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DIFFERENTIAL PRIVACY DYNAMIC ECONOMIC DISPATCH WITH DISTRIBUTED CONSENSUS BASED ON ADMM

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Abstract. *This paper addresses the economic dispatch problem with privacy protection in a smart grid. The smart grid consists of traditional generators (GTs), wind turbines (WTs), and photovoltaic stations (PVs). The objective is to meet energy demand and minimize the total cost while ensuring the security of users' data. To achieve this, an optimization method based on the alternating direction method of multipliers (ADMM) is proposed. Firstly, the cost functions for GTs, PVs, and WTs are established, taking account of both static and dynamic constraints of distributed energies. Next, the algorithm is utilized to solve the optimization problems, and the Newton method is applied to finding the zero points of functions that cannot be easily separated into variables. Additionally, differential privacy is employed to protect the cost functions from potential attackers attempting to steal them. Finally, the effectiveness of the optimization method and the confidentiality of the system are verified through simulations.*

Keywords: Smart grid, Distributed consensus, Economic dispatch, ADMM, Differential privacy

1. **Introduction.** In recent years, with the rapid development of power technology, smart grids, as an important tool under the background of energy transformation, have attracted widespread attention. Economic dispatch, as one of the core technologies of smart grids, is crucial for the economic and stable operation of smart grids [1]. Most current research on economic dispatch in smart grids is conducted under static constraints [2]. However, with the implementation of the national new energy strategy, the integration of renewable energy sources such as wind and solar energy has become an important development direction for smart grids. Considering the operational safety of smart grids and the unpredictability of renewable energy sources, dynamic constraints spanning multiple periods are more suitable for implementation in smart grids. In addition, the economic dispatch process of smart grids involves the transmission of a large amount of user information. Therefore, ensuring the security of users' private data is also an urgent issue that needs to be addressed in economic dispatch.

The centralized algorithm has been extensively employed in economic dispatch, encompassing both traditional and intelligent algorithms. [3] presented an economic dispatch framework that integrates with a finite-time domain optimization model. In [4], the model predictive control technique was utilized to determine the optimal operation of the smart grid by extending the evaluation and recourse horizon. [5] proposed a deep learning method that facilitates resource allocation coordination among equipment. However,

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the centralized algorithm is not suitable for smart grids with decentralized energy well [6].

Compared to centralized algorithms, distributed economic dispatch algorithms have received more attention. [7] introduced a distributed primal-dual consensus algorithm to address the economic dispatch problem involving quadratic cost functions. [8] proposed a distributed dispatch strategy specifically designed for smart grids that incorporate wind energy. The algorithm proposed in [9] combined ADMM and consensus algorithms to solve the problem of economic dispatch with distributed sources. In [10], the authors utilized ADMM and consensus theory to address the dynamic economic dispatch problem. In [11], the paper proposed a distributed control method to develop PV generators-energy storage coordination. [12] proposed a fixed-time event-triggered distributed secondary voltage controller to achieve fixed-time restoration of voltage to the reference value. The above literature studied the distributed economic dispatch problem of smart grids under various operating conditions. However, the aforementioned distributed algorithms do not take privacy protection into consideration.

Privacy protection technology is an emerging technology in smart grids that caters to user needs. In the privacy protection strategy for economic dispatch, differential privacy is the most common and relatively mature privacy protection method. Differential privacy mainly involves adding appropriate noise to the data that needs to be communicated to encrypt the data while ensuring the accuracy of the economic dispatch algorithm. In [13], the authors proposed a fully distributed algorithm based on bias tracking to handle constrained economic dispatch problems. This algorithm protects the differential privacy of the cost function by using decaying Laplace noise to mask the state and direction. The algorithm is applicable when the cost function is strongly convex. Wang et al. proposed an economic dispatch algorithm based on heterogeneous privacy-preserving consensus, which consists of two stages in [14]. The first stage protects consumer demand through a differential privacy algorithm. The second stage involves a privacy-preserving incremental cost consensus process, which protects the privacy of power output and price sensitivity. In [15], on the basis of complete decentralization, random noise was added to the transmitted data, and combined with an event-triggered mechanism, privacy protection for the economic dispatch process was achieved. In [16], the authors designed a distributed economic dispatch method suitable for time-varying topologies, also adopting the method of adding noise to protect privacy. Xu et al. established a privacy-preserving distributed optimization algorithm by combining sub-gradient consistent iterations [17]. This algorithm not only protects the privacy of the optimization but also guarantees the optimality of the solution with certain bias bounds. In [18], under a consistency framework, state decomposition and Laplace noise are embedded into the distributed optimization algorithm to enhance privacy protection capabilities. Subsequently, through Lyapunov stability theory and stochastic analysis, sufficient conditions for the consistency coupling strength of the algorithm and the convergence accuracy caused by noise are revealed. Zhao et al. developed an economic dispatch method for time-varying directional communication, also considering line loss issues [19]. Further consideration of stricter communication conditions adds conditional noise to auxiliary variables, thereby achieving privacy protection effects. However, research on privacy protection in economic dispatch of smart grids mostly considers cost functions that are strongly convex or real-time economic dispatch problems without time-coupled constraints. There is also limited research on problems involving renewable energy sources.

This paper considers generating nodes in smart grids including photovoltaic (PV), wind turbines (WT), and traditional generators (GT). While addressing the dynamic economic dispatch problem, we also pay attention to privacy protection during the information transmission process. The contributions of this paper can be summarized as follows.

- The research object of this paper is a smart grid with distributed renewable energy sources, which closely reflects the actual situation. Unlike previous studies, the cost function in this paper is not limited to being quadratic.
- In this paper, the cost of environmental protection is introduced when dispatch distributed energies, and dynamic constraints are also taken into account.
- In the process of information transmission, we add Laplace noise to the information sent by neighbors to prevent attackers from inferring the cost function based on stolen information.

The remaining sections of the paper are organized as follows. In Section 2, we present a model of a smart grid with distributed energy, including the cost functions and constraints associated with each energy source. In Section 3, we propose an economic dispatch algorithm based on ADMM, which incorporates distributed privacy protection and a projection method. In Section 4, we validate the effectiveness of the optimization method and examine the confidentiality of communication between neighboring nodes through simulations. Finally, Section 5 concludes the paper.

2. **Problem Statement.** In this section, we first give the cost model of photovoltaic power generation and describe the wind power model. Finally, the energy dispatch model of smart grid with photovoltaic wind power is presented.

2.1. **The model of PVs power.** The cost of photovoltaic power generation includes direct cost and deviation cost. The direct cost model of photovoltaic power generation in a dispatch period is as follows:

$$F_{PVd} = \sum_{t=1}^{T} \sum_{i=1}^{n} C_{PVd} \cdot P_{PVi}^{t} \tag{1}$$

C_{PVd} represents the direct operating cost of unit electric energy generated by grid connected photovoltaic power plants. P_{PVi} is the actual power of photovoltaic power station i. When the power grid dispatch plan is formulated one day in advance, it is necessary to predict the dispatch power in the T period of the next day. The forecast cost of photovoltaic power generation is shown as follows:

$$F_{PVp} = \sum_{t=1}^{T} \sum_{i=1}^{n} C_{PVo} \left(P_{PVi}^{t} - P_{PVpi}^{t} \right)^2 \tag{2}$$

In Formula (2), C_{PVo} is the deviation cost, and P_{PVpi}^{t} is predictive value. Then, $F_{PV}^{t} = F_{PVp}^{t} + F_{PVd}^{t}$.

2.2. **The model of wind power.** Wind power output is closely related to wind speed, and Weibull distribution can better describe the randomness of wind power. The Weibull model of wind speed is as follows:

$$f_V(v) = \frac{\kappa}{c} \left(\frac{v}{c} \right)^{(\kappa-1)} \exp\left(-\left(\frac{v}{c} \right)^{\kappa} \right) \tag{3}$$

where κ and c represent scale parameters and shape parameters of Weibull distribution, and v represents wind speed. The randomness of wind speed causes the randomness of wind power output. Define the available power of the jth turbine in the t dispatch period as P_{Wja}^{t}. The cost model of wind power can be expressed as

$$F_{Wj}^{t} = C_{Wjd} \cdot P_{Wj}^{t} + C_{Wjo} E_{Wjo} + C_{Wju} E_{Wju} \tag{4}$$

where P_{Wj}^t is actual dispatch power of the jth turbine. C_{Wjd} is the direct cost coefficient for wind power generation. C_{Wju} and C_{Wjo} are the penalty cost that the dispatched wind energy is greater or less than the actual wind energy at the current time. Define average of the gap between dispatched power and actual power:

$$\begin{aligned} E_{Wjo} &= \int_{P_{Wj}^t}^{P_{Wjr}} \left(p^t - P_{Wj}^t\right) f_{p^t}\left(p^t\right) d\left(p^t\right) \\ E_{Wju} &= \int_0^{P_{Wj}^t} \left(P_{Wj}^t - p^t\right) f_{p^t}\left(p^t\right) d\left(p^t\right) \end{aligned} \tag{5}$$

where p^t is P_{Wja}^t. $f_{p^t}(p^t)$ is the probability of P_{Wja}^t at the current time, and P_{Wjr} is rated power. Therefore, the cost of wind power is

$$F_W = \sum_{t=1}^{T} \sum_{j=1}^{m} F_{Wj}^t \tag{6}$$

2.3. **The model of generators.** This paper studies the smart grid including wind power, light power and traditional generators. Considering a total load demand of smart grid, we suppose that there are n PVs, m WTs and l GTs and demand is D^t. The complete model is as follows.

For traditional generators:

$$F_{Gq}^t = c_q + b_q P_{Gq}^t + a_q \left(P_{Gq}^t\right)^2 \tag{7}$$

where a_q, b_q and c_q are cost parameters. Considering pollutant emission of GTs units, the environmental protection costs:

$$F_{GEq}^t = \gamma_q + \beta_q P_{Gq}^t + \alpha_q \left(P_{Gq}^t\right)^2 \tag{8}$$

where γ_q, β_q and α_q are pollutant emission coefficients of the node q. Total cost of GTs is

$$F_G = \sum_{t=1}^{T} \sum_{q=1}^{l} \left(F_{Gq}^t + F_{GEq}^t\right) \tag{9}$$

The constraints of P_{Gq}^t include power constraints and climbing constraints:

$$\begin{gathered} P_{Gq}^{\min} \le P_{Gq} \le P_{Gq}^{\max} \\ -\Delta P_{Gq}^u \le P_{Gq}^t - P_{Gq}^{t-1} \le \Delta P_{Gq}^o \end{gathered} \tag{10}$$

where ΔP_{Gq}^u and ΔP_{Gq}^o are two climbing constraints, which limit the scope of growth and decline.

Based on the above description, define the vector of output power of all units in the dispatch period: $P = \left[P_{PV1}^T, \ldots, P_{PVn}^T, P_{W1}^T, \ldots, P_{Wm}^T, P_{G1}^T, \ldots, P_{Gl}^T\right]$. The total cost of the system is defined as follows: $F^t\left(P^t\right) = F_{PV}^t + F_W^t + F_G^t$.

3. **Model Optimization.**

In this section, define the following two closed convex sets:

$$\Omega_1 = \left\{ (P_{PV}, P_W, P_G) \left| \sum_{i=1}^{n} P_{PV}^t + \sum_{j=1}^{m} P_W^t + \sum_{q=1}^{l} P_G^t = D^t \right. \right\} \tag{11}$$

Let $X = \left[P_{PV}^T, P_W^T, P_G^T\right]^T$. In order to use ADMM, define variables $Y = \left[P_{YPV}^T, P_{YW}^T, P_{YG}^T\right]^T$, vector X and vector Y are equal. Vector Y belongs to set:

$$\begin{aligned} \Omega_2 = \{&(P_{YPV}, P_{YW}, P_{YG}) \,| 0 \le P_{PVi} \le P_{PVi}^{\max},\ i = 1, \ldots, n; \\ &0 \le P_{Wj} \le P_{Wjr},\ j = 1, \ldots, m;\ P_{Gq}^{\min} \le P_{Gq} \le P_{Gq}^{\max},\ q = 1, \ldots, l; \end{aligned}$$

$$-\Delta P_{Gq}^{u} \leq P_{Gq}^{t} - P_{Gq}^{t-1} \leq \Delta P_{Gq}^{o},\ \forall t = 2, \ldots, T\} \tag{12}$$

To adapt to the ADMM algorithm, we transform the previously mentioned coupling constraints and local constraint sets into indicator functions and incorporate the obtained indicator functions into the objective function. In order to simplify the notation, we represent $F^t(P^t)$ by $F(X)$ in the following formula:

$$\begin{aligned} &\min F(X) + \delta_{\Omega_1}(X) + \delta_{\Omega_2}(Y) \\ &\text{s.t. } X = Y \end{aligned} \tag{13}$$

where $\delta_{\Omega_1}(X)$ and $\delta_{\Omega_2}(Y)$ are the indicator functions of sets Ω_1 and Ω_2. If $X \in \Omega_1$, $\delta_{\Omega_1}(X) = 0$; else, $\delta_{\Omega_1}(X) = \infty$. Similarly, if $Y \in \Omega_2$, $\delta_{\Omega_2}(Y) = 0$; else, $\delta_{\Omega_2}(Y) = \infty$.

Then, use ADMM to solve the above problems:

$$X_{k+1}^{t} = \arg\min_{X_t} \left(F^t\left(X^t\right) + \delta_{\Omega_1}\left(X^t\right) + \frac{\rho}{2}\left\|X^t - Y_k^t + u_k^t\right\|_2^2 \right) \tag{14a}$$

$$Y_{k+1}^{t} = P_{\Omega_2}\left(X_{k+1}^{t} + u_k^t\right) \tag{14b}$$

$$u_{k+1}^{t} = u_k^t + X_{k+1}^{t} - Y_{k+1}^{t} \tag{14c}$$

where P_{Ω_2} is projection operator and u^t is extension function of dual variable.

3.1. ***X* internal circulation.** This section will focus on handling Formula (14a). Rewrite Formula (14a) into the following optimized form:

$$\begin{aligned} &\min_{X^t \in R^N} \mathcal{F}\left(X^t\right) \\ &\text{s.t. } 1^T X^t = D^t \end{aligned} \tag{15}$$

where $\mathcal{F}(X^t) = F^t(X^t) + \frac{\rho}{2}\left\|X^t - Y_k^t + u_k^t\right\|_2^2$.

Then, the function of each agent in one period:

$$\mathcal{F}_i\left(X_i^t\right) = \begin{cases} F_{PVi}^t + \frac{\rho}{2}\left(P_{PVi}^t + u_i^{k,t} - P_{YPVi}^{k,t}\right)^2, & \forall i \in \Omega_{PV} \\ F_{Wi}^t + \frac{\rho}{2}\left(P_{Wi}^t + u_i^{k,t} - P_{YWi}^{k,t}\right)^2, & \forall i \in \Omega_W \\ F_{Gi}^t + \frac{\rho}{2}\left(P_{Gi}^t + u_i^{k,t} - P_{YGi}^{k,t}\right)^2, & \forall i \in \Omega_G \end{cases} \tag{16}$$

where Ω_{PV}, Ω_W and Ω_G represent respectively set of PVs, WTs and GTs.

Assuming that there are N nodes in the smart grid, the total energy demand of the smart grid at time t is D^t, and the energy demand of each node is $D_i^t = D^t/N$. Define a new form:

$$\begin{aligned} &\min_{X^t \in R^N} \mathcal{F}\left(X^t\right) \\ &\text{s.t. } \sum_{i=1}^{N} X_i^t = \sum_{i=1}^{N} D_i^t \end{aligned} \tag{17}$$

For optimization problem (17), the first-order optimal condition can be expressed as follows:

$$\begin{aligned} &\nabla \mathcal{F}\left(X^{t,*}\right) = \lambda^* \\ &\sum_{i=1}^{N} X_i^{t,*} = \sum_{i=1}^{N} D_i^t \end{aligned} \tag{18}$$

where λ^* is the optimal Lagrangian multiplier, and the above formula means that under the optimal power allocation, the first derivative of all node cost functions tends to be the optimal Lagrangian multiplier, and the optimal power allocation satisfies supply and demand constraints.

The differential privacy method refers to adding random noise to the original data to ensure the utility and privacy of the data. This means that adding noise should not affect the normal use of user data in the system while controlling the risk of data leakage for any individual user. In this section, privacy protection is achieved by adding Laplace noise to the broadcasted information. According to the first-order optimal condition shown in Formula (18), based on the Laplace noise mechanism and distributed consensus algorithm, X_i^t updates according to the following:

$$\lambda_i^t(k+1) = \sum_{j=1}^{N} w_{ij} z_{\lambda j}^t(k) - \alpha\theta_i^t(k) \tag{19a}$$

$$P_{PVi}^t(k+1) = \frac{\rho\big(P_{YPVi}^t(k) - u_i^t(k)\big) + 2C_{PVo}P_{PVpi}^t - C_{PVd} + \lambda_i^t(k+1)}{2C_{PVo} + \rho} \tag{19b}$$

$$P_{Gi}^t(k+1) = \frac{\rho\big(P_{YGi}^t(k) - u_i^t(k)\big) - \beta_i - b_i + \lambda_i^t(k+1)}{2(a_i + \alpha_i) + \rho} \tag{19c}$$

$$C_{Wid} + C_{Wio}\dot{E}_{Wio} + C_{Wiu}\dot{E}_{Wiu} + \rho P_{Wi}^t(k+1) = \lambda_i^t(k+1) - \rho\left(u_i^t(k) - P_{YWi}^t(k)\right) \tag{19d}$$

$$\theta_i^t(k+1) = \sum_{i=1}^{N} w_{ij} z_{\theta j}^t(k) + X_i^t(k+1) - X_i^t(k) \tag{19e}$$

where w_{ij} is an element in a doubly random matrix. Double random matrices can be randomly generated based on the topology. And $z_{\lambda j}^t = \lambda_j^t(k) + \zeta_j^t(k)$, $z_{\theta j}^t = \theta_j^t(k) + \eta_j^t(k)$. In the above update methods, the updates of Formulas (19a) and (19e) are distributed, so it is necessary to encrypt the communication information during the update to ensure its privacy. $\zeta_j^t(k)$ and $\eta_j^t(k)$ represent the added Laplace random noise, which is mutually independent between agents and between iterations. Meanwhile, from Formulas (19a) and (19e), it can be seen that in order for λ_i^t and θ_i^t to converge accurately, the noise added in communication needs to ensure convergence to 0. Therefore, the noise $\zeta_j^t(k) \sim Lap\left(0, b_{\zeta j,t}(k)\right)$, scale parameter $b_{\zeta j,t}(k) = \gamma_{\zeta j,t}/(k^{p_{\zeta j,t}})$, noise $\eta_j^t(k) \sim Lap(0, b_{\eta j,t}(k))$, $b_{\eta j,t}(k) = \gamma_{\eta j,t}/(k^{p_{\eta j,t}})$, and parameters $\gamma_{\zeta j,t}$, $p_{\zeta j,t}$, $\gamma_{\eta j,t}$, $p_{\eta j,t}$ used in this paper are greater than 0. As the number of iterations increases, the scale parameter will gradually reach 0. In Equation (19d), for differential variables $\dot{E}_{Wiu}$ and $\dot{E}_{Wio}$:

$$\begin{aligned} \dot{E}_{Wiu} &= \exp\left(-\frac{v_{out}^\kappa}{c^\kappa}\right) - \exp\left(-\frac{v_j^\kappa}{c^\kappa}\right) \\ \dot{E}_{Wio} &= 1 + \exp\left(-\frac{v_{out}^\kappa}{c^\kappa}\right) - \exp\left(-\frac{v_j^\kappa}{c^\kappa}\right) \end{aligned} \tag{20}$$

where $v_j = v_{in} + (v_r - v_{in})\, P_{Wi}^t / P_{Wir}$, v_{in}, v_r and v_{out} are respectively cut-in speed, rated speed and cut-out speed. According to Formula (19d), variable P_{Wi}^t cannot be separated. What is more, it is easy to get that

$$\ddot{E}_{Wiu} = \ddot{E}_{Wio} = \left(\frac{\kappa v_j^{\kappa-1}}{c^\kappa}\right)\left(\frac{v_r - v_{in}}{P_{Wir}}\right)\exp\left(-\frac{v_j^\kappa}{c^\kappa}\right) > 0$$

from $\ddot{E}_{Wiu}$ we can cover Formula (19d) form $\Phi\left(P_{Wi}^t(k+1)\right) = 0$. It can be transformed into the problem of finding zero point of $\Phi\left(P_{Wi}^t(k+1)\right)$. Newton iterative method can solve the problem above. For $\Phi(x)$, define initial value x_0, and the maximum number of iterations $i_{\max}$:

Step 1. $x_{k+1} = x_k - \frac{\Phi(x_k)}{\dot{\Phi}(x_k)}$;

Step 2. If $x_{k+1} - x_k < e^{-3}$ or $k > i_{\max}$, break; else, return to Step 1.

3.2. ***Y* and *u*-update.** After obtaining the optimal X value, continue Formula (21) to update Y. Before that, we analyze the convex set in Formula (12) firstly. Since GTs have climbing constraint, the power constraints of thermal power units are not fixed, in the adjacent dispatch period. The update process of Y is described as follows:

$$Y_i^t(k+1) = \begin{cases} X_i^{t,\min}, & X_i^t(k+1) + u_i^t(k) \le X_i^{t,\min} \\ X_i^t(k+1) + u_i^t(k), & X_i^{t,\min} \le X_i^t(k+1) + u_i^t(k) \le X_i^{t,\max} \\ X_i^{t,\max}, & X_i^t(k+1) + u_i^t(k) \ge X_i^{t,\max} \end{cases} \quad (21)$$

For the convenience of description, we define $X_i^{t,\min}$ and $X_i^{t,\max}$:

$$X_i^{t,\min} = \begin{cases} \max\left(P_G^{\min}, P_{Gi}^{t-1} + \Delta P_{Gq}^u\right), & i \in \Omega_G \\ 0, & i \in \Omega_{PV}, i \in \Omega_W \end{cases}$$

$$X_i^{t,\max} = \begin{cases} \min\left(P_G^{\max}, P_{Gi}^{t-1} + \Delta P_{Gi}^o\right), & i \in \Omega_G \\ P_{Wir}, & i \in \Omega_W \\ P_{PVpi}, & i \in \Omega_{PV} \end{cases}$$

After $X_i^t(k+1)$ and $Y_i^t(k+1)$ are obtained, $u_i^t(k+1)$ can be obtained by Formular (14c). The update of Y and u takes advantage of local information instead of distribution.

4. **Case Study.** This section will verify the effectiveness of the above algorithm through case simulations. The case study tests a power system with five generating nodes, as shown in Figure 1. The generating units consist of three traditional generators GTs1, GTs2, GTs3, one photovoltaic generator PV1, and one wind turbine system WT1. The total dispatch time $T = 24$. Given some specific parameters, such as the direct cost C_{PVd} and deviation cost C_{PVo} of the photovoltaic generator, the rated wind speed v_r, cut-in speed v_{in}, cut-out speed v_{out}, rated power P_{Wjr}, direct cost C_{Wjd}, and deviation costs C_{Wjo} and C_{Wju} of the wind turbine, all are 6.1. The ramping constraints of the wind turbine are both 30. In addition, define $\rho = 0.8$, $Y(0) = 0$ and $u(0) = 0$. For the parameters of the traditional generator, the predicted power of the photovoltaic generation, the energy demand of the total dispatch time, and the wind power prediction parameters κ and c within the total dispatch time are shown in Table 1 and Table 2.

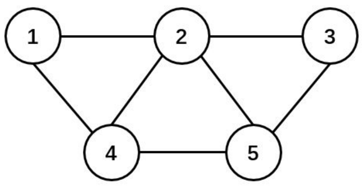


Figure 1. Power system topology

In the case simulation section, we first conduct a simulation example of the alternating direction multiplier method at time $t = 1$. Figure 2 shows the iteration results of the primal-dual residual. After 30 iterations, both the primal and dual residuals converge to 0, verifying the convergence of the economic dispatch algorithm. Figure 3 displays the dispatch results for the four selected time periods, showing the optimal allocation of power output from generating nodes while meeting the demand. Due to power constraints, in subplot (a), the power output of the GTs3 generating node is constrained to

TABLE 1. Generator parameters

Parameters	GTs1	GTs2	GTs3
α_q	0.03	0.05	0.07
β_q	3.5	3	2.5
γ_q	90	70	50
a_q	0.01	0.01	0.01
b_q	1	1	1
c_q	10	10	10
$P_{Gq}^{\min}$	17	15	18
$P_{Gq}^{\max}$	40	35	30
ΔP_{Gq}^{u}	16	14	15
ΔP_{Gq}^{o}	16	14	15

TABLE 2. Running parameters

Period	Demand	c	κ	PVi
1	90	8	2	0
2	85	7	2.05	0
3	70	7	2.1	0
4	80	7.5	2.15	0
5	95	7.5	2.12	0
6	100	6.5	2.07	3
7	110	5	2.1	6
8	105	5.5	2.11	9
9	110	4.5	2.13	11
10	110	4.8	1.95	13
11	120	4	1.8	14
12	120	4	1.85	16
13	115	4.5	2.05	17
14	110	3.6	2.15	18
15	105	3.6	2.07	16
16	95	3.6	2.1	10
17	110	4.3	2.13	6
18	110	4.5	2.1	3
19	120	5.1	2.2	0
20	100	5.1	2.17	0
21	110	6.2	2.15	0
22	100	7.4	2.11	0
23	90	8	2.10	0
24	90	7	2.06	0

18. Subsequently, a simulation analysis of the economic dispatch for the entire time period is conducted. Figure 4 shows the economic dispatch results for 24 time periods, with subplot (a) displaying the optimal power allocation of generating nodes in each period. Due to power constraints, at multiple times, GTs1 is constrained to the maximum power

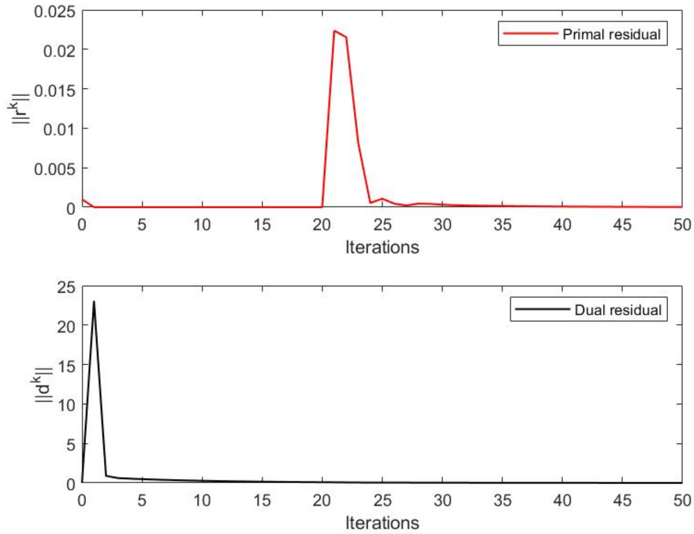
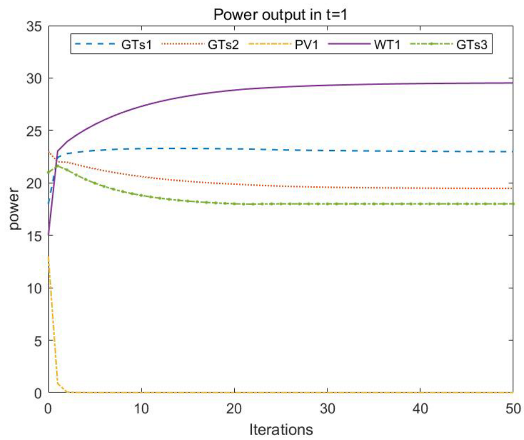
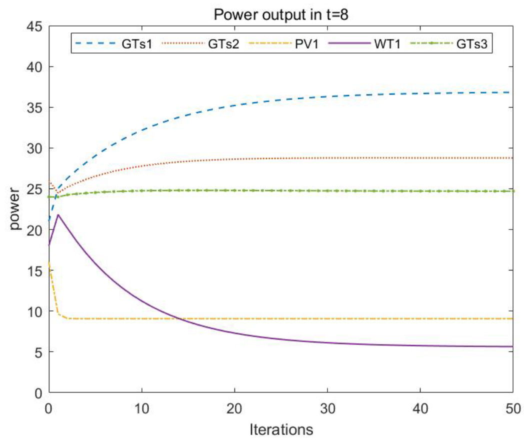


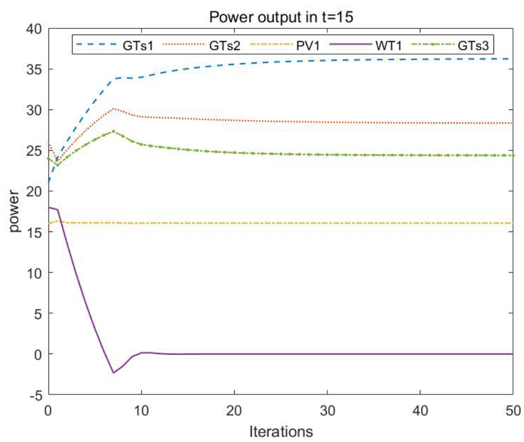
FIGURE 2. Primal and dual residual iteration



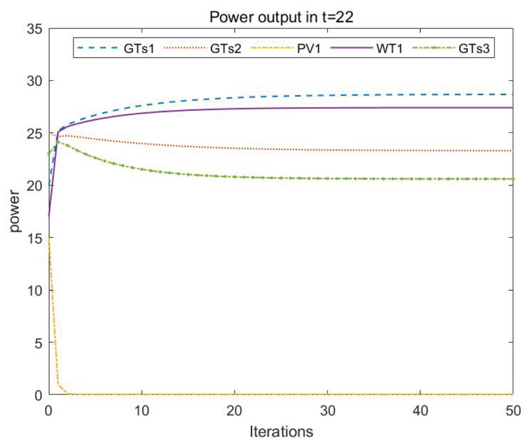
(a) Dispatch result at $t = 1$



(b) Dispatch result at $t = 8$

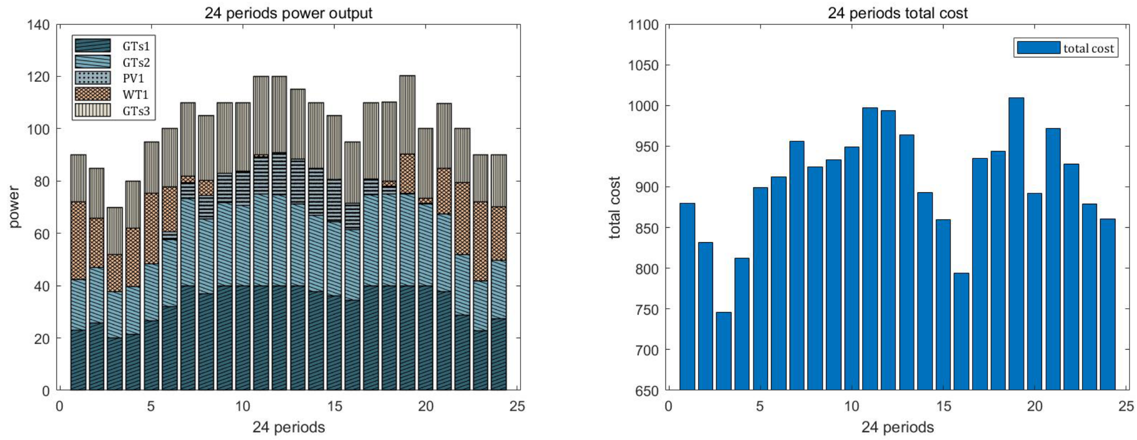


(c) Dispatch result at $t = 15$



(d) Dispatch result at $t = 22$

FIGURE 3. Economic dispatch result for four time periods



(a) Power dispatch at 24 periods (b) Total cost at 24 periods

FIGURE 4. Economic dispatch results during 24 periods

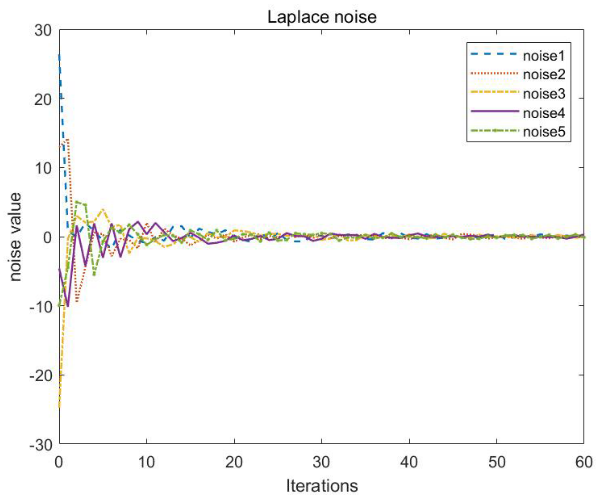
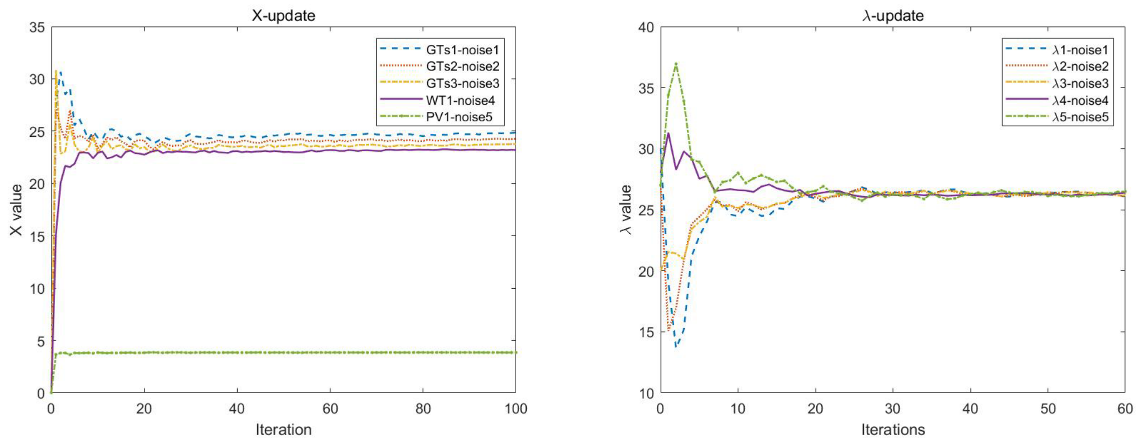
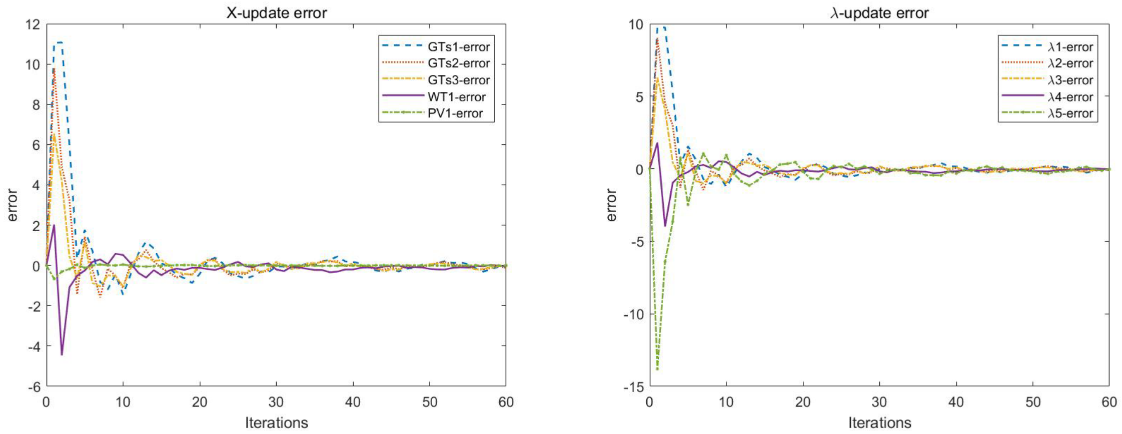


FIGURE 5. Laplace noise curve



(a) X-value update at $t = 6$ (b) λ-value update at $t = 6$

FIGURE 6. Update status of X at $t = 6$



(a) X-value noise error at $t = 6$ (b) λ-value noise error at $t = 6$

FIGURE 7. Update status of noise error at $t = 6$

of 40, GTs3 is constrained to the minimum power of 18, and wind and photovoltaic power are also limited to the minimum power. Subplot (b) shows the total cost of dispatch for the 24 time periods, with the bars representing the optimal dispatch cost for each period. Finally, the effect of adding noise is validated. Figure 5 shows the iteration graph of the added Laplace noise, with the amplitude of the noise decreasing as the number of iterations increases. A randomly selected X-update at time $t = 6$ is shown in Figure 6, where subplot (a) shows the iteration results of the power variables under the effect of Laplace noise in the update. Subplot (b) displays the iteration graph of the Lagrange multiplier after adding Laplace noise. Figure 7 shows the iteration of error in the power values and Lagrange multipliers with and without added noise. Both noise errors eventually tend to 0, indicating that the convergence accuracy can be guaranteed after adding noise. These case studies verify the effectiveness of adding noise.

5. **Conclusions.** This paper introduces an algorithm for economic dispatch problems that considers privacy protection in smart grid. We use the proposed method to dispatch the output of distributed energy resources to minimize total costs while meeting demand. In simulations, we validate the effectiveness of the algorithm. In the future, more practical smart grids models will be considered, such as smart grids with energy storage devices and diverse distributed energy resources. It is also necessary to research more effective privacy protection algorithms.

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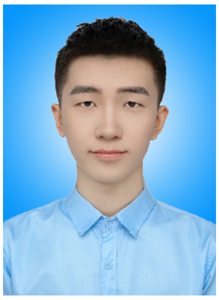

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