

## TRACKING SYNCHRONIZATION CONTROL OF MULTIPLE ROBOT MANIPULATORS IN TASK SPACE VIA INTERMITTENT CONTROL

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**ABSTRACT.** *Tracking synchronization of multiple robot manipulators with a directed graph via intermittent control is investigated in this paper. For given a desired trajectory in task space, the end-effectors of the manipulators can be synchronized to the desired trajectory via aperiodically intermittent control. First, the joint position is transformed into the end-effector position in the task space. Second, the controlled multiple robotic systems with tracking error vectors are obtained based on the designed intermittent control. Third, some tracking synchronization criteria are deduced by constructing Lyapunov function. Finally, the tracking synchronization of six 2-link revolute manipulators is taken for an application example. And numerical simulations verify the effectiveness and feasibility of the designed control strategy. The main contribution of this paper is to study the intermittent control rather than continuous control for multiple robot manipulators in task space instead of joint space.*

**Keywords:** Tracking synchronization, Multiple robot manipulators, Task space, Intermittent control

1. **Introduction.** Control problem of nonlinear dynamical systems is one of the hot topics among scientists from various fields of science [1-5]. Among them, the control problem of robot manipulators described by Lagrangian systems is widely used in automatic mechanical device in the field of mechanical technology, which can simulate the action of human arms and assist human workers to complete various complex tasks such as assistance, assembly and transportation quickly and efficiently. Consequently, it plays an increasingly important role in medical applications, manufacturing, space exploration, and other fields.

Due to the requirements of efficiency and quality in practical application, as well as the flexibility and operational limitations of a single robot in performing tasks, two or more robots perform the same activities synchronously, which is more suitable for people's needs [5, 6]. Therefore, the problems of synchronization and control protocols for multiple robot manipulators have been investigated [7-15]. Tracking control schemes for robot manipulators with deadzone robust compensator via adaptive control were discussed [7]. Several modifications of an admittance force control scheme were presented and derived from force tracking impedance functions to give a force tracking capability to position-controlled robot manipulators [8]. Impulsive practical tracking synchronization of networked robot manipulators described by uncertain Lagrangian systems was investigated [9]. The distributed backstepping control was used to discuss the synchronization problems of uncertain networked Lagrangian systems with directed communication topologies [10]. The sliding mode control algorithm was utilized to study a distributed coordinated

tracking problem for networked Euler-Lagrange systems [11]. Synchronized control with neuro-agents was developed for multiple robotic manipulators [12]. Intermittent control was presented to discuss the robot described by Lagrangian system [13-15]. Despite the interesting results on the synchronization and control problems of robot manipulators cited above, most of the existing works were concentrated on joint space [7-15]. However, the synchronization control schemes of the robot manipulators considered in task space are more interesting, since robot manipulators are usually applied to useful practical tasks, which are specified in terms of a trajectory expressed in Cartesian coordinates to be tracked by the end-effector in task space.

Many interesting results were available on the synchronization and control problems of robot manipulators in task space [16-21]. Adaptive Jacobian controller was designed to make robot end-effector converge to a desired trajectory with uncertain kinematics and dynamics [16, 17]. Tracking problem of networked redundant robotic systems with a dynamic leader was discussed by designing distributed control in task space [18]. Based on passivation framework, the task-space tracking and synchronization of the networked robotic agents were investigated by introducing a weighted Lyapunov-Krasovskii function [19]. An adaptive control algorithm was proposed to guarantee the task-space synchronization of networked robotic manipulators in the presence of dynamic uncertainties and time-varying communication delays [20]. An adaptive backstepping control scheme was presented to investigate task-space tracking for robot manipulators [21]. It should be pointed out that the control laws developed in the above existing works were concentrated on the continuous control [16-21]. In recent years, intermittent control as an important discontinuous control was designed for robot manipulators in task space [22-24]. Unfortunately, the designed controllers needed to receive the information of the desired trajectory continuously [22]. Besides, the desired trajectory given in [23] can only be constant rather than time-varying. In addition, only a single robot manipulator rather than multiple robot manipulators was studied in [23, 24]. Thus, it leads to some limitations in practical application. It seems that the results on the task-space synchronization for multiple robot manipulators via intermittent control have not attracted enough attention. It is well known that intermittent control as an important discontinuous control has been widely applied to investigating the control problems of dynamical systems in many fields [25, 26].

Motivated by the aforementioned comments, this paper is to study task-space tracking synchronization of multiple robot manipulators via intermittent control. Some general criteria are deduced to make all robot manipulators synchronize to a desired trajectory in task space. The obtained results are illustrated by tracking synchronization of six 2-link revolute manipulators. There are three main contributions of this work. First, the control problem of the robot manipulators is studied in task space instead of joint space, which is very different from [7-15] researched on joint space. Second, the designed control is intermittent control rather than continuous control. This is different from the continuous control proposed in the existing works [16-21]. Third, compared with [23, 24], the multiple robots rather than a single robot are considered.

The rest of this paper is organized as follows. In Section 2, some mathematical preliminaries are presented. In Section 3, model description and intermittent control problem formulation are depicted. In Section 4, tracking synchronization criteria are derived. In Section 5, application examples and simulation results are given to illustrate the effectiveness of the designed control strategy. Finally, conclusive remarks are given in Section 6.

## 2. Preliminaries.

**2.1. Notations.** In this paper, the following notations and definitions will be used. Let  $R^n$  be a set of  $n \times 1$  real vectors,  $R^{n \times n}$  be a set of  $n \times n$  real matrices,  $I_n \in R^{n \times n}$  be the  $n$ -order identify matrix,  $0_{n \times n} \in R^{n \times n}$  be the  $n$ -order matrix with all zeros, and  $\text{diag}(\gamma_1, \gamma_2, \dots, \gamma_n) \in R^{n \times n}$  be the diagonal matrix with diagonal entries  $\gamma_i$  ( $i = 1, 2, \dots, n$ ). For the vector  $x \in R^n$ ,  $x^T$  stands for its transpose, and the norm of  $x = (x_1, x_2, \dots, x_n)^T$  is defined as  $\|x\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$ . For  $D \in R^{n \times n}$ ,  $D^T$  stands for its transpose, and the norm of  $D \in R^{n \times n}$  is defined as  $\|D\| = \sqrt{\lambda_{\max}(D^T D)}$ , where  $\lambda_{\max}(\cdot)$  denotes the maximum eigenvalue of the matrix. The symmetric part of  $D$  is denoted by  $D^S = \frac{(D+D^T)}{2}$ .  $D^{-1}$ ,  $D^*$  and  $|D|$  are the inverse matrix, the adjoint matrix and the determinant of  $D$ , respectively. The symbol  $\otimes$  is used to indicate the Kronecker product of two matrices.

**2.2. Graph theory.** For multi-agent network systems, the network topology of information exchange between individuals is usually described by a graph. A directed graph is a graph consisting of a number of nodes and the edges connecting two nodes. Let  $\mathcal{G}_A = (\mathcal{V}, \mathcal{E}, A)$  denote a directed graph with a set of nodes  $\mathcal{V} = \{1, 2, \dots, N\}$ , a set of edges  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ , and a weighted adjacency matrix  $A = [a_{ij}] \in R^{N \times N}$ . A directed edge in a directed graph is denoted by  $e_{ij} = (i, j)$ .  $(i, j) \in \mathcal{E}$  means the node  $i$  can receive information from node  $j$ , where  $i$  is called the child node of  $j$ , while  $j$  is called the parent node of  $i$ . The element  $a_{ij}$  of the adjacency matrix  $A$  is positive if and only if there is a directed  $e_{ij} \in \mathcal{E}$ ; otherwise,  $a_{ij} = 0$ . We usually assume that  $a_{ii} = 0$  for all  $i \in \mathcal{V}$ . Let  $\text{deg}_{\text{in}}(i) = a_{i1} + a_{i2} + \dots + a_{iN}$  and  $\text{deg}_{\text{out}}(i) = a_{1i} + a_{2i} + \dots + a_{Ni}$  be the in-degree and out-degree of node  $i$ , respectively.  $\text{deg}_{\text{out}}(i) - \text{deg}_{\text{in}}(i)$  represents the degree difference for the  $i$ th node [27]. The Laplacian matrix  $L_A = [l_{ij}] \in R^{N \times N}$  associated with the adjacency matrix  $A$  is defined as  $l_{ii} = a_{1i} + a_{2i} + \dots + a_{Ni}$  and  $l_{ij} = -a_{ij}$  where  $i \neq j$  [27-29]. Usually, the Laplacian matrix of a digraph is asymmetric.

A directed path from node  $i$  to node  $j$  is a sequence of edges  $(i, i_1), (i_1, i_2), \dots, (i_l, j)$  in the directed graph  $\mathcal{G}_A = (\mathcal{V}, \mathcal{E}, A)$  with distinct nodes. A directed graph has a directed spanning tree if there exists at least one node called root which has a directed path to all other nodes [27].

### 2.3. Instrumental lemmas.

**Lemma 2.1.** [25] *If  $X$  and  $Y$  are real matrices with appropriate dimensions, then there exists  $\varepsilon > 0$  such that*

$$X^T Y + Y^T X \leq \varepsilon X^T X + \frac{1}{\varepsilon} Y^T Y.$$

**Lemma 2.2.** [24] *If the non-negative function  $y(t)$ ,  $t \in [t_0, +\infty)$  satisfies the following*

$$\dot{y}(t) \leq -ay(t) + b, \quad t \geq t_0,$$

*where  $a > 0$  and  $b > 0$ , then the following holds*

$$y(t) \leq y(t_0) \exp\{-a(t - t_0)\} + \frac{b}{a}, \quad t \geq t_0.$$

**Lemma 2.3.** [24] *If the non-negative function  $y(t)$ ,  $t \in [t_0, +\infty)$  satisfies the following*

$$\dot{y}(t) \leq ay(t) + b, \quad t \geq t_0,$$

*where  $a > 0$  and  $b > 0$ , then the following inequality holds*

$$y(t) \leq \left( y(t_0) + \frac{b}{a} \right) \exp\{a(t - t_0)\} - \frac{b}{a}, \quad t \geq t_0.$$

**Lemma 2.4.** [26] *If the non-negative function  $y(t)$  satisfies the following conditions for any  $m = 0, 1, 2, \dots$ ,*

$$y(t) \leq \begin{cases} y(t_{2m}) \exp\{-a(t - t_{2m})\} + \mu, & t_{2m} \leq t \leq t_{2m+1}, \\ (y(t_{2m+1}) + \gamma) \exp\{\xi(t - t_{2m+1})\} - \gamma, & t_{2m+1} < t < t_{2m+2}, \end{cases}$$

*also,  $\omega_1 = \inf_m(t_{2m+1} - t_{2m})$ ,  $\omega_2 = \sup_m(t_{2m+2} - t_{2m})$ ,  $\omega_1 > 0$ ,  $\omega_1 < \omega_2 < +\infty$ ,  $\mu > 0$ ,  $\gamma > 0$ ,  $\xi > 0$  and  $\rho = a\omega_1 - \xi(\omega_2 - \omega_1) > 0$ , then the following inequality holds*

$$y(t) \leq y(0) \exp\{\rho\} \exp\left\{-\left(\frac{\rho}{\omega_2}\right)t\right\} + \frac{\kappa_0}{1 - \exp\{-\rho\}} + \mu,$$

*for  $t \geq 0$ , where  $\kappa_0 = (\mu + \gamma) \exp\{\xi(\omega_2 - \omega_1)\} - \gamma$ .*

### 3. Problem Formulation.

**3.1. Multiple robotic manipulators.** A network composed of different multiple robot manipulators with n-link revolute joints can be described by the following Lagrangian system [28-30]

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \quad i = 1, 2, \dots, N, \quad (1)$$

where  $q_i = (q_{i1}, q_{i2}, \dots, q_{in})^T \in R^n$  represents the generalized coordinates in the joint space of the  $i$ th agent (node  $i$ ),  $\dot{q}_i = (\dot{q}_{i1}, \dot{q}_{i2}, \dots, \dot{q}_{in})^T \in R^n$  and  $\ddot{q}_i = (\ddot{q}_{i1}, \ddot{q}_{i2}, \dots, \ddot{q}_{in})^T \in R^n$  are the joint-space velocity and acceleration,  $\tau_i = (\tau_{i1}, \tau_{i2}, \dots, \tau_{in})^T \in R^n$  is the generalized force vector,  $M_i(q_i) : R^n \rightarrow R^{n \times n}$  is the symmetric positive-definite inertia matrix,  $C_i(q_i, \dot{q}_i)\dot{q}_i$  is the Coriolis and centrifugal terms, and  $g_i(q_i)$  is the vector of gravitational force. It is reasonable to assume that the Lagrangian equation satisfies the following properties [28-30].

**Property 3.1.** (*Skew-symmetry*) *The matrix  $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$  is skew-symmetry, implying that  $x^T [\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)] x = 0$  for arbitrary  $x \in R^n$ .*

**Property 3.2.** (*Boundedness*) *There exist positive constants  $\underline{M}_i$ ,  $\bar{M}_i$  and  $k_{iC}$ , such that for all  $q_i \in R^n$ ,  $\underline{M}_i \leq \|M_i(q_i)\| \leq \bar{M}_i$ ,  $\|C_i(q_i, \dot{q}_i)\| \leq k_{iC}\|\dot{q}_i\|$ .*

If we take  $\tau_i = g_i(q_i)$ , then the dynamics of robotic system (1) can be rewritten as

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = 0. \quad (2)$$

According to [24], the velocity  $\dot{q}_i$  in system (2) is bounded.

Let  $x_i \in R^n$  denote the end-effector position of the agent in the task space. According to the nonlinear mapping  $h_i : R^n \rightarrow R^n$ , the joint position  $q_i$  can be transformed into the end-effector position  $x_i$  in the task space:

$$x_i = h_i(q_i). \quad (3)$$

Differentiating (3) with respect to time gives the relation between the task-space velocity  $\dot{x}_i$  and the joint-space velocity  $\dot{q}_i$  as

$$\dot{x}_i = J_i(q_i)\dot{q}_i, \quad (4)$$

where  $J_i(q_i) = \partial h_i(q_i)/\partial q_i \in R^{n \times n}$  is the Jacobian matrix. For the n-link revolute manipulator, the elements of  $h_i(q_i)$  are the trigonometric functions of  $q_i = (q_{i1}, q_{i2}, \dots, q_{in})^T \in R^n$ , implying that the elements of  $x_i$ ,  $J_i(q_i)$  and  $J_i^*(q_i)$  are bounded. As a result,  $\|x_i\|$ ,  $\|J_i(q_i)\|$  and  $\|J_i^*(q_i)\|$  are bounded. Together with  $\dot{q}_i$  bounded,  $\|\dot{x}_i\|$  and  $\left\|\dot{J}_i(q_i)\right\|$  are both bounded, where  $\dot{J}_i(q_i)$  denotes the derivative of  $t$  for each element of  $J_i(q_i)$ . The Jacobian matrix

considered here has full rank. Thus,  $|J_i(q_i)| \neq 0$ . Therefore, we can get that the norm of  $J_i^{-1}(q_i) = \frac{J_i^*(q_i)}{|J_i(q_i)|}$  is also bounded.

The desire position in task space is assumed to be  $x_d(t) \in R^n$  with  $\|x_d\|$ ,  $\|\dot{x}_d\|$  and  $\|\ddot{x}_d\|$  bounded. We will design the controllers  $u_i(t)$  ( $i = 1, 2, \dots, N$ ) to make the end-effectors of the multiple robotic manipulators track the desired trajectory  $x_d(t)$  with a desired tracking error bound. Before moving on, we introduce the following signals

$$r_i = J_i^{-1}(\dot{x}_d - \lambda(x_i - x_d)) = J_i^{-1}(\dot{x}_d - \lambda\tilde{x}_i), \tag{5}$$

$$s_i = J_i^{-1}(\dot{x}_i - \dot{x}_d + \lambda\tilde{x}_i) = \dot{q}_i - r_i, \tag{6}$$

where  $\lambda > 0$ ,  $\tilde{x}_i = x_i - x_d$  is the tracking error vector. And we can get  $\dot{q}_i = s_i + r_i$  and  $\ddot{q}_i = \dot{s}_i + \dot{r}_i$ .

On the basis of the robotic system (2), we consider the controlled multiple robot manipulators as

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = \sum_{j=1}^N c(t)a_{ij}(s_j(t) - s_i(t)) + u_i(t), \tag{7}$$

where  $i = 1, 2, \dots, N$ , the control  $u_i(t)$  is to be designed,  $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \in R^n$  and  $\dot{x}_i = (\dot{x}_{i1}, \dot{x}_{i2}, \dots, \dot{x}_{in})^T \in R^n$  are the end-effector position and velocity, respectively.  $c(t) \geq 0$  is the coupling strength.  $a_{ij}$  is the  $(i, j)$ th entry of the adjacency matrix  $A \in R^{N \times N}$ . Let  $L_A = [l_{ij}] \in R^{N \times N}$  be the Laplacian matrix with  $\mathcal{G}_A$ . Then the multiple robotic system (7) can be rewritten as

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = - \sum_{j=1}^N c(t)l_{ij}s_j(t) + u_i(t). \tag{8}$$

**3.2. Intermittent control.** In this subsection,  $u_i(t)$  is designed as an intermittent control to make the multiple robot manipulators achieve tracking synchronization in task space. That is, for given any desired trajectory  $x_d(t) \in R^n$  and initial values of system (8), there exist constants  $T_0 > 0$  and  $\varepsilon_* > 0$  such that  $\|\tilde{x}_i\| < \varepsilon_*$  for all  $t > T_0$ .  $\varepsilon_*$  is called as the tracking error bound. Without loss of generality, we can rearrange the order of all agents and select the first  $l$  nodes to be pinned. The aperiodically intermittent control  $u_i(t)$  is designed as

$$u_i(t) = \begin{cases} -k_i J_i^{-1}(\dot{\tilde{x}}_i + \lambda\tilde{x}_i) - \lambda J_i^T \tilde{x}_i, & 1 \leq i \leq l, t \in [t_{2m}, t_{2m+1}], \\ 0, & l + 1 \leq i \leq N, t \in [t_{2m}, t_{2m+1}], \\ 0, & 1 \leq i \leq N, t \in (t_{2m+1}, t_{2m+2}), \end{cases} \tag{9}$$

where  $m = 0, 1, 2, \dots$ ,  $\lambda > 0$  and  $k_i > 0$  ( $i = 1, 2, \dots, l$ ) are the constants of the control gains to be designed. The time span  $[t_{2m}, t_{2m+1}]$  is called the work time, while  $(t_{2m+1}, t_{2m+2})$  is called the rest time.

In addition, the coupling strength  $c(t)$  is designed as

$$c(t) = \begin{cases} c, & t_{2m} \leq t \leq t_{2m+1}, \\ 0, & t_{2m+1} < t < t_{2m+2}, \end{cases}$$

where  $c$  is a constant to be designed, and  $m = 0, 1, 2, \dots$ . It is worth noting that  $c(t) > 0$  if and only if  $t \in [t_{2m}, t_{2m+1}]$ . Otherwise,  $c(t) = 0$ . Letting  $\omega_1 = \inf_m(t_{2m+1} - t_{2m})$ ,  $\omega_2 = \sup_m(t_{2m+2} - t_{2m})$ , the values of  $\omega_1$  and  $\omega_2$  are taken to satisfy  $\omega_1 > 0$  and  $\omega_1 < \omega_2 < +\infty$ .

In the case with  $t \in [t_{2m}, t_{2m+1}]$ , substituting  $\dot{q}_i = s_i + r_i$ ,  $r_i = J_i^{-1}(\dot{x}_d - \lambda\tilde{x}_i)$  into the controlled multiple robot system (8), we have

$$\begin{cases} M_i(q_i)\dot{s}_i = -\Delta_i - C_i(q_i, \dot{q}_i)s_i - c \sum_{j=1}^N l_{ij}s_j - k_i s_i - \lambda J_i^T \tilde{x}_i, & 1 \leq i \leq l, \\ M_i(q_i)\dot{s}_i = -\Delta_i - C_i(q_i, \dot{q}_i)s_i - c \sum_{j=1}^N l_{ij}s_j, & l+1 \leq i \leq N, \end{cases} \quad (10)$$

where  $\Delta_i = M_i(q_i)\dot{r}_i + C_i(q_i, \dot{q}_i)r_i$ . Let  $\bar{M} = \max\{\bar{M}_1, \bar{M}_2, \dots, \bar{M}_N\}$  and  $k_C = \max\{k_{1C}, k_{2C}, \dots, k_{NC}\}$ . By using the inequalities in Property 3.2, we have

$$\|\Delta_i\| \leq \|M_i(q_i)\|\|\dot{r}_i\| + \|C_i(q_i, \dot{q}_i)\|\|r_i\| \leq \bar{M}\|\dot{r}_i\| + k_C\|\dot{q}_i\|\|r_i\|.$$

Due to the boundedness of  $\|x_i\|$ ,  $\|x_d\|$ ,  $\|\dot{x}_d\|$  and  $\|J_i^{-1}\|$ , it can be obtained from (5) that  $\|r_i\|$  is bounded. By differentiating both sides of (5), we have  $\dot{r}_i = -J_i^{-1}[\lambda(\dot{x}_i - \dot{x}_d) - \ddot{x}_d + \dot{J}_i r_i]$ .  $\|\dot{r}_i\|$  is bounded due to the boundedness of  $\|r_i\|$ ,  $\|\dot{x}_i\|$ ,  $\|\dot{x}_d\|$ ,  $\|\ddot{x}_d\|$ ,  $\|J_i^{-1}\|$  and  $\|\dot{J}_i\|$ . This means that  $\|\Delta_i\|$  is bounded. We let  $\|\Delta_i\| \leq \delta_i$  with  $\delta_i > 0$ .

Let  $M(q) = \text{diag}(M_1(q_1), M_2(q_2), \dots, M_N(q_N))$ ,  $C(q, \dot{q}) = \text{diag}(C_1(q_1, \dot{q}_1), C_2(q_2, \dot{q}_2), \dots, C_N(q_N, \dot{q}_N))$ ,  $s = (s_1^T, s_2^T, \dots, s_N^T)^T$ ,  $\tilde{x} = (\tilde{x}_1^T, \tilde{x}_2^T, \dots, \tilde{x}_N^T)^T$ ,  $K = \text{diag}(k_1, k_2, \dots, k_l, 0, \dots, 0)$ ,  $J^T = \text{diag}(J_1^T(q_1), J_2^T(q_2), \dots, J_N^T(q_N))$ ,  $\Delta = (\Delta_1^T, \Delta_2^T, \dots, \Delta_N^T)^T$ . Then, the controlled robotic system (10) can be rewritten as

$$M(q)\dot{s} = -\Delta - [(cL_A + K) \otimes I_n]s - C(q, \dot{q})s - \lambda J^T \Lambda \tilde{x}, \quad (11)$$

where  $\Lambda = \begin{pmatrix} I_{l \times l} & 0_{l \times (N-l)} \\ 0_{(N-l) \times l} & 0_{(N-l) \times (N-l)} \end{pmatrix} \otimes I_n \in R^{Nn \times Nn}$ .

In the case with  $t \in (t_{2m+1}, t_{2m+2})$ , we have  $u_i(t) = 0$  ( $1 \leq i \leq N$ ). Then the multiple robot manipulators in the vector form can be derived as

$$M(q)\dot{s} = -\Delta - C(q, \dot{q})s. \quad (12)$$

Based on the above discussion, we can obtain the closed-loop dynamic system

$$\begin{cases} M(q)\dot{s} = -\Delta - [(cL_A + K) \otimes I_n]s - C(q, \dot{q})s - \lambda J^T \Lambda \tilde{x}, & t_{2m} \leq t \leq t_{2m+1}, \\ M(q)\dot{s} = -\Delta - C(q, \dot{q})s, & t_{2m+1} < t < t_{2m+2}. \end{cases} \quad (13)$$

**4. Tracking Synchronization Criteria.** Subsequently, some synchronization criteria are deduced to make the end-effector positions of the multiple robot manipulators track the desired trajectory  $x_d(t)$  with a desired tracking error bound. Based on the dynamical system (13), the control gains  $k_i$  ( $i = 1, 2, \dots, l$ ),  $\lambda$  and the coupling strength  $c$  will be designed. Let  $\underline{M} = \min\{\underline{M}_1, \underline{M}_2, \dots, \underline{M}_N\}$ ,  $\delta = \max\{\delta_1, \delta_2, \dots, \delta_N\}$  and  $\bar{J} = \max\{\bar{J}_1, \bar{J}_2, \dots, \bar{J}_N\}$ , which  $\bar{J}_i$  is the bound of  $\|J_i(q_i)\|$ .

**Theorem 4.1.** *The control gains  $k_i$  ( $i = 1, 2, \dots, l$ ),  $\lambda$  and the coupling strength  $c$  are designed as*

(A1)  $\lambda_{\min}(cL_A^S + K - \frac{\varepsilon}{2}I_N) \geq \frac{1}{2}\bar{M}\xi_1$ ; (A2)  $\lambda \geq \frac{1}{2}\xi_1$ ,  
in which  $\varepsilon$  is an arbitrary positive number, and  $\xi_1$  is chosen to satisfy

$$\rho = \xi_1\omega_1 - \xi_2(\omega_2 - \omega_1) > 0, \quad \xi_2 \geq \max\left\{\frac{(4\varepsilon + \bar{J}^2)}{4\underline{M}}, 2\lambda\right\}. \quad (14)$$

Then the end-effector positions of the multiple robot manipulators described by system (8) can track the desired trajectory  $x_d(t)$  with the tracking error bound

$$\varepsilon_* = \varepsilon_0 + \sqrt{\frac{2}{\lambda} \left( \frac{v_0}{1 - \exp\{-\rho\}} + \frac{\delta^2}{2\varepsilon\xi_1} \right)}, \tag{15}$$

in which  $\varepsilon_0$  is arbitrary small positive number, and

$$v_0 = \frac{\delta^2}{2\varepsilon} \left( \frac{1}{\xi_1} + \frac{1}{\xi_2} \right) \exp\{\xi_2(\omega_2 - \omega_1)\} - \frac{\delta^2}{2\varepsilon\xi_2}. \tag{16}$$

**Proof:** For the networked system (13), the Lyapunov function candidate is chosen as

$$V(t) = \frac{1}{2}s^T M(q)s + \frac{1}{2}\lambda\tilde{x}^T \Lambda^T \tilde{x}. \tag{17}$$

Then the derivative of  $V(t)$  with respect to time  $t \in [t_{2m}, t_{2m+1}]$  along the trajectory of the first equation of system (13) equals

$$\begin{aligned} \dot{V}(t) &= s^T M(q)\dot{s} + \frac{1}{2}s^T \dot{M}(q)s + \lambda\tilde{x}^T \Lambda^T \dot{\tilde{x}} \\ &= s^T \{-\Delta - C(q, \dot{q})s - [(cL_A + K) \otimes I_n]s - \lambda J^T \Lambda \tilde{x}\} + \frac{1}{2}s^T \dot{M}(q)s + \lambda\tilde{x}^T \Lambda^T \dot{\tilde{x}} \\ &= -s^T \Delta - s^T [(cL_A + K) \otimes I_n]s - \lambda s^T J^T \Lambda \tilde{x} + \lambda\tilde{x}^T \Lambda^T (Js - \lambda\tilde{x}) \\ &= -s^T \Delta - s^T [(cL_A + K) \otimes I_n]s - \lambda^2 \tilde{x}^T \Lambda^T \tilde{x} \\ &\leq -s^T [(cL_A^S + K) \otimes I_n]s - \lambda^2 \tilde{x}^T \Lambda^T \tilde{x} + \frac{\varepsilon}{2}s^T s + \frac{1}{2\varepsilon}\|\Delta\|^2 \\ &\leq -s^T \left[ \left( cL_A^S + K - \frac{\varepsilon}{2}I_N \right) \otimes I_n \right] s - \lambda^2 \tilde{x}^T \Lambda^T \tilde{x} + \frac{\delta^2}{2\varepsilon}, \end{aligned} \tag{18}$$

where Lemma 2.1 and the skew-symmetric Property 3.1 are used. From conditions (A1) and (A2), we have

$$\dot{V}(t) \leq -\xi_1 \left( \frac{\bar{M}}{2}s^T s + \frac{1}{2}\lambda\tilde{x}^T \Lambda \tilde{x} \right) + \frac{\delta^2}{2\varepsilon} \leq -\xi_1 V(t) + \frac{\delta^2}{2\varepsilon}. \tag{19}$$

By Lemma 2.2, one has

$$V(t) \leq V(t_{2m}) \exp\{-\xi_1(t - t_{2m})\} + \frac{\delta^2}{2\varepsilon\xi_1}, \tag{20}$$

for  $t \in [t_{2m}, t_{2m+1}]$ .

When  $t \in (t_{2m+1}, t_{2m+2})$ , the derivative of  $V(t)$  can be obtained from the second equation of system (13) as follows:

$$\begin{aligned} \dot{V}(t) &= s^T M(q)\dot{s} + \frac{1}{2}s^T \dot{M}(q)s + \lambda\tilde{x}^T \Lambda^T \dot{\tilde{x}} \\ &= s^T [-\Delta - C(q, \dot{q})s] + \frac{1}{2}s^T \dot{M}(q)s + \lambda\tilde{x}^T \Lambda^T \dot{\tilde{x}} \\ &= -s^T \Delta + \lambda\tilde{x}^T \Lambda^T Js - \lambda^2 \tilde{x}^T \Lambda^T \tilde{x} \\ &\leq \frac{\varepsilon}{2}s^T s + \frac{1}{2\varepsilon}\|\Delta\|^2 + \frac{\bar{J}^2}{8}s^T s + \lambda^2 \tilde{x}^T \Lambda^T \tilde{x} \\ &\leq s^T \left[ \left( \frac{\varepsilon}{2} + \frac{\bar{J}^2}{8} \right) \otimes I_n \right] s + \lambda^2 \tilde{x}^T \Lambda^T \tilde{x} + \frac{\delta^2}{2\varepsilon}. \end{aligned} \tag{21}$$

Therefore, it can be further deduced from  $\xi_2 \geq \max \left\{ \frac{4\varepsilon + \bar{J}^2}{4\bar{M}}, 2\lambda \right\}$  that

$$\dot{V}(t) \leq \xi_2 \left( \frac{M}{2}s^T s + \frac{1}{2}\lambda\tilde{x}^T \Lambda^T \tilde{x} \right) + \frac{\delta^2}{2\varepsilon} \leq \xi_2 V(t) + \frac{\delta^2}{2\varepsilon}. \tag{22}$$

By Lemma 2.3, we have

$$V(t) \leq \left( V(t_{2m+1}) + \frac{\delta^2}{2\varepsilon\xi_2} \right) \exp\{\xi_2(t - t_{2m+1})\} - \frac{\delta^2}{2\varepsilon\xi_2}, \quad (23)$$

for  $t \in (t_{2m+1}, t_{2m+2})$ .

From inequalities (20) and (23), we can derive the following inequality by Lemma 2.4.

$$V(t) \leq V(0) \exp\{\rho\} \exp\left\{-\left(\frac{\rho}{\omega_2}\right)t\right\} + \frac{v_0}{1 - \exp\{-\rho\}} + \frac{\delta^2}{2\varepsilon\xi_1}, \quad (24)$$

for  $t \geq 0$ , where  $\rho$  and  $v_0$  are shown in expressions (14) and (16). Thus, one has

$$\begin{aligned} \frac{1}{2}\lambda\tilde{x}^T\Lambda^T\tilde{x} &\leq \frac{1}{2}\lambda\|\tilde{x}\|^2 \\ &\leq V(t) \\ &\leq V(0) \exp\{\rho\} \exp\left\{-\left(\frac{\rho}{\omega_2}\right)t\right\} + \frac{v_0}{1 - \exp\{-\rho\}} + \frac{\delta^2}{2\varepsilon\xi_1} \end{aligned} \quad (25)$$

which leads to

$$\|\tilde{x}\| \leq \sqrt{\frac{2}{\lambda}V(0) \exp\{\rho\} \exp\left\{-\left(\frac{\rho}{2\omega_2}\right)t\right\}} + \sqrt{\frac{2}{\lambda}\left(\frac{v_0}{1 - \exp\{-\rho\}} + \frac{\delta^2}{2\varepsilon\xi_1}\right)}, \quad (26)$$

for  $t \geq 0$ . Thus, there exists a constant  $T_0 > 0$  such that

$$\|\tilde{x}\| \leq \varepsilon_0 + \sqrt{\frac{2}{\lambda}\left(\frac{v_0}{1 - \exp\{-\rho\}} + \frac{\delta^2}{2\varepsilon\xi_1}\right)} = \varepsilon_*, \quad (27)$$

for any  $t \geq T_0$ . Therefore, the end-effector positions of the multiple robot manipulators described by system (8) for given any initial values can track the desired trajectory  $x_d(t)$  with the tracking error bound  $\varepsilon_*$ . This completes the proof.  $\square$

**Remark 4.1.** *We do not have to assume that the diagraph  $\mathcal{G}_A$  contains a directed spanning tree. According to the method of node selection provided in [27], the agent with zero in-degree node must be pinned. Besides, one should pin the remaining agents in descending order according to their degree differences.*

**Remark 4.2.** *It is necessary to understand that the conditions of Theorem 4.1 are sufficient but not necessary. Therefore, although the conditions in Theorem 4.1 are not satisfied, the end-effector positions of system (8) may also track the desired trajectory under the intermittent controller (9). This will be further illustrated by the numerical examples.*

**5. Application Examples and Simulation Results.** As a direct application, we consider the tracking synchronization of six 2-link revolute manipulators. The 2-link revolute manipulator model is shown in Figure 1, whose dynamics can be described by [30]

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \quad i = 1, 2, 3, 4, 5, 6,$$

where  $q_i = (q_{i1}, q_{i2})^T \in \mathbb{R}^2$ .

For the  $i$ th manipulator, the masses of each manipulator link are denoted by  $m_{i1}$  and  $m_{i2}$ , respectively.  $I_{i1}$  and  $I_{i2}$  denote the moments of inertia about the center of mass. The lengths of each link are  $l_{i1}$  and  $l_{i2}$  while  $l_{ic1}$  and  $l_{ic2}$  denote the distance from the previous joint to the center of the next link. The inertial matrix is defined as

$$M_i(q_i) = \begin{pmatrix} \theta_{i1} + 2\theta_{i2} \cos q_{i2} & \theta_{i3} + \theta_{i2} \cos q_{i2} \\ \theta_{i3} + \theta_{i2} \cos q_{i2} & \theta_{i3} \end{pmatrix},$$

where  $\theta_{i1} = I_{i1} + I_{i2} + m_{i1}l_{ic1}^2 + m_{i2}(l_{i1}^2 + l_{ic2}^2)$ ,  $\theta_{i2} = m_{i2}l_{i1}l_{ic2}$  and  $\theta_{i3} = I_{i2} + m_{i2}l_{ic2}^2$ . The matrix  $C_i(q_i, \dot{q}_i)$  is

$$C_i(q_i, \dot{q}_i) = \begin{pmatrix} -\theta_{i2}\dot{q}_{i2} \sin q_{i2} & -\theta_{i2}(\dot{q}_{i1} + \dot{q}_{i2}) \sin q_{i2} \\ \theta_{i2}\dot{q}_{i1} \sin q_{i2} & 0 \end{pmatrix}.$$

The nonlinear mapping  $h_i$  from the joint configuration variable  $q_i$  to the end-effector position  $x_i$  is given by

$$x_i = \begin{pmatrix} l_{i1} \cos q_{i1} + l_{i2} \cos(q_{i1} + q_{i2}) \\ l_{i1} \sin q_{i1} + l_{i2} \sin(q_{i1} + q_{i2}) \end{pmatrix}.$$

The Jacobian matrix from joint space to Cartesian space for the 2-link robot is given by

$$J_i(q_i) = \begin{pmatrix} -l_{i1} \sin q_{i1} - l_{i2} \sin(q_{i1} + q_{i2}) & -l_{i2} \sin(q_{i1} + q_{i2}) \\ l_{i1} \cos q_{i1} + l_{i2} \cos(q_{i1} + q_{i2}) & l_{i2} \cos(q_{i1} + q_{i2}) \end{pmatrix}.$$

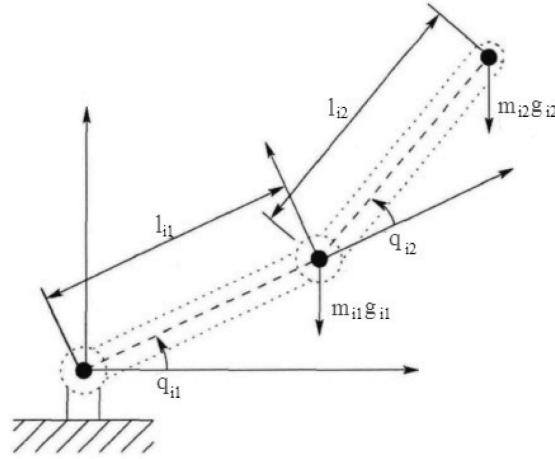


FIGURE 1. The 2-link revolute manipulator model

It is assumed that the desired trajectory in task space is given by  $x_d(t) = (2 \sin(3t + 1), \cos(2t + 6))^T$ . The desired trajectory is time-varying rather than constant, which is different from [23]. In the following simulations, the physical parameters of the first, third and fifth manipulator are taken as  $m_{11} = 2.2$  kg,  $m_{12} = 1.8$  kg,  $l_{11} = 1.5$  m,  $l_{12} = 1.2$  m,  $l_{1c1} = 0.6$  m,  $l_{1c2} = 0.5$  m,  $I_{11} = \frac{m_{11}l_{1c1}^2}{3}$ ,  $I_{12} = \frac{m_{12}l_{1c2}^2}{3}$ . The physical parameters of the second, fourth and sixth manipulator are taken as  $m_{21} = (1 - 0.1)m_{11}$ ,  $m_{22} = (1 + 0.1)m_{12}$ ,  $l_{21} = (1 - 0.1)l_{11}$ ,  $l_{22} = (1 + 0.1)l_{12}$ ,  $l_{2c1} = (1 - 0.05)l_{1c1}$ ,  $l_{2c2} = (1 + 0.05)l_{1c2}$ ,  $I_{21} = \frac{m_{21}l_{2c1}^2}{3}$ ,  $I_{22} = \frac{m_{22}l_{2c2}^2}{3}$ . Thus,  $\bar{M} = 8.8$ ,  $\underline{M} = 0.15$  and  $\bar{J} = 2.8$  are obtained. In fact, the physical parameters  $m_{i1}$ ,  $m_{i2}$ ,  $l_{i1}$  and  $l_{i2}$  can be arbitrarily selected in this simulation. Since  $l_{ic1}$  and  $l_{ic2}$  denote the distance from the previous joint to the center of the next link, we take  $l_{ic1} < l_{i1}$  and  $l_{ic2} < l_{i2}$ . Additionally, referring to [22], the moments of inertia about the center of mass can be taken as  $I_{i1} = \frac{m_{i1}l_{ic1}^2}{3}$  and  $I_{i2} = \frac{m_{i2}l_{ic2}^2}{3}$ . Of course, these physical parameters can be reasonably selected based on the actual objects of the robotic manipulators in practical engineering applications.

The interaction diagram of the Lagrangian network consisting of six 2-link revolute manipulators is shown in Figure 2, which does not have a directed spanning tree. It is worth noting that six manipulators with interaction diagram instead of a single manipulator are taken as examples, which is different from [23, 24]. The degree differences of

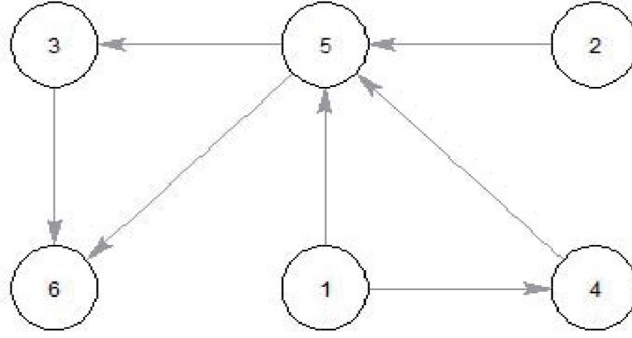


FIGURE 2. Network structure of six manipulators

the six agents are equal to 2, 1, 0, 0, -1, -2, respectively. From Figure 2, the Laplacian matrix  $L_A$  can be obtained as

$$L_A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{pmatrix}.$$

Assume that the aperiodically intermittent control exists on the following time span  $[0, 0.6] \cup [1.15, 1.8] \cup [2.2, 2.8] \cup [3.35, 4] \cup [4.4, 5] \cup [5.55, 6.2] \cup \dots$ . These time intervals are referred to the work time of the controller  $u_i(t)$  for  $1 \leq i \leq l$  according to the designed intermittent controller (9). And  $u_i(t) = 0$  in the rest time  $(0.6, 1.15) \cup (1.8, 2.2) \cup (2.8, 3.35) \cup (4, 4.4) \cup (5, 5.55) \cup \dots$ . This is obviously different from the continuous control designed in the existing works [16-21]. For comparison, we rewrite the intermittent controller (9) into the corresponding continuous controller as follows:

$$u_i(t) = \begin{cases} -k_i J_i^{-1} (\dot{\tilde{x}}_i + \lambda \tilde{x}_i) - \lambda J_i^T \tilde{x}_i, & 1 \leq i \leq l, \\ 0, & l+1 \leq i \leq N, \end{cases} \quad (28)$$

where the tracking error vectors  $\tilde{x}_i = x_i - x_d$  and  $\dot{\tilde{x}}_i = \dot{x}_i - \dot{x}_d$  are taken as the same as those in (9). It can be seen from (28) that the continuous controller  $u_i(t)$  needs to continuously receive the desired position  $x_d(t)$  and velocity  $\dot{x}_d(t)$  for  $t \geq 0$ . In contrast, the intermittent controller (9) only needs to receive the information about the desired trajectory in the work time  $[0, 0.6] \cup [1.15, 1.8] \cup [2.2, 2.8] \cup [3.35, 4] \cup [4.4, 5] \cup [5.55, 6.2] \cup \dots$ , which is also different from [22]. This can greatly reduce the amount of information transmission. It is well known that the desired trajectory transmission may be discontinuous because of a wide variety of environmental factors including the device constraints, the external abrupt disturbance, the restriction of the network communication channels and so on. Thus, intermittent controller is often more effective than continuous controller in practical applications.

Next, we will obtain the control gains  $k_i$  ( $i = 1, 2, 3, 4, 5, 6$ ),  $\lambda$  and the coupling strength  $c$  based on Theorem 4.1. According to the above work time, we have  $\omega_1 = 0.6$  and  $\omega_2 = 1.15$ . Taking  $\varepsilon = 10$ ,  $\xi_2 \geq 79.7$  is obtained from the second expression of (14). If  $\xi_2 = 80$ ,  $\xi_1 > 73.3$  is obtained from the first expression of (14). Then the control gain  $\lambda \geq 37$  is derived from the synchronization criteria (A2) for choosing  $\xi_1 = 74$ . Choosing  $c = 8$ , if five agents are allowed to be pinned (i.e.,  $l = 5$ ),  $k_i = 335$  ( $i = 1, 2, 3, 4, 5$ ) can be obtained from criteria (A1) in Theorem 4.1 to satisfy  $\lambda_{\min}(cL_A^S + K - \frac{\varepsilon l N}{2}) = 326.8 \geq \frac{M\xi_1}{2} = 325.6$ .

Based on the above date, we take  $k_i = 335$  ( $i = 1, 2, 3, 4, 5$ ),  $\lambda = 37$  and  $c = 8$ , then the conditions in Theorem 4.1 can be satisfied. The simulation results by pinning the first five agents are presented in Figures 3 and 4. They show that the six 2-link revolute manipulators can synchronize and track the desired trajectory  $x_d(t)$  with the tracking errors  $\|x_i(t) - x_d(t)\| \leq 0.03$  in task space. Therefore, our present intermittent control strategy in task space is effective and feasible.

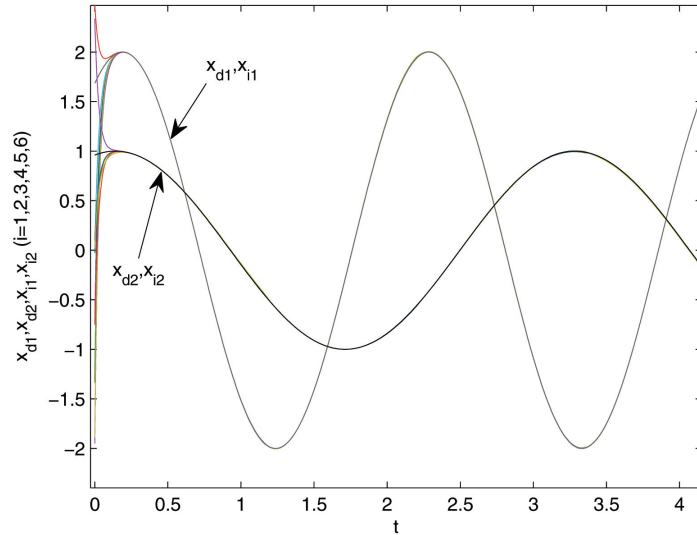


FIGURE 3. Six 2-link revolute manipulators track the desired trajectory when  $k_1 = k_2 = k_3 = k_4 = k_5 = 335$ ,  $k_6 = 0$ ,  $c = 8$  and  $\lambda = 37$

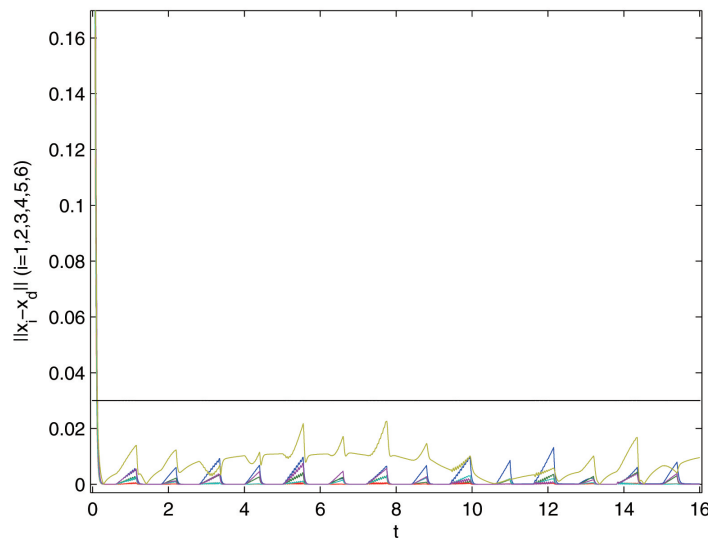


FIGURE 4. Evolution of  $\|x_i(t) - x_d(t)\|$  ( $i = 1, 2, 3, 4, 5, 6$ ) under the intermittent control with  $k_1 = k_2 = k_3 = k_4 = k_5 = 335$ ,  $k_6 = 0$ ,  $c = 8$  and  $\lambda = 37$

If we choose the first three nodes as the pinning nodes (i.e.,  $l = 3$ ), we take  $\lambda = 25$ ,  $c = 5$  and  $k_1 = k_2 = k_3 = 200$ , which are not taken to satisfy the conditions in Theorem 4.1. Fortunately, the six 2-link revolute manipulators can also synchronize and track the desired trajectory  $x_d(t)$  with a small tracking error in task space, as shown in Figure 5. Thus, the conditions in Theorem 4.1 are sufficient but not necessary, just as stated in Remark 4.2.

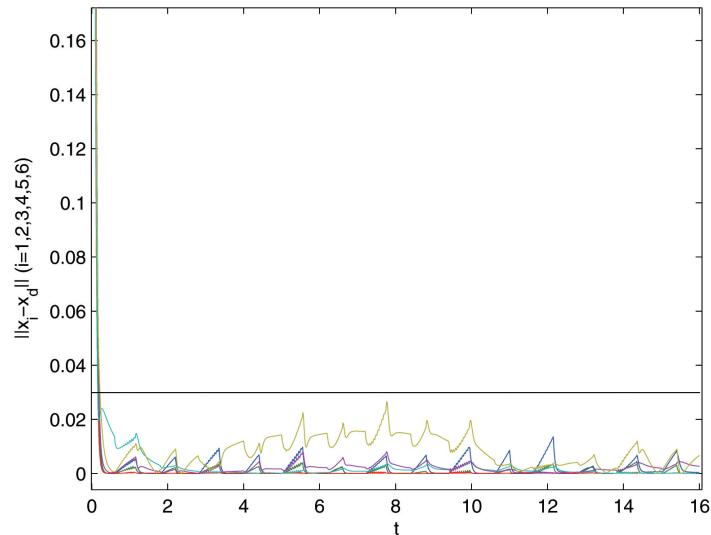


FIGURE 5. Evolution of  $\|x_i(t) - x_d(t)\|$  ( $i = 1, 2, 3, 4, 5, 6$ ) under the intermittent control with  $k_1 = k_2 = k_3 = 200$ ,  $k_4 = 0$ ,  $k_5 = 0$ ,  $k_6 = 0$ ,  $c = 5$  and  $\lambda = 25$

**6. Conclusion.** In this paper, by designing the intermittent control law, task-space tracking synchronization of multiple robot manipulators has been investigated. The intermittent control is studied in task space, which is very different from most of the existing works on the control of robot manipulators. For a given desired trajectory in task space, some algebraic criteria are derived to make the end-effector positions of the multiple robotic manipulators track the trajectory with a desired tracking error under the intermittent control law. The results are applied to the tracking synchronization of six different 2-link revolute manipulators. The simulation results show the effectiveness of the control strategy. It should be pointed out that the task-space control problems of multiple robot manipulators in the presence of dynamic uncertainties and communication delays by discontinuous control law are more interesting and challenging topic for future research in this direction. In addition, more discontinuous control strategies such as impulsive control, sampling control and event-triggered control will be studied to discuss the task-space control problems for robot manipulators.

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