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LOW SENSITIVITY CONTROL FOR NON MINIMUM-PHASE SYSTEMS USING DOUBLE FEEDBACK CONTROL

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Abstract. *In this paper, we consider a design method of the control system by using double feedback control with robust stability and low sensitivity for single-input/single-output non-minimum-phase systems having a varying number of unstable poles. It is well known that the uncertainty of the plant often negatively affects the stability of the control system. According to several studies, to maintain the stability of the control system for the large uncertainty, a low-sensitivity control guaranteeing robust stability must be made, which is currently difficult. By contrast, a control system with low sensitivity is not always unstable for uncertainty. For a class of uncertainty where the low sensitivity guarantees robust stability, Yu et al. have proposed a new control structure, which is called the double feedback control structure, to make a control system robustly stable and reduce the effects of the uncertainty on the output for the single-input and single-output minimum-phase system. However, there are no studies regarding the design method of a control system using a double feedback control structure for the single-input and single-output non-minimum-phase systems. In this paper, we provide evidence of the control system using double feedback control that the low-sensitivity control guarantees robust stability and provide a design procedure for the control system using double feedback control. In addition, we illustrate two numerical examples to show the effects of the proposed design procedure.*

Keywords: Non-minimum-phase system, Low-sensitivity control, Sensitivity control, Robust stability, Uncertainty, System having a varying number of unstable poles

1. Introduction.

A nominal plant of a plant is often equal to its plant. An error between a nominal plant and a plant is called an uncertainty. The uncertainty often makes the control system unstable, and the control system loses a low sensitivity in the meaning of the disturbance attenuation, and so on. This makes it an important problem to derive an analytical method, a stability condition, and the control system design for systems with uncertainty. This problem is called a robust stabilization problem. In the robust stabilization problem, it is important to clarify a necessary and sufficient condition to make a control system robustly stable for the uncertainty, which is called the robust stability condition. Doyle and Stein built the basic solution to this problem [1]. In [1],

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Doyle and Stein showed the necessary and sufficient conditions for the multiplicative uncertainty and the additive uncertainty. Chen and Desoer derived a complete proof of the solution in [2]. Kimura considered the robust stabilizability problem for single-input and single-output systems [4]. Vidyasagar and Kimura expanded the result by Kimura [4] for multiple-input and multiple-output systems [5].

According to [1, 2, 3], to keep stability for the large uncertainty, the complementary sensitivity function must be small. By contrast, making the complementary sensitivity function of the control system small, the control system yielded lower performance about low-sensitivity characteristics in the meaning of disturbance attenuation property and so on. To produce the control system with high disturbance attenuation property, we must make the sensitivity function of the control system small. Also, to make a control system have low sensitivity, the sensitivity function of the control system must be small. Because the sum of the sensitivity function and the complementary sensitivity function is equal to 1, we cannot obtain either low sensitivity or high robust stability characteristics for a large uncertainty.

By contrast, the low-sensitivity control does not always make a system unstable for uncertainty. Maeda and Vidyasagar considered this problem an infinite gain margin problem [10, 11]. Nogami et al. clarified the condition that a high-gain controller does not make the control system unstable and also proposed a design method [12]. Doyle et al. considered this low-sensitivity control problem from another viewpoint; there exists a class of uncertainty that has low sensitivity, making the system robustly stable [14]. In [14], Doyle et al. clarified a robust stabilization problem that the low-sensitivity control guarantees robust stability. This result is suitable for high-performance robust control system design.

However, there exists a remaining problem that a class of uncertainty proposed by Doyle et al. cannot apply to systems having a varying number of unstable poles in the closed right half plane. There are several applications for systems with a varying number of unstable poles in the closed right half plane. For example, there is a large flexible spacecraft. The number of unstable poles of the large flexible spacecraft can and does change as the configuration of the spacecraft changes [5]. The problem of obtaining a robust stability condition for the system having a varying number of unstable poles in the closed right half plane is overcome by Yamada [18]. Yamada gave a robust stability condition that the low-sensitivity control guarantees robust stability for systems having a varying number of unstable poles in the closed right half plane.

To design a control system with low sensitivity, it is important to consider the control structure of the control system. The internal model control structure [19] is known as an effective control structure for low-sensitivity control. However, the internal model control structure cannot be applied to unstable systems. Zhou and Ren overcame this problem and proposed a new control structure named the generalized internal model control [20]. The generalized internal model control structure has several applications: an automotive electric power steering system [21]; active front steering system [22]; fault-tolerant control systems [23, 24]; magnetic suspension systems [25]; and mechatronic systems [26]. Okajima et al. proposed a model error compensator structure from another viewpoint; if the modeling gap between the nominal model and the actual plant can be made small by using an additional compensator, the desired response given by the design methods for the nominal model would be achieved even if the modeling gap exists [27]. The model error compensator structure applies for several systems [28, 29, 30].

As a structure of the control system with low sensitivity, the control system with a double feedback loop is known [31, 32, 33]. Anwar and Pan proposed the PID controller design technique of the control system with a double feedback loop with the inner loop containing a stabilizing gain and the outer loop containing the parameters of the PID controller [31]. This control system with a double feedback loop can reduce the effect of measurement noise. By contrast, Yu et al. proposed an effective control structure for low-sensitivity control design called the double feedback control structure from another viewpoint; using a control structure with low sensitivity, the influence of the uncertainty on the output can be reduced. The double feedback control structure proposed by Yu et al. has robust stability for a class of uncertainty proposed by Yamada and can reduce the influence of the uncertainty on the output less than a two-degree-of-freedom control system. In other words, for a class of uncertainty in [18], the control system using a double feedback structure makes the output follow the reference input. Koyama et al. showed evidence of the control system using a double feedback control structure that the low-sensitivity control guarantees robust stability and proposed a design procedure for its control system [33]. Using a result of [32, 33], we can construct the control system with robust stability and low sensitivity. However, Yu et al. did not consider a design method for the control system using a double feedback structure for a non-minimum-phase system.

In this paper, we show the robust stability condition of the double feedback control system for single-input/single-output time-invariant non-minimum-phase plants having varying number of unstable poles. In addition, we propose a design method for double feedback control systems that reduces the influence of uncertainty on the output. This paper is organized as follows. In Section 2, we explain the preliminary results of two-degree-of-freedom control systems and the problem considered in this paper. In Section 3, the proof of robust stability condition of the double feedback control systems is clarified. In Section 4, a design method for the double feedback control system with robust stability and low-sensitivity characteristics is presented. In Section 5, a design procedure for the double feedback control system with robust stability and low-sensitivity characteristics is shown. In Section 6, we show two numerical examples to illustrate the effectiveness of the proposed method. Section 7 gives concluding remarks.

Notation

R	the set of real numbers.
C	the set of complex numbers.
$R(s)$	the set of real relational functions with s.
RH_∞	the set of stable proper real relational functions.
$\|\cdot\|_\infty$	H_∞ norm.

2. **Preliminary Results and Problem Formulation.** In this section, we explain the preliminary results of a two-degree-of-freedom control system and a control system by using a double feedback control and the problem considered in this paper. This is because the control system using double feedback control is based on the results of the two-degree-of-freedom control system. To explain the problem considered in this paper, some results of designing the two-degree-of-freedom control system are presented.

An example of a two-degree-of-freedom control system is presented in Figure 1. Here, $r(s) \in R(s)$ is the reference input, $y(s) \in R(s)$ is the output, $G(s) \in R(s)$ is a single-input/single-output non-minimum-phase strictly proper plant, $F_1(s) \in RH_\infty$ is a feed-forward controller, and $C_1(s) \in R(s)$ is a stabilizing controller for the plant $G(s)$.

The nominal plant of $G(s)$ is denoted as $F_0(s) \in R(s)$. Assuming that $F_0(s) \in R(s)$ is a non-minimum-phase strictly proper system, the relationship between $G(s)$ and $F_0(s)$ is written in the form

$$G(s) = F_0(s)\left(1 + \Delta(s)\right), \tag{1}$$

where $\Delta(s) \in R(s)$ is an uncertainty.

The uncertainty $\Delta(s) \in R(s)$ often affects the low-sensitivity characteristics of the control system shown in Figure 1 in the meaning of the disturbance attenuation and so

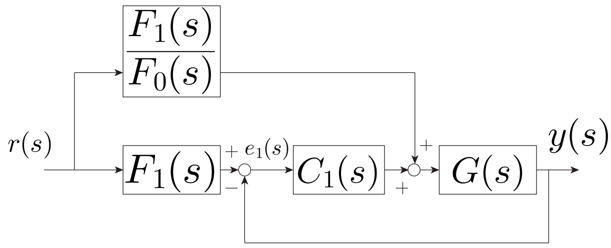

FIGURE 1. The two-degree-of-freedom control system

on because the uncertainty $\Delta(s)$ remained on the output $y(s)$. Thus, there is an influence of the uncertainty $\Delta(s)$ on the output $y(s)$, and it remains on the output $y(s)$. To show the influence of $\Delta(s)$ on $y(s)$, the transfer function from $r(s)$ to $y(s)$ is provided as

$$y(s) = F_1(s)\left(1 + H_1(s)\right) r(s), \tag{2}$$

where $H(s)$ is a function to show the influence of $\Delta(s)$ on $y(s)$ written in the form

$$H_1(s) = \frac{\frac{1}{1+C_1(s)F_0(s)}\frac{\Delta(s)}{1+\Delta(s)}}{1 - \frac{1}{1+C_1(s)F_0(s)}\frac{\Delta(s)}{1+\Delta(s)}}. \tag{3}$$

From (3), when $\Delta(s)$ exists, the uncertainty $\Delta(s)$ remains as $H_1(s)$ on the output $y(s)$. To reduce the influence of $\Delta(s)$ on $y(s)$, the function $1/1 + C_1(s)F_0(s)$ must yield a small value. The function

$$S_1(s) = \frac{1}{1 + C_1(s)F_0(s)}, \tag{4}$$

is called the sensitivity function of the double feedback control system. When the influence of the uncertainty $\Delta(s)$ on the output $y(s)$ is reduced, the control system is a control system with low-sensitivity characteristics.

The control system with low sensitivity often is made to be robustly stable for the uncertainty $\Delta(s)$. According to [18], we can obtain a necessary and sufficient condition for the control system with low sensitivity to be robustly stable for the following class of uncertainty $\Delta(s)$.

Definition 2.1. *$G(s)$ is called the elementary of the set Ω if the following expressions hold.*

- *$\Delta(s)$ satisfies*

$$\left|\frac{\Delta(j\omega)}{1 + \Delta(j\omega)}\right| < |W(j\omega)| \quad (\omega \in R), \tag{5}$$

where $W(s) \in R(s)$ satisfies

$$\lim_{\omega \to \infty} |W(j\omega)| < 1. \tag{6}$$

- *The number of zeros in the closed right half plane of $F_0(s)$ is equal to that of $G(s)$.*

According to [18], from Definition 2.1 and the assumption that $G(s)$ and $F_0(s)$ are strictly proper, we have the following theorem.

Theorem 2.1. [18] *If $G(s)$ and $F_0(s)$ hold (6), then the relative degree of $F_0(s)$ is equal to that of $G(s)$.*

According to [18, 32], the robust stability condition of the two-degree-of-freedom control system in Figure 1 is summarized as the following theorem.

Theorem 2.2. [18, 32] *Assume that $C_1(s)$ stabilizes $F_0(s) \in R(s)$ and $F_1(s)/F_0(s) \in RH_\infty$.*

The control system in Figure 1 is robustly stable for $G(s) \in \Omega$ if and only if

$$\|S_1(s)W(s)\|_\infty < 1. \tag{7}$$

Theorem 2.2 states the following expressions:

- Making the sensitivity function a small value, the control system shown in Figure 1 is robustly stable.
- From Definition 2.1, the robust stability condition does not depend on the number of poles in the closed right plane.

Satisfying Theorem 2.2, we can construct a control system with robust stability and low sensitivity. However, the controller $C_1(s)$ satisfying Theorem 2.2 does not always achieve the low-sensitivity characteristics of the control system shown in Figure 1 on the desired frequency range. This is because the sensitivity function $S_1(s)$ often has a large value on the frequency range, making the gain of $W(s)$ small.

Yu et al. expanded the two-degree-of-freedom control system shown in Figure 1 and proposed the control system to have a new double feedback control system [32]. The double feedback control structure is a structure in the two-degree-of-freedom control system that is included in the two-degree-of-freedom control system. Because the double feedback control system has some controllers, we can give several characteristics for each of the controllers. The double feedback control system is shown in Figure 2. Here, $G(s) \in R(s)$ is a strictly proper single-input/single-output non-minimum-phase system, that is, $G(s)$ has a zero in the closed right half plane, $F_1(s) \in RH_\infty$, $F_2(s) \in RH_\infty$, $C_1(s) \in R(s)$, and $C_2(s) \in R(s)$ are the controllers, $r(s)$ is a reference input, and $y(s)$ is an output.

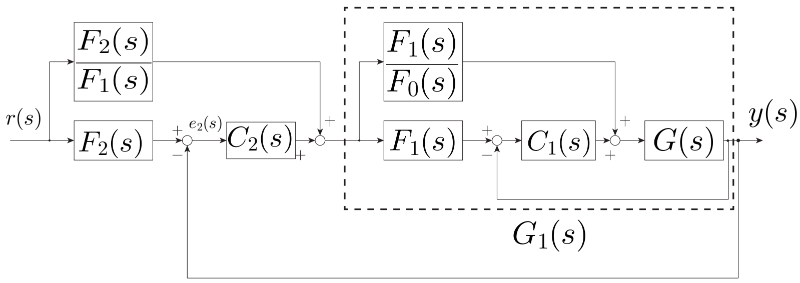

FIGURE 2. The double feedback control system

The transfer function of the double feedback control system shown in Figure 2 from $r(s)$ to $y(s)$ is written in the form

$$y(s) = F_2(s)\left(1 + H_2(s)\right) r(s), \tag{8}$$

where

$$H_2(s) = \frac{S_1(s)S_2(s)\frac{\Delta(s)}{1+\Delta(s)}}{1 - S_1(s)S_2(s)\frac{\Delta(s)}{1+\Delta(s)}}. \tag{9}$$

Here, $S_2(s)$ is a function described as

$$S_2(s) = \frac{1}{1 + F_1(s)C_2(s)}. \tag{10}$$

The uncertainty $\Delta(s)$ to $y(s)$ of the double feedback control system shown in Figure 2 can be reduced by making the function denoted as

$$S(s) = S_1(s)S_2(s) \tag{11}$$

small. The function $S(s)$ is called the sensitivity function of the double feedback control system shown in Figure 2. The sensitivity function of the double feedback control system shown in Figure 2 differs from that of the two-degree-of-freedom control system shown in Figure 1 in that not only $S_1(s)$ but also $S_2(s)$ is involved.

In this paper, we show proof of the necessary and sufficient conditions to make low-sensitivity control guarantee robust stability of the control system shown in Figure 2 and propose a design procedure of the control system with low sensitivity and robust stability shown in Figure 2. This is similar to the results in [33]. However, because the plant and the nominal plant are assumed to be minimum-phase systems in [32, 33], the results of [32, 33] cannot be applied to the non-minimum-phase system, which has some zeros in the closed right half plane. The following are some applications of the non-minimum-phase system: the control of a four-wheel steering vehicle [34]; the control of V/STOL Aircraft [35]; and the control of large flexible spacecraft such as the Hubble space telescope [36].

The problem considered in this paper expands the problem in [33]. The necessary and sufficient condition to make the control system robustly stable is applicable for non-minimum-phase systems.

3. Robust Stability Condition of the Double Feedback Control System.

In this section, a robust stability condition of the double feedback control system in Figure 2 for $G(s) \in \Omega$ and evidence of its robust stability condition is clarified.

The robust stability condition of the double feedback control system shown in Figure 2 for $G(s) \in \Omega$ is summarized as follows.

Theorem 3.1. [32] *Assume that $C_1(s)$ stabilizes $F_0(s) \in R(s)$, $F_1(s)/F_0(s) \in RH_\infty$, $C_2(s)$ stabilizes $F_1(s) \in RH_\infty$, and $F_2(s)/F_1(s) \in RH_\infty$. The control system in Figure 2 is stable for $G(s) \in \Omega$ if and only if*

$$\|S_1(s)S_2(s)W(s)\|_\infty < 1. \tag{12}$$

The proof of Theorem 3.1 requires the following lemma.

Lemma 3.1. *It is assumed that $F_0(s)$ has q-th zero in the closed right half plane and the p_m-th number of the pole in the closed right half plane and that $G(s)$ has q-th zero in the closed right half plane and the p-th number of the pole in the closed right half plane. Then, the Nyquist plot of $1 + \Delta(j\omega)$ for $-\infty \leq \omega \leq \infty$ encircles the origin $(0, 0)$ $p - p_m$ times in the counterclockwise direction* [18].

Using Lemma 3.1, we prove Theorem 3.1.

Proof: The characteristic polynomial of the double feedback control system in Figure 2 is given by

$$1 + \left\{ \left(\frac{F_1(s)}{F_0(s)} + C_1(s)F_1(s) \right) C_2(s) + C_1(s) \right\} G(s). \tag{13}$$

From the assumption that $F_1(s) \in RH_\infty$ and $F_0(s)$ and $G(s)$ have the same number of zeros in the closed right half plane if the Nyquist plot of the characteristic polynomial in (13) for $-\infty \leq \omega \leq \infty$ encircles the origin $(0, 0)$ $p+p_{c1}+p_{c2}$ times in the counterclockwise direction, then the control system in Figure 2 is stable. The characteristic polynomial in (13) is rewritten as

$$\begin{aligned} &1 + \left\{ \left(\frac{F_1(s)}{F_0(s)} + C_1(s)F_1(s) \right) C_2(s) + C_1(s) \right\} G(s) \\ &= (1 + F_1(s)C_2(s))(1 + F_0(s)C_1(s))(1 + \Delta(s)) \left(1 - S_2(s)S_1(s)\frac{\Delta(s)}{1 + \Delta(s)} \right). \end{aligned} \tag{14}$$

From the assumption that $C_1(s)$ stabilizes $F_0(s)$, the Nyquist plot of $1 + C_1(s)F_0(s)$ for $-\infty \leq \omega \leq \infty$ encircles the origin $(0, 0)$ $p_m + p_{c1}$ times in the counterclockwise direction, where p_m is the number of unstable poles of $F_0(s)$ and p_{c1} is that of $C_1(s)$. Because $C_2(s)$ stabilizes $F_1(s) \in RH_\infty$, the Nyquist plot of $1 + C_2(s)F_1(s)$ for $-\infty \leq \omega \leq \infty$ encircles the origin $(0, 0)$ p_{c2} times in the counterclockwise direction, where p_{c2} is the number of unstable poles of $C_2(s)$. From Lemma 3.1, the Nyquist plot of $1 + \Delta(s)$ for $-\infty \leq \omega \leq \infty$ encircles the origin $(0, 0)$ $p - p_m$ times in the counterclockwise direction. Thus, the necessary and sufficient condition that the double feedback control system in Figure 2 is stable for $G(s) \in \Omega$ is that the Nyquist plot of $1 - S_1(s)S_2(s)\Delta(s)/(1 + \Delta(s))$ does not encircle the origin $(0, 0)$ any times.

The remaining problem is to prove the necessary and sufficient condition that $1 - S_1(s)S_2(s)\Delta(s)/(1 + \Delta(s))$ does not encircle the origin any times is equivalent to (12). We adopt the same procedure as in [13] to prove this.

First, sufficiency is shown, that is, we show that if $\|S_1(s)S_2(s)W(s)\|_\infty < 1$, then the Nyquist plot of $1 - S_1(s)S_2(s)\Delta(s)/(1 + \Delta(s))$ encircles the origin zero times. It is clear that the Nyquist plot of $1 - S_1(s)S_2(s)\Delta(s)/(1 + \Delta(s))$ never encircles the origin for any $\Delta(s)$. Thus, sufficiency has been proved.

Next, necessity is shown, that is, we show that if $\|S_1(s)S_2(s)W(s)\|_\infty > 1$, then there exists $\Delta(s) \in \Omega$ to make the Nyquist plot of $1 - S_1(s)S_2(s)\Delta(s)/(1 + \Delta(s))$ encircle the origin $(0, 0)$. From the assumption of $\|S_1(s)S_2(s)W(s)\|_\infty \geq 1$, there exist ω and $\epsilon > 0$ satisfying

$$|S_1(j\omega)S_2(j\omega)W(j\omega)| = 1 + \epsilon. \tag{15}$$

If we set

$$\frac{\Delta(j\omega)}{1 + \Delta(j\omega)} = \frac{|W(j\omega)|}{1 + \epsilon} < |W(j\omega)|, \tag{16}$$

then we have

$$1 - S_1(j\omega)S_2(j\omega)\frac{\Delta(j\omega)}{1 + \Delta(j\omega)} = 0. \tag{17}$$

This implies that the Nyquist plot of $1 - S_1(s)S_2(s)\Delta(s)/(1 + \Delta(s))$ passes through the origin. Therefore, for this $\Delta(s) \in \Omega$, the system in Figure 2 is unstable. Thus, necessity has been proved.

We have thus proved Theorem 3.1. □

4. **Control Characteristics for the Double Feedback Control System.** In this section, we consider a design method of the double feedback control system shown in Figure 2 to make the influence of uncertainty $\Delta(s)$ on the output $y(s)$ smaller than that of the two-degree-of-freedom control system shown in Figure 1. To make the double feedback control system robustly stable for the class of uncertainty Ω, the double feedback control system shown in Figure 2 must satisfy Theorem 3.1. For the double feedback control system shown in Figure 2 with robust stability and low sensitivity, the influence of $\Delta(s)$ on $y(s)$ of the double feedback system shown in Figure 2 can be smaller than that of the two-degree-of-freedom control system shown in Figure 1.

To compare the influence of $\Delta(s)$ on $y(s)$ of the double feedback control system shown in Figure 2 and that of the two-degree-of-freedom control system shown in Figure 1, let $F_2(s)$ be $F_2(s) = F_1(s)$. The influence of $\Delta(s)$ on $y(s)$ of the double feedback control system shown in Figure 2 and that of the two-degree-of-freedom control system shown in Figure 1 can be compared by considering the function denoted as

$$\frac{H_2(s)}{H_1(s)} = \frac{S_2(s)\left(1 - S_1(s)\frac{\Delta(s)}{1+\Delta(s)}\right)}{1 - S_2(s)S_1(s)\frac{\Delta(s)}{1+\Delta(s)}} = 1 + \frac{1 - S_2(s)}{1 - S_2(s)S_1(s)\frac{\Delta(s)}{1+\Delta(s)}}. \tag{18}$$

From (18), if the gain of $H_2(s)/H_1(s)$ is close to 1, the influence of $\Delta(s)$ on $y(s)$ of the double feedback control system shown in Figure 2 is equal to that of the two-degree-of-freedom control system shown in Figure 1. By contrast, if the gain of $H_2(s)/H_1(s)$ is less than 1, the influence of $\Delta(s)$ on $y(s)$ of the double feedback control system shown in Figure 2 is smaller than that of the two-degree-of-freedom control system shown in Figure 1. From (18), to make the gain of $H_2(s)/H_1(s)$ close to 0 on the frequency range $\hat{\omega} \in [0, \omega_1]$ ($\omega_1 \in R$), $S_2(s)$ must be $|S_2(j\omega)| \approx 0, \omega \in R$.

From the assumption that $F_1(s)/F_0(s) \in RH_\infty$, $F_1(s) \in RH_\infty$ is written as

$$F_1(s) = N_m(s)\hat{Q}_1(s), \tag{19}$$

where $\hat{Q}_1(s)$ is an arbitrary strictly stable function and $N_m(s)$ is a coprime factor of $F_0(s)$ on RH_∞ satisfying

$$F_0(s) = \frac{N_m(s)}{D_m(s)}, \tag{20}$$

with $D_m(s) \in RH_\infty$. Because $N_m(s)$ is a non-minimum-phase, as $F_1(s) \in RH_\infty$ is a non-minimum-phase system and $C_2(s)$ stabilizes $F_1(s)$, the stabilizing controller $C_2(s)$ can be described as

$$C_2(s) = \frac{Q_2(s)}{1 - Q_2(s)F_1(s)}, \tag{21}$$

where $Q_2(s) \in RH_\infty$ is any function. From (21), $S_2(s)$ can be rewritten in the form

$$S_2(s) = 1 - Q_2(s)F_1(s). \tag{22}$$

5. **Design Procedure.** In this section, a design procedure for the double feedback control system in Figure 2 is presented.

A design procedure for the double feedback control system with robust stability and low-sensitivity characteristics satisfying Theorem 3.1 is summarized as follows.

Step 1 Obtain a coprime factor $N_m(s) \in RH_\infty$ and $D_m(s) \in RH_\infty$ of the nominal plant $F_0(s)$ on RH_∞ satisfying (20).

Step 2 $N_m(s)$ can be factorized as follows:

$$N_m(s) = N_{mo}(s)N_{mi}(s), \tag{23}$$

where $N_{mo}(s) \in RH_\infty$ is an outer function and $N_{mi}(s) \in RH_\infty$ is an inner function on RH_∞ satisfying $N_{mi}(0) = 1$.

Step 3 Using $N_{mo}(s)$, design $Q_{f1}(s)$ satisfying

$$Q_{f1}(s) = \frac{1}{N_{mo}(s)}q_{f1}(s), \tag{24}$$

where $q_{f1}(s)$ is a function described as

$$q_{f1}(s) = \frac{1}{(1 + \tau_{q1}s)^{\alpha_{q1}}}, \tag{25}$$

where $\tau_{q1} \in \mathbb{R}$ is an arbitrary real number and $\alpha_{q1} \in \mathbb{N}$ is an arbitrary integer to make $Q_{f1}(s)$ proper.

Step 4 Using $Q_1(s)$, design $F_1(s)$ as follows:

$$F_1(s) = N_m(s)Q_{f1}(s), \\ = N_{mi}(s)q_{f1}(s). \tag{26}$$

Step 5 Design $C_1(s)$ to make $S_1(s)$ satisfy (7) by using the Riccati equation-based H_∞ control.

Step 6 $F_2(s)$ is settled to make $F_2(s)$ be $F_2(s) = F_1(s)$.

Step 7 Design $Q_2(s) \in RH_\infty$ satisfying

$$Q_2(s) = \frac{1}{q_{f1}(s)} q_2(s), \tag{27}$$

where $q_2(s)$ is a function described as

$$q_2(s) = \frac{1}{(1 + \tau_{q2}s)^{\alpha_{q2}}}, \tag{28}$$

where $\tau_{q2} \in \mathbb{R}$ is an arbitrary real number and $\alpha_{q2} \in \mathbb{N}$ is an arbitrary integer to make $Q_2(s)$ proper.

Step 8 Using $Q_2(s)$, $C_2(s)$ is settled as

$$C_2(s) = \frac{Q_2(s)}{1 - Q_2(s)F_1(s)}. \tag{29}$$

6. **Numerical Examples.** In this section, we show two numerical examples to illustrate the effectiveness of the proposed method. The effectiveness of the proposed method is shown by comparing the error between the reference input and the output of the two-degree-of-freedom control system shown in Figure 1 and that of the double feedback control system shown in Figure 2. To show the effectiveness of the proposed method, we present two numerical examples as follows.

1) $G(s)$ and $F_0(s)$ have some of the same poles and zeros in the closed right half plane.
2) $G(s)$ has different poles and zeros in the closed right half plane from $F_0(s)$.

6.1. ***G(s)* and *F*$_0$*(s)* have some of the same poles and zeros in the closed right half plane.** In this subsection, we present a numerical example of the case that $G(s)$ and $F_0(s)$ have some of the same poles and zeros in the closed right half plane.

Consider the problem of designing a robustly stabilizing controller for the set $G(s) \in \Omega$, where

$$F_0(s) = \frac{0.5(s-50)(s+1)}{(s+2)(s+3)(s-3)}, \tag{30}$$

and

$$W(s) = \frac{0.20833(s+73.72)(s+16.28)}{(s+79.87)(s+0.1252)}. \tag{31}$$

The nominal plant $F_0(s)$ described as (30) has zero at $s = 2$ and is a non-minimum-phase system. For the non-minimum-phase system such as $F_0(s)$ described as (30), the results of [33] do not always apply for the non-minimum-phase system having varying numbers of poles in the closed right half plane. The proposed design method in this paper can be applied to a non-minimum-phase system.

First, we design $F_1(s)$ and $C_2(s)$. From (30), obtain $N_m(s)$ and $D_m(s)$ satisfying (20) as follows:

$$N_m(s) = \frac{0.5(s-50)}{(s+2)(s+3)}, \tag{32}$$

and

$$D_m(s) = \frac{s-3}{s+1}. \tag{33}$$

From (32), factors $N_{mi}(s)$ and $N_{mo}(s)$ satisfying (23) are given as

$$N_{mi}(s) = \frac{-s+50}{s+50}, \tag{34}$$

and

$$N_{mo}(s) = \frac{-0.5(s+50)}{(s+3)(s+2)}. \tag{35}$$

Using $N_{mo}(s)$, $Q_{f1}(s)$ is designed as

$$Q_{f1}(s) = \frac{2(s+3)(s+2)}{s+50} q_{f1}(s), \tag{36}$$

where $q_{f1}(s)$ is a function described as

$$q_{f1}(s) = \frac{1}{(0.001s+1)^2}. \tag{37}$$

Using $Q_{f1}(s)$ in (36) and $N_m(s)$ in (32), $F_1(s)$ is obtained as

$$F_1(s) = \frac{s-50}{s+50} q_{f1}(s). \tag{38}$$

$C_1(s)$ satisfying Theorem 2.2 is designed using a linear matrix inequality. $C_1(s)$ is obtained as

$$C_1(s) = \frac{-77938.5328(s+71.84)(s+2.999)(s+2.473)(s+2)}{(s+2059)(s+79.87)(s+64.38)(s+1)(s-0.172)}. \tag{39}$$

To confirm that the designed $C_1(s)$ satisfies Theorem 2.2, the gain plots of $S_1(s)$ and $1/W(s)$ are shown in Figure 3. Here, the solid line shows the gain plot of $S_1(s)$ and the dashed line shows the gain plot of $1/W(s)$. Because the gain plot of $S_1(s)$ is less than that of $1/W(s)$ over the whole frequency range, Figure 3 shows that $C_1(s)$ satisfies (7).

Next, we design $F_2(s)$ and $C_1(s)$. To compare the influence of uncertainty on the output of the control system shown in Figure 2 and that of the two-degree-of-freedom control

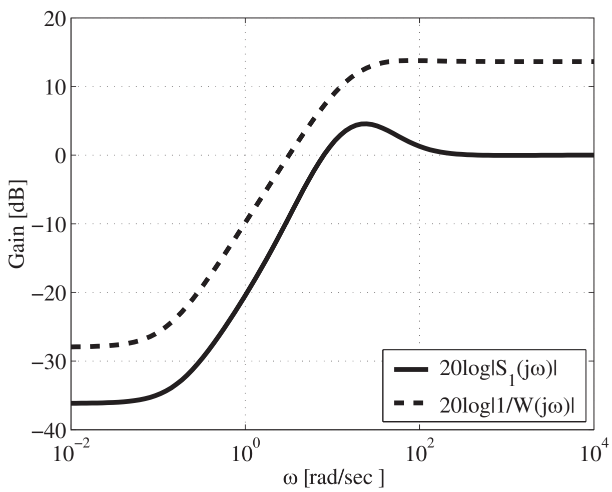

FIGURE 3. The gain plot of $S_1(s)$ and that of $1/W(s)$

system shown in Figure 1, $F_2(s)$ is chosen as $F_2(s) = F_1(s)$. $Q_2(s)$ in (27) is chosen as

$$Q_2(s) = 1, \tag{40}$$

where $q_2(s)$ in (28) is chosen as

$$q_2(s) = \frac{1}{(0.001s+1)^2}. \tag{41}$$

Using $Q_2(s)$ in (40) and $F_1(s)$ in (38), $C_2(s)$ is designed as

$$C_2(s) = \frac{(s+50)(0.001s+1)^2}{s(s^2+2050s+2.1\times 10^6)}. \tag{42}$$

To confirm that $C_2(s)$ satisfies Theorem 3.1, the gain plots of $S_2(s)S_1(s)$ and $1/W(s)$ are shown in Figure 4. Here, the solid line shows the gain plot of $S_2(s)S_1(s)$ and the dashed line shows the gain plot of $1/W(s)$. Because the gain plot of $S_1(s)S_2(s)$ is less than that of $1/W(s)$ over the whole frequency range, Figure 4 shows that $C_2(s)$ satisfies (12).

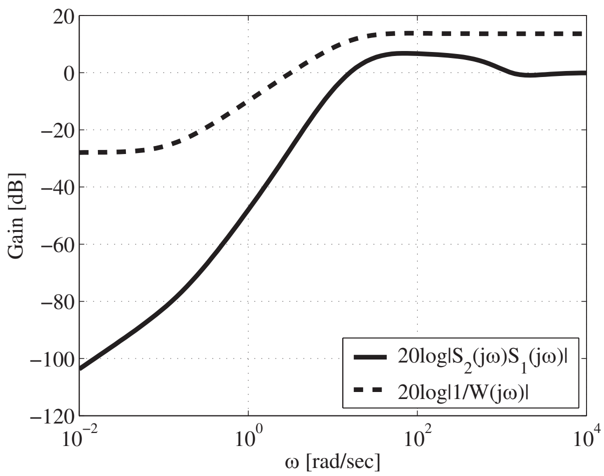

FIGURE 4. The gain plot of $S_2(s)S_1(s)$ and that of $1/W(s)$

Let $G(s)$ be

$$G(s) = \frac{0.45(s-50)(s+1)}{(s-0.06)(s-3.5)(s+6)}. \tag{43}$$

From (30) and (43), $G(s)$ has the same poles and zeros as $F_0(s)$ in the closed right half plane. In addition, the number of poles of the plant $G(s)$ in the closed right half plane is not equal to that of the nominal plant $F_0(s)$. The fact that $\Delta(s)$ is included in the set Ω is confirmed by showing gain plots of $\Delta(s)/(1+\Delta(s))$ and $W(s)$. The gain plots of $\Delta(s)/(1+\Delta(s))$ and $W(s)$ are shown as Figure 5. Here, the solid line shows the gain plot of $\Delta(s)/(1+\Delta(s))$ and the dashed line shows the gain plot of $W(s)$. Because the gain plot of $\Delta(s)/(1+\Delta(s))$ is less than that of $W(s)$, the uncertainty $\Delta(s)$ is included in the class Ω. Furthermore, the relative degree of $F_0(s)$ is equal to that of $G(s)$. Therefore, the plant $G(s)$ in (43) is an elementary of the set Ω.

To illustrate the effectiveness of the robust stability, the responses of the output using designed controllers from the two-degree-of-freedom control system shown in Figure 1 and the double feedback control system shown in Figure 2 for the reference input

$$r(t) = \sin(t) \tag{44}$$

are shown in Figure 6 and Figure 7.

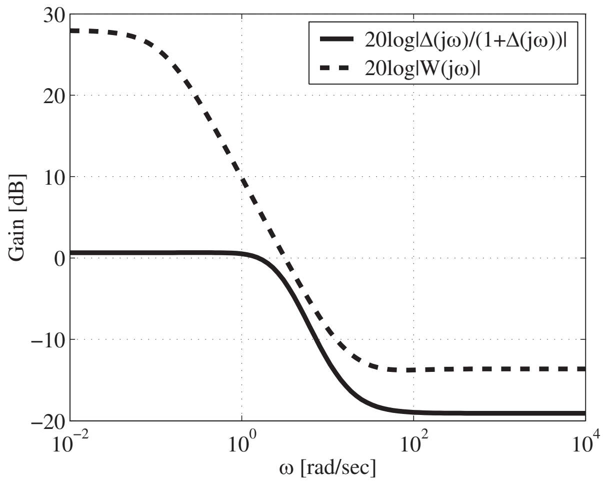

Figure 5. The gain plots of $\Delta(s)/(1 + \Delta(s))$ and $W(s)$

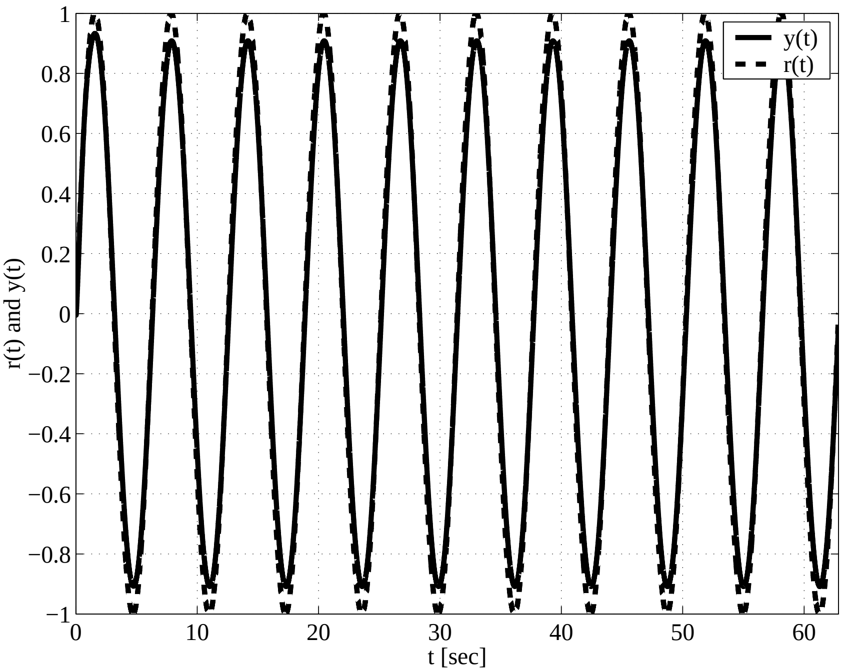

Figure 6. The response of the output from the two-degree-of-freedom control system in Figure 1 and the reference input $r(t)$

Figure 6 shows the response of the output $y(t)$ and the reference input $r(t)$ of the two-degree-of-freedom control system shown in Figure 1. In Figure 6, the solid line shows the response of the output $y(t)$ from the two-degree-of-freedom control system shown in Figure 1 satisfying Theorem 2.2 and the dashed line shows the reference input $r(t)$.

From Figure 6, we understand that the two-degree-of-freedom control system shown in Figure 1 satisfying Theorem 2.2 is stable for the class $G(s) \in \Omega$.

Figure 7 shows the response of the output $y(t)$ from the double feedback control system shown in Figure 2 satisfying Theorem 3.1 and the reference input $r(t)$. Here, the solid line shows the response of the output $y(t)$ from the double feedback control system shown in Figure 2 satisfying Theorem 3.1, and the dashed line shows the reference input $r(t)$. Figure 7 shows that the two-degree-of-freedom control system shown in Figure 2 satisfying Theorem 3.1 is stable for the class $G(s) \in \Omega$.

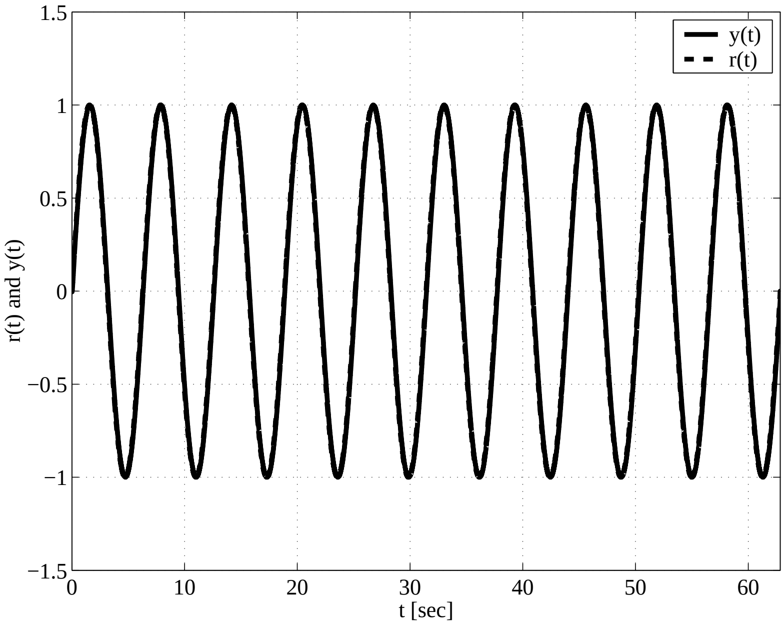

FIGURE 7. The response of the output from the double feedback control system Figure 2 and the reference input $r(t)$

To illustrate the effectiveness of robust performances, we show the error of the output $y(t)$ and the reference input $r(t)$ of the two-degree-of-freedom control system shown in Figure 1 and the double feedback control system shown in Figure 2.

The error $e(t) = r(t) - y(t)$ of the two-degree-of-freedom control system shown in Figure 1 and the double feedback control system shown in Figure 2 is shown in Figure 8. Here, the solid line shows the error $e(t) = r(t) - y(t)$ of the double feedback control system

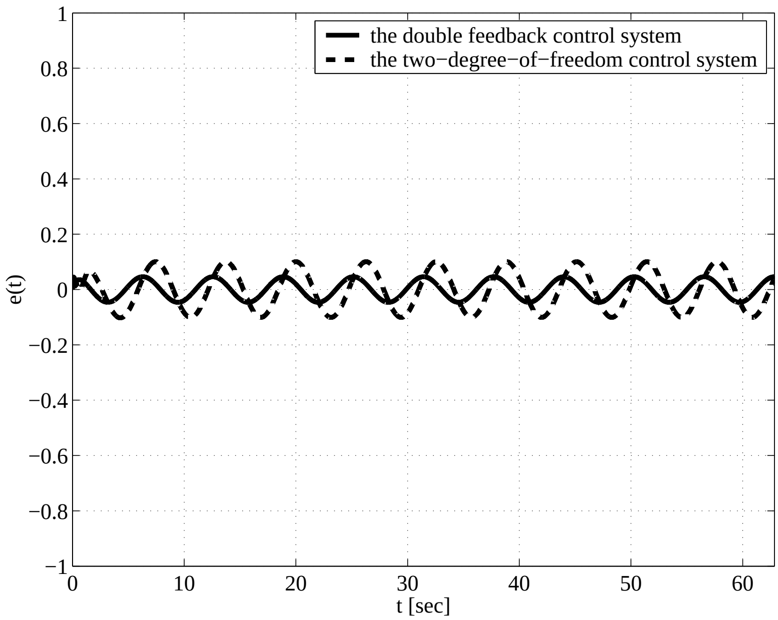

FIGURE 8. The error $e(t) = r(t) - y(t)$ of the two-degree-of-freedom control system shown in Figure 1 and the double feedback control system shown in Figure 2

shown in Figure 2, and the dashed line shows the error $e(t) = r(t) - y(t)$ of the two-degree-of-freedom control system shown in Figure 1. From Figure 8, the error between the reference input $r(t)$ and the output $y(t)$ of the double feedback control system shown in Figure 2 is smaller than that of the two-degree-of-freedom control system shown in Figure 1.

In this way, it is shown that by using the proposed design method of the double feedback control system in Figure 2, we can design the double feedback control system in Figure 1 with robust stability and low-sensitivity characteristics.

6.2. **$G(s)$ has different poles and zeros in the closed right half plane from $F_0(s)$.** In this subsection, we present a numerical example of the case that $G(s)$ has different poles and zeros in the closed right half plane from $F_0(s)$.

Consider the problem of designing a robustly stabilizing controller for the set $G(s) \in \Omega$, where

$$F_0(s) = \frac{0.5(s-30)(s+1)}{(s+2)(s+3)(s-3)}, \tag{45}$$

and

$$W(s) = \frac{0.20833(s+73.72)(s+16.28)}{(s+79.87)(s+0.1252)}. \tag{46}$$

First, we design $F_1(s)$ and $C_2(s)$. From (45), obtain $N_m(s)$ and $D_m(s)$, satisfying (20) as follows:

$$N_m(s) = \frac{0.5(s-30)(s+1)}{(s+4)(s+3)(s+2)}, \tag{47}$$

and

$$D_m(s) = \frac{s-3}{s+4}. \tag{48}$$

From (47), factors $N_{mi}(s)$ and $N_{mo}(s)$ satisfying (23) are given as

$$N_{mi}(s) = \frac{-s+30}{s+30}, \tag{49}$$

and

$$N_{mo}(s) = \frac{-0.5(s+30)(s+1)}{(s+4)(s+3)(s+2)}. \tag{50}$$

Using $N_{mo}(s)$, $Q_{f1}(s)$ is designed as

$$Q_{f1}(s) = \frac{-2(s+4)(s+3)(s+2)}{(s+30)(s+1)} q_{f1}(s), \tag{51}$$

where $q_{f1}(s)$ is a function described as

$$q_{f1}(s) = \frac{1}{(0.01s+1)^2}. \tag{52}$$

Using $Q_{f1}(s)$ in (51) and $N_m(s)$ in (47), $F_1(s)$ is obtained as

$$F_1(s) = \frac{-s+30}{s+30} q_{f1}(s). \tag{53}$$

$C_1(s)$ satisfying Theorem 2.2 is designed using a linear matrix inequality. $C_1(s)$ is obtained as

$$C_1(s) = \frac{-9973363.5362(s+83.34)(s+3)(s+2.589)(s+2)}{(s+3877)(s+2758)(s+79.84)(s+0.9996)(s-0.05127)}. \tag{54}$$

To confirm that the designed $C_1(s)$ satisfies Theorem 2.2, the gain plots of $S_1(s)$ and $1/W(s)$ are shown in Figure 9. Here, the solid line shows the gain plot of $S_1(s)$ and the dashed line shows the gain plot of $1/W(s)$. Because the gain plot of $S_1(s)$ is less than that of $1/W(s)$ over the whole frequency range, Figure 9 shows that $C_1(s)$ satisfies (7).

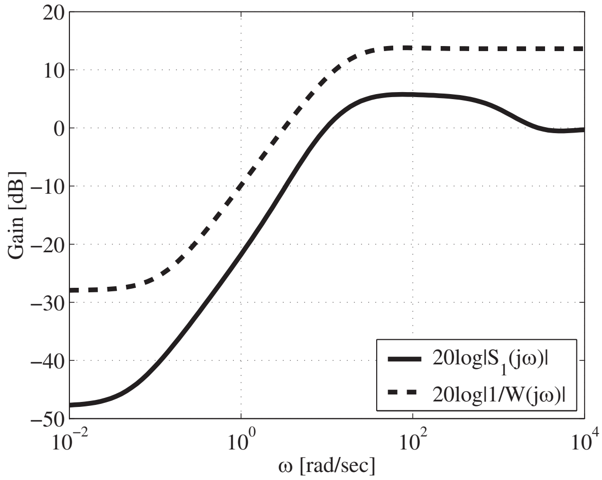

FIGURE 9. The gain plots of $S_1(s)$ and $1/W(s)$

Next, we design $F_2(s)$ and $C_1(s)$. To compare the influence of uncertainty on the output of the control system shown in Figure 2 and that of the two-degree-of-freedom control system shown in Figure 1, $F_2(s)$ is chosen as $F_2(s) = F_1(s)$. $Q_2(s)$ in (27) is chosen as

$$Q_2(s) = 1, \tag{55}$$

where $q_2(s)$ in (28) is chosen as

$$q_2(s) = \frac{1}{(0.001s + 1)^2}. \tag{56}$$

Using $Q_2(s)$ in (40) and $F_1(s)$ in (53), $C_2(s)$ is designed as

$$C_2(s) = \frac{(s + 30)(s + 1000)^2}{s(s^2 + 2030s + 2.06 \times 10^6)}. \tag{57}$$

To ensure $C_2(s)$ satisfies Theorem 3.1, the gain plots of $S_2(s)S_1(s)$ and $1/W(s)$ are shown in Figure 10. Here, the solid line shows the gain plot of $S_2(s)S_1(s)$, and the dashed line shows the gain plot of $1/W(s)$. Because the gain plot of $S_1(s)S_2(s)$ is less than that of $1/W(s)$ over the whole frequency range, Figure 10 shows that $C_2(s)$ satisfies (12).

Let $G(s)$ be

$$G(s) = \frac{0.45(s - 28.3)(s + 1)}{(s - 0.06)(s - 3.5)(s + 6)}. \tag{58}$$

From (45) and (58), $G(s)$ does not have the same poles and zeros as $F_0(s)$ in the closed right half plane. The gain plots of $\Delta(s)/(1 + \Delta(s))$ and $W(s)$ are shown as Figure 11. Here, the solid line shows the gain plot of $\Delta(s)/(1+\Delta(s))$ and the dashed line shows the gain plot of $W(s)$. Because the gain plot of $\Delta(s)/(1 + \Delta(s))$ is less than that of $W(s)$, the uncertainty $\Delta(s)$ is included in the class Ω. Furthermore, the relative degree of $F_0(s)$ is equal to that of $G(s)$. Therefore, the plant $G(s)$ in (58) is an elementary of the set Ω.

Using the designed controllers, the responses of the output from the two-degree-of-freedom control system shown in Figure 1 and the double feedback control system shown

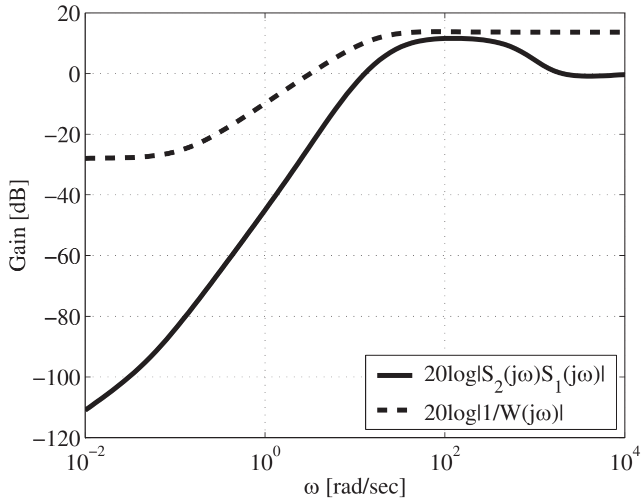

FIGURE 10. The gain plots of $S_2(s)S_1(s)$ and $1/W(s)$

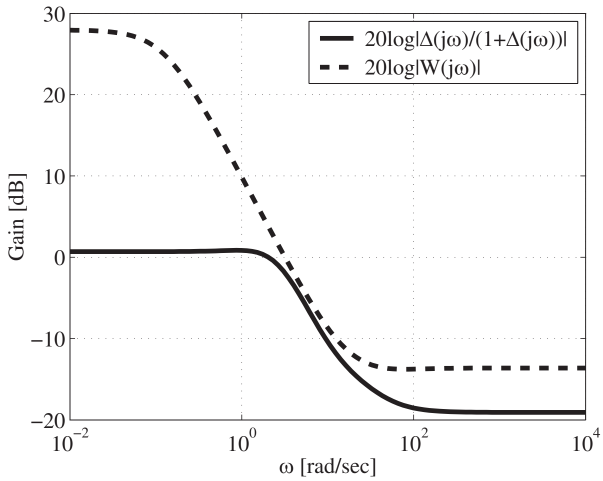

FIGURE 11. The gain plots of $\Delta(s)/(1+\Delta(s))$ and $W(s)$

in Figure 2 for the reference input

$$r(t) = \sin(t) \tag{59}$$

are shown in Figure 12 and Figure 13, respectively.

Figure 12 and Figure 13 are presented to show the effectiveness of robust stability. Figure 12 shows the response of the output $y(t)$ from the two-degree-of-freedom control system shown in Figure 1, satisfying Theorem 2.2 and the reference input $r(t)$. Here, the solid line shows the response of the output $y(t)$ from the two-degree-of-freedom control system shown in Figure 1, satisfying Theorem 2.2, and the dashed line shows the reference input $r(t)$. From Figure 12, we understand that the two-degree-of-freedom control system shown in Figure 1, satisfying Theorem 2.2, is stable for the class $G(s) \in \Omega$.

Figure 13 shows the response of the output $y(t)$ from the double feedback control system shown in Figure 2, satisfying Theorem 3.1 and the reference input $r(t)$. Here, the solid line shows the response of the output $y(t)$ from the double feedback control system shown in Figure 2 satisfying Theorem 3.1, and the dashed line shows the reference input $r(t)$. From Figure 13, we understand that the two-degree-of-freedom control system shown in Figure 2 satisfying Theorem 3.1 is stable for the class $G(s) \in \Omega$.

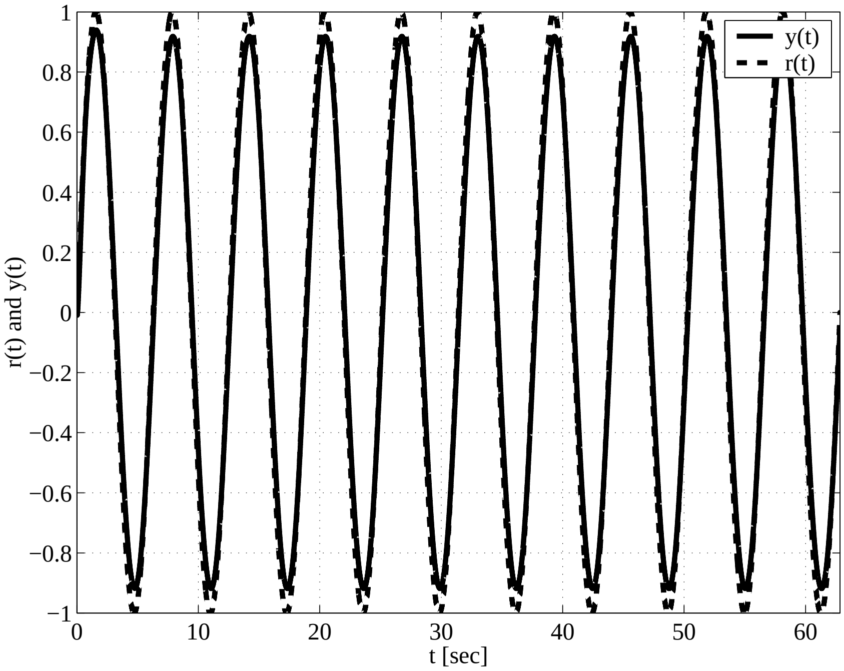

FIGURE 12. The response of the output from the two-degree-of-freedom control system in Figure 1 and the reference input $r(t)$

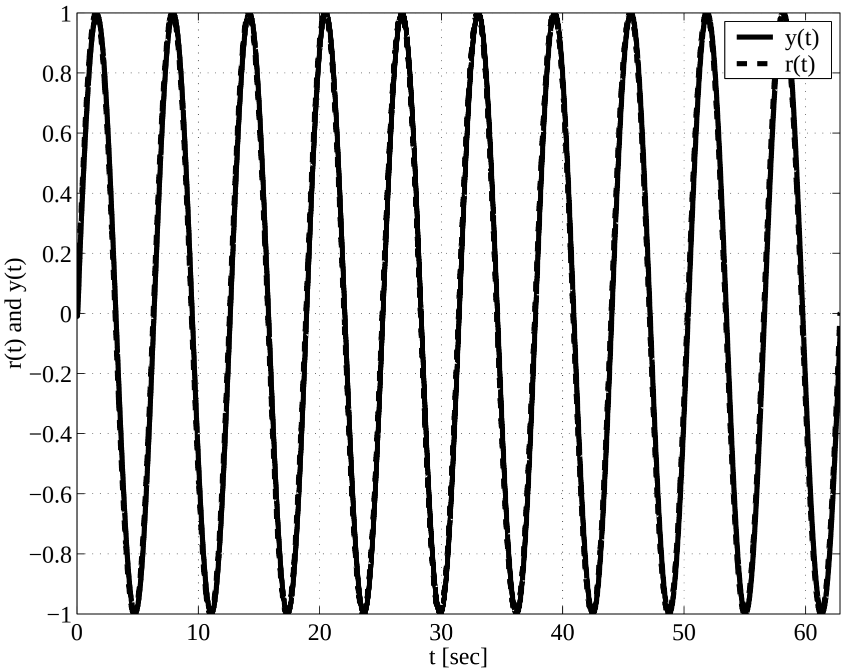

FIGURE 13. The response of the output from the double feedback control system Figure 2 and the reference input $r(t)$

To illustrate the effectiveness of robust performances, we show the error of the output $y(t)$ and the reference input $r(t)$ of the two-degree-of-freedom control system shown in Figure 1 and the double feedback control system shown in Figure 2.

The error $e(t) = r(t) - y(t)$ of the two-degree-of-freedom control system shown in Figure 1 and the double feedback control system shown in Figure 2 is shown in Figure 14. Here, the solid line shows the error $e(t) = r(t) - y(t)$ of the double feedback control

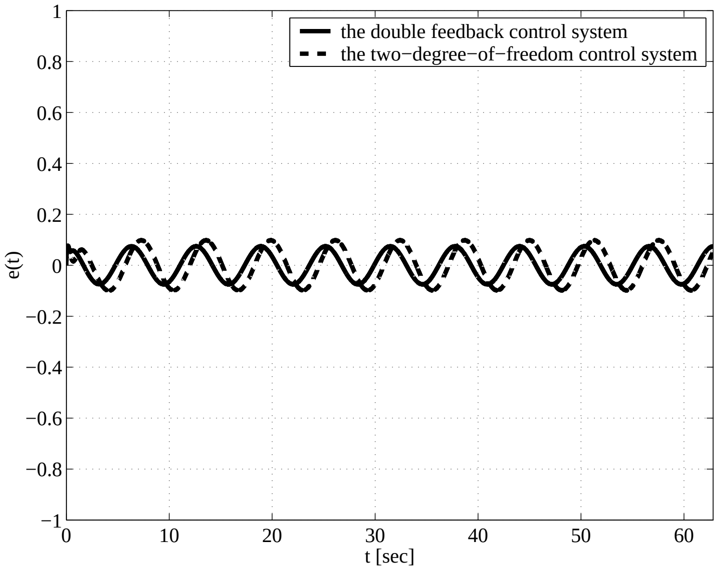

FIGURE 14. The error $e(t) = r(t) - y(t)$ of the two-degree-of-freedom control system shown in Figure 1 and the double feedback control system shown in Figure 2

system shown in Figure 2 and the dashed line shows the error $e(t) = r(t) - y(t)$ of the two-degree-of-freedom control system shown in Figure 1. Figure 14 shows that the error between the reference input $r(t)$ and the output $y(t)$ of the double feedback control system shown in Figure 2 is smaller than that of the two-degree-of-freedom control system shown in Figure 1.

In this way, it is shown that by using the proposed design method of the double feedback control system in Figure 2, we can design the double feedback control system in Figure 1 with robust stability and low-sensitivity characteristics.

7. **Conclusion.** In this paper, we have proposed a control structure called double feedback control structure for non-minimum systems and have shown a complete proof of the robust stability condition of the control system using a double feedback control system. In addition, a design method and a design procedure for the control system using double feedback control are described. A numerical example is presented to show the effectiveness of the proposed method. Note that the proposed design method of the control system using double feedback control is only considered to increase a robust performance such as disturbance attenuation on a low-frequency range. Some elements of the uncertainty often have a high-frequency range component caused by the sensor noise. To reduce the influence of the high-frequency range components on the output, in general, the complementary sensitivity function must be small. Because making the complementary sensitivity function small causes the sensitivity function to become larger, it is difficult to make a control system with low sensitivity reduce the influence of the high-frequency range components on the output. However, the double feedback control structure can make a control system with low sensitivity reduce the influence of the high-frequency components of the uncertainty on the output. Using the results in this paper, we will consider a design method for a control system using a double feedback control structure to reduce the influence of the high-frequency components of the uncertainty on the output in another article.

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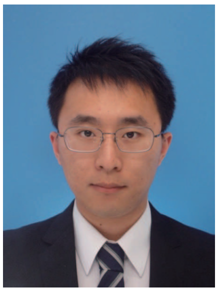

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