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OPTIMAL SOLVING MULTI-DEPOT VEHICLE ROUTING PROBLEM BY MODERN METAHEURISTIC ALGORITHM

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Abstract. *The multi-depot vehicle routing problem (MDVRP) is a generalized form of the vehicle routing problem (VRP) and travelling salesman problem (TSP). Considered as one of the NP-hard problems, the MDVRP consists of multiple vehicles for transporting goods to travel in and out of many warehouses (locations). The objective is to determine the optimal vehicle route in order to minimize the total distance satisfying the particular constraints and criteria. In this paper, the application of the modern metaheuristic algorithm (MoMA) to optimally solving the MDVRP is presented. As one of the new hybrid metaheuristic optimization search techniques, the MoMA combines with two types of the random process drawn from the uniform and Lévy distributions for generating the feasible solutions. Moreover, the automatic adjustable search radius mechanism (ASRM) is also utilized to balance the intensification (exploitation) and diversification (exploration) as well as to speed up the search process. The MoMA is applied to solving ten selected real-world MDVRP consisting of approximately 50-200 locations. Results obtained by the MoMA will be compared with those obtained by the genetic algorithm (GA), particle swarm optimization (PSO) and cuckoo search (CS). As results, it was found that the MoMA can provide optimal solutions of all ten selected MDVRP with shorter total distance than the original CS, PSO and GA, respectively.*
Keywords: Multi-depot vehicle routing problem, Modern metaheuristic algorithm, Modern optimization, Adjustable search radius mechanism, NP-hard problem

1. **Introduction.** The multi-depot vehicle routing problem (MDVRP) is one of the real-world logistic engineering problems focusing on the pickup and/or delivery of products from several depots to many customers. The MDVRP arises as a generalization of the vehicle routing problem (VRP) and travelling salesman problem (TSP), where vehicles depart from and return to one of multiple depots [1-3]. The MDVRP can be considered as a class of combinatorial optimization problems and also the NP-hard problems [4-7]. In general, there is a set of service locations (customers) to be served by a set of vehicles from a set of depots established in different places. The objective of the MDVRP is to minimize total distance in order to minimize the overall costs and to maximize the customers' demand by optimizing the sequence of locations visited by each vehicle (optimal vehicle route), satisfying such conditions and criteria as distance, time, and cost involved in the operation. Consisting of a fleet of vehicles, the MDVRP includes different service requirements (pickup and/or delivery of products) at each location, different capacities and time constraints of each vehicle in the fleet [8-10]. In the MDVRP, vehicles leave from one of the depots, serve customers along the routes and return to the depot where they

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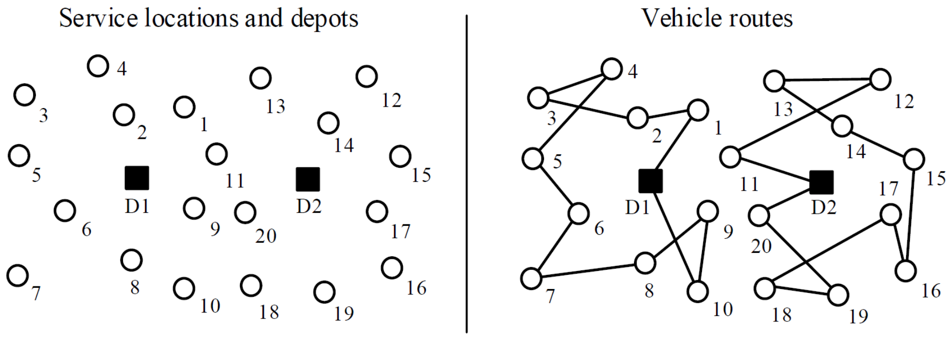

FIGURE 1. The MDVRP consisting of 2 depots and 20 service locations

leave after completion of their routes. In addition, each location will be visited exactly once by any vehicle in a fleet. For example, assume that in the considered MDVRP there are 2 depots and 20 service locations (excluding 2 depots) as can be visualized by Figure 1, where ○ stands for the service locations and ■ stands for the depots D1 and D2.

Following the literature, many variants of MDVRP are studied to address the variety of conditions in real-world applications, for example, the capacitated MDVRP (CMDVRP), the MDVRP with time windows (MDVRPTW), the heterogeneous fleet MDVRP (HMD-VRP), the MDVRP with pickup and delivery (MDVRPPD), and the generalized vehicle routing problem for multi-depot with pickup and delivery requests (GVRP-MDPDR) [1-3]. Based on the modern optimization approach, the MDVRP and its variants can be considered as a class of NP-hard problems which consume a great deal of computational time to find optimal solutions for large problems. The MDVRP can be efficiently solved by the efficient metaheuristic optimization search techniques, such as tabu search [11-13], genetic algorithm (GA) [14,15], particle swarm optimization (PSO) [16-20], ant colony optimization (ACO) [21-24], cuckoo search (CS) [25-27], artificial bee colony (ABC) [28], iterated local search (ILS) [29] and sector combination optimization (SCO) [30]. However, these metaheuristic techniques do not guarantee optimal solutions, but they generally promise a near optimal solution within a reasonable solution search time.

Recently, a novel hybrid metaheuristic optimization search technique named the modern metaheuristic algorithm (MoMA) has been proposed for function minimization in 2023 [31]. Algorithms of the MoMA combine with two types of the random process drawn from the uniform distribution and the Lévy distribution to generate the feasible solutions. Moreover, the automatic adjustable search radius mechanism (ASRM) is utilized to balance the intensification (exploitation) and diversification (exploration) properties as well as to speed up the search process. The MoMA was tested against several benchmark optimization problems to perform its effectiveness and search performance. Once comparing with other well-known metaheuristics including GA, PSO and CS, the MoMA was superior to other existing metaheuristic algorithms for function minimization [31]. From comparison with state-of-the-art studies and its advantages over existing well-known metaheuristic algorithms, the MoMA possesses few search parameters. This makes the MoMA algorithm not complicate and ease of use. In this paper, the MoMA is thus applied to optimally solving ten selected real-world MDVRP consisting of approximately 50-200 locations. Results obtained by the MoMA will be compared with those obtained by GA, PSO and CS to perform its effectiveness.

This paper consists of five sections. After an introduction is presented in Section 1, the remainder of this paper is arranged as follows. The problem formulation including the MDVRP model, objective function and constrained functions are provided in Section

2. The MoMA algorithm and the MoMA-based MDVRP optimization are described in Section 3. Experimental results and discussions are illustrated in Section 4. Finally, conclusions and future research are given in Section 5.

2. Problem Formulation. Regarding the original VRP firstly introduced in 1959 by Dantzig and Ramser [4], the VRP is generally defined by a graph $G = (V, \varepsilon, C)$ based on the graph theory, where $V = (v_0, \ldots, v_n)$ is the set of vertices which represent the locations, $\varepsilon = \{(v_i, v_j)|(v_i, v_j) \in V^2, i \neq j\}$ is the arc set which represents distances and $C = \{C_{ij}|(v_i, v_j) \in \varepsilon\}$ is the cost matrix defined over ε which represents traveling times or traveling costs. The MVRP and MDVRP models will be presented as follows.

2.1. **MVRP model.** The MVRP having a single depot can be modeled as follows [1,32]. Assuming there are N locations (customers) and K vehicles in a fleet, the distance between the i-th and the j-th locations is represented by d_{ij}. In the symmetric case, $d_{ij} = d_{ji}$, for all locations (i, j). They can be displayed by the distance matrix d: $n \times n \to \Re$ between the locations. All vehicles will start at the same depot. They will take a route such that each location except the depot is visited by exactly one vehicle. Finally, all vehicles will return to the depot at the end of the tour. The decision variables $\delta_{ijk} = 1$ if and only if the vehicle k travels from the i-th location to the j-th location; otherwise, $\delta_{ijk} = 0$. T_{ijk} is the traveling time of the vehicle k from the i-th location to the j-th location. T_{ijk} can be calculated by the relation between the average vehicle's speed and the working time, and $T_{\max}$ is the maximum working time of each vehicle.

$$\text{Minimize } Z(\cdot) = \sum_{i\in N}\sum_{j\in N}\sum_{k\in K} d_{ij}\delta_{ijk}, \quad i \neq j \tag{1}$$

$$\text{Subject to } \sum \delta_{ijk} = 1, \quad i = 1,\ \forall 2 \le j \le N,\ \forall 1 \le k \le K,\ i \neq j \tag{2}$$

$$\sum \delta_{jik} = 1, \quad \forall 2 \le i \le N,\ j = 1,\ \forall 1 \le k \le K,\ i \neq j \tag{3}$$

$$\sum_{j\in N}\sum_{k\in K} \delta_{ijk} = 1, \quad \forall 2 \le i \le N \tag{4}$$

$$\sum_{i\in N}\sum_{k\in K} \delta_{ijk} = 1, \quad \forall 2 \le j \le N \tag{5}$$

$$\sum_{i\in N} \delta_{irk} = \sum_{j\in N} \delta_{rjk}, \quad \forall 2 \le r \le N,\ \forall 1 \le k \le K \tag{6}$$

$$u_i - u_j + (N-K)\sum_{k\in K}\delta_{irk} \le N - K - 1, \quad \forall 2 \le i, j \le N,\ i \neq j \tag{7}$$

$$\sum_{i\in N}\sum_{j\in N} T_{ijk} \le T_{\max}, \quad \forall 1 \le k \le K,\ i \neq j \tag{8}$$

The objective function $Z(\cdot)$ of the MVRP is stated in (1) to minimize the total traveling distances satisfying the constrained functions as stated in (2)-(8). The constrained function in (2) ensures that all vehicles will leave the depot exactly once. The constrained function in (3) ensures that all vehicles will return to the depot exactly once. The constrained function in (4) ensures that all locations (except the depot) will be left by only one vehicle exactly once. The constrained function in (5) ensures that all locations (except the depot) will be arrived by only one vehicle exactly once. The constrained function in (6) ensures that the amount of time that all vehicles spend for visiting all locations equals the amount of time that all locations are left. The constrained function in (7) ensures that no sub-tours exist (degenerate routes that do not include the depot), by using $N-1$

as dummy variables of $u_2, \ldots, u_n$. Finally, the constrained function in (8) ensures that each vehicle spends the working time within its defined maximum working time.

2.2. **MDVRP model.** The MVRP having several depots can be modeled as follows [1,2,33,34]. Let N be a set of nodes, $N = N_c \cup N_d$, where N_c is a set of locations (customers), and N_d is a set of depots with K vehicles in a fleet. Let F be the number of vehicles available in each depot. That is $|K| = F|N_d|$. f_i is the number of vehicles used in the i-th depot. The distance between the i-th and the j-th locations is represented by d_{ij} as the MVRP. All vehicles will start at any depot. Then, they will take a route such that each location except the depot is visited by exactly one vehicle. At the end of the tour, all vehicles will return to the depot where they depart. The decision variables $\delta_{ijk} = 1$ if and only if the vehicle k travels from the i-th location to the j-th location; otherwise, $\delta_{ijk} = 0$. Also, the decision variables $z_{ij} = 1$ if and only if the j-th depot is assigned to the i-th location; otherwise, $z_{ij} = 0$. The objective function $Z(\cdot)$ of the MDVRP is stated in (9) to minimize the total traveling distances satisfying the constrained functions as stated in (10)-(19).

$$\text{Minimize } Z(\cdot) = \sum_{i\in N}\sum_{j\in N}\sum_{k\in K} d_{ij}\delta_{ijk}, \quad i \neq j \tag{9}$$

$$\text{Subject to } \sum_{j\in N}\sum_{k\in K} \delta_{ijk} = 1, \quad \forall i \in N_c,\ i \neq j \tag{10}$$

$$\sum_{j\in N} \delta_{jik} = \sum_{j\in N} \delta_{ijk}, \quad \forall i \in N,\ \forall k \in K,\ i \neq j \tag{11}$$

$$\delta_{ijk} = 0, \quad \forall i, j \in N_d,\ \forall k \in K \tag{12}$$

$$\sum_{i\in N_d}\sum_{j\in N_c} \delta_{ijk} \leq 1, \quad \forall k \in M \tag{13}$$

$$\sum_{j\in N_d} z_{ij} = 1, \quad \forall i \in N_c \tag{14}$$

$$\sum_{j\in N_c}\sum_{k\in K} \delta_{ijk} = f_i, \quad \forall i \in N_d \tag{15}$$

$$\sum_{j\in N_c}\sum_{k\in K} T_{ijk} \leq T_{\max}, \quad \forall k \in K,\ i \neq j \tag{16}$$

$$\sum_{k\in K} \delta_{ijk} \leq z_{ij}, \quad \forall i \in N_c,\ \forall j \in N_d \tag{17}$$

$$\sum_{k\in K} \delta_{jik} \leq z_{ji}, \quad \forall i \in N_c,\ \forall j \in N_d \tag{18}$$

$$\sum_{k\in K} \delta_{ijk} + z_{ir} + \sum_{m\in N_d} z_{jm} \leq 2, \quad \forall i, j \in N_c,\ i \neq j,\ m \neq r,\ \forall r \in N_d \tag{19}$$

The constrained function in (10) ensures that each location is served exactly once. The constrained function in (11) ensures that the number of entering arcs is equal to the number of leaving arcs for each node. The constrained function in (12) ensures that a vehicle should leave and enter the same depot. The constrained function in (13) ensures that one vehicle only travels in one route. The constrained function in (14) guarantees that each location is assigned to one depot. The constrained function in (15) defines the number of vehicles used for each depot. The constrained function in (16) ensures that each vehicle spends the working time within its defined maximum working time. Finally, the constrained functions in (17)-(19) ensure that no sub-tours exist (prohibit infeasible routes).

3. MoMA Algorithm for MDVRP Optimization.

In this section, the MoMA algorithm is briefly described. Then, the MoMA-based MDVRP optimization is elaborately illustrated as follows.

3.1. MoMA algorithm.

As one of the hybrid metaheuristic optimization search techniques, the MoMA [31] utilizes the random processes drawn from the uniform distribution and the Lévy distribution for generating the elite solutions in each search iteration. In addition, to balance the intensification (exploitation) and diversification (exploration) properties and speed up the search process, the ASRM mechanism is conducted in the MoMA algorithm to automatically reduce the search radius.

The MoMA algorithm is represented by the pseudo code as shown in Figure 2 [31]. After initialization, the search radius R will be calculated by using (20), where R_t is the

```
Initialized:
- Objective function Z(x), x = (x_1,...,x_d)^T,
- Search space Θ,
- Number of initial solutions N_s,
- Search radius R_0 = Θ (100% of Θ), α ∈ [0, 1].
- Counters ii = jj = kk = t = CT = 1.
- Uniformly random initial solution X_ii, ii = 1,2,...,N_s within Θ.
- Evaluate Z(X_ii) then rank X_ii and store in set ℵ.
- Let x_0 = X_jj as selected initial solution.
- X_global = X_local = x_0.
- Maximum iteration T_max
- Maximum total iteration CT_max
while (jj ≤ N_s or Termination criteria: TC)
    while (t ≤ T_max);
        Activate ASRM to calculate R_t.
        if mod(t,2) ≠ 0 (Odd iterations → Population-based);
            - Set number of feasible solutions n = N_p.
            - Random x_kk, kk = 1,2,...,N_p around x_0 within R_t by Lévy distribution.
        else (Even iterations → Trajectory-based);
            - Set number of feasible solutions n = 1.
            - Random x_kk, kk = 1 around x_0 within R_t by uniform distribution.
        end
        Evaluate Z(x_kk) and set the best one as x*. Set kk = 1.
        if Z(x*) < Z(x_0);
            Update x_0 = x*.
        end
        Update t = t + 1 and CT = CT + 1.
    end
    if Z(x_0) < Z(X_local);
        Update X_local = x_0.
    end
    - Update jj = jj + 1 and set t = 1.
    - Set x_0 = X_jj as selected initial solution.
end
if Z(X_local) < Z(X_global);
    - Update X_global = X_local.
    - Report best solutions found.
end
```

FIGURE 2. Pseudo code of MoMA algorithm

search radius at the tth iteration, R_0 is the initial search radius and α is the decreasing factor. Then, it goes into the search loop. In odd iterations, the MoMA performs the population-based manner. The feasible solutions will be set as $n = N_p$ and the random process drawn from the Lévy distribution in (21)-(24) is activated to generate the feasible solutions. In (21)-(24), L is the random process drawn from the Lévy distribution, $\Gamma(\lambda)$ is the standard Gamma function, s is step-size, U and V are Gaussian distributions and σ^2 is variance. In even iterations, the MoMA becomes the trajectory-based manner. The feasible solution $n = 1$ is set and the random process drawn from the uniform distribution in (25) and (26) is invoked to generate the feasible solution. In (25) and (26), a and b are the ranges of random and $\bar{x}$ is mean. In each iteration, R will be reduced to balance the intensification and diversification. The search process of the MoMA in the search loop will be iteratively proceeded and stopped when the termination criteria (TC) are met

$$R_t = R_0 e^{-\alpha t}, \quad 0 < \alpha < 1 \tag{20}$$

$$L \sim \frac{\lambda\Gamma(\lambda)\sin(\pi\lambda/2)}{\pi}\frac{1}{s^{1+\lambda}}, \quad (s \gg s_0 > 0) \tag{21}$$

$$\Gamma(\lambda) = \int_0^\infty t^{\lambda-1}e^{-t}dt \tag{22}$$

$$s = \frac{U}{|V|^{1/\lambda}}, \quad U \sim N(0,\sigma^2), \quad V \sim N(0,1) \tag{23}$$

$$\sigma^2 = \left[\frac{\Gamma(1+\lambda)}{\lambda\Gamma[(1+\lambda)/2]} \cdot \frac{\sin(\pi\lambda/2)}{2^{(\lambda-1)/2}}\right]^{1/\lambda} \tag{24}$$

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

$$\bar{x} = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12} \tag{26}$$

3.2. **MoMA-based MDVRP optimization.** The MoMA algorithm is applied to optimally solving the MDVRP. The MoMA-based MDVRP optimization can be described as follows.

Step-0 For MDVRP: initialize the objective function $Z(\cdot)$ of the MDVRP as stated in (9) and the constrained functions as stated in (10)-(19), initialize entire search space Θ (number of locations N_c and their correspondingly distances d_{ij}), number of vehicles K in the fleet, and number of depots N_d.
For MoMA algorithm: initialize the number of initial solutions N_s, decreasing factor α and number of feasible solutions N_p. Counters $ii = jj = kk = t = CT = 1$. The maximum iteration $T_{\max}$ and the maximum total iteration $CT_{\max}$ are set.
Pre-process: uniformly random initial solution $\boldsymbol{X}_{ii}$, $ii \in N_s$ within Θ. Then, evaluate $Z(\boldsymbol{X}_{ii})$ in (9) satisfying the constrained functions in (10)-(19). Rank $\boldsymbol{X}_{ii}$ and store in set $\aleph$. Let $\boldsymbol{x}_0 = \boldsymbol{X}_{jj}$ (the selected initial solution), $\boldsymbol{X}_{global} = \boldsymbol{X}_{local} = \boldsymbol{x}_0$.

Step-1 If $jj \le N_s$, go to Step-2. Otherwise (TC are met), go to Step-10.

Step-2 If $t \le T_{\max}$, go to Step-3. Otherwise, go to Step-9.

Step-3 Activate ASRM mechanism to calculate R_t by using (20).

Step-4 If mod$(t, 2) \ne 0$ (Odd iterations: population-based), go to Step-5. Otherwise, go to Step-6.

Step-5 Set the number of feasible solutions $n = N_p$. Random $\boldsymbol{x}_{kk}$, $kk \in N_p$, around $\boldsymbol{x}_0$ within R_t by random from the Lévy distribution in (21)-(24).

Step-6 Set the number of feasible solution $n = 1$ (Even iterations: trajectory-based). Random $\boldsymbol{x}_{kk}$, $kk = 1$, around $\boldsymbol{x}_0$ within R_t by random from the uniform distribution in (25) and (26).

Step-7 Evaluate $Z(\boldsymbol{x}_{kk})$ in (9) satisfying the constrained functions in (10)-(19). Then set $\boldsymbol{x}^*$ as the best solution among $\boldsymbol{x}_{kk}$, and set $kk = 1$.

Step-8 If $Z(\boldsymbol{x}^*) < Z(\boldsymbol{x}_0)$, update $\boldsymbol{x}_0 = \boldsymbol{x}^*$. Update $t = t + 1$, $CT = CT + 1$. Then, go back to Step-2.

Step-9 If $Z(\boldsymbol{x}_0) < Z(\boldsymbol{X}_{local})$, update $\boldsymbol{X}_{local} = \boldsymbol{x}_0$. Update $jj = jj + 1$ and set $t = 1$. Set $\boldsymbol{x}_0 = \boldsymbol{X}_{jj}$ (as the new selected initial solution). Then, go back to Step-1.

Step-10 Post-process: If $Z(\boldsymbol{X}_{local}) < Z(\boldsymbol{X}_{global})$, update $\boldsymbol{X}_{global} = \boldsymbol{X}_{local}$. Terminate the search process, and report the best solutions found.

4. **Experimental Results and Discussions.** For this study, the objective of the MoMA-based MDVRP optimization is to minimize total distance. Therefore, the load capacity of each vehicle, pickup/delivery time requirements, and traffic situation are neglected. In addition, the symmetric case, $d_{ij} = d_{ji}$, is assumed. Ten real-world MDVRP consisting of approximately 50-200 locations are selected from [35-37]. Ten selected MDVRP are detailed as summarized in Table 1. For example, the 51 service locations and its depots of MDVRP#1 (EIL51) are displayed in Figure 3, where ◯ stands for the service locations and ■ stands for the depots.

Table 1. Ten selected real-world MDVRP problems

Problems	**Names**	**Nunber of locations**	**Optimal tour for one vehicle (km.)**	**Comment**
MDVRP#1	EIL51	51	426	Eilon
MDVRP#2	BERLIN52	52	7,542	Groetschel
MDVRP#3	EIL76	76	538	Eilon
MDVRP#4	GR96	96	55,209	Groetschel
MDVRP#5	KROB100	100	22,141	Nelson
MDVRP#6	BIER127	127	118,282	Reinelt
MDVRP#7	CH150	150	6,528	Churritz
MDVRP#8	BRG180	180	1,950	Rinaldi
MDVRP#9	RAT195	195	2,323	Pulleyblank
MDVRP#10	D198	198	15,780	Reinelt

To solve ten selected real-world MDVRP in Table 1, the MoMA algorithm was coded by MATLAB version 2018b run on Intel(R) Core(TM) i5-3470 CPU@3.60GHz, 4.0GB-RAM. For each MDVRP, 50 trial-runs are executed to search for their best solutions. The depot's locations are arbitrary defined. The daily working time of any vehicle is defined as no longer than 8 hr. Therefore, $T_{\max} = 8$ hr. is set as the maximum working time in (16). 80 km/hr. is approximated for the average speed of all vehicles. Thus, the overall distance of each vehicle must not exceed 640 km/day. Also, each depot should serve at least 10 locations. These data are used to define the number of depots N_d. The number of vehicles K in a fleet has to be equal to or greater than the number of depots N_d for each MDVRP problem.

In this work, the search parameters of the MoMA are set from the preliminary studies against ten selected real-world MDVRP with different ranges of parameters, i.e., number of

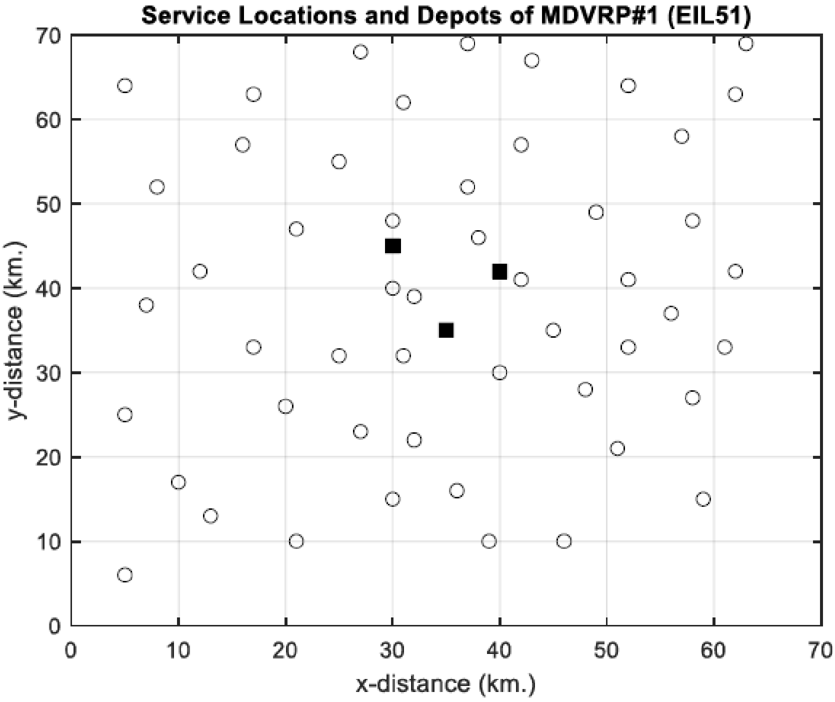

Figure 3. Service locations and depots of MDVRP#1 (EIL51)

initial solutions $N = 5, 10, 15, 20, \ldots, 50$, decreasing factor of the ASRM mechanism $\alpha = 0.005, 0.01, 0.015, 0.02, \ldots, 0.5$ and number of feasible solutions $N_p = 5, 10, 15, 20, \ldots, 50$. From the preliminary studies, the best parameters for all selected MDVRP are $N = 25$ to 45, $\alpha = 0.01$ to 0.025 and $N_p = 25$ to 40. Thus, $N = 40$, $\alpha = 0.02$ and $N_p = 35$ are set for the MoMA algorithm in order to solve all ten selected MDVRP. For comparison with GA, PSO and CS, the search parameters of those algorithms are also preliminary studied as the MoMA and then set as follows. For GA [38], number of populations = 40 (fixed as N_p of MoMA), crossover probability = 0.95 and mutation probability = 0.05. For PSO [39], number of swarms = 40 (fixed as N_p of MoMA), cognitive learning rate = 2.0 and social learning rate = 2.0. For CS [32,40], number of nests (or cuckoos) = 40 (fixed as N_p of MoMA) and discovery probability = 0.25. For all algorithms, maximum total iteration (or maximum generation) $CT_{\max} = 10{,}000$ is set. 50-trial runs with different initial solutions depending on the random process are conducted to search for the optimal solutions of the MDVRP problems.

Over 50-trial runs, the convergent rates of the MoMA for the MDVRP#1 (EIL51) are depicted in Figure 4 as the example. The convergent rates of other MDVRP problems are omitted because they have a similar form to those in Figure 4. From Figure 4, it can be observed that the MoMA performs the high robustness with different initial solutions. Results of ten selected real-world MDVRP optimization obtained by the GA, PSO, CS and MoMA are summarized in Table 2. As an example, the optimal tours of the MDVRP#1 (EIL51) obtained by the GA, PSO, CS and MoMA are plotted in Figures 5-8, where ◯ stands for the service locations, ■ stands for the depots, —— stands for the 1st vehicle route, -·-· stands for the 2nd vehicle route and ------- stands for the 3rd vehicle route, respectively. Results in Figures 5-8 and Table 2 are further analyzed as follows.

For the MDVRP#1 (EIL51), it possesses the number of depots $N_d = 3$ and the number of vehicles $K = 3$. From Figures 5-8 and Table 2, the GA provides the total distance of 164.36 km., the PSO performs the total distance of 162.35 km., the CS yields the total distance of 160.01 km., while the MoMA gives the total distance of 157.32 km.

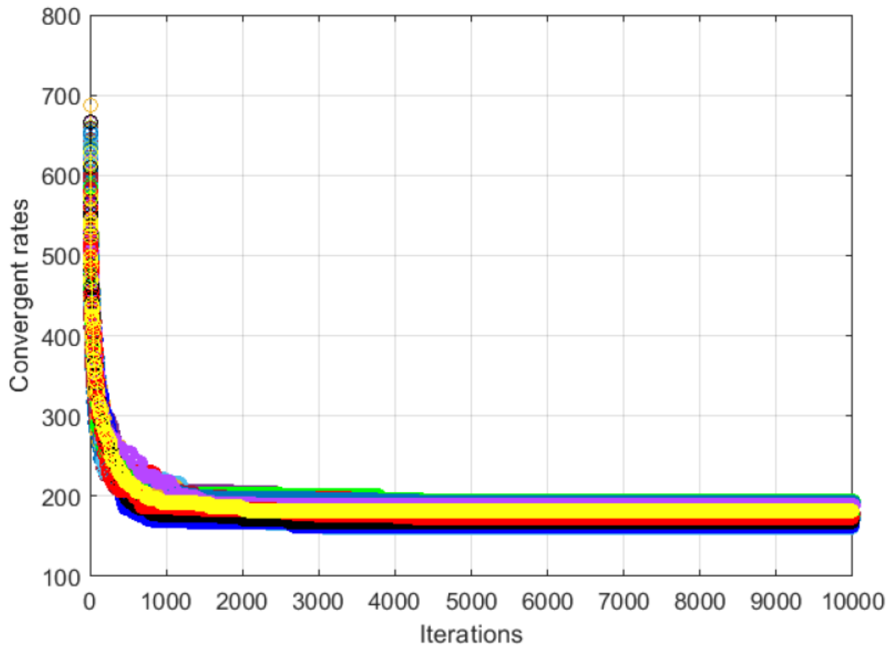

FIGURE 4. (color online) Convergent rates of MoMA for MDVRP#1 (EIL51) over 50-trial runs

TABLE 2. Optimal tours of MDVRP obtained by GA, PSO, CS and MoMA algorithms

Problems	No. of depots N_d	No. of vehicles K	Optimal tour (km.)			
			GA	PSO	CS	MoMA
MDVRP#1	3	3	164.36	162.35	160.01	157.32
MDVRP#2	3	3	1,592.25	1,506.64	1,458.93	1,446.56
MDVRP#3	5	6	267.10	250.12	227.48	215.75
MDVRP#4	7	8	26,864.54	26,206.48	25,102.39	24,624.18
MDVRP#5	8	9	15,403.38	13,310.37	12,017.04	11,256.21
MDVRP#6	10	12	68,012.05	66,998.02	66,430.22	65,767.54
MDVRP#7	12	15	1,658.76	1,490.56	1,286.42	1,230.53
MDVRP#8	14	17	957.27	918.13	847.06	812.23
MDVRP#9	15	20	1,515.59	1,324.48	1,106.67	1,054.54
MDVRP#10	16	22	7,508.37	7,393.65	7,102.08	6,861.29

For the MDVRP#2 (BERLIN52), it has $N_d = 3$ and $K = 3$. From Table 2, the GA provides the total distance of 1,592.25 km., the PSO performs the total distance of 1,506.64 km., the CS yields the total distance of 1,458.93 km., while the MoMA gives the total distance of 1,446.56 km.

For the MDVRP#3 (EIL76), it possesses $N_d = 5$ and $K = 6$. From Table 2, the GA provides the total distance of 267.10 km., the PSO performs the total distance of 250.12 km., the CS yields the total distance of 227.48 km., while the MoMA gives the total distance of 215.75 km.

For the MDVRP#4 (GR96), it has $N_d = 7$ and $K = 8$. From Table 2, the GA provides the total distance of 26,864.54 km., the PSO performs the total distance of 26,206.48 km., the CS yields the total distance of 25,102.39 km., while the MoMA gives the total distance of 24,624.18 km.

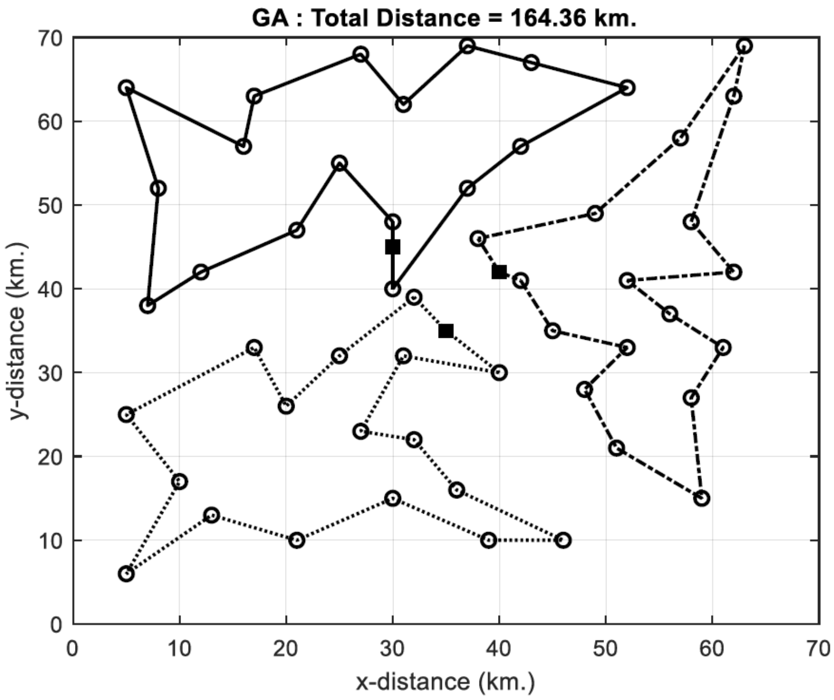

FIGURE 5. Optimal tour of MDVRP#1 (EIL51) obtained by GA

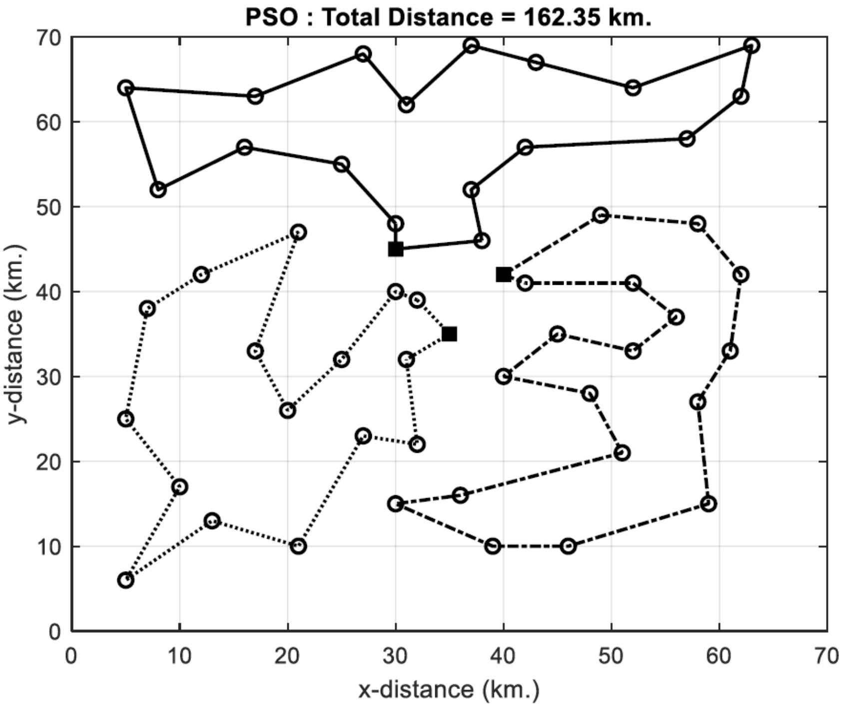

FIGURE 6. Optimal tour of MDVRP#1 (EIL51) obtained by PSO

For the MDVRP#5 (KROB100), it possesses $N_d = 8$ and $K = 9$. From Table 2, the GA provides the total distance of 15,403.38 km., the PSO performs the total distance of 13,310.37 km., the CS yields the total distance of 12,017.04 km., while the MoMA gives the total distance of 11,256.21 km.

For the MDVRP#6 (BIER127), it has $N_d = 10$ and $K = 12$. From Table 2, the GA provides the total distance of 68,012.05 km., the PSO performs the total distance of

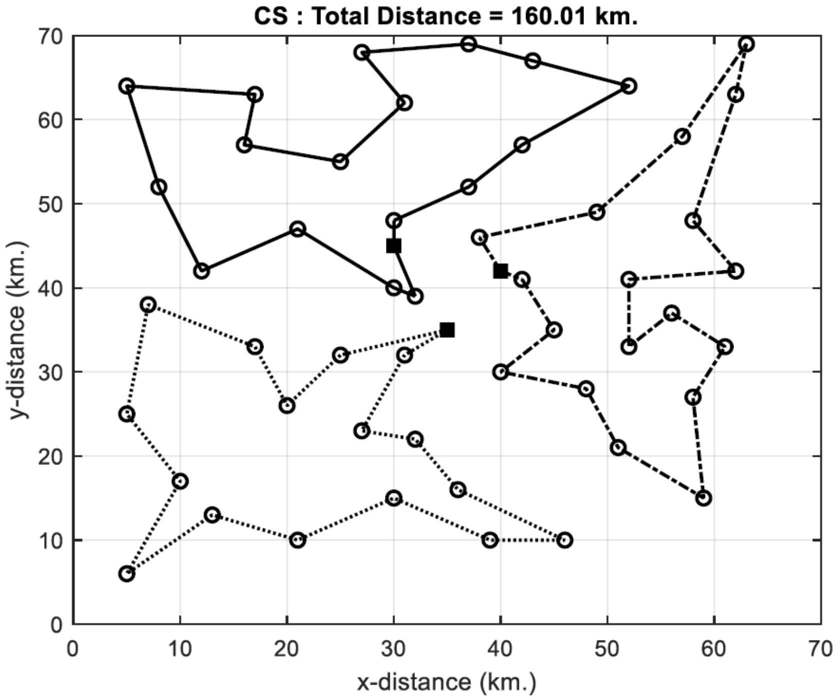

FIGURE 7. Optimal tour of MDVRP#1 (EIL51) obtained by CS

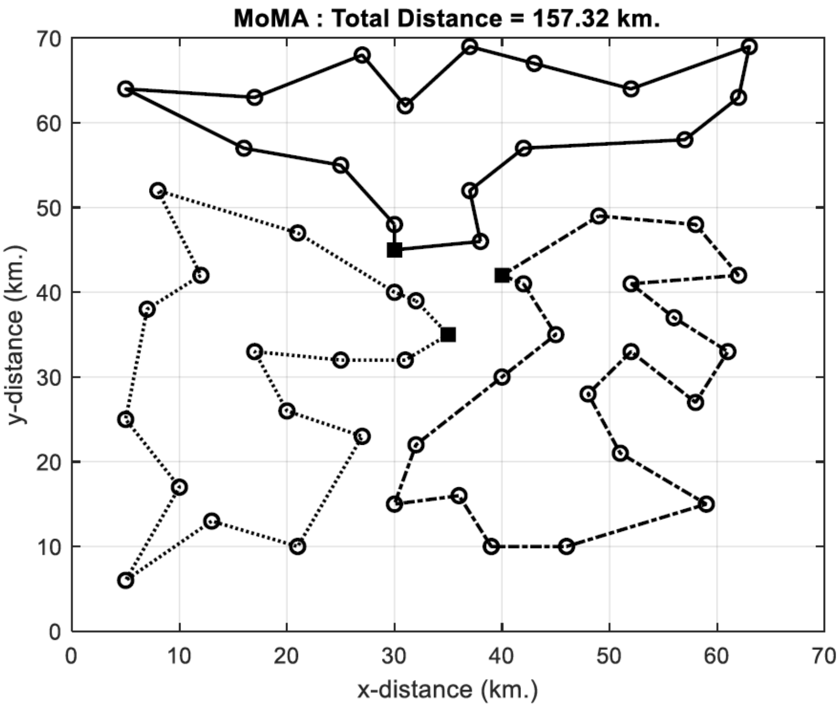

FIGURE 8. Optimal tour of MDVRP#1 (EIL51) obtained by MoMA

66,998.02 km., the CS yields the total distance of 66,430.22 km., while the MoMA gives the total distance of 65,767.54 km.

For the MDVRP#7 (CH150), it possesses $N_d = 12$ and $K = 15$. From Table 2, the GA provides the total distance of 1,658.76 km., the PSO performs the total distance of 1,490.56 km., the CS yields the total distance of 1,286.42 km., while the MoMA gives the total distance of 1,230.53 km.

For the MDVRP#8 (BRG180), it has $N_d = 14$ and $K = 17$. From Table 2, the GA provides the total distance of 957.27 km., the PSO performs the total distance of 918.13 km., the CS yields the total distance of 847.06 km., while the MoMA gives the total distance of 812.23 km.

For the MDVRP#9 (RAT195), it possesses $N_d = 15$ and $K = 20$. From Table 2, the GA provides the total distance of 1,515.59 km., the PSO performs the total distance of 1,324.48 km., the CS yields the total distance of 1,106.67 km., while the MoMA gives the total distance of 1,054.54 km.

For the MDVRP#10 (D198), it has $N_d = 16$ and $K = 22$. From Table 2, the GA provides the total distance of 7,508.37 km., the PSO performs the total distance of 7,393.65 km., the CS yields the total distance of 7,102.08 km., while the MoMA gives the total distance of 6,861.29 km.

From overall results of all ten selected real-world MDVRP summarized in Table 2, it was found that the PSO can yield shorter total distance than the GA. The CS can give shorter total distance than the PSO, while the MoMA can provide shorter total distance than the CS, PSO and GA, respectively.

The total distances (TD) in Table 2 are converted into percentage decrease of TD (PDTD) of the PSO, CS and MoMA with-respect-to the GA by using the relation stated in (27)-(29) for comparison as summarized in Table 3. From Table 3, it can be noticed that PSO, CS and MoMA can averagely decrease the PDTD by 5.89%, 12.31% and 15.12%, respectively, once compared with the GA. From Tables 2 and 3, it was found that the MoMA can provide optimal solutions of all ten selected MDVRP with shorter total distance than the CS, PSO and GA, respectively.

$$\mathrm{PDTD}_{\mathrm{PSO}} = 100 \times \left(\frac{\mathrm{TD}_{\mathrm{GA}} - \mathrm{TD}_{\mathrm{PSO}}}{\mathrm{TD}_{\mathrm{GA}}} \right) \tag{27}$$

$$\mathrm{PDTD}_{\mathrm{CS}} = 100 \times \left(\frac{\mathrm{TD}_{\mathrm{GA}} - \mathrm{TD}_{\mathrm{CS}}}{\mathrm{TD}_{\mathrm{GA}}} \right) \tag{28}$$

$$\mathrm{PDTD}_{\mathrm{MoMA}} = 100 \times \left(\frac{\mathrm{TD}_{\mathrm{GA}} - \mathrm{TD}_{\mathrm{MoMA}}}{\mathrm{TD}_{\mathrm{GA}}} \right) \tag{29}$$

Table 3. PDTD of PSO, CS and MoMA for MDVRP with-respect-to the GA

Problems	**PDTD for MDVRP with-respect-to the GA (%)**			
	GA	**PSO**	**CS**	**MoMA**
MDVRP#1	0.0000	1.2229	2.6466	4.2833
MDVRP#2	0.0000	5.3767	8.3731	9.1499
MDVRP#3	0.0000	6.3572	14.8334	19.2250
MDVRP#4	0.0000	2.4495	6.5594	8.3395
MDVRP#5	0.0000	13.5880	21.9844	26.9238
MDVRP#6	0.0000	1.4910	2.3258	3.3002
MDVRP#7	0.0000	10.1401	22.4469	25.8163
MDVRP#8	0.0000	4.0887	11.5129	15.1514
MDVRP#9	0.0000	12.6096	26.9809	30.4205
MDVRP#10	0.0000	1.5279	5.4112	8.6181
Averages	0.0000	5.8852	12.3075	15.1228

5. **Conclusions.** The application of the MoMA to optimally solve the MDVRP problems has been proposed in this paper. As one of the new hybrid metaheuristic optimization search techniques, the MoMA combines with two types of the random process, i.e., uniform and Lévy distributions, to generate the feasible solutions. It was associated with the ASRM to balance the intensification and diversification as well as to speed up the search process. In this work, ten selected real-world MDVRP consisting of approximately 50-200 locations have been selected for testing the MoMA search performance based on the modern optimization context. Results obtained by the MoMA have been compared with those obtained by the GA, PSO and CS. From experimental results, it was found that the MoMA can provide the optimal tours of all ten selected real-world MDVRP with shorter total distance than the CS, PSO and GA, respectively. This can be concluded that the MoMA can give optimal solutions of the MDVRP, satisfactory. For the future research, the MoMA will be applied to solving the more practical MDVRP such as CMDVRP and the MDVRP with pickup/delivery problem and time windows or MDVRP-PDPTW to address the variety of conditions in real-world applications in order to minimize total distance and balance the number of vehicles, number of service locations and traveling times under traffic situation.

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