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IMMERSION AND INVARIANCE ADAPTIVE ROBUST CONTROL FOR A CLASS OF NONLINEAR SYSTEMS WITH UNCERTAINTIES AND DISTURBANCES

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Abstract. *This paper presents an immersion and invariance adaptive robust control framework for a class of nonlinear systems with uncertainties and external disturbances. Sliding mode control law with continuous reaching law is used to avoid chattering. A novel immersion and invariance (I&I) adaptive law is developed to estimate the time-varying lumped uncertainty. In the proposed adaptive approach, there is an additional term that can shape the dynamics of the estimation errors. By ingeniously designing this additional item, the two proposed adaptive robust control approaches enable the tracking error to converge to zero in finite time. The salient feature of the proposed control is to suppress time-varying uncertainty in the controlled system without using a large learning rate in the adaptive laws. Simulation results of robot manipulator show the effectiveness of the proposed method.*

Keywords: Adaptive robust control, Sliding mode, Nonlinear system, Finite time convergence, Immersion and invariance, Robot manipulator

1. **Introduction.** Disturbances and uncertainties widely exist in all real-world plant [1-4], which seriously affects the performance of control system or causes system instability [5,6]. Various systems such as flexible-link robots, hydraulic servo systems, and synchronous machines are nonlinear systems with disturbances and uncertainties. In recent years, control problems of uncertain nonlinear systems have attracted much attention.

Adaptive control methodologies seemed to be a natural solution for the problem [7]. Adaptive control can take advantage of the so-called adaptive rate to provide estimations of unknown parameters or disturbances at each instant, and then update control gains, system parameters or disturbances in control law to achieve perfect performance [8]. The sensitivity method and the MIT adaptive rule are often used to design the adaptive laws of the various control schemes. With the emergence of state space technology and stability theory based on Lyapunov, Lyapunov synthesis method has become one of the most widely studied adaptive control methods. For the past decades, many different kinds of adaptive control have been developed using Lyapunov design approach [9-15]. It is pointed out that small disturbances could easily make these adaptive schemes go unstable. That is to say, adaptive control is sensitive to unmodeled dynamics, disturbances,

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and measurement noises, which not only lead to performance degradation but also cause some unpredictable instability phenomena [16]. Several modified adaptive laws [17-20] were proposed and analyzed to guarantee states and performance error boundedness in the presence of "reasonable" unmodeled dynamics and bounded disturbances. Lyapunov synthesis method based adaptive law need construct Lyapunov function. There is not a systematic approach to construct a Lyapunov function [21]. The common practice is empirical and trial-and-error at present. Sometimes, it is difficult to choose a Lyapunov function for some complex nonlinear systems. Moreover, these adaptive laws are difficult to guarantee that the estimation error converges to zero in the presence of unmodeled dynamics and disturbances. Immersion and invariance (I&I) methodology provides a new design way of adaptive law [22]. Adaptive control based on immersion and invariance can avoid choosing Lyapunov function. The salient feature of the immersion and invariance adaptive control is the introduction of an additional term in the adaptive law. In this way, I&I adaptive control not only contains the classical "integral action", but also achieves "proportional action". This kind of adaptive law was called as proportional-integral (PI) adaptation that was potentially superior to purely integral adaptation. In recent years, I&I adaptive control has obtained extensive attention, and applied to various fields [23-27]. However, both adaptive laws based on Lyapunov synthesis method and I&I adaptive law satisfy assumption that uncertainty and disturbance must be constant or slow-varying. For time-varying uncertainty and disturbance, sliding mode control (SMC) is an effective and promising robust control method [28]. It has been extensively investigated for various systems in the presence of uncertainties and disturbances. However, the SMC suffers from the undesired chattering resulted from the discontinuous control law, which is very harmful for actuators used in practical systems. A common way to counter the chattering is to add a boundary layer around the sliding manifold and use continuous control inside the boundary [29]. To suppress the chattering, a great many approaches were developed [30-33]. In [34], a continuous sliding mode control was proposed using a kind of continuous reaching law to achieve finite-time convergence of the state on the sliding manifold. However, uncertainties and disturbances are not considered in the continuous sliding mode control.

Inspired by continuous reaching law and immersion and invariance (I&I) methodology, an adaptive robust control framework was proposed in this paper. To the best of the author's knowledge, very few attempts have been made to design an adaptive robust control for nonlinear system with time-varying uncertainties by combining I&I methodology and sliding mode control. The proposed adaptive law does not require assumption that uncertainty and disturbance must be constant or slow-varying. It is robust by designing flexibly the additional terms in adaptive law. Compared with classical sliding mode control, it overcomes the chattering problem caused by the discontinuous sign function. It is proved by Lyapunov theory approach that both tracking error and estimation error converge to zero in finite time.

This paper is organized as follows. Preliminaries and problem formulation are presented in Section 2. Section 3 gives controller design and responding stability analysis. Section 4 presents simulation results. Section 5 draws the conclusion.

2. **Problem Statement and Preliminaries.**

A single-input uncertain nonlinear dynamic system is described by

$$\dot{x} = f(x) + g(x)u + d(t) \tag{1}$$

where $x \in R^n$ is a state vector, $u \in R$ is a control input, $d(t) \in R$ is a matched disturbance, $f(x) \in R^n$ is differentiable and known vector field, and $g(x) \in R$ is a smooth function.

Assume that a sliding variable $\sigma = \sigma(x,t) \in R$ whose relative degree with respect to input u equals one is designed such that the desirable compensated dynamics are achieved in the sliding mode $\sigma = 0$.

The dynamics of the sliding variable σ is given as

$$\dot{\sigma} = \frac{\partial \sigma}{\partial t} + \frac{\partial \sigma}{\partial x} f(x) + \frac{\partial \sigma}{\partial x} d(t) + \frac{\partial \sigma}{\partial x} g(x) u = a(x,t) + b(x,t) u \tag{2}$$

where $a(x,t) = \frac{\partial \sigma}{\partial t} + \frac{\partial \sigma}{\partial x} f(x) + \frac{\partial \sigma}{\partial x} d(t)$, $b(x,t) = \frac{\partial \sigma}{\partial x} g(x)$.

The functions $b(x,t) \in R$ and $a(x,t) \in R$ are uncertain. They are written as

$$\begin{aligned} a(x,t) &= a_0(x,t) + \Delta a(x,t) \\ b(x,t) &= b_0(x,t) + \Delta b(x,t) \end{aligned} \tag{3}$$

where $b_0(x,t)$ and $a_0(x,t)$ are known functions and $\Delta b(x,t)$ and $\Delta a(x,t)$ are perturbations. The dynamics of the sliding variable σ is represented as

$$\dot{\sigma} = a_0(x,t) + b_0(x,t) u + \Delta \tag{4}$$

where $\Delta = \Delta a(x,t) + \Delta b(x,t)$ represents the lumped uncertainty.

Before further discussion, the following basic assumptions are presented.

Assumption 2.1. *The function $b_0(x,t)$ satisfies the constraints $b_0(x,t) > 0$ and $\Delta b(x,t)$ satisfies the constraints $\Delta b(x,t) > -|b(x,t)|$ such that the control gain $b(x,t)$ is positive.*

Assumption 2.2. *The lumped uncertainty $\Delta \in R^n$ is unknown. It is differentiable, and the first derivative is bounded, i.e., $\left|\dot{\Delta}\right| \le \delta_1$, where δ_1 is positive numbers.*

The objective of this paper is to propose a design framework of adaptive robust control, such that the system error-states converge to zero at a finite-time in the presence of unknown continuous disturbance and uncertainties.

Definition 2.1. [35] *Consider a time-invariant nonlinear system*

$$\dot{x} = f(x), \ f(0) = 0, \ x \in R^n \tag{5}$$

where $f : U_0 \to R^n$ is continuous in an open neighborhood U_0 of the origin. The equilibrium point $x = 0$ of the system is (locally) finite-time stable if the two following conditions are satisfied: 1) The equilibrium point $x = 0$ of the system is asymptotically stable in an open neighborhood U of the origin, $U \subseteq U_0$; 2) For any initial state $x_0 \in U \backslash \{0\}$, there exists a settling time $T > 0$ such that every state $x(t, x_0)$ of the system (5) is in $U \backslash \{0\}$ for $t \in [0, T)$, at the same time, it satisfies $\lim_{t \to T} x(t, x_0) = 0$ and $x(t, x_0) = 0$ for $t \ge T$. Specially, if $U = R^n$, the equilibrium point $x = 0$ of the system is globally finite-time stable.

Lemma 2.1. [36] *Given the nonlinear system (1), suppose that $V(x)$ is a smooth positive definite function on $U \subset R^n$ and satisfies the differential inequality*

$$\dot{V}(x) + \mu V^{\chi}(x) \le 0 \quad \forall x \in U \backslash \{0\} \tag{6}$$

where $\mu > 0$ and $0 < \chi < 1$ are real numbers. Then, the origin of the system (5) is globally finite-time stable, i.e., the trajectory of the system converges to the origin from any initial state x_0 in finite time and the settling time $t(x_0)$ satisfies $t(x_0) \le V^{1-\chi}(x_0)/\mu(1-\chi)$.

3. I&I Adaptive Robust Control Design.

To achieve finite-time convergence, the reaching law is designed as

$$\dot{\sigma} = -k_1 |\sigma|^{\frac{1}{2}} sign(\sigma) \tag{7}$$

where k_1 is a positive coefficient, and $sign(\cdot)$ is the standard sign function.

According to the dynamics (6) and the reaching law (7), the continuous sliding mode control law is designed as

$$u = [b_0(x,t)]^{-1}\left(-a_0(x,t) - k_1|\sigma|^{\frac{1}{2}}sign(\sigma) - \Delta\right) \tag{8}$$

Using $\hat{\Delta} + \beta(\sigma)$ instead of lumped disturbance Δ, the control law for the system (6) is designed as

$$u = [b_0(x,t)]^{-1}\left(-a_0(x,t) - k_1|\sigma|^{\frac{1}{2}}sign(\sigma) - \left(\hat{\Delta} + \beta(\sigma)\right)\right) \tag{9}$$

Different from the estimators based on the Lyapunov synthesis adaptive approach, the adaptive method introduces an additional term $\beta(\sigma)$ to shape the dynamics of the estimation errors.

To obtain adaptive law, the estimation error is then defined as

$$z = \Delta - \left(\hat{\Delta} + \beta(\sigma)\right) \tag{10}$$

The derivate of estimation error z is obtained as

$$\begin{aligned}\dot{z} &= \dot{\Delta} - \dot{\hat{\Delta}} - \frac{\partial\beta}{\partial\sigma}\dot{\sigma} \\ &= \dot{\Delta} - \dot{\hat{\Delta}} - \frac{\partial\beta}{\partial\sigma}\left(a_0(x,t) + b_0(x,t)u + \Delta\right) \\ &= \dot{\Delta} - \dot{\hat{\Delta}} - \frac{\partial\beta}{\partial\sigma}\left(a_0(x,t) + b_0(x,t)u + \hat{\Delta} + \beta(\sigma) + z\right)\end{aligned} \tag{11}$$

Substituting (9) and (10) into (11) yields

$$\dot{z} = \dot{\Delta} - \dot{\hat{\Delta}} - \frac{\partial\beta}{\partial\sigma}\left(-k_1|\sigma|^{\frac{1}{2}}sign(\sigma) + z\right) \tag{12}$$

The adaptive law is designed as

$$\dot{\hat{\Delta}} = \frac{\partial\beta}{\partial\sigma}\left(k_1|\sigma|^{\frac{1}{2}}sign(\sigma)\right) \tag{13}$$

Substituting (13) into (12), the dynamics of the estimation errors is rewritten as

$$\dot{z} = -\frac{\partial\beta}{\partial\sigma}z + \dot{\Delta} \tag{14}$$

The function $\beta(\sigma)$ is designed as

$$\frac{\partial\beta}{\partial\sigma} = k_2\varphi'(\sigma) \tag{15}$$

where $\varphi'(\sigma) = \frac{1}{2}|\sigma|^{-\frac{1}{2}}$, k_2 is a positive coefficient.

The dynamics of the estimation errors is rewritten as

$$\dot{z} = -k_2\varphi'(\sigma)z + \dot{\Delta} \tag{16}$$

It is clear from the construction of the adaptive rate that, besides the classical "integral action" of the parameter estimator, through the action of $\beta(\sigma)$ we have introduced in the control law a "proportional" term.

Using (10), substituting (9) into (6) yields

$$\dot{\sigma} = -k_1|\sigma|^{\frac{1}{2}}sign(\sigma) + z \tag{17}$$

An auxiliary variable is defined as

$$\varphi(\sigma) = |\sigma|^{\frac{1}{2}}sign(\sigma) \tag{18}$$

Defining a new vector $\varepsilon = [\ \varphi(\sigma)\ \ z\ ]^T$, Equations (16) and (17) constitute a new dynamic system. The dynamic model is expressed in compact form as

$$\dot{\varepsilon} = \varphi'(\sigma)A_1\varepsilon + B\bar{\Delta} \tag{19}$$

where $A_1 = \begin{bmatrix} -k_1 & 1 \\ 0 & -k_2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\bar{\Delta} = \frac{\dot{\Delta}}{\varphi'(\sigma)}$.

Theorem 3.1. *Considering the dynamic system (6) where the upper bound of the lumped disturbance is supposed to be known, the control law is designed as (9) with adaptive law (13), the sliding variable* σ *and the estimation error* z *will then converge to the origin in finite time, if the gains* k_1 *and* k_2 *are chosen so that there exists a suitable matrix* $P_1 = P_1^T$ *such that*

$$A_1^T P_1 + P_1 A_1 + P_1 B B^T P_1 + 4\delta_1^2 C^T C + \omega_1 I = 0 \tag{20}$$

where I *is a unit matrix,* ω_1 *is a positive number,* $C = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$.

Proof: A Lyapunov function can be written as

$$V_1 = \varepsilon^T P_1 \varepsilon \tag{21}$$

According to Assumption 2.2, it follows that

$$\bar{\Delta}_1 = \frac{\dot{\Delta}}{\varphi'(\sigma)} \le \frac{\delta_1}{\varphi'(\sigma)|\varphi(\sigma)|}|\varphi(\sigma)| \le \frac{\delta_1}{\varphi'(\sigma)|\varphi(\sigma)|}|C\varepsilon| \tag{22}$$

Note that

$$\varphi'(\sigma)|\varphi(\sigma)| = \frac{1}{2}|\sigma|^{-\frac{1}{2}} \left| |\sigma|^{\frac{1}{2}} sign(\sigma) \right| = \frac{1}{2} \tag{23}$$

Substituting (23) into (22) yields

$$\bar{\Delta}_1 \le 2\delta_1 |C\varepsilon| \tag{24}$$

Considering Inequality (24), the time derivative of V_1 is given by

$$\begin{aligned} \dot{V}_1 &= \dot{\varepsilon}^T P_1 \varepsilon + \varepsilon^T P_1 \dot{\varepsilon} \\ &= \varphi'(\sigma) \left(\varepsilon^T \left(A_1^T P_1 + P_1 A_1 \right) \varepsilon + 2\varepsilon^T P_1 B \bar{\Delta}_1 \right) \\ &\le \varphi'(\sigma) \left(\varepsilon^T \left(A_1^T P_1 + P_1 A_1 \right) \varepsilon + \varepsilon^T P_1 B B^T P_1 \varepsilon + \left| \bar{\Delta}_1 \right|^2 \right) \\ &\le \varphi'(\sigma) \left(\varepsilon^T \left(A_1^T P_1 + P_1 A_1 \right) \varepsilon + \varepsilon^T P_1 B B^T P_1 \varepsilon + 4\delta_1^2 \varepsilon^T C^T C \varepsilon \right) \\ &\le \varphi'(\sigma) \varepsilon^T \left(A_1^T P_1 + P_1 A_1 + P_1 B B^T P_1 + 4\delta_1^2 C^T C \right) \varepsilon \end{aligned} \tag{25}$$

The following inequality for quadratic forms holds

$$\lambda_{\min}(P_1)\|\varepsilon\|^2 \le \varepsilon^T P_1 \varepsilon \le \lambda_{\max}(P_1)\|\varepsilon\|^2 \tag{26}$$

According to the definition of Euclidean norm, it follows that

$$\|\varepsilon\|^2 = [\varphi(\sigma)]^2 + z^2 \ge |\sigma| + z^2 \ge |\sigma| \tag{27}$$

According to Inequality (26) and Inequality (27), the following inequality holds

$$|\sigma| \le \|\varepsilon\|^2 \le \frac{V_1}{\lambda_{\min}(P_1)} \tag{28}$$

From Inequality (28), one obtains

$$|\sigma|^{-\frac{1}{2}} \ge \left(\frac{V_1}{\lambda_{\min}(P_1)} \right)^{-\frac{1}{2}} \tag{29}$$

Using (26), the equality $\varphi'(\sigma) = \frac{1}{2}|\sigma|^{-\frac{1}{2}}$ and (29), Inequality (25) is then rewritten as

$$\dot{V}_1 \le -\omega_1 \frac{1}{2}|\sigma|^{-\frac{1}{2}} \varepsilon^T \varepsilon \le -\omega_1 \frac{1}{2} \left(\frac{V_1}{\lambda_{\min}(P_1)} \right)^{-\frac{1}{2}} \frac{V_1}{\lambda_{\max}(P_1)} = -\tau_1 V_1^{\frac{1}{2}} \tag{30}$$

where $\tau_1 = \frac{\omega_1 \sqrt{\lambda_{\min}(P)}}{2\lambda_{\max}(P)}$.

Consequently, according to Lemma 2.1, if the coefficients are properly chosen so that the condition (20) is satisfied, the sliding variable σ and the estimation error z will then converge to the origin in finite time.

Remark 3.1. [37] *Lyapunov function* V_1 *in (21) is continuous and differentiable everywhere except when* $e = 0$*. It is obvious that* V_1 *is positive definite and radially unbounded. It is proved that* $\dot{V}_1$ *is negative definite except when* $e = 0$*. Before arriving at the equilibrium point* $(e, z) = (0, 0)$*, the trajectories of system cannot stay on the line* $e = 0$*. And it will intersect the plane* $e = 0$ *when* $z \neq 0$ *before reaching the origin. This means that* $\dot{V}_1$ *exists almost everywhere and* V_1 *decreases until the system reaches the equilibrium. If the origin is reached at some time* T*, then the trajectory will stay there.*

4. **Simulations.**

In this section, numerical simulations of robot manipulator with one joint are implemented to evaluate the performance of the proposed immersion and invariance adaptive robust control.

Dynamic model of robot manipulator with one joint is given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x_1, x_2) + gu + d(t) \end{aligned} \tag{31}$$

where $f(x_1, x_2) = -a \cos x_1 - bx_2 + \Delta f$, $\Delta f = -\Delta a \cos x_1 - \Delta b x_2$ is uncertainties, and $d(t)$ is disturbance. The parameters in the nonlinear system (31) is set as $a = 54.4$, $b = 24$ and $g = 12$.

A sliding mode variable is defined as $s = \dot{e}_1 + ce_1$, where $e_1 = x_1 - x_{1r}$ is the tracking error, and x_{1r} is the desired signal. According to the design process in Section 3, the proposed I&I adaptive robust control (IIARC) is designed as

$$\begin{aligned} u &= \frac{1}{g}\left[-f(x_1, x_2) - c\dot{e}_1 - k_1|s|^{\frac{1}{2}}sign(s) - \left(\hat{\Delta} + \beta(s)\right)\right] \\ \dot{\hat{\Delta}} &= \frac{\partial \beta}{\partial s}\left(k_1|s|^{\frac{1}{2}}sign(s)\right), \quad \frac{\partial \beta}{\partial s} = \frac{1}{2}k_2|s|^{-\frac{1}{2}} \end{aligned} \tag{32}$$

To verify the effectiveness of the proposed adaptive robust controller (32), the Lyapunov based adaptive control (LBAC) and classical sliding mode control (CSMC) are selected for comparison. LBAC for the system (31) is given as

$$\begin{aligned} u &= \frac{1}{g}\left[-f(x_1, x_2) + \dot{x}_{1r} - e_1 - k_{a1}\dot{e}_1\right] \\ \dot{a} &= -\gamma_1 \dot{e}_1 \cos x_1, \quad b = -\gamma_2 \dot{e}_1 x_2 \end{aligned} \tag{33}$$

CSMC for the system (31) is given as

$$u = \frac{1}{g}\left[-f(x_1, x_2) - c\dot{e}_1 - k_{s1}s - k_{s2}sign(s)\right] \tag{34}$$

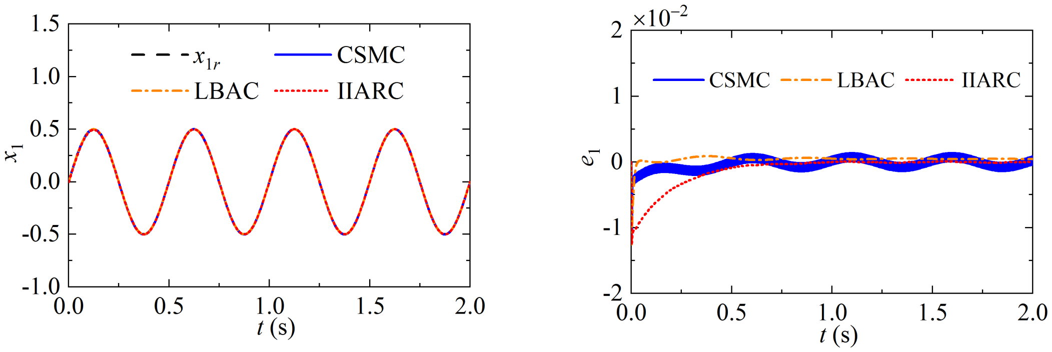
The first order Euler algorithm is used in Matlab/Simulink to solve the nonlinear dynamical equations with a sample time of 0.001 s. Simulations are implemented in the presence of different disturbances and uncertainties. The controller parameters are shown in Table 1.

Table 1. The parameters of three controllers

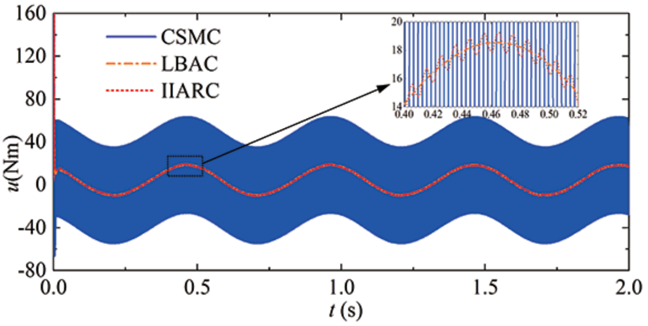
Controller	Parameters
IIARC	$c = 0.2$, $k_1 = 100$, $k_2 = 50$
LBAC	$k_{a1} = 1000$, $\gamma_1 = 10$, $\gamma_2 = 10$
CSMC	$c = 0.2$, $k_{s1} = 1000$, $k_{s2} = 400$

The desired signal $x_{1r} = 0.5\sin(2\pi t)$ is given into the system (31). Simulations under different working conditions are conducted.

Case 1: There are no uncertainties and disturbances in the system. The corresponding simulation results are given in Figure 1.



(a) Trajectory tracking (b) Tracking error



(c) Control input

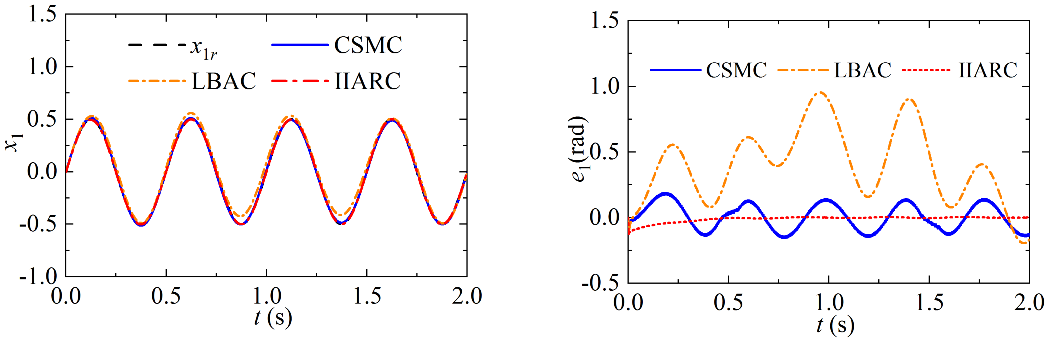
FIGURE 1. Tracking performance without uncertainties and disturbances

As can be seen from Figure 1, all closed-loop systems with different controllers have good tracking performance when there are no uncertainties and disturbances in the system. Compared with the other two methods, the proposed IIARC has the smallest tracking error (See Figure 1(b)). Figure 1(c) shows that CSMC suffers from the undesired chattering resulted from the discontinuous control law, which is very harmful for actuators used in practical systems.

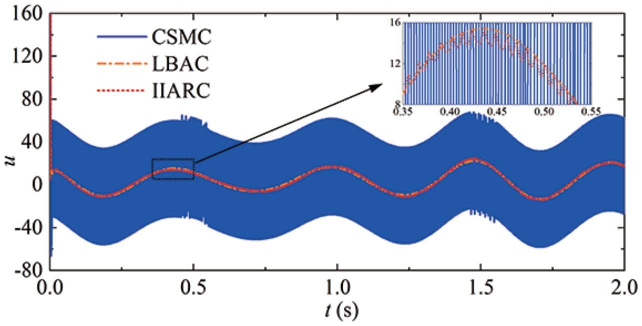
Case 2: There are uncertainties and disturbances in the system. The system parameters are set as $\Delta a = 5.44\sin(3\pi t)$ and $\Delta b = 2.4\sin(3\pi t)$. The disturbance is set as $d(t) = 50\sin(5\pi t)$. The corresponding simulation results are given in Figure 2.

Figure 2 shows that IIARC has best tracking performance in the presence of uncertainties and disturbances. CSMC has strong robustness resulted from the discontinuous control law, which causes undesired chattering. Figure 3 and Figure 4 show the convergence of adaptive parameters for LBAC and the lumped disturbance estimation of the proposed IIARC, respectively. From Figure 3, it is inferred that time-varying parameters and disturbances seriously affect parameter estimation for LBAC, which degrades system tracking performance. Figure 4 shows that the proposed IIARC has relatively good estimation ability of time-varying uncertainties and disturbances. Therefore, the proposed IIARC has strong robustness against uncertainties and disturbances.

5. **Conclusions.** An immersion and invariance adaptive robust control approach is proposed for a class of nonlinear systems with uncertainties and external disturbances. Sliding

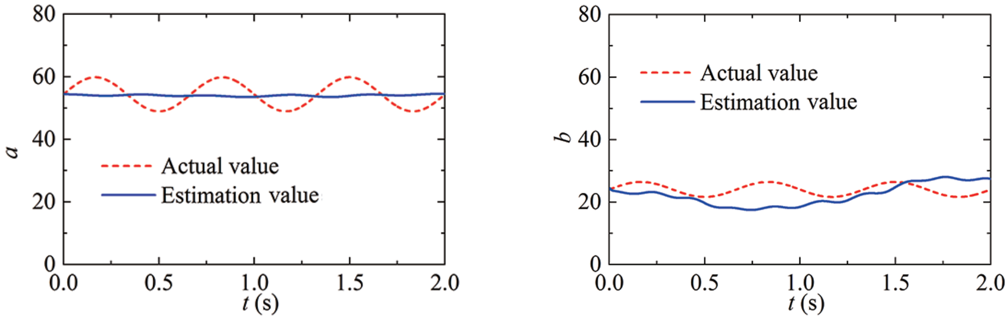


(a) Trajectory tracking (b) Tracking error



(c) Control input

FIGURE 2. Tracking performance in the presence of uncertainties and disturbances



(a) Parameter a (b) Parameter b

FIGURE 3. The convergence of adaptive parameters for LBAC

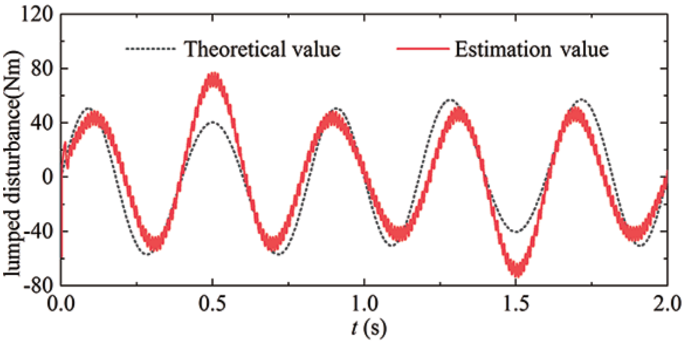


FIGURE 4. The lumped disturbance estimation of the proposed IIARC

mode control law with continuous reaching law is used to avoid chattering. A novel immersion and invariance (I&I) adaptive law is developed to estimate the time-varying lumped uncertainty. Simulations of robot manipulator verify the effectiveness of the proposed methods. The simulation results show that the proposed controller has good control performance and strong robustness against disturbances and uncertainties. In the future, the parameter adjustment method and physical experiment of the proposed control algorithm will be studied.

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