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OBSERVER BASED FINITE-TIME CONSENSUS BY INTEGRAL SLIDING MODE CONTROL FOR MULTIPLE PERMANENT MAGNET LINEAR SYNCHRONOUS MOTORS

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Abstract. *The finite-time tracking and synchronization performance of multi-motor systems is essential to ensuring safety, reliability, and quality in manufacturing systems' production processes due to the presence of unknown compound disturbance. This paper proposes a consensus tracking control method based on integral sliding mode control for achieving coordination control of multiple linear motor systems with disturbance. Firstly, the distributed consensus tracking control protocol is adopted to optimize tracking and synchronization performance. Secondly, an integrated sliding mode controller is introduced to enhance the stability and robustness of the system. Moreover, considering the potential impact of disturbances on the system, we propose a nonlinear disturbance observer to estimate and compensate for the disturbances. This observer serves as a feedforward compensation component for the controller. Then, the effectiveness of the proposed method is subsequently verified using the Lyapunov stability theory. The suggested algorithm demonstrates efficient performance in terms of synchronization control accuracy, disturbance immunity, and convergence, as illustrated by the simulation results in the end.*

Keywords: Permanent magnet linear synchronous motor, Finite-time consensus, Integral sliding mode control, Nonlinear disturbance observer, Cooperative control

1. **Introduction.** In recent years, linear motors have been widely used in industry for their high precision and low loss [1, 2]. The simple structure, high speed, high thrust, and other advantages make permanent magnet linear synchronous motor (PMLSM) very popular in the industry among various linear motors. With the development of modern industrial production and the improvement of people's requirements for product quality, it is increasingly difficult for traditional manufacturing technology to satisfy the needs of current production. To satisfy the requirements of intelligence, more scenes require multiple motors to work together, and relevant control technology has gradually become the key to restrict system performance [3, 4, 5]. At present, the single-motor control strategy cannot be directly applied to the problem of multi-motor cooperative control, which brings resistance to its application and promotion. Therefore, the cooperative control method of the multi-motor system needs to be studied systematically.

Most of the traditional synchronization control schemes adopt the parallel synchronous control strategy of relatively independent uncoupled connection structure [6, 7] and the master-slave control strategy of unidirectional coupling structure [8]. However, there are

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various disturbance in the operation of motors, so it is difficult to obtain better control effects of uncoupled and unidirectional coupling structures in this case. To solve this problem, many coupling control structures have been developed and combined with modern control theory, and a lot of research has made progress [9, 10, 11, 12]. However, as the number of motors increases, the parameter coupling results in a significant increase in the number of on-line calculation of the control scheme. Consequently, the velocity compensation model will be more complicated. Hence, it becomes imperative and holds great significance to explore a synchronization control method that offers superior synchronization performance.

In recent times, the control field has witnessed a surge of interest in cooperative control using multi-agent systems (MAS) [13, 14]. Combining multi-agent consistency theory with advanced control strategy to realize multi-motor cooperative control has become the subject of many researchers [15, 16, 17, 18]. To address the issue of coordination control in traction systems, the multi-agent technology is proposed in [13]. [15] aims to address the same problem and successfully adopts multi-agent technology to achieve speed synchronization. [16] presents a tracking algorithm for ensuring consistency between the master and slave in synchronous and coordinated control of multi-motor systems, and an observer-based variable structure consensus control strategy is employed to achieve precise synchronous control. [17] proposes a method that integrates the leader-tracking consensus algorithm of MAS with a centralized event-triggering scheme. This approach aims to achieve speed synchronization in a multi-motor system and reduce the communication loss caused by continuous control.

In practical applications, control systems such as multi-motor systems require a fast dynamic response and system errors converge to zero within a limited time [19]. The above method does not take the synchronization time of the system into account. In recent years, finite-time consensus control has gained much attention due to its ability to improve synchronization control accuracy and achieve faster convergence speed compared to infinite stable time [20, 21]. [20] studies nonlinear MAS with strict feedback, and designs an observer-based finite-time control method. [21] proposes integrated sliding mode control (ISMC) that combines both discontinuous and continuous approaches. This method builds upon the continuous homogeneous finite-time consistency protocol of nominal MAS and achieves accurate finite-time consistency without disturbances.

For a multi-motor system, the disturbance cannot be ignored, and sliding mode control (SMC) will inevitably bring chattering problems to the system, resulting in system instability. So far, many techniques have been proposed to estimate and compensate for disturbance [22, 23, 24, 25]. In [22], a non-singular fast terminal sliding mode (NFTSM) control strategy is proposed to achieve fast and accurate tracking performance for position adjustment in PMLSMs based on high-order super-twisting observer (HOSTO). In order to overcome the parameter mismatch problem in the current loop, a second-order supertwisted sliding mode observer is constructed, and the corresponding estimates are compensated into the closed-loop control by the feedforward method in [23]. The nonlinear disturbance observer (NDO) can significantly improve the performance of the controlled object by feedforward compensation of the predicted disturbance, and further reduce the buffeting caused by fractional order non-singular fast terminal sliding mode [25].

Inspired by multi-agent finite-time consistency control, a consensus tracking control method based on nonlinear disturbance observer is proposed in this paper. The contributions of this paper are as follows:

1) Based on multi-agent continuous homogeneous finite-time consistency protocol, a synchronization control strategy applied to multi-motor systems is proposed so that multi-motor systems can realize position synchronization in finite time;

2) The ISMC and the exponential approach rate function are used to improve the robustness of the system and make the sliding mode surface converge in finite time;

3) The NDO is used to estimate the load disturbance, reduce the influence of disturbance on position tracking, and suppress the influence of sliding mode surface buffeting on the system.

The structure of this article is as follows. In Section 2, we first analyze the model of PMLSM and provide its structure. We also introduce the relevant concepts of directed graph theory and present a lemma that applies to our study. Section 3 elucidates the derivation process and the stability of the method is proved. In order to show the superiority of the methods presented in this paper, the proposed algorithms are compared with the distributed PI algorithms in [26] in Section 4. It is proved that the proposed method has better tracking and synchronization performance. Finally, in Section 5, we summarize the main findings of our study and suggest potential avenues for future research.

2. Problem Description.

2.1. Dynamic model of PMLSM.

The PMLSM can be viewed as an unfolded version of the permanent magnet synchronous motor (PMSM), where Figure 1 presents a schematic depiction of the linear motor structures. These structures consist of a primary element comprising slider and moving windings, along with a secondary element consisting of a track and permanent magnets. The primary and secondary components are similar to the stator and rotor of a PMSM. When the current passes through the windings, it generates a horizontal magnetic field referred to as a traveling wave magnetic field. This field interacts with the magnetic field produced by the permanent magnet resulting in an electromagnetic thrust that propels the motor. Before delving into the mathematical model of the PMLSM, it is necessary to establish certain assumptions. Hysteresis and eddy current losses are disregarded, and a sinusoidal space distribution of the magnetic field is assumed while neglecting any magnetic flux distortion. Then, the voltage model of PMLSM in d-q coordinates can be written as

$$\begin{cases} u_d = R_s i_d + L_d \dfrac{di_d}{dt} - \dfrac{\pi v}{\tau} L_q i_q \\ u_q = R_s i_q + L_q \dfrac{di_q}{dt} + \dfrac{\pi v}{\tau} L_d i_d + \dfrac{\pi v}{\tau} \psi_f \end{cases} \tag{1}$$

In the equation, variables u_d, u_q denote voltages of the d-q axis, while i_d, i_q represent current of d-q axis. Other variables include τ which stands for pole pitch, L_d, L_q represent

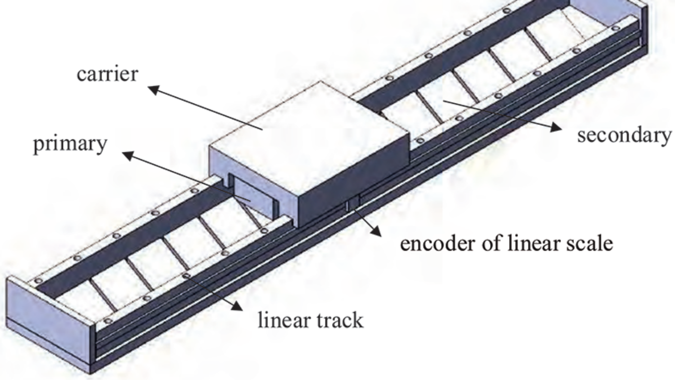

FIGURE 1. Structural diagram of PMLSM

the inductance of d-q axis, R_s which denotes the stator resistance, v indicates the linear motor speed, and ψ_f is the permanent magnet flux.

In this paper, $i_d^* = 0$ field-oriented control (FOC) method is adopted. Because the air gap of the linear motor is relatively large, it can be approximated as $L_d = L_q$. Electromagnetic thrust can be expressed as follows:

$$\begin{cases} F_e = K_f i_q \\ K_f = \dfrac{3\pi n_p}{2\tau}\psi_f \end{cases} \tag{2}$$

The dynamic equation of the PMLSM can be derived, where F_e denotes the electromagnetic thrust and n_p represents the pole pair number of the linear motor.

$$F_e = M\frac{dv}{dt} + Bv + d \tag{3}$$

By taking the mass of the motor as M into account, the viscous friction coefficient as B, and the system's total uncertainty as d, we can derive a simplified mathematical model for PMLSM on the d-q axis:

$$\begin{cases} \dfrac{dv}{dt} = \dfrac{K_f}{M}i_q - \dfrac{B}{M}v - \dfrac{d}{M} \\ \dfrac{di_d}{dt} = \dfrac{1}{L_d}u_d + \dfrac{\pi n_p}{\tau}vi_q - \dfrac{R_s}{L_d}i_d \\ \dfrac{di_q}{dt} = \dfrac{1}{L_q}u_q - \dfrac{\pi n_p}{\tau}vi_d - \dfrac{R_s}{L_q}i_q - \dfrac{\pi n_p \psi_f}{L_q\tau}v \end{cases} \tag{4}$$

For ease of expression, its dynamic equation can be rewritten as

$$\frac{dv}{dt} = A_m v + B_m i_q + D \tag{5}$$

where $A_m = -B/M$, $B_m = K_f/M$, $D = -d/M$.

2.2. **Graph theory.** In the context of a multi-motor system, there is a leader and N followers, where motor 0 represents the virtual leader and motor 1 to N are the followers, and each motor can be regarded as a single node in a multi-agent network. Data exchange between motors is facilitated by communication networks and can be effectively elucidated using concepts derived from graph theory [27, 28, 29].

Using undirected graph $G_n = \{V_n, E_n\}$ to represent multi-motor system topology of the network, the node set $V_n = \{V_1, V_2, \ldots, V_n\}$ represents N motors, edge set $E_n = \{(V_i, V_j)|V_i, V_j \in V_n\}$, $(i, j = 1, 2, \ldots, n)$ represents the connection between the motors. Definition figure of adjacency matrix $A(G) = (a_{ij})_{n\times n}$ is the undirected graph nodes V_i connection between V_j, when $(i, j) \in e$, $a_{ij} = a_{ji} = 1$; otherwise, $a_{ij} = 0$, define the neighborhood set of agent i as $N_i = \{V_j \in V_n : a_{ij} = 1, j \neq i\}$. Further, define the graph Laplacian matrix $L(G) = (l_{ij})_{n\times n}$, when $i = j$, $l_{ii} = \sum_{j=1}^{n} a_{ij}$; otherwise, $l_{ij} = -a_{ij}$.

Let us assume the weight matrix between the leader and follower is represented by the diagonal matrix $B = diag\{a_{10}, a_{20}, \ldots, a_{n0}\}$. In case $a_{i0} = 1$, it signifies that follower i has access to information about the leader. Alternatively, if $a_{i0} = 0$, it implies otherwise. The definition of Laplacian matrix $L(G)$ is a matrix that is both symmetric and semi-definite. A graph can be considered connected if there exists a path connecting any two distinct nodes within it.

2.3. **Some lemmas and assumptions.** For the convenience of proof, the correlation lemmas and assumptions are given.

Lemma 2.1. *$sig^{\alpha}(x) = sign(x)|x|^{\alpha}$, where $\alpha > 0$, $x \in R$, $sign(x)$ is a standard symbolic function.*

Lemma 2.2. *For any a, b, γ, if a nonnegative function satisfies*

$$\dot{V}(x) + \alpha V(x) + \beta V^{\gamma}(x) \leq 0, \quad 0 < \gamma < 1 \tag{6}$$

the system will converge in a finite time, and its convergence time is [30]

$$T \leq \frac{1}{\alpha(1-\gamma)} \ln \frac{\alpha V^{1-\gamma}(x_0) + \beta}{\beta} \tag{7}$$

where $a, b > 0$, $0 < \gamma < 1$.

Lemma 2.3. *For any $x_i \in R$, $i = 1, 2, \ldots, n$, $0 < p < 1$, the following inequality holds:*

$$(|x_1| + \cdots + |x_n|)^p \leq (|x_1|^p + \cdots + |x_n|^p) \tag{8}$$

Assumption 2.1. *In this multi-motor system, the leader is globally reachable.*

Assumption 2.2. *In the motor dynamics equation (5), the load disturbance D of the system is bounded, and there exists a constant $l > 0$ that $|D| \leq l$.*

3. Controller Design.

3.1. Homogeneous finite-time protocol.

The state-space equation can be designed for a second-order system, which comprises a virtual leader and N followers.

$$\begin{cases} \dot{x}_i = v_i, \ \dot{v}_i = u_i + d_i, \quad i = 1, 2, \ldots, N \\ \dot{x}_0 = v_0, \ \dot{v}_0 = 0 \end{cases} \tag{9}$$

The position, speed, disturbance, and control input of the followers are denoted by x_i, v_i, d_i, and u_i, respectively. Meanwhile, the position and speed of the leader are represented by x_0 and v_0, respectively. Specifically, the given reference signal is considered as the output signal of the leader in the proposed design.

Under the condition where $d_i = 0$, the multi-motor systems can achieve consistency tracking within a finite time $T > 0$ adopting the following consensus protocol.

$$u_i = -\sum_{j=0}^{N} a_{ij} sig^{\sigma_1}(x_i - x_j) - \sum_{j=0}^{N} a_{ij} sig^{\sigma_2}(v_i - v_j) \tag{10}$$

where $\sigma_1 \in (0, 1)$, $\sigma_2 = \frac{2\sigma_1}{1+\sigma_1}$, according to Assumption 2.1, leaders are globally reachable, and the proof is similar to that in [31]. Then we can get

$$\lim_{t \to T} |x_i(t) - x_0(t)| = 0, \quad \lim_{t \to T} |v_i(t) - v_0(t)| = 0 \tag{11}$$

3.2. Design of ISMC.

It can be seen from (5) that the dynamic equation of each motor is

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = A_m v_i + B_m i_{q_i} + D_i \end{cases} \tag{12}$$

Based on the closed-loop system finite-time consensus tracking protocol (10) and motor dynamics equation (12), the following ISMC can be designed

$$s_i = v_i - \int_0^t u_i d\tau = 0 \tag{13}$$

The control rate is designed as

$$i_{q_i}^* = \frac{1}{B_m}(u_{eq} + u_{sw}) \tag{14}$$

where

$$\begin{cases} u_{eq} = -A_m v_i - \sum_{j=0}^{N} a_{ij} sig^{\sigma_1}(x_i - x_j) - \sum_{j=0}^{N} a_{ij} sig^{\sigma_2}(v_i - v_j) \\ u_{sw} = -\lambda_1 sign(s_i) - \lambda_2 s_i \end{cases} \tag{15}$$

where λ_1 and λ_2 are normal numbers, and the selection of control parameters satisfied $0 < \delta < 1$, $\lambda_1 > l$. The buffeting phenomenon can be reduced, the speed error can be quickly converged, and the state error of the system will quickly converge to zero in a limited time.

Proof: First, taking the derivative of the sliding mode surface (13) and substituting the control rate function (15), we can get

$$\dot{s}_i = -\lambda_1 sign(s_i) - \lambda_2 s_i + D_i \tag{16}$$

Convert it to vector form

$$\dot{s} = -\lambda_1 sign(s) - \lambda_2 s + D \tag{17}$$

where $s = [s_1, s_2, \ldots, s_n]^T$, $D = [D_1, D_2, \ldots, D_n]^T$ are the column vectors of s_i and D_i, respectively.

Define Lyapunov function V_1 as follows:

$$V_1 = \frac{1}{2} s^T s \tag{18}$$

According to Lemma 2.3, the derivative of V_1 can be obtained as follows:

$$\begin{aligned} \dot{V}_1 &= s^T \dot{s} = \dot{s} = s^T(-\lambda_1 sign(s) - \lambda_2 s + D) \\ &= -\lambda_1 s^T sign(s) - \lambda_2 s^T s + D s^T \\ &\leq -2\lambda_2 V - \lambda_1 ||s||_1 + l||s||_1 \\ &\leq -2\lambda_2 V - (\lambda_1 - l)||s||_2 \\ &= -2\lambda_2 V - \sqrt{2}(\lambda_1 - l)V^{\frac{1}{2}} \end{aligned} \tag{19}$$

Because $\lambda_1 > l$, according to Lemma 2.2, Equation (18) can be rewritten as

$$\dot{V}_1 + 2\lambda_2 V_1 + \sqrt{2}(\lambda_1 - l)V_1^{\frac{1}{2}} \leq 0 \tag{20}$$

Based on Equation (20), the system state will converge to the sliding mode surface $s(t) = 0$ in a finite time. Consequently, this implies that the tracking error in speed for the PMLSM will approach zero in the following finite time

$$T \leq \frac{1}{\lambda_2} \ln\left[1 + \frac{\sqrt{2}\lambda_2}{\lambda_1 - l} V_1^{\frac{1}{2}}(0)\right] \tag{21}$$

Therefore, by designing a switching gain higher than the disturbance estimation error, the system finally reaches a stable state under the action of the proposed controller.

3.3. **Design of NDO.** In actual PMLSM operation, the disturbances often lead to fluctuations in speed and a decrease in tracking accuracy. To enhance the overall robustness of the system, it is necessary to incorporate an NDO for the estimation of unknown disturbances. From Equations (3) and (5), it can be seen that disturbance d is the set of system disturbance and uncertainty. Since the change rate of disturbance is relatively slow compared with the control frequency, it is assumed that F is slow time-varying, that is $\dot{d} = 0$, the NDO can be constructed using the following form.

$$\begin{cases} \dot{p} = -agp - a\left(agv + A_m v + B_m i_q\right) \\ \hat{d} = p + av \end{cases} \tag{22}$$

where $g = \frac{1}{m}$, $\hat{d}$ is the estimate of d, p is the observer internal state variable, a is the observer gain, and $a > 0$.

Thus, the finite time consistent integral sliding mode control rate of multi-motor based on nonlinear disturbance observer can be obtained as follows:

$$i_{q_i}^* = \frac{1}{B_m}\left(-A_m v_i - \sum_{j=0}^{N} a_{ij} sig^{\sigma_1}\left(x_i - x_j\right) - \sum_{j=0}^{N} a_{ij} sig^{\sigma_2}\left(v_i - v_j\right)\right)$$
$$- \lambda_1 sign(s_i) - \lambda_2 s_i - \hat{d}_i \tag{23}$$

Proof: First, the observation error is defined as

$$\tilde{d} = d - \hat{d} \tag{24}$$

and the derivative of $\tilde{d}$ can be obtained as follows:

$$\begin{aligned} \dot{\tilde{d}} &= \dot{d} - \dot{\hat{d}} = \dot{d} - \dot{p} - a\dot{v} \\ &= \dot{d} + agp + a\left(agv + A_m v + B_m i_q\right) - a\dot{v} \\ &= \dot{d} - a\left(\dot{v} - gp - agv - A_m v - B_m i_q\right) \\ &= \dot{d} - a\left(gd - g\hat{d}\right) \\ &= -ag\tilde{d} \end{aligned} \tag{25}$$

Define Lyapunov function V_2 as follows:

$$V_2 = V_1 + \frac{1}{2}\tilde{d}^2 \tag{26}$$

The derivative of V_1 can be obtained as follows:

$$\dot{V}_2 = \dot{V}_1 + \tilde{d}\dot{\tilde{d}} = \dot{V}_1 - ag\tilde{d}^2 \tag{27}$$

As known $g > 0$, if the appropriate gain is selected to make $a > 0$, we can get $\dot{V}_2 < 0$. Consequently, based on the principles of Lyapunov stability theory, the proposed controller ensures the asymptotic stability of the entire system. To visually represent the structure of the proposed controller, refer to Figure 2 for the corresponding block diagram.

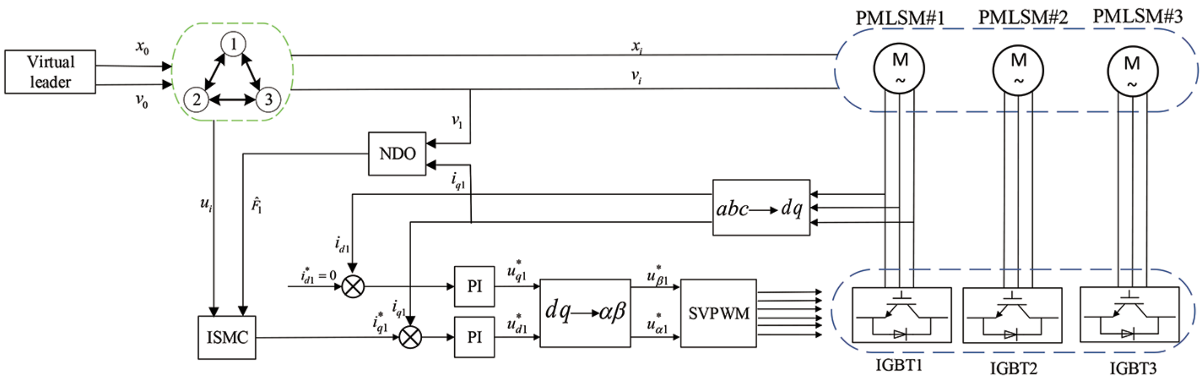

Figure 2. Block diagram of PMLSM drive system control structure

As can be seen from the figure, each motor not only receives a reference signal from the leader but also receives signal feedback from other trackers according to the communication topology between the motors. ISMC controller is designed for the position ring of each motor to output the control current signal. The traditional PID control is used to control the current loop, and the coordinate transformation of the output voltage control signal is modulated vector by SVPWM. The modulated voltage controls the IGBT to output a three-phase current to the linear motor. By feeding the motor speed signal and current signal to NDO, the load disturbance of the motor can be estimated and fed back to the position loop ISMC to suppress the disturbance.

4. **Simulations.** To verify the proposed algorithm, some simulations are performed in this section. Three PMLSMs are used in the simulation.

Let us denote the output signal of the virtual leader as the reference signal for the motor. Define the three followers as motor 1, motor 2, and motor 3. Figure 3 illustrates the communication interaction among these motors, while Table 1 provides the corresponding parameter settings for each motor. To showcase the superiority of the proposed control method, a comparison is made with a distributed PID control system in [26]. According to the directed graph, the synchronization error between the motors is taken into account, and the tracking error with the leader and the synchronization error between the motors are taken as the integrated position error signal input by the PID controller. The controller is designed as

$$u_i(t) = k_p \sum_{j=0}^{N} a_{ij}(x_i(t) - x_j(t)) + k_i \sum_{j=0}^{N} a_{ij} \int_0^t (x_i(\tau) - x_j(\tau))d\tau \tag{28}$$

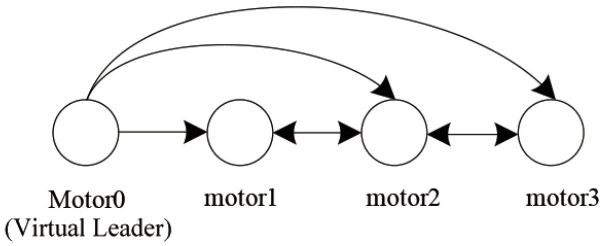

FIGURE 3. Topology structural diagram of the interaction between motors

TABLE 1. Motor parameters

Parameter	Symbol	Value
Resistance (Ω)	R	9.7
Inductance (mH)	L	43.3
Mover mass (kg)	M	3.2
Frictional coefficient (Ns/m)	B	5
Pole pith (m)	τ	0.027
Permanent magnet flux (Wb)	ψ_f	0.165
Pole pairs	n_p	2

The convergence time of the consensus protocol can be controlled by adjusting parameter σ_1, parameter λ_2 controls the speed at which the system approaches the sliding mode when the sliding mode surface s is large, when the sliding mode surface s is close to 0, the approaching velocity is λ_1, which can guarantee the system's arrival in finite time. The parameters for the designed controller are as follows: $\sigma_1 = 0.5$, $\sigma_2 = 2/3$, $\lambda_1 = 200$, $\lambda_2 = 20$, $a = -1000$.

We can get adjacency matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and diagonal matrix $B = diag(a_{10}, a_{20}, a_{30}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

4.1. **Performance evaluations of observers.** Three different disturbance signals, namely $d_1 = 5\sin(t)$, $d_2 = 20\cos(2t)$, $d_3 = 10\sin(t) + 5\cos(2t)$, are adopted to assess the tracking performance of the proposed controller across diverse conditions. The accuracy estimation of these disturbances directly impacts the subsequent controllers' effectiveness. Initially, the design and evaluation of disturbance observers take place. Subsequently, to validate the performance of these disturbance observers, three disturbance signals d_1, d_2, and d_3, specifically get introduced to follower motors 1, 2, and 3. As shown in Figure 4, NDO proves capable of accurately estimating disturbance information.

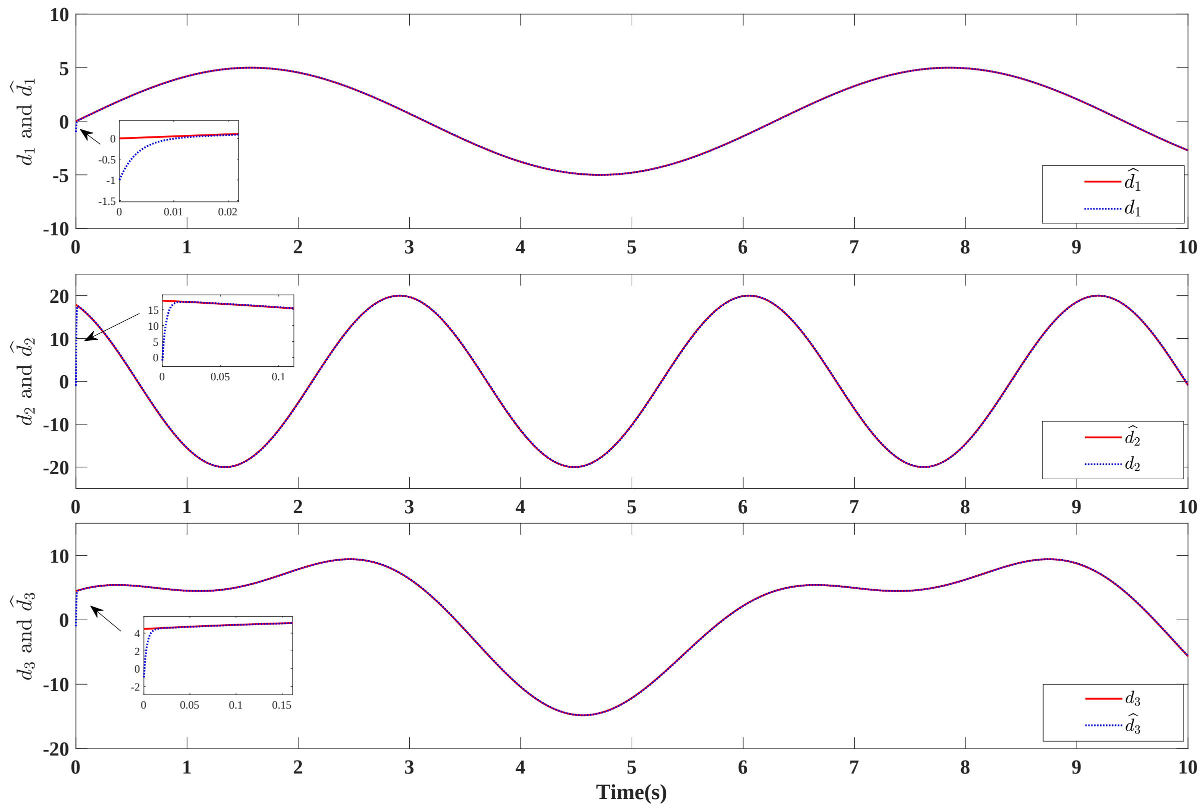

FIGURE 4. Distubrance of each PMLSM and its estimated value

4.2. **Performance evaluations of the proposed control.** To assess the synchronous control effectiveness of all motors when applying the consensus control protocol, we examine two distinct cases for validation.

Case 1: The reference position is set as a sinusoidal signal $x_0 = 0.3\sin(t)$, and the position tracking and error curves of the two controllers are presented in Figures 5 and 6. As we can see, under a load disturbance, the proposed method can track the given position reference signal after 0.2 s, and the position tracking error can be within 0.01 mm after stabilization. The tracking error of the multi-motor system controlled by PI is 0.03 mm after stabilization, and the tracking error waveform will have a slight oscillation when the

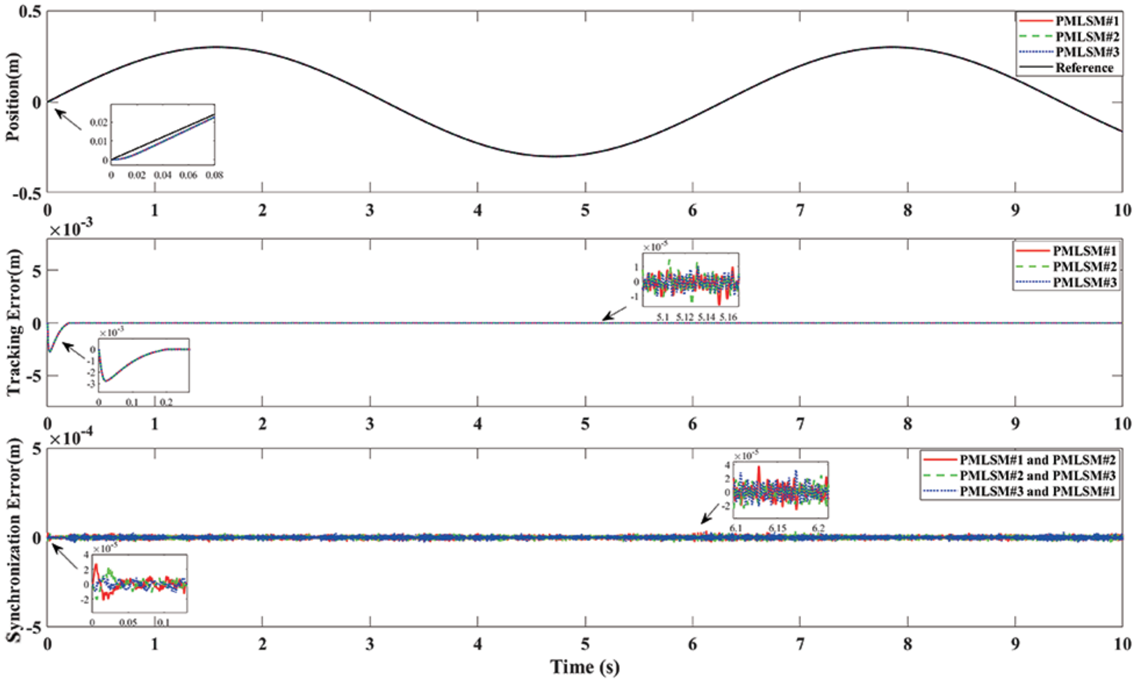

FIGURE 5. Proposed control for multi-PMLSMs in simulation case 1

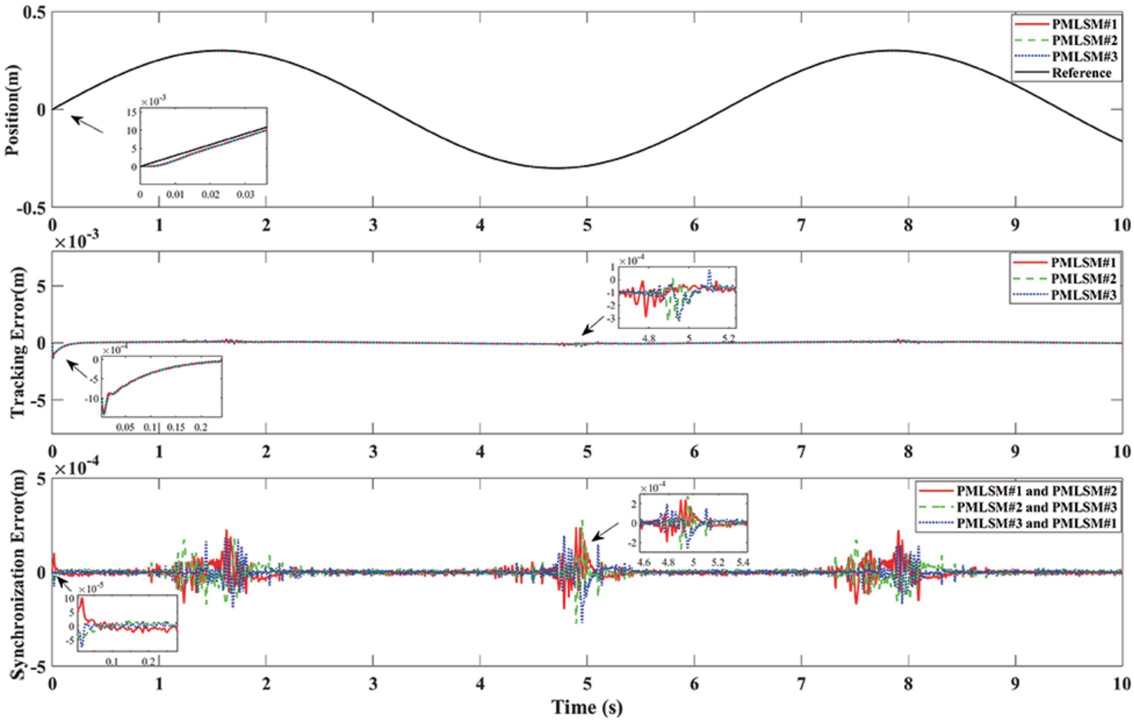

FIGURE 6. Distributed PID control for multi-PMLSMs in simulation case 1

sinusoidal signal reaches the amplitude. In terms of synchronization performance, it can be seen that the proposed method is superior to distributed PI control. In the amplitude of the sinusoidal signal, the synchronization error of the multi-motor system controlled by PI fluctuates greatly, and the maximum synchronization error can reach 0.2 mm, while the synchronization error of the proposed method is stable within 0.04 mm during the whole operation process.

Case 2: The control performance of the two control strategies is verified when the speed is a triangular reference signal which $T = 8$ s and amplitude is ± 0.3 m. The position tracking and error curves of the two controllers are presented in Figures 7 and 8. Under the triangular wave signal, the synchronization error of the proposed method

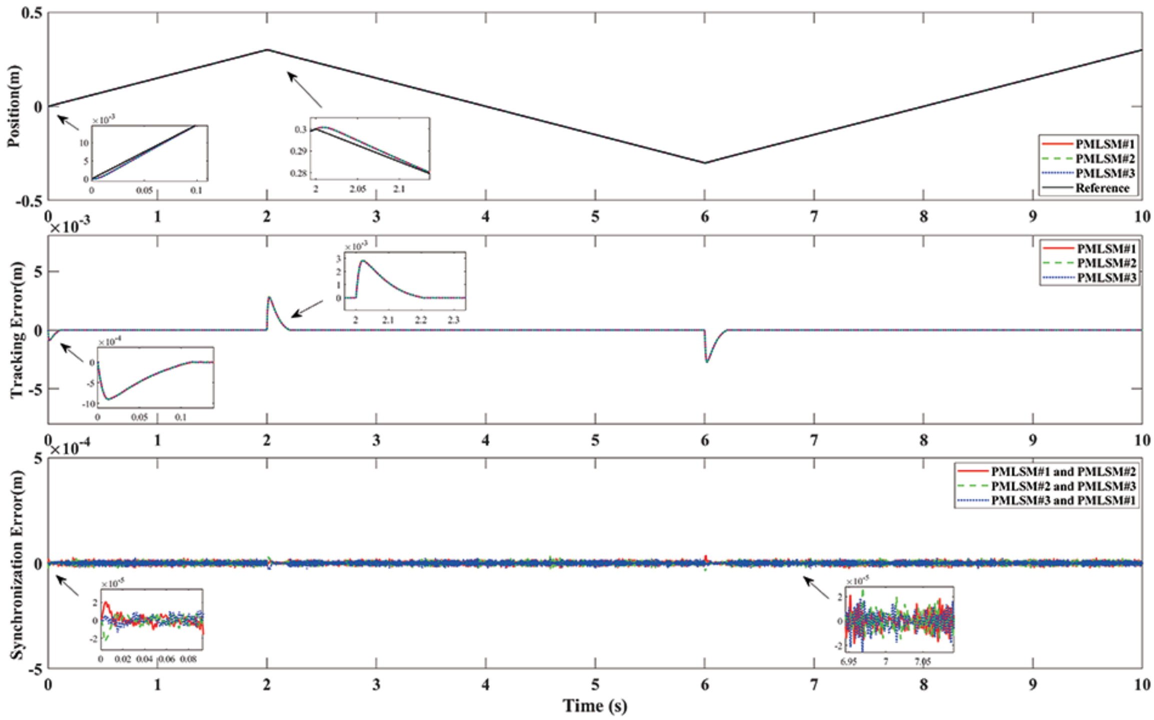

FIGURE 7. Proposed control for multi-PMLSMs in simulation case 2

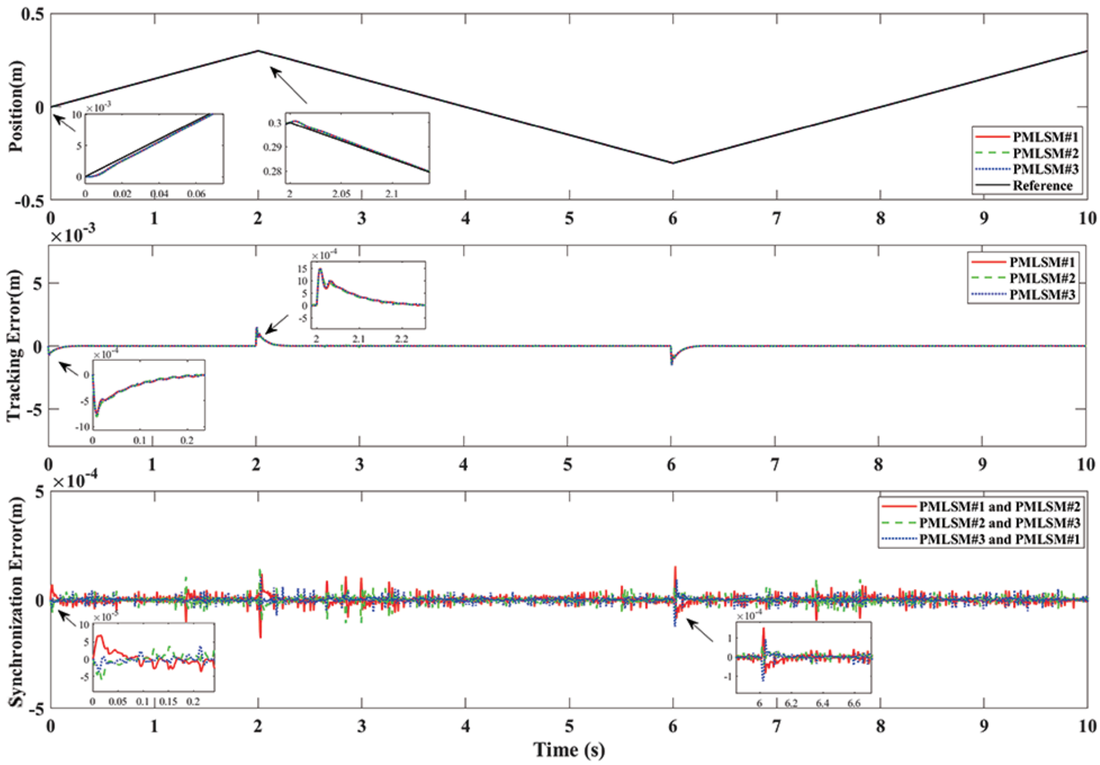

FIGURE 8. Distributed PID control for multi-PMLSMs in simulation case 2

is consistent with that of the sinusoidal signal, and can be stabilized within 0.04 mm. However, although the synchronization error waveform of PI control does not fluctuate greatly, the synchronization error value is still significantly larger than that of the proposed method in the whole operation stage, and the maximum synchronization error can reach 0.1 mm. In summary, the method proposed in this paper is superior to distributed PI control in tracking performance and synchronization performance of multi-motor systems.

5. **Conclusion.** This paper presents a novel approach for achieving synchronous coordination control in a multiple linear motors system. In this paper, we first adopt the multi-agent continuous homogeneous finite-time consistency protocol in order to approach the tracking error of the system to zero in the finite time. Then, we improve the robustness and steady-state performance of the system by applying the ISMC and introduce an NDO to observe and reduce the influence of perturbations on position tracking. Finally, simulation results show that this method has better synchronization control accuracy compared to alternative control techniques. As future work, the research direction is to build a multi-motor collaborative control experimental platform, by improving the sliding mode surface and the reaching law to speed up the adjustment time of the entire control system.

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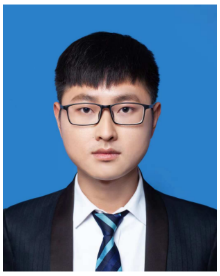

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