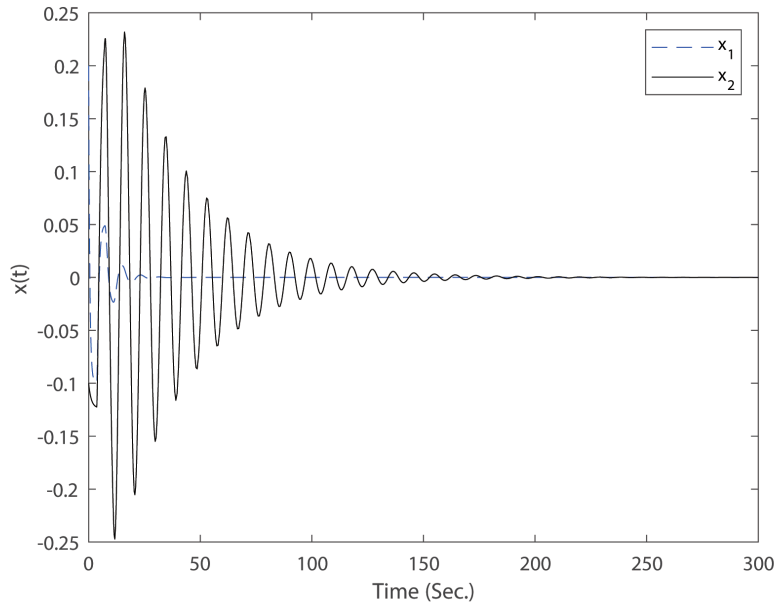


FIGURE 2. The state responses of the system (1)

the system (1) obtained by $\gamma(t, \mathcal{W}(t)) = 0.3 \tanh(\mathcal{W}(t))$ is robustly absolute stable. This shows the validity of the stability criterion obtained in Theorem 3.2.

Example 4.2. *In this example, we consider the following Chua’s circuit*

$$\begin{cases} \dot{\mathcal{X}} = \phi_1(\mathcal{U} - \mathcal{G}(\mathcal{X})), \\ \dot{\mathcal{U}} = \mathcal{X} - \mathcal{U} + \mathcal{Z}, \\ \dot{\mathcal{Z}} = -\phi_2\mathcal{U}. \end{cases} \tag{20}$$

Here, matrices ϕ_1 and ϕ_2 are known and determined by the internal components of the system (20). $\mathcal{G}(\mathcal{X}) = \varpi_1\theta_1 + 0.5(\varpi_0 - \varpi_1)(|\mathcal{X} + \phi_3| - |\mathcal{X} - \phi_3|)$ is a nonlinear characteristic. It can describe the electronic response of the nonlinear resistance and its shape is dependent on the specific configuration of its elements $\varpi_0 = -\frac{1}{7}$, $\varpi_1 = \frac{2}{7}$, $\phi_1 = 9$, $\phi_2 = 14.28$ and $\phi_3 = 1$.

In [28], for uncertain Lur’e systems, a master-slave synchronization scheme is designed using time-varying delayed feedback control on Chua’s circuit (20) with the equation.

$$\begin{aligned} \text{Master: } & \begin{cases} \dot{\mathcal{X}}(t) = \beta_1\mathcal{X}(t) + \beta_2g(\beta_3\mathcal{X}(t)), \\ \mathcal{U}(t) = \beta_4\mathcal{X}(t), \end{cases} \\ \text{Slave: } & \begin{cases} \dot{\mathcal{U}}(t) = \beta_1\mathcal{U}(t) + \beta_2g(\beta_3\mathcal{U}(t)) + \mathcal{Y}(t), \\ \mathcal{U}(t) = \beta_4\mathcal{U}(t), \end{cases} \\ \text{Controller: } & \mathcal{Y}(t) = -\beta_5(\mathcal{X}(t) - \mathcal{U}(t)) + \beta_6(\mathcal{U}(h_d) - \mathcal{U}(h_d)), \end{aligned} \tag{21}$$

where $g(\kappa) = 0.5(|\kappa + 1| - |\kappa - 1|) \in [0, 1]$, $\beta_3 = \beta_4 = [1 \ 0 \ 0]$ and

$$\beta_1 = \begin{bmatrix} -\phi_1\varpi_1 & \phi_1 & 0 \\ 1 & -1 & 1 \\ 0 & -\phi_2 & 0 \end{bmatrix}, \beta_2 = \begin{bmatrix} -\phi_1(\varpi_0 - \varpi_1) \\ 0 \\ 0 \end{bmatrix}.$$

Defining the $\mathcal{E} = \mathcal{X} - \mathcal{U}$, we can get the following synchronization error system:

$$\dot{\mathcal{E}}(t) = (\beta_1 + \beta_5)\mathcal{E}(t) - \beta_6\beta_4\mathcal{E}(h_d) + \beta_2\gamma(\beta_3\mathcal{E}(t)). \tag{22}$$

Assuming that the synchronous controller gains are designed by [31]

$$\beta_5 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \beta_6 = \begin{bmatrix} 6.0029 \\ 1.3367 \\ -2.1264 \end{bmatrix}.$$

The MADUBs h obtained by Corollary 3.1 are listed in Table 2. From Table 2, for the same method, MADUBs h increase with the increase of μ . Comparing the MADUBs in Table 2, Corollary 3.1 leads to less conservative MADUBs than [22, 23, 26, 28, 29, 34], however, higher computational complexity. Moreover, for $h_t = 0.2699$, Figure 3 describes the error state response of the error system (22) with synchronous controller control. It shows that the master-slave system is synchronous and that the error system is absolutely stable when the time-varying delay h_t is smaller than 0.2699.

TABLE 2. MADUBs h for various μ (Example 4.2)

μ	[28]	[23]	[34]	[26]	[29]	[22]	Corollary 3.1
0	0.1622	0.1745	0.1747	0.1771	0.1894	0.2638	0.2659
0.3	0.1591	0.1638	0.1710	0.1721	0.1894	0.2578	0.2699
0.6	0.1566	0.1698	0.1703	0.1715	0.1894	0.2540	0.2544
0.9	0.1527	0.1698	0.1703	0.1715	0.1893	0.2510	0.2517
NoVs	18	371	140	194	699	1563	3132

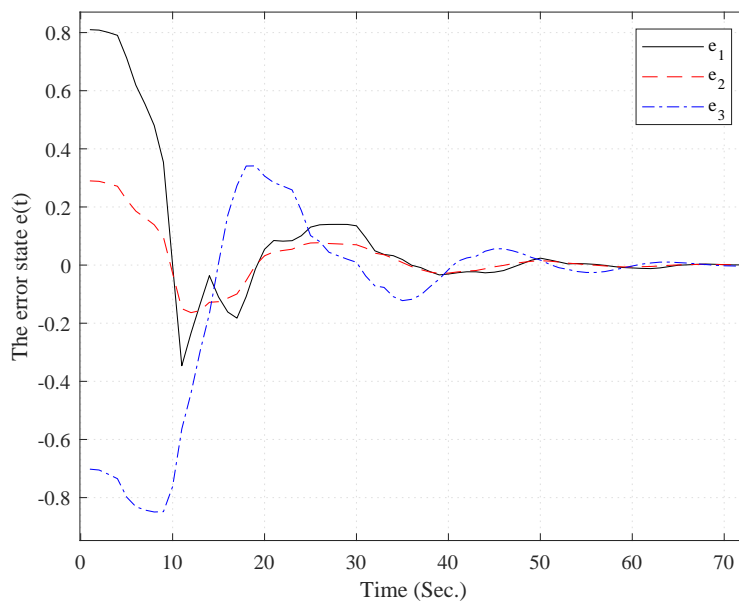


FIGURE 3. The error state responses of the error system (22)

5. Conclusion. A novel LKF with more delay-related information than some literature is constructed. For Lur'e systems with time-varying delays and sector-bounded nonlinearities, some robustly stability criteria are derived by the modified LKF. A novel negative definite inequality equivalent transformation lemma can equivalently convert the nonlinear matrix inequalities containing h_t^2 into the corresponding LMIs, which avoids the conservativeness introduced in the transformation. The advantages of the presented results have been demonstrated through two numerical examples provided finally. The stability criteria presented in this paper can be extended to the stability analysis of other systems. Therefore, developing the stability analysis of various systems with time-varying delays will be one of our future research works.

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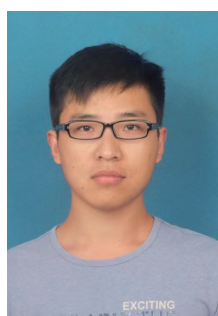
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