International Journal of Innovative Computing, Information and Control
Volume **19**, Number **5**, October **2023**
ICIC International ©2023 ISSN 1349-4198
pp. 1487–1500

OPTIMIZATION FOR I-SECTION STEEL BEAMS OF THREE WELDED PLATES WITH STRAIGHT HAUNCHES

Arnulfo Luévanos-Rojas
Instituto de Investigaciones Multidisciplinaria
Facultad de Contaduría y Administración
Universidad Autónoma de Coahuila, Unidad Torreón
Blvd. Revolución 151 Ote. CP 27000, Torreón, Coahuila, México
arnulfoluevanos@uadec.edu.mx

Received February 2023; revised May 2023

Abstract. *In this research work an optimal model for I-cross section steel beams is presented with straight haunches under the minimum cost criterion that takes into account the equations provided by the LRFD (Load and Resistance Factor Design) code. The beam is subjected to any type of vertical load and a moment at each end. This model presents the equation of the objective function (minimum cost for structural design) and its constraints consider the beams as compact profiles. Two numerical examples are analyzed, one for uniformly distributed load and the other for a concentrated load located in the center of the beam; for each example, the same load is considered and also the moment at support A (left support), and the moment at support B (right support) varies its value. Likewise, a comparison between the proposed model (variable section) and the current model (constant section) is presented. The results show that variable section beams are cheaper and lighter than constant section beams.*
Keywords: Optimal design, Minimum cost, Straight haunches, I-cross section steel beams, Uniformly distributed load, Concentrated load

1. **Introduction.** The optimal design of structures has been the subject of numerous studies in the field of structural design. The goal of a designer is to develop an "optimal solution" for structural design under certain considerations. An optimal solution generally involves the most economical structure without impairing the functional purposes of the structure's service.

Generally, the best optimal design is that meets the project criteria. Typically, there is some kind of objective function that can be calculated from the variables that define a design.

Welded I-section beams are frequently used in the process of building steel frame structures. For this reason, special attention is paid to I-section steel beams works that present theoretical and experimental analysis [1-7].

The optimal design of beams has aroused great interest from several researchers in recent years. These studies are shown as follows. Ostwald et al. proposed the optimal design of the simply supported beams for two I-sections shapes subjected to pure bending under the criterion of cross-section area deflection of the beam [8]. Rahmanian et al. presented a review for the optimal design of constant sections reinforced concrete beams under the criterion of minimum cost or weight [9]. Erdal presented the optimal design for two types of open web beams with hexagonal openings (castellated beams) and with circular openings (cellular beams) under the criterion of minimum weights [10]. Erdal studied the effect of reinforcements on failure analysis for the optimal design of perforated steel beams

DOI: 10.24507/ijicic.19.05.1487

taking account of the (BS 5950) code [11]. Alekseytsev and Ali proposed an approach for the optimization of welded hybrid I-section beams based on the modification of the particle swarm method, and this is carried out by a search for a solution on discrete variable parameters, which take the size of the steel sheets rolled and steel grades [12]. Ho-Huu et al. developed a procedure for the optimal design of composite laminated lightweight beams combining the finite element method and a global optimization algorithm called Jaya [13]. Ozbasaran and Yilmaz presented the optimization in the shape of the doubly symmetric tapered wing and/or web of the I-beams and analyzed the contribution of the flange and/or web taper and the location of the inflection point to the economic design [14]. Bolideh et al. used the teaching-learning-based optimization algorithm to present an approach to design a multidimensional continuous reinforced concrete beam applying the constraints proposed by the (ACI318-14) code [15]. Talaslioglu proposed an optimal design of tubular lattice beams to minimize the total weight and joint displacement, and maximize load capacity in accordance with API RP2A-LRFD design codes [16]. Talaslioglu estimated the optimal design of geometrically nonlinear lattice beams using the member and joint related design constraints [17]. Su et al. investigated an optimal path for logistics distribution of fresh products under a new retail mode [18].

Some mathematical models have been investigated for rectangular section reinforced concrete beams with parabolic and straight haunches for a uniformly distributed load and for a concentrated load located at any point on the beam [19-24], and for steel I-section beams with straight haunches have been investigated for a uniformly distributed load and for a concentrated load located at any point on the beam [25,26]. However, these works present only the fixed-end moments, the carry-over factors and the stiffness factors used in the structural analysis.

The recently published works on the topic of optimal cost design of rectangular section reinforced concrete beams with straight haunches by Luévanos-Rojas et al. [27], and with parabolic haunches by García-Canales et al. [28], and for T-section reinforced concrete beams with straight haunches by López-Chavarría et al. [29] can be referred to.

Therefore, the state of the art review clearly shows that there is no close relationship to the topic for optimal design or minimum cost for I-section steel beams with straight haunches that is addressed in this document.

This research paper shows an optimal model for I-section steel beams of three welded plates with straight haunches (General case) for the optimal design or minimum cost that takes into account the equations provided by the LRFD (Load and Resistance Factor Design) code [30]. The beam is subjected to any type of vertical load and a moment at each end. This model presents the equation of the objective function (minimum cost for structural design) and its constraints consider the beams as compact profiles. Two numerical examples are analyzed, one for uniformly distributed load (unsupported length in negative moment zone and supported length in the positive momentum zone) and the other for concentrated load located in the center of the span (length supported in the center of the span where the concentrated load is located); for each example the same load is considered and the moment at support A (left support), and the moment at support B (right support) varies its value. For each example four cases are presented and each case shows six types. A comparison between the new model (variable section) and the current design (constant section) is also presented. The results show that variable section beams are cheaper and lighter than constant section beams.

The paper is organized as follows. Section 2 describes the formulation of the optimal model according to the equations provided by the LRFD (Load and Resistance Factor Design) code [30]. Subsection 2.1 shows the general principles of the I-section steel beams. Subsection 2.2 presents the equations of the mathematical model for the I-section steel beams of three welded plates with straight haunches. Section 3 presents the numerical problems. Subsection 3.1 shows the numerical example 1 for uniformly distributed load. Subsection 3.2 presents the numerical example 2 for concentrated load. Section 4 shows the results. Section 5 presents the conclusions to complete the paper.

2. **Formulation of the Optimal Model.** The main contributions by other researchers on the optimal design are for reinforced concrete beams with straight and parabolic haunches, and for I-sections of constant sections. This paper shows an optimal design for I-section steel beams of three welded plates with straight haunches, which is the main contribution of this document.

2.1. **General principles.** Figure 1 presents a beam of cross I-section with straight haunches subjected to any type of vertical load (uniformly distributed, triangular, concentrated load), and at a moment at each end, these are obtained from the structural analysis.

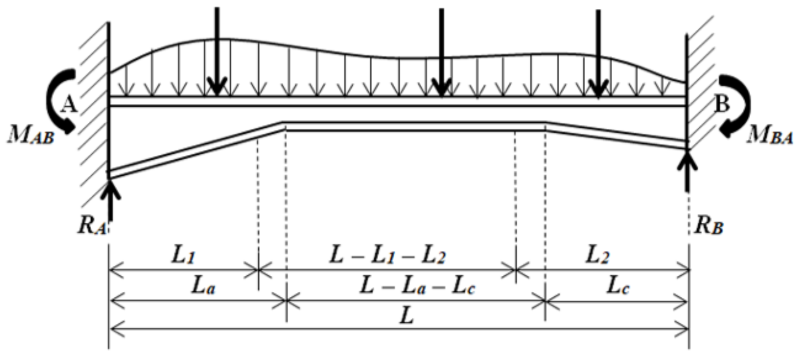

FIGURE 1. Any type of load for I-section steel beams with straight haunches

Reactions "R_A and R_B" in their respective supports are obtained by sum of moments in one of the supports, and subsequently by sum of static forces the other reaction is obtained. The moment equation "M_x" at a distance "x" is generated from support A to obtain: The maximum positive moment "$M_{\max}$" is obtained by deriving "M_x" with respect to "x" (dM_x/dx) and this equation is made equal to zero to find the position of the maximum moment "$x_{\max}$", and subsequently, "$x_{\max}$" is substituted in the equation for "M_x" to obtain the maximum positive moment "$M_{\max}$". Distance "L_1" is the inflection point from support A and distance "L_2" is the inflection point from support B which are obtained by setting the equation of "M_x" to zero. "L_a" is the horizontal distance of the beam from support A to the constant section (left haunches), and "L_c" is the horizontal distance of the beam from support B to the constant section (right haunches).

The basic equations for the design according to the code are (LRFD 2009).

The moment for compact profiles is obtained:

$$\begin{aligned} M_n &= M_p = F_y Z \leq 1.5 M_y \\ M_u &= \phi_b M_n \end{aligned} \tag{1}$$

where M_n is the nominal moment, M_p is the plastic moment, M_y is the yield moment, M_u is the ultimate moment, F_y is the yield stress, Z is the plastic section modulus of the cross section, ϕ_b is the bending resistance factor and accounting to the code (LRFD 2009) it is 0.90.

For the sections to be compact, the width to thickness ratios of the flanges and height to thickness of the webs of sections I and C are limited to the following maximum values, taken from Table B51 of the code (LRFD 2009).

For the flange:

$$\frac{b}{t_f} \leq 0.38\sqrt{\frac{E}{F_y}} \tag{2}$$

For the web:

$$\frac{d}{t_w} \leq 3.76\sqrt{\frac{E}{F_y}} \tag{3}$$

When the unsupported length L_b of the beam is $L_b \leq L_p$, it is considered that the beam has total lateral support and therefore its bending capacity is $M_n = M_p$. The parameter L_p is obtained:

$$L_p = 1.74 r_y \sqrt{\frac{E}{F_y}} \tag{4}$$

where r_y is the minimum radius of gyration.

For the shear of compact profiles:

$$\begin{aligned} V_n &= 0.6 F_y A_w \\ V_u &= \phi_v V_n \end{aligned} \tag{5}$$

where V_u is the ultimate shear, V_n is the nominal shear, ϕ_v is the resistance factor (shear is 0.90), d is the height of the web, t_w is the thickness of the web, A_w is the area of the web, E is the modulus of elasticity of the material (structural steel is 199.95 GPa).

The properties of an I-section are

$$Z_x = t_f b_f (d_x + t_f) + \frac{t_w {d_x}^2}{4} \tag{6}$$

$$r_y = \sqrt{\frac{2 t_f {b_f}^3 + d_x {t_w}^3}{12(2 t_f b_f + t_w d_x)}} \tag{7}$$

$$A_w = t_w d_x \tag{8}$$

where Z_x is the plastic section modulus around the X axis, t_f is the thickness of the flange, b_f is the width of the flange, d_x is the total height of the web at a distance "x".

2.2. **Mathematical model.** Figure 2 shows the geometric properties for an I-section beam to observe the dimensions in general.

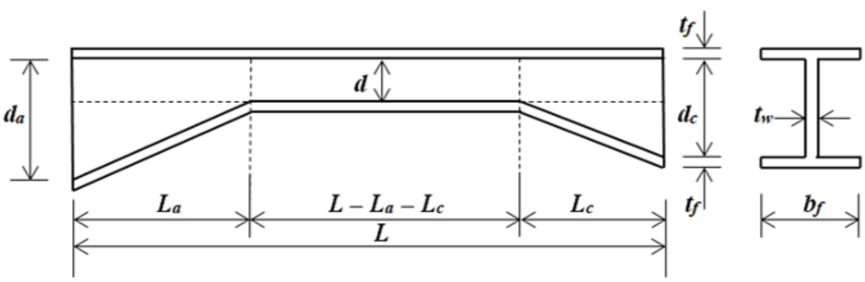

FIGURE 2. Geometric properties for an I-section beam with straight haunches

The equation of the total cost for a steel beam with straight haunches C_t is presented in terms of the cost of steel per unit volume C_s. The total cost of the I-section steel beam is

$$C_t = C_s V_s \tag{9}$$

where V_s is steel volume.

The volume of steel is

$$V_s = t_f b_f \left[\sqrt{{L_a}^2 + (d_a - d)^2} + 2L - L_a - L_c + \sqrt{{L_c}^2 + (d_c - d)^2} \right] + t_w \left[\frac{(d_a - d)L_a}{2} + dL + \frac{(d_c - d)L_c}{2} \right] \tag{10}$$

Substituting Equation (10) into Equation (9) the general equation of the total cost of steel is obtained (Objective function):

$$C_t = C_s \left\{ t_f b_f \left[\sqrt{{L_a}^2 + (d_a - d)^2} + 2L - L_a - L_c + \sqrt{{L_c}^2 + (d_c - d)^2} \right] + t_w \left[\frac{(d_a - d)L_a}{2} + dL + \frac{(d_c - d)L_c}{2} \right] \right\} \tag{11}$$

The constraint functions in general for each section are obtained as follows. The first is obtained by substituting Equation (6) into Equation (1). The second is obtained by substituting Equation (7) into Equation (4). The third is obtained by substituting Equation (8) into Equation (5). The fourth is obtained using Equation (4).

For support A

$$\frac{M_{uAB}}{\phi_b F_y} \leq t_f b_f (d_a + t_f) + \frac{t_w {d_a}^2}{4} \tag{12}$$

$$L_1 \leq 1.74 \sqrt{\frac{E}{F_y}} \sqrt{\frac{2 t_f {b_f}^3 + d_a {t_w}^3}{12(2 t_f b_f + t_w d_a)}} \tag{13}$$

$$V_{uA} \leq \phi_v 0.6 F_y t_w d_a \tag{14}$$

$$\frac{d_a}{t_w} \leq 3.76 \sqrt{\frac{E}{F_y}} \tag{15}$$

where M_{uAB} is the factored moment that occurs at support A, V_{uA} is the factored shear force that occurs at support A.

For the central part

$$\frac{M_{u\max}}{\phi_b F_y} \leq t_f b_f (d + t_f) + \frac{t_w d^2}{4} \tag{16}$$

$$L_b \leq 1.74 \sqrt{\frac{E}{F_y}} \sqrt{\frac{2 t_f {b_f}^3 + d {t_w}^3}{12(2 t_f b_f + t_w d)}} \tag{17}$$

$$V_{ua}, V_{uc} \leq \phi_v 0.6 F_y t_w d \tag{18}$$

$$\frac{d}{t_w} \leq 3.76 \sqrt{\frac{E}{F_y}} \tag{19}$$

where $M_{u\max}$ is the positive maximum factored moment, V_{ua} and V_{uc} are the factored shear forces located in section changes.

For support B

$$\frac{M_{uBA}}{\phi_b F_y} \leq t_f b_f (d_c + t_f) + \frac{t_w {d_c}^2}{4} \tag{20}$$

$$L_2 \leq 1.74 \sqrt{\frac{E}{F_y}} \sqrt{\frac{2 t_f {b_f}^3 + d_c {t_w}^3}{12(2 t_f b_f + t_w d_c)}} \tag{21}$$

$$V_{uB} \leq \phi_v 0.6 F_y t_w d_c \tag{22}$$

$$\frac{d_c}{t_w} \leq 3.76\sqrt{\frac{E}{F_y}} \tag{23}$$

where M_{uBA} is the factored moment that occurs at support B, V_{uB} is the factored shear force that occurs at support B.

For three intervals, Equation (2) is used:

$$\frac{b_f}{t_f} \leq 0.38\sqrt{\frac{E}{F_y}} \tag{24}$$

The results should be assumed non-negative.

3. **Numerical Problems.**

Two numerical examples are presented to verify the proposed model. Each example is developed for four different cases.

1) Case 1 considers $L_a = L_1$ and $L_c = L_2$, i.e., the inflection points coincide with the section changes.

2) Case 2 takes into account $L_a = L_c = L/4$.

3) Case 3 considers $L_a = L_c \geq L_1, L_2$.

4) Case 4 takes into account that the section is constant. The data for the two examples are $E = 199.95$ GPa, $F_y = 248$ MPa.

Case 1 is the ideal case because the section changes and it is where the inflection points occur or it changes from negative to positive moment and from positive to negative. Cases 2 and 3 are presented for aesthetic reasons because $L_a = L_c$. Case 4 is shown to observe the advantages of the beams with straight haunches over the beams with constant sections.

3.1. **Example 1.**

Design an I-section beam with straight haunches subjected to a uniformly distributed load. Figure 3 shows the diagram of shear forces and moments in general for a beam subjected to a uniformly distributed load and a moment at each end. The unsupported length of the beam flange in the region for negative moment is considered equal to the length of the negative moment because the compression flange is completely free at the bottom of the beam, and the beam is fully laterally supported by the slab in the region of positive moment, because the compression flange is at the top of the beam.

The basic data are $L = 12.00$ m, $w_u = 60$ kN/m, $M_{uAB} = -1000$ kN-m, $M_{uBA} = 0$ (type 1), $0.2M_{uAB}$ (type 2), $0.4M_{uAB}$ (type 3), $0.6M_{uAB}$ (type 4), $0.8M_{uAB}$ (type 5), M_{uAB} (type 6).

Table 1 shows the results obtained from the diagram of shear forces and moments (see Figure 3).

The constraints imposed on the decision variables are $0 \leq b_f$, $0 \leq d_a$, $10 \leq d$, $0 \leq d_c$, $1.27 \leq t_f$, $1.27 \leq t_w$.

Now, substituting this information in Equations (11) to (24) the objective function and the constraint functions are obtained.

The optimal solution is obtained using MAPLE-15 software.

Tables 2, 3, 4 and 5 show the optimal solution for cases 1, 2, 3 and 4.

3.2. **Example 2.**

Design an I-section beam with straight haunches subjected to a concentrated load. Figure 4 shows the diagram of shear forces and moments in general for a beam subjected to a concentrated load located in the center of the beam and a

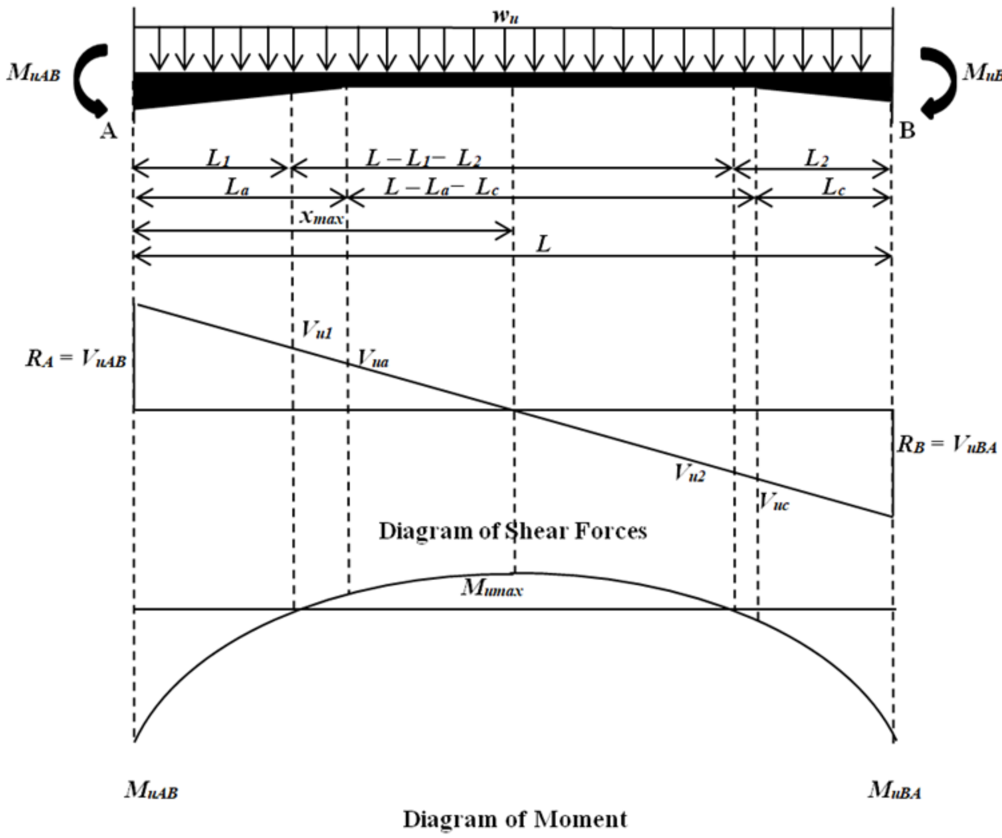

FIGURE 3. Beam subjected to a uniformly distributed load with straight haunches

TABLE 1. Example 1

Type	M_{uBA} (kN-m)	V_{uAB} (kN)	V_{u1} (kN)	V_{ua} (kN)	V_{u2} (kN)	V_{uc} (kN)	V_{uBA} (kN)	$x_{\max}$ (m)	$M_{u\max}$ (kN-m)	L_1 (m)	L_2 (m)
1	0	443.33	276.53	263.33	276.67	96.67	276.67	7.39	637.85	2.78	0.00
2	−200	426.67	249.07	246.67	248.93	113.33	293.33	7.11	517.06	2.96	0.74
3	−400	410.00	219.20	230.00	219.40	130.00	310.00	6.83	400.83	3.18	1.51
4	−600	393.33	186.33	213.33	186.27	146.67	326.67	6.56	289.24	3.45	2.34
5	−800	376.67	148.07	196.67	147.73	163.33	343.33	6.27	182.34	3.81	3.26
6	−1000	360.00	97.80	180.00	97.80	180.00	360.00	6.00	80.00	4.37	4.37

where V_{ua} is the shear force at $L_a = L/4 = 3.00$ m, V_{uc} is the shear force at $L_c = L/4 = 3.00$ m.

TABLE 2. Case 1: $L_a = L_1$ and $L_c = L_2$

Type	b_f (cm)	t_f (cm)	t_w (cm)	L_a (m)	L_c (m)	d_a (cm)	d (cm)	d_c (cm)	C_t
1	24.90	2.31	1.27	2.78	0.00	57.43	39.01	39.01	$0.201C_s$
2	25.89	2.40	1.27	2.96	0.74	54.54	30.23	17.25	$0.199C_s$
3	27.09	2.51	1.27	3.18	1.51	51.15	21.70	21.65	$0.202C_s$
4	28.57	2.65	1.27	3.45	2.34	47.21	13.69	29.28	$0.212C_s$
5	30.57	2.83	1.27	3.81	3.26	42.33	10.00	34.25	$0.236C_s$
6	33.75	3.13	1.27	4.37	4.37	35.52	10.00	35.52	$0.283C_s$

TABLE 3. Case 2: $L_a = L/4$, $L_c = L/4$ and $d_a = d_c$

Type	b_f (cm)	t_f (cm)	t_w (cm)	L_a (m)	L_c (m)	d_a (cm)	d (cm)	d_c (cm)	C_t
1	24.90	2.31	1.27	3.00	3.00	57.43	39.01	57.43	$0.202C_s$
2	25.89	2.40	1.27	3.00	3.00	54.54	30.23	54.54	$0.204C_s$
3	27.09	2.51	1.27	3.00	3.00	51.15	21.70	51.15	$0.208C_s$
4	28.57	2.65	1.27	3.00	3.00	47.21	13.69	47.21	$0.216C_s$
5	30.57	2.83	1.27	3.00	3.00	42.33	11.56	42.33	$0.237C_s$
6	33.75	3.13	1.27	3.00	3.00	35.52	10.58	35.52	$0.279C_s$

TABLE 4. Case 3: $L_a = L_c \geq L_1, L_2$ and $d_a = d_c$

Type	b_f (cm)	t_f (cm)	t_w (cm)	L_a (m)	L_c (m)	d_a (cm)	d (cm)	d_c (cm)	C_t
1	24.90	2.31	1.27	2.78	2.78	57.43	39.01	57.43	$0.202C_s$
2	25.89	2.40	1.27	2.96	2.96	54.54	30.23	54.54	$0.204C_s$
3	27.09	2.51	1.27	3.18	3.18	51.15	21.70	51.15	$0.208C_s$
4	28.57	2.65	1.27	3.45	3.45	47.21	13.69	47.21	$0.216C_s$
5	30.57	2.83	1.27	3.81	3.81	42.33	10.00	42.33	$0.239C_s$
6	33.75	3.13	1.27	4.37	4.37	35.52	10.00	35.52	$0.283C_s$

TABLE 5. Case 4: $L_a = L_c = 0$ and $d_a = d = d_c$

Type	b_f (cm)	t_f (cm)	t_w (cm)	L_a (m)	L_c (m)	d_a (cm)	d (cm)	d_c (cm)	C_t
1	24.90	2.31	1.27	0.00	0.00	57.43	57.43	57.43	$0.225C_s$
2	25.89	2.40	1.27	0.00	0.00	54.54	54.54	54.54	$0.232C_s$
3	27.09	2.51	1.27	0.00	0.00	51.15	51.15	51.15	$0.241C_s$
4	28.57	2.65	1.27	0.00	0.00	47.21	47.21	47.21	$0.254C_s$
5	30.57	2.83	1.27	0.00	0.00	42.33	42.33	42.33	$0.272C_s$
6	33.75	3.13	1.27	0.00	0.00	35.52	35.52	35.52	$0.307C_s$

moment at each end. The beam has a lateral support in the center of the beam, where the concentrated load is located.

The basic data are $L = 12.00$ m, $P_u = 50$ kN, $M_{uAB} = -1000$ kN-m, $M_{uBA} = 0$ (type 1), $0.2M_{uAB}$ (type 2), $0.4M_{uAB}$ (type 3), $0.6M_{uAB}$ (type 4), $0.8M_{uAB}$ (type 5), M_{uAB} (type 6).

Table 6 shows the results obtained from the diagram of shear forces and moments (see Figure 4).

The constraints imposed on the decision variables are $0 \leq b_f$, $0 \leq d_a$, $0 \leq d$, $0 \leq d_c$, $1.27 \leq t_f$, $1.27 \leq t_w$.

Now, substituting this information in Equations (11) to (24) the objective function and the constraint functions are obtained.

The optimal solution is obtained using MAPLE-15 software.

Tables 7, 8, 9 and 10 show the optimal solution for cases 1, 2, 3 and 4.

4. **Results.**

Example 1 (Tables 2 to 5) shows the following. For all cases of the 6 types, the same values are presented for b_f, t_f, d_a, t_w, d (except in case 2 of types 5 and 6 because the shears are greater than those that occur at the inflection points, and case 4

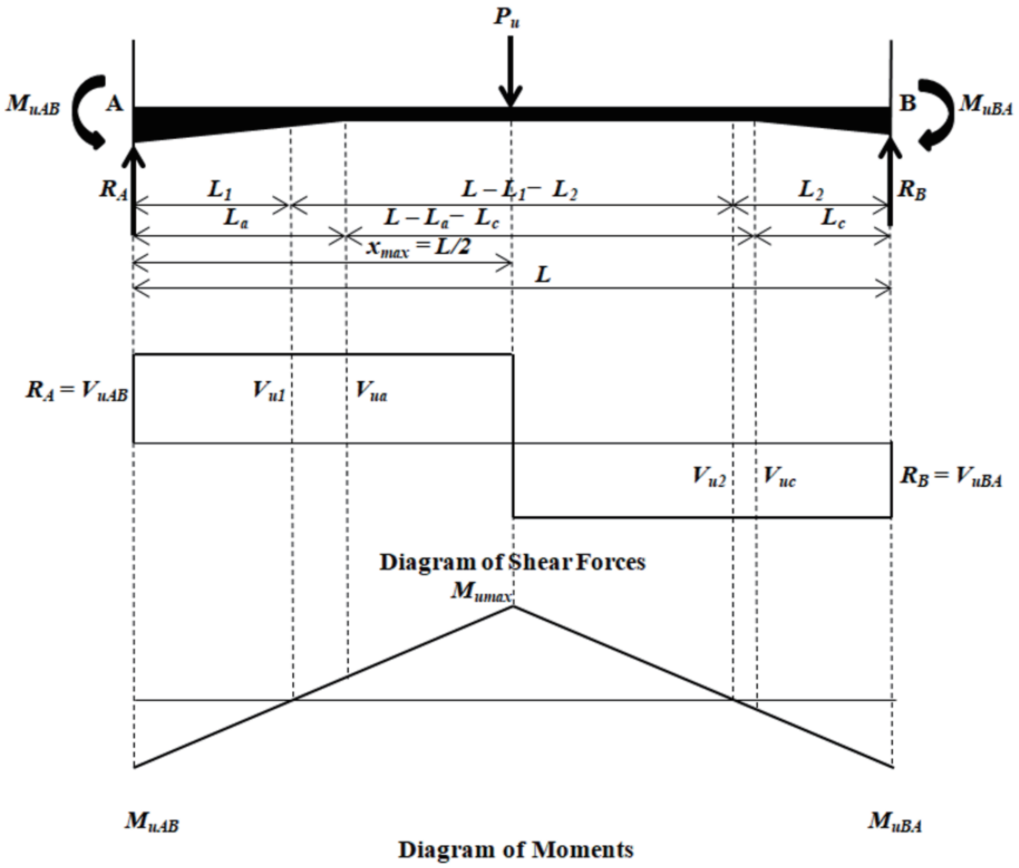

FIGURE 4. Beam subjected to a concentrated load with straight haunches

TABLE 6. Example 2

Type	M_{uBA} (kN-m)	V_{uAB} (kN)	V_{u1} (kN)	V_{ua} (kN)	V_{u2} (kN)	V_{uc} (kN)	V_{uBA} (kN)	$x_{\max}$ (m)	$M_{u\max}$ (kN-m)	L_1 (m)	L_2 (m)
1	0	333.33	333.33	333.33	166.67	166.67	166.67	6.00	999.98	3.00	0
2	−200	316.67	316.67	316.67	183.33	183.33	183.33	6.00	900.02	3.16	1.09
3	−400	300	300	300	200	200	200	6.00	800.00	3.33	2.00
4	−600	283.33	283.33	283.33	216.67	216.67	216.67	6.00	699.98	3.53	2.77
5	−800	266.67	266.67	266.67	233.33	233.33	233.33	6.00	600.02	3.75	3.43
6	−1000	250	250	250	250	250	250	6.00	500.00	4.00	4.00

TABLE 7. Case 1: $L_a = L_1$ and $L_c = L_2$

Type	b_f (cm)	t_f (cm)	t_w (cm)	L_a (m)	L_c (m)	d_a (cm)	d (cm)	d_c (cm)	C_t
1	43.60	4.04	1.27	3.00	0	20.62	20.62	20.62	$0.454C_s$
2	36.71	3.40	1.27	3.16	1.09	30.15	27.02	10.78	$0.341C_s$
3	31.15	2.89	1.27	3.33	2.00	41.01	33.11	16.13	$0.266C_s$
4	29.01	2.69	1.27	3.53	2.77	46.09	33.06	28.47	$0.240C_s$
5	30.24	2.80	1.27	3.75	3.43	43.11	26.33	34.93	$0.249C_s$
6	31.64	2.93	1.27	4.00	4.00	39.91	19.86	39.91	$0.263C_s$

TABLE 8. Case 2: $L_a = L/4$, $L_c = L/4$ and $d_a = d_c$

Type	b_f (cm)	t_f (cm)	t_w (cm)	L_a (m)	L_c (m)	d_a (cm)	d (cm)	d_c (cm)	C_t
1	43.60	4.04	1.27	3.00	3.00	20.62	20.62	20.62	$0.454C_s$
2	36.71	3.40	1.27	3.00	3.00	30.15	27.02	30.15	$0.342C_s$
3	31.15	2.89	1.27	3.00	3.00	41.01	33.11	41.01	$0.269C_s$
4	29.01	2.69	1.27	3.00	3.00	46.09	33.06	46.09	$0.243C_s$
5	30.24	2.80	1.27	3.00	3.00	43.11	26.33	43.11	$0.250C_s$
6	31.64	2.93	1.27	3.00	3.00	39.91	19.86	39.91	$0.261C_s$

TABLE 9. Case 3: $L_a = L_c \geq L_1$, L_2 and $d_a = d_c$

Type	b_f (cm)	t_f (cm)	t_w (cm)	L_a (m)	L_c (m)	d_a (cm)	d (cm)	d_c (cm)	C_t
1	43.60	4.04	1.27	3.00	3.00	20.62	20.62	20.62	$0.454C_s$
2	36.71	3.40	1.27	3.16	3.16	30.15	27.02	30.15	$0.342C_s$
3	31.15	2.89	1.27	3.33	3.33	41.01	33.11	41.01	$0.270C_s$
4	29.01	2.69	1.27	3.53	3.53	46.09	33.06	46.09	$0.244C_s$
5	30.24	2.80	1.27	3.75	3.75	43.11	26.33	43.11	$0.252C_s$
6	31.64	2.93	1.27	4.00	4.00	39.91	19.86	39.91	$0.263C_s$

TABLE 10. Case 4: $L_a = L_c = 0$ and $d_a = d = d_c$

Type	b_f (cm)	t_f (cm)	t_w (cm)	L_a (m)	L_c (m)	d_a (cm)	d (cm)	d_c (cm)	C_t
1	43.60	4.04	1.27	0.00	0.00	20.62	20.62	20.62	$0.454C_s$
2	36.71	3.40	1.27	0.00	0.00	30.15	30.15	30.15	$0.346C_s$
3	31.15	2.89	1.27	0.00	0.00	41.01	41.01	41.01	$0.279C_s$
4	29.01	2.69	1.27	0.00	0.00	46.09	46.09	46.09	$0.258C_s$
5	30.24	2.80	1.27	0.00	0.00	43.11	43.11	43.11	$0.269C_s$
6	31.64	2.93	1.27	0.00	0.00	39.91	39.91	39.91	$0.283C_s$

because the section is constant), d_c (except in case 1 because the length of the haunches coincide with the inflection points). When the moment at support B "M_{BA}" increases, C_t increases for all four cases (except in case 1 and type 1 because the inflection point coincides with support B, and $d_c = d$).

Example 2 (Tables 7 to 10) shows the following. For all the cases of the 6 types, the same values are presented for b_f, t_f, d_a, t_w, d (except in case 4 because the section is constant), d_c (except in case 1 because the length of the haunches coincide with the inflection points). When the moment at support B "M_{BA}" increases, C_t decreases until type 4 and from this type it increases for the four cases (because the flange thickness increases).

Figures 5 and 6 show the comparison between the volumes of the I-section beams with straight haunches of the two examples.

Figure 5 shows the following. The smallest volume is presented in case 1 for the 6 types (except in type 6, the smallest volume is found in case 2 because the values of L_a and L_c are less than cases 1 and 3). The savings for each type with respect to the constant section (case 4) are 11.94% for type 1 (case 1), 16.58% for type 2 (case 1), 19.31% for

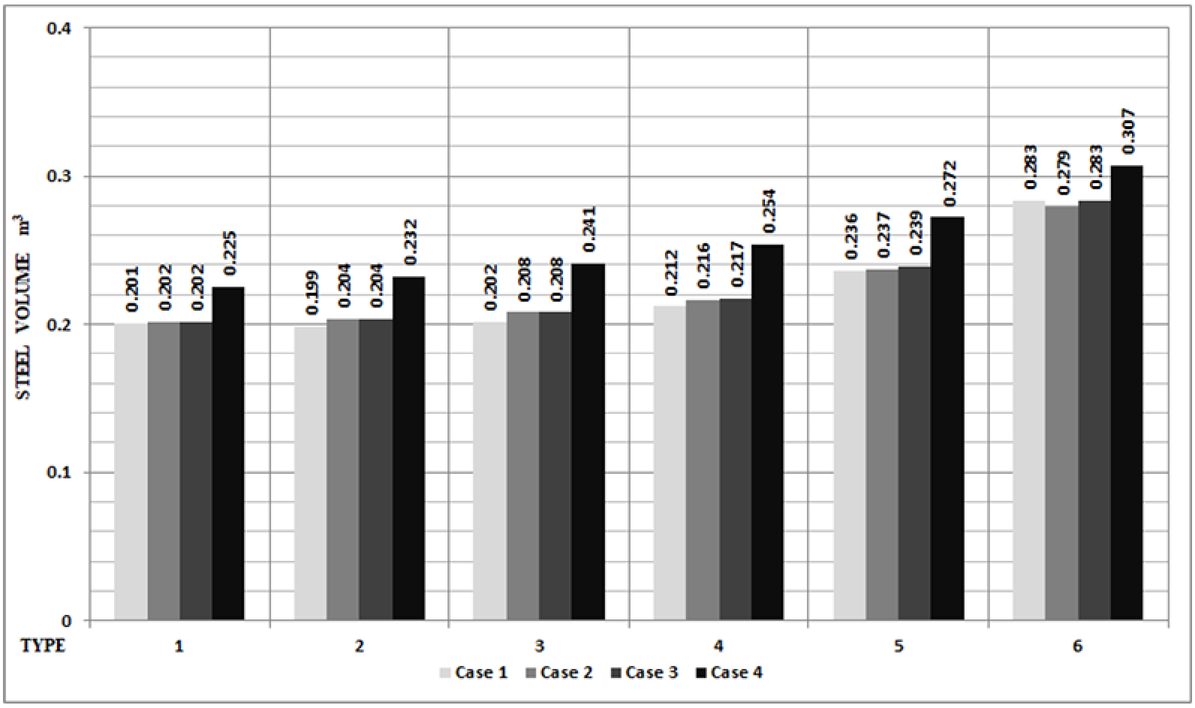

Figure 5. Comparison for example 1 (Uniformly distributed load)

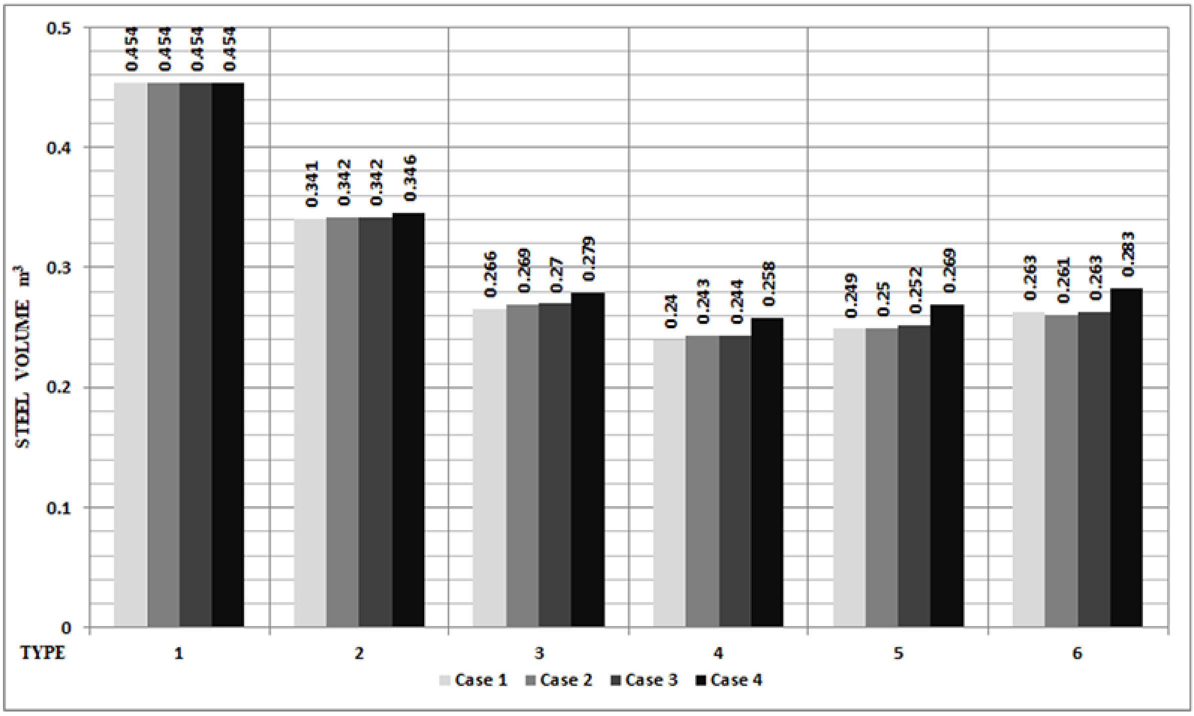

Figure 6. Comparison for example 2 (Concentrated load)

type 3 (case 1), 19.81% for type 4 (case 1), 15.25% for type 5 (case 1), 10.04% for type 6 (case 2).

Figure 6 shows the following. The smallest volume is presented in case 1 for the 6 types (except in type 6, the smallest volume is found in case 2 because the values of L_a and L_c are less than cases 1 and 3). Type 1 has a constant section, because the negative moment at support A is practically the same as the positive moment and the negative moment at support B is zero. The savings for each type with respect to the constant section (case 4) are 0% for type 1 (case 1 to 3), 1.47% for type 2 (case 1), 4.89% for type 3 (case 1), 7.50% for type 4 (case 1), 8.03% for type 5 (case 1), 8.43% for type 6 (case 2).

5. **Conclusions.** The document presented in this work shows an optimal model for I-section steel beams with straight haunches (non-prismatic beams) for the optimal design or minimum cost that takes into account the equations provided by the LRFD (Load and Resistance Factor Design) code (LRFD 2009).

This work presents two practical numerical examples of the new model. Example 1 is developed under the following information: $L = 12.00$ m, $w_u = 60$ kN/m, $M_{uAB} = -1000$ kN-m, $M_{uBA} = 0$ (type 1), $0.2M_{uAB}$ (type 2), $0.4M_{uAB}$ (type 3), $0.6M_{uAB}$ (type 4), $0.8M_{uAB}$ (type 5), M_{uAB} (type 6). Example 2 is designed under the following data: $L = 12.00$ m, $P_u = 50$ kN located in $L/2$, $M_{uAB} = -1000$ kN-m, $M_{uBA} = 0$ (type 1), $0.2M_{uAB}$ (type 2), $0.4M_{uAB}$ (type 3), $0.6M_{uAB}$ (type 4), $0.8M_{uAB}$ (type 5), M_{uAB} (type 6). The thickness of the web t_w and the flange thicknesses t_f are limited to 1.27 cm.

The main findings are the following.

The I-section steel beams with straight haunches are cheaper and have lower weight compared to constant I-section steel beams, because they have less volume of steel (see Figures 5 and 6).

Another advantage of I-section steel beams with straight haunches over constant section is the following. If the I-section steel beams with straight haunches are lighter than constant section beams, the structural members that support beams such as columns and footings can be more economical and have greater savings in materials.

The model presented can be adapted to optimize simply supported steel beams taking into account the critical moments and shear forces that occur in the intervals of $0 \leq x \leq L/4$ and $3L/4 \leq x \leq L$, to obtain smaller sections near the supports.

The following or future investigations can be the following:

1) Optimal design of I-section steel beams with parabolic haunches (non-prismatic beams);

2) Optimal design of steel beams with straight or parabolic haunches for other cross sections such as "T", "C" and "L";

3) Optimal design of the complete frame taking into account the beams, slabs, columns and footings.

REFERENCES

[1] Y. Yang and E. M. Lui, Behavior and design of steel I-beams with inclined stiffeners, *Steel and Composite Structures*, vol.12, no.3, pp.183-205, 2012.

[2] J. S. Park and Y. S. Park, Inelastic lateral-torsional buckling strengths of stepped I-beams subjected to general loading condition, *Structural Engineering and Mechanics*, vol.48, no.2, pp.275-289, 2013.

[3] D. Gąska, T. Haniszewski and J. Margielewicz, I-beam girders dimensioning with numerical modelling of local stresses in wheel-supporting flanges, *Mechanika*, vol.23, no.3, pp.347-352, 2017.

[4] X.-X. Qin, H.-B. Liu, C.-L. Wu and Z.-W. Gu, Semi-analytical solution of horizontally composite curved I-beam with partial slip, *Steel and Composite Structures*, vol.27, no.1, pp.1-12, 2018.

[5] M. Rezaiee-Pajand, A. R. Masoodi and A. Alepaighambar, Lateral-torsional buckling of functionally graded tapered I-beams considering lateral bracing, *Steel and Composite Structures*, vol.28, no.4, pp.403-414, 2018.

[6] A. R. M. Garoosi, M. T. Roudsari, M. Torkaman, S. Bonyadirad, A. Saeedmanesh, K. Soleimani, H. R. Lotfi, R. Jowkar and A. Alipour, Introducing new rigid I-beam to box column connections by using a stiffened channel link, *Steel and Composite Structures*, vol.41, no.4, pp.533-548, 2021.

[7] M. Rezaiee-Pajand, A. R. Masoodi and A. Alepaighambar, Critical buckling moment of functionally graded tapered mono-symmetric I-beam, *Steel and Composite Structures*, vol.39, no.5, pp.509-614, 2021.

[8] M. Ostwald, K. Magnucki and M. Rodak, Bicriteria optimal design of open cross sections of cold-formed thin-walled beams, *Steel and Composite Structures*, vol.7, no.1, pp.53-70, 2007.

[9] I. Rahmanian, Y. Lucet and S. Tesfamariam, Optimal design of reinforced concrete beams: A review, *Computers and Concrete*, vol.13, no.4, pp.457-482, 2014.

[10] F. Erdal, The comparative analysis of optimal designed web expanded beams via improved harmony search method, *Structural Engineering and Mechanics*, vol.54, no.4, pp.665-691, 2015.

[11] F. Erdal, Effect of stiffeners on failure analyses of optimally designed perforated steel beams, *Steel and Composite Structures*, vol.22, no.1, pp.183-201, 2016.

[12] A. Alekseytsev and M. A. Ali, Optimization of hybrid I-beams using modified particle swarm method, *Magazine of Civil Engineering*, vol.83, no.7, pp.175-185, 2018.

[13] V. Ho-Huu, T. Vo-Duy, D. Duong-Gia and T. Nguyen-Thoi, An efficient procedure for lightweight optimal design of composite laminated beams, *Steel and Composite Structures*, vol.27, no.3, pp.297-310, 2018.

[14] H. Ozbasaran and T. Yilmaz, Shape optimization of tapered I-beams with lateral-torsional buckling, deflection and stress constraints, *Journal of Constructional Steel Research*, vol.143, pp.119-130, 2018.

[15] A. Bolideh, H. G. Arab and M. R. Ghasemi, A fast and robust procedure for optimal detail design of continuous RC beams, *Computers and Concrete*, vol.24, no.4, pp.313-327, 2019.

[16] T. Talaslioglu, Design optimization of tubular lattice girders, *Advanced Steel Construction*, vol.15, no.3, pp.274-287, 2019.

[17] T. Talaslioglu, Design optimization of lattice girders according to member and joint-related design constraints, *Advanced Steel Construction*, vol.17, no.2, pp.181-198, 2021.

[18] J. Su, F. Zhang, S. Chen, X. Yu and S. Suntrayuth, Path optimization of fresh products logistics distribution under new retail mode, *International Journal of Innovative Computing, Information and Control*, vol.18, no.2, pp.511-523, 2022.

[19] A. Luévanos-Rojas and J. Montoya-Ramirez, Mathematical model for rectangular beams of variable cross section of symmetrical linear shape for uniformly distributed load, *International Journal of Innovative Computing, Information and Control*, vol.10, no.2, pp.545-564, 2014.

[20] A. Luévanos-Rojas, R. Luévanos-Rojas, I. Luévanos-Soto, R. G. Luévanos-Vazquez and O. A. Ramirez-Luévanos, Mathematical model for rectangular beams of variable cross section of symmetrical linear shape for concentrated load, *International Journal of Innovative Computing, Information and Control*, vol.10, no.3, pp.851-881, 2014.

[21] A. Luévanos-Rojas, S. López-Chavarría and M. Medina-Elizondo, Modelling for mechanical elements of rectangular members with straight haunches using software: Part 1, *International Journal of Innovative Computing, Information and Control*, vol.12, no.3, pp.973-985, 2016.

[22] A. Luévanos-Rojas, S. López-Chavarría and M. Medina-Elizondo, Modelling for mechanical elements of rectangular members with straight haunches using software: Part 2, *International Journal of Innovative Computing, Information and Control*, vol.12, no.4, pp.1027-1041, 2016.

[23] F. Velázquez-Santillán, A. Luévanos-Rojas, S. López-Chavarría and M. Medina-Elizondo, Modeling for beams of rectangular cross section with parabolic haunches: Part 1, *Computacin y Sistemas*, vol.23, no.2, pp.547-556, 2019.

[24] R. Sandoval-Rivas, A. Luévanos-Rojas, S. López-Chavarría and M. Medina-Elizondo, Modeling for beams of rectangular cross section with parabolic haunches: Part 2, *Computación y Sistemas*, vol.23, no.3, pp.1115-1124, 2019.

[25] A. Luévanos-Rojas, Modeling for beams of cross section "I" subjected to a uniformly distributed load with straight haunches, *Ingeniería Mecánica Tecnología y Desarrollo*, vol.5, no.2, pp.281-292, 2015.

[26] I. Luévanos-Soto and A. Luévanos-Rojas, Modeling for fixed-end moments of I-sections with straight haunches under concentrated load, *Steel and Composite Structures*, vol.23, no.5, pp.297-610, 2017.

[27] A. Luévanos-Rojas, S. López-Chavarría, M. Medina-Elizondo and V. V. Kalashnikov, Optimal design of reinforced concrete beams for rectangular sections with straight haunches, *Journal of Construction*, vol.19, no.1, pp.90-102, 2020.

[28] E. García-Canales, A. Luévanos-Rojas, S. López-Chavarría and M. Medina-Elizondo, Minimum cost for reinforced concrete rectangular beams with parabolic haunches, *Computación y Sistemas*, vol.24, no.3, pp.1063-1073, 2020.

[29] S. López-Chavarría, A. Luévanos-Rojas, M. Medina-Elizondo, M. García-Galván, R. Sandoval-Rivas, J. R. Garay-Gallardo and J. B. Rivera-Mendoza, Optimal cost design of reinforced concrete T-beams with straight haunches, *International Journal of Innovative Computing, Information and Control*, vol.18, no.3, pp.739-754, 2022.

[30] LRFD Manual of Steel Construction (Load Resistance Factor Design), *Structural Members Specifications & Codes*, (3rd Edition), AISC (American Institute Steel Construction), 2009.

Author Biography

Arnulfo Luévanos-Rojas received his B.Sc. degree in Civil Engineering (1981), Master in Science with Specialization in Structures (1983), Master in Science with Specialization in Planning and Construction of Works (2000), Master in Administration (2004), and Doctor in Engineering with Specialization in Planning Systems and Construction (2009). He was professor and researcher of the Facultad de Ingeniería, Ciencias y Arquitectura, Gomez Palacio Campus of the Universidad Juarez del Estado de Durango from 2006 to 2015, and of the Facultad de Contaduría y Administración, Torreón campus of the Universidad Autónoma de Coahuila since 2015 to date. He has published more than 116 papers in journals indexed in the Web of Science. His research interests are mathematical models applied to Engineering and Administration. He is member of the National System of Researchers of Mexico (Level I from 2016-2022 and Level II from 2023-2027).