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NEW RESULTS ON INTUITIONISTIC FUZZY SETS IN IUP-ALGEBRAS

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Abstract. *The concept of fuzzy sets was introduced by Zadeh in 1965. After that, Atanassov introduced the concept of intuitionistic fuzzy sets in 1986, a generalization of fuzzy sets. In this paper, we apply the concept of intuitionistic fuzzy sets to IUP-algebras and introduce the notions of intuitionistic fuzzy IUP-subalgebras, intuitionistic fuzzy IUP-ideals, intuitionistic fuzzy IUP-filters, and intuitionistic fuzzy strong IUP-ideals. Their basic properties are investigated. Upper t-(strong) level subsets and lower t-(strong) level subsets of an intuitionistic fuzzy set are derived.*
Keywords: IUP-algebra, Intuitionistic fuzzy set, Intuitionistic fuzzy IUP-subalgebra, Intuitionistic fuzzy IUP-ideal, Intuitionistic fuzzy IUP-filter, Intuitionistic fuzzy strong IUP-ideal

1. **Introduction.** Zadeh [1] commenced the concept of fuzzy sets in 1965, an important concept. After that, Atanassov [2] introduced the notion of intuitionistic fuzzy sets in 1986, which is a generalization of fuzzy sets. On the generalizations of intuitionistic fuzzy sets and their application to numerous logical algebras, for example, in 2008, Akram et al. [3] introduced the notion of interval-valued intuitionistic fuzzy ideals of K-algebras. In 2013, Tripathy et al. [4] introduced the notion of intuitionistic fuzzy lattices and intuitionistic fuzzy Boolean algebras. In 2015, Kesorn et al. [5] introduced the concept of intuitionistic fuzzy sets in UP (BCC)-algebras. In 2017, Senapati et al. [6] introduced the notion of representation of UP (BCC)-algebras in an interval-valued intuitionistic fuzzy environment. Tarsuslu and Yorulmaz [7] introduced the notion of H-intuitionistic fuzzy sets. In 2018, Senapati et al. [8] introduced the concept of intuitionistic fuzzy BG-subalgebras of BG-algebras. Sunday et al. [9] introduced the notion of difference and symmetric difference for intuitionistic fuzzy sets. Jun et al. [10] introduced the notion of

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cubic interval-valued intuitionistic fuzzy sets in BCK/BCI-algebras. In 2020, Touqeer [11] introduced the notion of intuitionistic fuzzy soft set theoretic approaches to α-ideals in BCI-algebras. In 2021, Mostafa et al. [12] introduced the notion of crossing intuitionistic KU-ideals on KU-algebras. In 2022, Rajesh et al. [13] introduced the concept of certain level operators over temporal intuitionistic fuzzy sets. Kaewprasert et al. [14] introduced the notion of intuitionistic cubic sets in UP (BCC)-algebras. Deva and Felix [15] introduced the notion of introducing interpolative Boolean algebra into intuitionistic fuzzy sets. Amigo et al. [16] introduced the notion of multipolar intuitionistic fuzzy ideals in B-algebras. Derseh et al. [17] introduced the notion of t-intuitionistic fuzzy structures on PMS-ideals of a PMS-algebra. Iampan et al. [18] introduced the notion of interval-valued intuitionistic fuzzy subalgebras and ideals of Hilbert algebras. In 2023, Iampan et al. [19] introduced the notion of intuitionistic hesitant fuzzy UP (BCC)-filters of UP (BCC)-algebras. Derseh et al. [20] introduced the notion of t-intuitionistic fuzzy PMS-subalgebras of PMS-algebras. Iampan et al. [21] introduced the notions of intuitionistic fuzzy subalgebras and intuitionistic fuzzy ideals of Hilbert algebras. Khamrot et al. [22, 23] introduced the concepts of intuitionistic fuzzy comparative and implicative UP (BCC)-filters of UP (BCC)-algebras. Iampan et al. [24] presented the concepts of intuitionistic $\mathscr{N}$-fuzzy subalgebras and intuitionistic $\mathscr{N}$-fuzzy ideals of Hilbert algebras. Senapati et al. [25] introduced the concept of intuitionistic fuzzy translation to intuitionistic fuzzy subalgebras in BG-algebras. Iampan et al. [26, 27] introduced the notions of intuitionistic hesitant fuzzy subalgebras, ideals, and deductive systems of Hilbert algebras.

From reviewing the literature, it can be seen that the study of intuitionistic fuzzy sets has been studied by many researchers and is being studied continuously. Since IUP-algebras were released in 2022 by Iampan et al. [28] and are an interesting new algebraic system, our research team is interested in studying intuitionistic fuzzy sets in IUP-algebras. Therefore, we will apply intuitionistic fuzzy sets to IUP-subalgebras, IUP-filters, IUP-ideals, and strong IUP-ideals in IUP-algebras and find their properties and relationships. We will also study the relationship between level subsets and their intuitionistic fuzzy sets. We have divided this article's content into four sections. Section 1 will discuss related research and the inspiration for this article. Section 2 introduces the definition of IUP-algebras, providing examples and important properties. We will also review the definitions of IUP-subalgebras, IUP-filters, IUP-ideals, and strong IUP-ideals to contribute to the definition of intuitionistic fuzzy IUP-subalgebras, intuitionistic fuzzy IUP-filters, intuitionistic fuzzy IUP-ideals, and intuitionistic fuzzy strong IUP-ideals. Section 3 reviews the definitions of fuzzy sets and intuitionistic fuzzy sets, introduces the definitions of intuitionistic fuzzy IUP-subalgebras, intuitionistic fuzzy IUP-filters, intuitionistic fuzzy IUP-ideals, and intuitionistic fuzzy strong IUP-ideals, and gives examples. After that, we will find the important properties of the four concepts and show their generalizations. This section's main result is to show the relationship between characteristic functions, level sets, and their intuitionistic fuzzy sets. Section 4 summarizes the results of the study and recommends further studies and extensions of this research.

2. **Preliminaries.**

This section reviews the concept of IUP-algebras along with necessary definitions and examples.

Definition 2.1. [28] *An algebra $X = (X; \cdot, 0)$ of type $(2, 0)$ is called an IUP-algebra, where X is a nonempty set, $\cdot$ is a binary operation on X, and 0 is a fixed element of X if it satisfies the following axioms:*

$$(\forall x \in X)(0 \cdot x = x) \qquad \text{(IUP-1)}$$

$$(\forall x \in X)(x \cdot x = 0) \qquad \text{(IUP-2)}$$

$$(\forall x, y, z \in X)((x \cdot y) \cdot (x \cdot z) = y \cdot z) \tag{IUP-3}$$

For convenience, we refer to X as an IUP-algebra $X = (X; \cdot, 0)$ until otherwise specified.

Example 2.1. *Let $X = \{0, 1, 2, 3, 4, 5\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	3	0	4	5	1	2
2	2	5	0	4	3	1
3	1	4	5	0	2	3
4	5	3	1	2	0	4
5	4	2	3	1	5	0

Then $X = (X; \cdot, 0)$ is an IUP-algebra.

Example 2.2. [28] *Let $(G; \cdot, e)$ be a group with the identity element e such that all element is the inverse of itself. Then $(G; \cdot, e)$ is an IUP-algebra.*

Example 2.3. [28] *Let X be a set and $\mathcal{P}(X)$ means the power set of X. It follows from Example 2.2 that $(\mathcal{P}(X); \triangle, \emptyset)$ is an IUP-algebra where the binary operation $\triangle$ is defined as the symmetric difference of any two sets.*

Example 2.4. [28] *Let $(G; \cdot, e)$ be a group with the identity element e. Define a binary operation $\cdot$ on G by*

$$(\forall x, y \in G)\left(x \cdot y = yx^{-1}\right). \tag{1}$$

Then $(G; \cdot, e)$ is an IUP-algebra.

For further study and examples of IUP-algebras, see [28, 29, 30, 31].

In X, the following assertions are valid (see [28]).

$$(\forall x, y \in X)((x \cdot 0) \cdot (x \cdot y) = y) \tag{2}$$
$$(\forall x \in X)((x \cdot 0) \cdot (x \cdot 0) = 0) \tag{3}$$
$$(\forall x, y \in X)((x \cdot y) \cdot 0 = y \cdot x) \tag{4}$$
$$(\forall x \in X)((x \cdot 0) \cdot 0 = x) \tag{5}$$
$$(\forall x, y \in X)(x \cdot ((x \cdot 0) \cdot y) = y) \tag{6}$$
$$(\forall x, y \in X)(((x \cdot 0) \cdot y) \cdot x = y \cdot 0) \tag{7}$$
$$(\forall x, y, z \in X)(x \cdot y = x \cdot z \Leftrightarrow y = z) \tag{8}$$
$$(\forall x, y \in X)(x \cdot y = 0 \Leftrightarrow x = y) \tag{9}$$
$$(\forall x \in X)(x \cdot 0 = 0 \Leftrightarrow x = 0) \tag{10}$$
$$(\forall x, y, z \in X)(y \cdot x = z \cdot x \Leftrightarrow y = z) \tag{11}$$
$$(\forall x, y \in X)(x \cdot y = y \Rightarrow x = 0) \tag{12}$$
$$(\forall x, y, z \in X)((x \cdot y) \cdot 0 = (z \cdot y) \cdot (z \cdot x)) \tag{13}$$
$$(\forall x, y, z \in X)(x \cdot y = 0 \Leftrightarrow (z \cdot x) \cdot (z \cdot y) = 0) \tag{14}$$
$$(\forall x, y, z \in X)(x \cdot y = 0 \Leftrightarrow (x \cdot z) \cdot (y \cdot z) = 0) \tag{15}$$

the right and the left cancellation laws hold (16)

Definition 2.2. [28] *A nonempty subset S of X is called*

(i) *an IUP-subalgebra of X if it satisfies the following condition:*

$$(\forall x, y \in S)(x \cdot y \in S) \tag{17}$$

(*ii*) *an IUP-filter of* X *if it satisfies the following conditions:*

$$\text{the constant } 0 \text{ of } X \text{ is in } S \tag{18}$$

$$(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S) \tag{19}$$

(*iii*) *an IUP-ideal of* X *if it satisfies the condition* (18) *and the following condition:*

$$(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S) \tag{20}$$

(*iv*) *a strong IUP-ideal of* X *if it satisfies the following condition:*

$$(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S) \tag{21}$$

From [28], we know that the concept of IUP-filters is a generalization of IUP-ideals and IUP-subalgebras, and IUP-ideals and IUP-subalgebras are generalizations of strong IUP-ideals. In an IUP-algebra X, we have that strong IUP-ideals coincide with X itself. We get the diagram of the special subsets of IUP-algebras, which is shown in Figure 1.

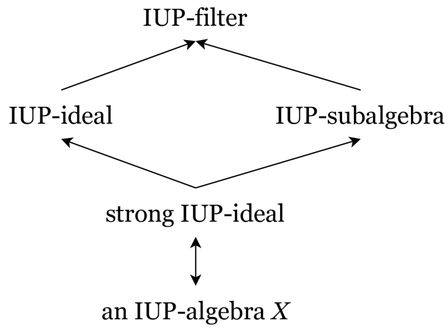

FIGURE 1. Special subsets of IUP-algebras

3. **Main Results.** In this section, we introduce the concepts of intuitionistic fuzzy IUP-subalgebras, intuitionistic fuzzy IUP-filters, intuitionistic fuzzy IUP-ideals, and intuitionistic fuzzy strong IUP-ideals of IUP-algebras, along with examples supporting the theorem.

Definition 3.1. [1] *A fuzzy set (briefly, FS) in a nonempty set* X *(or a fuzzy subset of* X*) is an arbitrary function* $f : X \to [0, 1]$*, where* $[0, 1]$ *is the unit segment of the real line.*

Definition 3.2. [2] *An intuitionistic fuzzy set (briefly, IFS) in a nonempty set* X *is an object* F *having the form*

$$F = \{(x, \mu_F(x), \gamma_F(x)) | x \in X\}, \tag{22}$$

where the fuzzy sets $\mu_F : X \to [0, 1]$ *and* $\gamma_F : X \to [0, 1]$ *denote the degree of membership and the degree of nonmembership, respectively,*

$$(\forall x \in X)(0 \le \mu_F(x) + \gamma_F(x) \le 1). \tag{23}$$

An IFS $F = \{(x, \mu_F(x), \gamma_F(x)) | x \in X\}$ *in a nonempty set* X *can be identified to an ordered pair* (μ_F, γ_F) *in* $[0, 1]^X \times [0, 1]^X$*. For the sake of simplicity, we shall use the symbol* $F = (\mu_F, \gamma_F)$ *for the IFS* $F = \{(x, \mu_F(x), \gamma_F(x)) | x \in X\}$*. If* $B \subseteq X$*, the characteristic functions* μ_{F_B} *and* γ_{F_B} *are functions of* X *into* $\{0, 1\}$ *defined as follows:*

$$\mu_{F_B}(x) = \begin{cases} 1 & \text{if } x \in B \\ 0 & \text{otherwise} \end{cases}$$

$$\gamma_{F_B}(x) = \begin{cases} 0 & \textit{if } x \in B \\ 1 & \textit{otherwise} \end{cases}$$

By the definition of the characteristic function, μ_{F_B} *and* γ_{F_B} *are functions of* X *into* $\{0,1\} \subset [0,1]$. *Therefore, the IFS* $F_B = (\mu_{F_B}, \gamma_{F_B})$ *is defined as the characteristic IFS of* B *in* X.

Definition 3.3. *Let* f *be a fuzzy set in a nonempty set* X. *The fuzzy set* $\overline{f}$ *defined by* $\overline{f}(x) = 1 - f(x)$ *for all* $x \in X$ *is called the complement of* f *in* X.

Definition 3.4. *An IFS* $F = (\mu_F, \gamma_F)$ *in* X *is called*

(*i*) *an intuitionistic fuzzy IUP-subalgebra of* X *if it satisfies the following properties:*

$$(\forall x, y \in X)(\mu_F(x \cdot y) \geq \min\{\mu_F(x), \mu_F(y)\}) \tag{24}$$

$$(\forall x, y \in X)(\gamma_F(x \cdot y) \leq \max\{\gamma_F(x), \gamma_F(y)\}) \tag{25}$$

(*ii*) *an intuitionistic fuzzy IUP-filter of* X *if it satisfies the following properties:*

$$(\forall x \in X)(\mu_F(0) \geq \mu_F(x)) \tag{26}$$

$$(\forall x \in X)(\gamma_F(0) \leq \gamma_F(x)) \tag{27}$$

$$(\forall x, y \in X)(\mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\}) \tag{28}$$

$$(\forall x, y \in X)(\gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\}) \tag{29}$$

(*iii*) *an intuitionistic fuzzy IUP-ideal of* X *if it satisfies the conditions* (26) *and* (27) *and the following properties:*

$$(\forall x, y, z \in X)(\mu_F(x \cdot z) \geq \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\}) \tag{30}$$

$$(\forall x, y, z \in X)(\gamma_F(x \cdot z) \leq \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\}) \tag{31}$$

(*iv*) *an intuitionistic fuzzy strong IUP-ideal of* X *if it satisfies the following properties:*

$$(\forall x, y \in X)(\mu_F(x \cdot y) \geq \mu_F(y)) \tag{32}$$

$$(\forall x, y \in X)(\gamma_F(x \cdot y) \leq \gamma_F(y)) \tag{33}$$

Lemma 3.1. *Every intuitionistic fuzzy IUP-subalgebra of* X *satisfies* (26) *and* (27).

Proof: Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X. Let $x \in X$. Then

$$\begin{aligned} \mu_F(0) &= \mu_F(x \cdot x) && \text{(by (IUP-2))} \\ &\geq \min\{\mu_F(x), \mu_F(x)\} && \text{(by (24))} \\ &= \mu_F(x), \\ \gamma_F(0) &= \gamma_F(x \cdot x) && \text{(by (IUP-2))} \\ &\leq \max\{\gamma_F(x), \gamma_F(x)\} && \text{(by (25))} \\ &= \gamma_F(x). \end{aligned}$$

Hence, $F = (\mu_F, \gamma_F)$ satisfies (26) and (27). □

Theorem 3.1. *Intuitionistic fuzzy strong IUP-ideals and constant IFSs coincide.*

Proof: We see that every constant IFS is an intuitionistic fuzzy strong IUP-ideal. Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong IUP-ideal of X. Let $x \in X$. Then

$$\begin{aligned} \mu_F(0) &= \mu_F(x \cdot x) && \text{(by (IUP-2))} \\ &\geq \mu_F(x), && \text{(by (32))} \end{aligned}$$

$$\begin{aligned}
\gamma_F(0) &= \gamma_F(x \cdot x) && \text{(by (IUP-2))} \\
&\leq \gamma_F(x), && \text{(by (33))} \\
\mu_F(x) &= \mu_F((x \cdot 0) \cdot 0) && \text{(by (5))} \\
&\geq \mu_F(0), && \text{(by (32))} \\
\gamma_F(x) &= \gamma_F((x \cdot 0) \cdot 0) && \text{(by (5))} \\
&\leq \gamma_F(0). && \text{(by (33))}
\end{aligned}$$

Therefore, $\mu_F(x) = \mu_F(0)$ and $\gamma_F(x) = \gamma_F(0)$, that is, $F = (\mu_F, \gamma_F)$ is constant of X. Hence, intuitionistic fuzzy strong IUP-ideals and constant IFSs coincide. □

Lemma 3.2. *Let B be a nonempty subset of X. Then the constant 0 of X is in B if and only if the characteristic IFS F_B satisfies* (26) *and* (27)*.*

Proof: If $0 \in B$, then $\mu_{F_B}(0) = 1$ and $\gamma_{F_B}(0) = 0$. Thus, $\mu_{F_B}(0) = 1 \geq \mu_{F_B}(x)$ and $\gamma_{F_B}(0) = 0 \leq \gamma_{F_B}(x)$ for all $x \in X$, that is, F_B satisfies (26) and (27).

Conversely, assume that F_B satisfies (26) and (27). Then $\mu_{F_B}(0) \geq \mu_{F_B}(x)$ for all $x \in X$. Since B is a nonempty subset of X, we let $a \in B$. Then $\mu_{F_B}(0) \geq \mu_{F_B}(a) = 1$, so $\mu_{F_B}(0) = 1$. Hence, $0 \in B$. □

Theorem 3.2. *A nonempty subset B of X is an IUP-subalgebra of X if and only if the characteristic IFS F_B is an intuitionistic fuzzy IUP-subalgebra of X.*

Proof: Assume that B is an IUP-subalgebra of X. Let $x, y \in X$.

Case 1: Suppose $x, y \in B$. Then $\mu_{F_B}(x) = 1$ and $\mu_{F_B}(y) = 1$. Since B is an IUP-subalgebra of X, we have $x \cdot y \in B$. Thus, $\mu_{F_B}(x \cdot y) = 1 \geq \min\{1, 1\} = \min\{\mu_{F_B}(x), \mu_{F_B}(y)\}$.

Case 2: Suppose $x \notin B$ or $y \notin B$. Then $\mu_{F_B}(x) = 0$ or $\mu_{F_B}(y) = 0$. Thus, $\mu_{F_B}(x \cdot y) \geq 0 = \min\{\mu_{F_B}(x), \mu_{F_B}(y)\}$.

Case 1′: Suppose $x, y \in B$. Then $\gamma_{F_B}(x) = 0$ and $\gamma_{F_B}(y) = 0$. Since B is an IUP-subalgebra of X, we have $x \cdot y \in B$. Thus, $\gamma_{F_B}(x \cdot y) = 0 \leq \max\{0, 0\} = \max\{\gamma_{F_B}(x), \gamma_{F_B}(y)\}$.

Case 2′: Suppose $x \notin B$ or $y \notin B$. Then $\gamma_{F_B}(x) = 1$ or $\gamma_{F_B}(y) = 1$. Thus, $\gamma_{F_B}(x \cdot y) \leq 1 = \max\{\gamma_{F_B}(x), \gamma_{F_B}(y)\}$.

Hence, the characteristic IFS F_B is an intuitionistic fuzzy IUP-subalgebra of X.

Conversely, assume that the characteristic IFS F_B is an intuitionistic fuzzy IUP-subalgebra of X. Let $x, y \in B$. Then $\mu_{F_B}(x) = 1$ and $\mu_{F_B}(y) = 1$. By (24), we have $\mu_{F_B}(x \cdot y) \geq \min\{\mu_{F_B}(x), \mu_{F_B}(y)\} = \min\{1, 1\} = 1$. Thus, $\mu_{F_B}(x \cdot y) = 1$, that is, $x \cdot y \in B$. Hence, B is an IUP-subalgebra of X. □

Theorem 3.3. *A nonempty subset B of X is an IUP-filter of X if and only if the characteristic IFS F_B is an intuitionistic fuzzy IUP-filter of X.*

Proof: Assume that B is an IUP-filter of X. Since $0 \in B$, it follows from Lemma 3.2 that μ_{F_B} and γ_{F_B} satisfy (26) and (27), respectively. Next, let $x, y \in X$.

Case 1: Suppose $x \cdot y \in B$ and $x \in B$. Since B is an IUP-filter of X, we have $y \in B$. Thus, $\mu_{F_B}(y) = 1 \geq \min\{\mu_{F_B}(x \cdot y), \mu_{F_B}(x)\}$.

Case 2: Suppose $x \cdot y \notin B$ or $x \notin B$. Then $\mu_{F_B}(x \cdot y) = 0$ or $\mu_{F_B}(x) = 0$. Thus, $\mu_{F_B}(y) \geq 0 = \min\{\mu_{F_B}(x \cdot y), \mu_{F_B}(x)\}$.

Case 1′: Suppose $x \cdot y \in B$ and $x \in B$. Since B is an IUP-filter of X, we have $y \in B$. Thus, $\gamma_{F_B}(y) = 0 \leq \max\{\gamma_{F_B}(x \cdot y), \gamma_{F_B}(x)\}$.

Case 2′: Suppose $x \cdot y \notin B$ or $x \notin B$. Then $\gamma_{F_B}(x \cdot y) = 1$ or $\gamma_{F_B}(x) = 1$. Thus, $\gamma_{F_B}(y) \leq 1 = \max\{\gamma_{F_B}(x \cdot y), \gamma_{F_B}(x)\}$.

Hence, the characteristic IFS F_B is an intuitionistic fuzzy IUP-filter of X.

Conversely, assume that the characteristic IFS F_B is an intuitionistic fuzzy IUP-filter of X. Since μ_{F_B} satisfies (26), it follows from Lemma 3.2 that $0 \in B$. Next, let $x, y \in X$ be such that $x \cdot y \in B$ and $x \in B$. Then $\mu_{F_B}(x \cdot y) = 1$ and $\mu_{F_B}(x) = 1$. Thus, $\min\{\mu_{F_B}(x \cdot y), \mu_{F_B}(x)\} = 1$. By (28), we have $\mu_{F_B}(y) \geq \min\{\mu_{F_B}(x \cdot y), \mu_{F_B}(x)\} = 1$, that is, $\mu_{F_B}(y) = 1$. Hence, $y \in B$, so B is an IUP-filter of X. □

Theorem 3.4. *A nonempty subset B of X is an IUP-ideal of X if and only if the characteristic IFS F_B is an intuitionistic fuzzy IUP-ideal of X.*

Proof: Assume that B is an IUP-ideal of X. Since $0 \in B$, it follows from Lemma 3.2 that μ_{F_B} and γ_{F_B} satisfy (26) and (27), respectively. Next, let $x, y, z \in X$.

Case 1: Suppose $x \cdot (y \cdot z) \in B$ and $y \in B$. Since B is an IUP-ideal of X, we have $x \cdot z \in B$. Thus, $\mu_{F_B}(x \cdot z) = 1 \geq \min\{\mu_{F_B}(x \cdot (y \cdot z)), \mu_{F_B}(y)\}$.

Case 2: Suppose $x \cdot (y \cdot z) \notin B$ or $y \notin B$. Then $\mu_{F_B}(x \cdot (y \cdot z)) = 0$ or $\mu_{F_B}(y) = 0$. Thus, $\mu_{F_B}(x \cdot z) \geq 0 = \min\{\mu_{F_B}(x \cdot (y \cdot z)), \mu_{F_B}(y)\}$.

Case 1′: Suppose $x \cdot (y \cdot z) \in B$ and $y \in B$. Since B is an IUP-ideal of X, we have $x \cdot z \in B$. Thus, $\gamma_{F_B}(x \cdot z) = 0 \leq \max\{\gamma_{F_B}(x \cdot (y \cdot z)), \gamma_{F_B}(y)\}$.

Case 2′: Suppose $x \cdot (y \cdot z) \notin B$ or $y \notin B$. Then $\gamma_{F_B}(x \cdot (y \cdot z)) = 1$ or $\gamma_{F_B}(y) = 1$. Thus, $\mu_{F_B}(x \cdot z) \leq 1 = \max\{\gamma_{F_B}(x \cdot (y \cdot z)), \gamma_{F_B}(y)\}$.

Hence, the characteristic IFS F_B is an intuitionistic fuzzy IUP-ideal of X.

Conversely, assume that the characteristic IFS F_B is an intuitionistic fuzzy IUP-ideal of X. Since μ_{F_B} satisfies (26), it follows from Lemma 3.2 that $0 \in B$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in B$ and $y \in B$. Then $\mu_{F_B}(x \cdot (y \cdot z)) = 1$ and $\mu_{F_B}(y) = 1$. Thus, $\min\{\mu_{F_B}(x \cdot (y \cdot z)), \mu_{F_B}(y)\} = 1$. By (30), we have $\mu_{F_B}(x \cdot z) \geq \min\{\mu_{F_B}(x \cdot (y \cdot z)), \mu_{F_B}(y)\} = 1$, that is, $\mu_{F_B}(x \cdot z) = 1$. Hence, $x \cdot z \in B$, so B is an IUP-ideal of X. □

Theorem 3.5. *A nonempty subset B of X is a strong IUP-ideal of X if and only if the characteristic IFS F_B is an intuitionistic fuzzy strong IUP-ideal of X.*

Proof: It is straightforward by Theorem 3.1. □

Theorem 3.6. *Every intuitionistic fuzzy strong IUP-ideal of X is an intuitionistic fuzzy IUP-subalgebra of X.*

Proof: It is straightforward by Theorem 3.1. □

Example 3.1. *Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:*

·	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	0	5	4	3	2
2	4	5	0	1	2	3
3	3	2	1	0	5	4
4	2	3	4	5	0	1
5	5	4	3	2	1	0

Then $X = (X; \cdot, 0)$ is an IUP-algebra. We define an IFS $F = (\mu_F, \gamma_F)$ on X as follows:

$$\mu_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.5 & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \end{pmatrix}$$

$$\gamma_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.2 & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \end{pmatrix}$$

Then $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X. Since $F = (\mu_F, \gamma_F)$ is not constant, it follows from Theorem 3.1 that $F = (\mu_F, \gamma_F)$ is not an intuitionistic fuzzy strong IUP-ideal of X.

Theorem 3.7. *Every intuitionistic fuzzy IUP-ideal of X is an intuitionistic fuzzy IUP-filter of X.*

Proof: Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-ideal of X. Then (26) and (27) hold. Let $x, y \in X$. Then

$$\begin{aligned}
\mu_F(y) &= \mu_F(0 \cdot y) && \text{(by (IUP-1))}\\
&\geq \min\{\mu_F(0 \cdot (x \cdot y)), \mu_F(x)\} && \text{(by (30))}\\
&= \min\{\mu_F(x \cdot y), \mu_F(x)\}, && \text{(by (IUP-1))}\\
\gamma_F(y) &= \gamma_F(0 \cdot y) && \text{(by (IUP-1))}\\
&\leq \max\{\gamma_F(0 \cdot (x \cdot y)), \gamma_F(x)\} && \text{(by (31))}\\
&= \max\{\gamma_F(x \cdot y), \gamma_F(x)\}. && \text{(by (IUP-1))}
\end{aligned}$$

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-filter of X. □

Example 3.2. *Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:*

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	2	0	1	4	5	3
2	1	2	0	5	3	4
3	3	4	5	0	1	2
4	4	5	3	2	0	1
5	5	3	4	1	2	0

Then $X = (X; \cdot, 0)$ is an IUP-algebra. We define an IFS $F = (\mu_F, \gamma_F)$ on X as follows:

$$\mu_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.8 & 0.2 & 0.2 & 0.8 & 0.2 & 0.2 \end{pmatrix}$$

$$\gamma_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.1 & 0.6 & 0.6 & 0.1 & 0.6 & 0.6 \end{pmatrix}$$

Then $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-filter of X. Since $\mu_F(4 \cdot 1) = \mu_F(5) = 0.2 \not\geq 0.8 = \min\{0.8, 0.8\} = \min\{\mu_F(0), \mu_F(3)\} = \min\{\mu_F(4 \cdot 4), \mu_F(3)\} = \min\{\mu_F(4 \cdot (3 \cdot 1)), \mu_F(3)\}$. Hence, $F = (\mu_F, \gamma_F)$ is not an intuitionistic fuzzy IUP-ideal of X.

Theorem 3.8. *Every intuitionistic fuzzy strong IUP-ideal of X is an intuitionistic fuzzy IUP-ideal of X.*

Proof: It is straightforward by Theorem 3.1. □

Example 3.3. *Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:*

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	5	0	4	2	3	1
2	3	4	0	1	5	2
3	2	3	5	0	1	4
4	4	2	1	5	0	3
5	1	5	3	4	2	0

Then $X = (X; \cdot, 0)$ is an IUP-algebra. We define an IFS $F = (\mu_F, \gamma_F)$ on X as follows:

$$\mu_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.9 & 0.7 & 0.7 & 0.7 & 0.9 & 0.7 \end{pmatrix}$$

$$\gamma_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.1 & 0.2 & 0.2 & 0.2 & 0.1 & 0.2 \end{pmatrix}$$

Then $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-ideal of X. Since $F = (\mu_F, \gamma_F)$ is not constant, it follows from Theorem 3.1 that $F = (\mu_F, \gamma_F)$ is not an intuitionistic fuzzy strong IUP-ideal of X.

Theorem 3.9. *Every intuitionistic fuzzy IUP-subalgebra of X is an intuitionistic fuzzy IUP-filter of X.*

Proof: Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X. By Lemma 3.1, we have $F = (\mu_F, \gamma_F)$ satisfies (26) and (27). Let $x, y \in X$. Then

$$\begin{aligned} \mu_F(y) &= \mu_F(0 \cdot y) && \text{(by (IUP-1))} \\ &= \mu_F((x \cdot 0) \cdot (x \cdot y)) && \text{(by (IUP-3))} \\ &\geq \min\{\mu_F(x \cdot 0), \mu_F(x \cdot y)\} && \text{(by (24))} \\ &\geq \min\{\min\{\mu_F(x), \mu_F(0)\}, \mu_F(x \cdot y)\} && \text{(by (24))} \\ &= \min\{\mu_F(x \cdot y), \mu_F(x)\}, && \text{(by (26))} \\ \gamma_F(y) &= \gamma_F(0 \cdot y) && \text{(by (IUP-1))} \\ &= \gamma_F((x \cdot 0) \cdot (x \cdot y)) && \text{(by (IUP-3))} \\ &\leq \max\{\gamma_F(x \cdot 0), \gamma_F(x \cdot y)\} && \text{(by (25))} \\ &\leq \max\{\max\{\gamma_F(x), \gamma_F(0)\}, \gamma_F(x \cdot y)\} && \text{(by (25))} \\ &= \max\{\gamma_F(x), \gamma_F(x \cdot y)\}. && \text{(by (27))} \end{aligned}$$

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-filter of X. □

Example 3.4. *Let $\mathbb{R}^*$ be the set of all nonzero real numbers. Then $(\mathbb{R}^*; \cdot, 1)$ is an IUP-algebra, where $\cdot$ is the binary operation on $\mathbb{R}^*$ defined by $x \cdot y = \frac{y}{x}$ for all $x, y \in \mathbb{R}^*$. Let $S = \{x \in \mathbb{R}^* | x \geq 1\}$. Then S is an IUP-ideal and an IUP-filter of $\mathbb{R}^*$ but it is not an IUP-subalgebra of $\mathbb{R}^*$. From Theorems 3.2, 3.4, and 3.3, we have the characteristic IFS F_S is an intuitionistic fuzzy IUP-ideal and an intuitionistic fuzzy IUP-filter of $\mathbb{R}^*$ but it is not an intuitionistic fuzzy IUP-subalgebra of $\mathbb{R}^*$.*

Example 3.4 has demonstrated that an intuitionistic fuzzy IUP-ideal is not an intuitionistic fuzzy IUP-subalgebra in general. Therefore, Example 3.5 will illustrate that an intuitionistic fuzzy IUP-subalgebra is not an intuitionistic fuzzy IUP-ideal in general as well. Consequently, both examples highlight that an intuitionistic fuzzy IUP-ideal and an intuitionistic fuzzy IUP-subalgebra are not generally associated with each other.

Example 3.5. *Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:*

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	0	4	5	2	3
2	5	4	0	1	3	2
3	3	2	1	0	5	4
4	4	5	3	2	0	1
5	2	3	5	4	1	0

Then $X = (X; \cdot, 0)$ is an IUP-algebra. We define an IFS $F = (\mu_F, \gamma_F)$ on X as follows:

$$\mu_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 0.1 & 0.1 & 0.1 & 0.6 & 0.1 \end{pmatrix}$$

$$\gamma_F = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0.7 & 0.7 & 0.7 & 0.4 & 0.7 \end{pmatrix}$$

Then $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X. Since $\mu_F(1 \cdot 5) = \mu_F(3) = 0.1 \not\geq 0.6 = \min\{1, 0.6\} = \min\{\mu_F(0), \mu_F(4)\} = \min\{\mu_F(1 \cdot (4 \cdot 5)), \mu_F(4)\}$, we have $F = (\mu_F, \gamma_F)$ is not an intuitionistic fuzzy IUP-ideal of X.

The study revealed a relationship between the four concepts: intuitionistic fuzzy IUP-ideals and intuitionistic fuzzy IUP-subalgebras are generalizations of intuitionistic fuzzy strong IUP-ideals of IUP-algebras, where intuitionistic fuzzy strong IUP-ideals of IUP-algebras can only be a constant IFS. Intuitionistic fuzzy IUP-filters are a generalization of intuitionistic fuzzy IUP-ideals and intuitionistic fuzzy IUP-subalgebras. We summarize the relationship between these four concepts, which is shown in Figure 2.

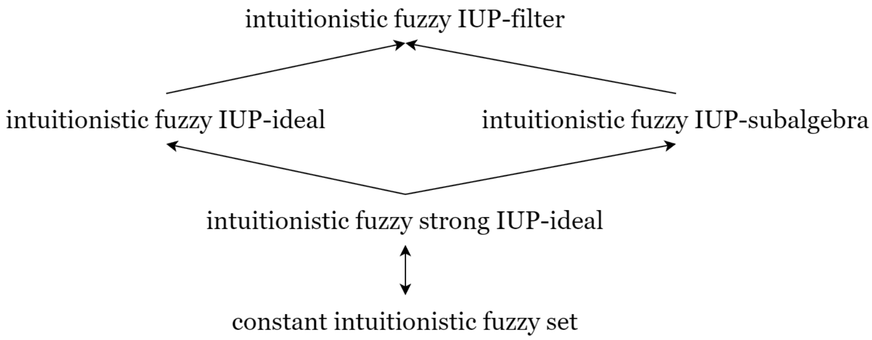

Figure 2. IFSs in IUP-algebras

From the definition of the complement and the properties of the min and max operations, we can immediately prove the following four theorems.

Theorem 3.10. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X if and only if the FSs μ_F and $\overline{\gamma}_F$ satisfy (24), and the FSs γ_F and $\overline{\mu}_F$ satisfy (25).*

Theorem 3.11. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-filter of X if and only if the FSs μ_F and $\overline{\gamma}_F$ satisfy (26) and (28), and the FSs γ_F and $\overline{\mu}_F$ satisfy (27) and (29).*

Theorem 3.12. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-ideal of X if and only if the FSs μ_F and $\overline{\gamma}_F$ satisfy (26) and (30), and the FSs γ_F and $\overline{\mu}_F$ satisfy (27) and (31).*

Theorem 3.13. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong IUP-ideal of X if and only if the FSs μ_F, $\overline{\mu}_F$, γ_F, and $\overline{\gamma}_F$ are constant.*

Theorem 3.14. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X if and only if the IFSs $\Box F = (\mu_F, \overline{\mu}_F)$ and $\Diamond F = (\overline{\gamma}_F, \gamma_F)$ are intuitionistic fuzzy IUP-subalgebras of X.*

Proof: It is straightforward by Theorem 3.10. □

Theorem 3.15. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-filter of X if and only if the IFSs $\Box F = (\mu_F, \overline{\mu}_F)$ and $\Diamond F = (\overline{\gamma}_F, \gamma_F)$ are intuitionistic fuzzy IUP-filters of X.*

Proof: It is straightforward by Theorem 3.11. □

Theorem 3.16. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-ideal of X if and only if the IFSs $\Box F = (\mu_F, \overline{\mu}_F)$ and $\Diamond F = (\overline{\gamma}_F, \gamma_F)$ are intuitionistic fuzzy IUP-ideals of X.*

Proof: It is straightforward by Theorem 3.12. □

Theorem 3.17. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong IUP-ideal of X if and only if the IFSs $\Box F = (\mu_F, \overline{\mu}_F)$ and $\Diamond F = (\overline{\gamma}_F, \gamma_F)$ are intuitionistic fuzzy strong IUP-ideals of X.*

Proof: It is straightforward by Theorem 3.13. □

Definition 3.5. *Let f be a fuzzy set in a nonempty set X. For any $t \in [0, 1]$, the sets $U(f; t) = \{x \in X | f(x) \geq t\}$ and $U^+(f; t) = \{x \in X | f(x) > t\}$ are called an upper t-level subset and an upper t-strong level subset of f, respectively. The sets $L(f; t) = \{x \in X | f(x) \leq t\}$ and $L^-(f; t) = \{x \in X | f(x) < t\}$ are called a lower t-level subset and a lower t-strong level subset of f, respectively.*

Theorem 3.18. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X if and only if for all $t, s \in [0, 1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or IUP-subalgebras of X.*

Proof: Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X. Let $t \in [0, 1]$ be such that $U(\mu_F; t) \neq \emptyset$. Let $x, y \in U(\mu_F; t)$. Then $\mu_F(x) \geq t$ and $\mu_F(y) \geq t$. Thus, $\min\{\mu_F(x), \mu_F(y)\} \geq t$. By (24), we have $\mu_F(x \cdot y) \geq \min\{\mu_F(x), \mu_F(y)\} \geq t$. Thus, $x \cdot y \in U(\mu_F; t)$. Hence, $U(\mu_F; t)$ is an IUP-subalgebra of X.

Let $s \in [0, 1]$ be such that $L(\gamma_F; s) \neq \emptyset$. Let $x, y \in L(\gamma_F; s)$. Then $\gamma_F(x) \leq s$ and $\gamma_F(y) \leq s$. Thus, $\max\{\gamma_F(x), \gamma_F(y)\} \leq s$. By (25), we have $\gamma_F(x \cdot y) \leq \max\{\gamma_F(x), \gamma_F(y)\} \leq s$. Thus, $x \cdot y \in L(\gamma_F; s)$. Hence, $L(\gamma_F; s)$ is an IUP-subalgebra of X.

Conversely, assume that for all $t, s \in [0, 1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or IUP-subalgebras of X. Let $x, y \in X$. Let $t = \min\{\mu_F(x), \mu_F(y)\}$. Then $\mu_F(x) \geq t$ and $\mu_F(y) \geq t$. Thus, $x, y \in U(\mu_F; t) \neq \emptyset$. By the assumption, we have $U(\mu_F; t)$ is an IUP-subalgebra of X. By (17), we have $x \cdot y \in U(\mu_F; t)$. Thus, $\mu_F(x \cdot y) \geq t = \min\{\mu_F(x), \mu_F(y)\}$.

Let $x, y \in X$. Let $s = \max\{\gamma_F(x), \gamma_F(y)\}$. Then $\gamma_F(x) \leq s$ and $\gamma_F(y) \leq s$. Thus, $x, y \in L(\gamma_F; s) \neq \emptyset$. By the assumption, we have $L(\gamma_F; s)$ is an IUP-subalgebra of X. By (17), we have $x \cdot y \in L(\gamma_F; s)$. Thus, $\gamma_F(x \cdot y) \leq s = \max\{\gamma_F(x), \gamma_F(y)\}$.

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X. □

Theorem 3.19. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-filter of X if and only if for all $t, s \in [0, 1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or IUP-filters of X.*

Proof: Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-filter of X. Let $t \in [0, 1]$ be such that $U(\mu_F; t) \neq \emptyset$. Let $a \in U(\mu_F; t)$. Then $\mu_F(a) \geq t$. By (26), we have $\mu_F(0) \geq \mu_F(a) \geq t$. Thus, $0 \in U(\mu_F; t)$. Let $x, y \in X$ be such that $x \cdot y \in U(\mu_F; t)$ and $x \in U(\mu_F; t)$. Then $\mu_F(x \cdot y) \geq t$ and $\mu_F(x) \geq t$. Thus, $\min\{\mu_F(x \cdot y), \mu_F(x)\} \geq t$. By (28), we have $\mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\} \geq t$. Thus, $y \in U(\mu_F; t)$. Hence, $U(\mu_F; t)$ is an IUP-filter of X.

Let $s \in [0, 1]$ be such that $L(\gamma_F; s) \neq \emptyset$. Let $b \in L(\gamma_F; s)$. Then $\gamma_F(b) \leq s$. By (27), we have $\gamma_F(0) \leq \gamma_F(b) \leq s$. Thus, $0 \in L(\gamma_F; s)$. Let $x, y \in X$ be such that $x \cdot y \in L(\gamma_F; s)$ and $x \in L(\gamma_F; s)$. Then $\gamma_F(x \cdot y) \leq s$ and $\gamma_F(x) \leq s$. Thus, $\max\{\gamma_F(x \cdot y), \gamma_F(x)\} \leq s$. By (29), we have $\gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\} \leq s$. Thus, $y \in L(\gamma_F; s)$. Hence, $L(\gamma_F; s)$ is an IUP-filter of X.

Conversely, assume that for all $t, s \in [0, 1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or IUP-filters of X. Let $x \in X$. Let $t = \mu_F(x)$. Then $\mu_F(x) \geq t$. Thus, $x \in U(\mu_F; t) \neq \emptyset$. By the assumption, we have $U(\mu_F; t)$ is an IUP-filter of X. By (18), we have $0 \in U(\mu_F; t)$. Thus, $\mu_F(0) \geq t = \mu_F(x)$. Let $x, y \in X$. Let $t = \min\{\mu_F(x \cdot y), \mu_F(x)\}$. Then $\mu_F(x \cdot y) \geq t$ and $\mu_F(x) \geq t$. Thus, $x \cdot y, x \in U(\mu_F; t) \neq \emptyset$. By the assumption, we have $U(\mu_F; t)$ is an IUP-filter of X. By (19), we have $y \in U(\mu_F; t)$. Thus, $\mu_F(y) \geq t = \min\{\mu_F(x \cdot y), \mu_F(x)\}$.

Let $x \in X$. Let $s = \gamma_F(x)$. Then $\gamma_F(x) \leq s$. Thus, $x \in L(\gamma_F; s) \neq \emptyset$. By the assumption, we have $L(\gamma_F; s)$ is an IUP-filter of X. By (18), we have $0 \in L(\gamma_F; s)$. Thus, $\gamma_F(0) \leq s = \gamma_F(x)$. Let $x, y \in X$. Let $s = \max\{\gamma_F(x \cdot y), \gamma_F(x)\}$. Then $\gamma_F(x \cdot y) \leq s$ and $\gamma_F(x) \leq s$. Thus, $x \cdot y, x \in L(\gamma_F; s) \neq \emptyset$. By the assumption, we have $L(\gamma_F; s)$ is an IUP-filter of X. By (19), we have $y \in L(\gamma_F; s)$. Thus, $\gamma_F(y) \leq s = \max\{\gamma_F(x \cdot y), \gamma_F(x)\}$.

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-filter of X. □

Theorem 3.20. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-ideal of X if and only if for all $t, s \in [0, 1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or IUP-ideals of X.*

Proof: Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-ideal of X. Let $t \in [0, 1]$ be such that $U(\mu_F; t) \neq \emptyset$. Let $a \in U(\mu_F; t)$. Then $\mu_F(a) \geq t$. By (26), we have $\mu_F(0) \geq \mu_F(a) \geq t$. Thus, $0 \in U(\mu_F; t)$. Let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in U(\mu_F; t)$ and $y \in U(\mu_F; t)$. Then $\mu_F(x \cdot (y \cdot z)) \geq t$ and $\mu_F(y) \geq t$. Thus, $\min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\} \geq t$. By (30), we have $\mu_F(x \cdot z) \geq \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\} \geq t$. Thus, $x \cdot z \in U(\mu_F; t)$. Hence, $U(\mu_F; t)$ is an IUP-ideal of X.

Let $s \in [0, 1]$ be such that $L(\gamma_F; s) \neq \emptyset$. Let $b \in L(\gamma_F; s)$. Then $\gamma_F(b) \leq s$. By (27), we have $\gamma_F(0) \leq \mu_F(b) \leq s$. Thus, $0 \in L(\gamma_F; s)$. Let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in L(\gamma_F; s)$ and $y \in L(\gamma_F; s)$. Then $\gamma_F(x \cdot (y \cdot z)) \leq s$ and $\gamma_F(y) \leq s$. Thus, $\max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\} \leq s$. By (31), we have $\gamma_F(x \cdot z) \leq \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\} \leq s$. Thus, $x \cdot z \in L(\gamma_F; s)$. Hence, $L(\gamma_F; s)$ is an IUP-ideal X.

Conversely, assume that for all $t, s \in [0, 1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or IUP-ideals of X. Let $x \in X$. Let $t = \mu_F(x)$. Then $\mu_F(x) \geq t$. Thus, $x \in U(\mu_F; t) \neq \emptyset$. By the assumption, we have $U(\mu_F; t)$ is an IUP-ideal of X. By (18), we have $0 \in U(\mu_F; t)$. Thus, $\mu_F(0) \geq t = \mu_F(x)$. Let $x, y, z \in X$. Let $t = \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\}$. Then $\mu_F(x \cdot (y \cdot z)) \geq t$ and $\mu_F(y) \geq t$. Thus, $x \cdot (y \cdot z), y \in U(\mu_F; t) \neq \emptyset$. By the assumption, we have $U(\mu_F; t)$ is an IUP-ideal of X. By (20), we have $x \cdot z \in U(\mu_F; t)$. Thus, $\mu_F(x \cdot z) \geq t = \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\}$.

Let $x \in X$. Let $s = \gamma_F(x)$. Then $\gamma_F(x) \leq s$. Thus, $x \in L(\gamma_F; s) \neq \emptyset$. By the assumption, we have $L(\gamma_F; s)$ is an IUP-ideal of X. By (18), we have $0 \in L(\gamma_F; s)$. Thus, $\gamma_F(0) \leq s = \gamma_F(x)$. Let $x, y, z \in X$. Let $s = \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\}$. Then $\gamma_F(x \cdot (y \cdot z)) \leq s$ and $\gamma_F(y) \leq s$. Thus, $x \cdot (y \cdot z), y \in L(\gamma_F; s) \neq \emptyset$. By the assumption, we have $L(\gamma_F; s)$ is an IUP-ideal of X. By (20), we have $x \cdot z \in L(\gamma_F; s)$. Thus, $\gamma_F(x \cdot z) \leq s = \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\}$.

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-ideal of X. □

Theorem 3.21. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong IUP-ideal of X if and only if for all $t, s \in [0, 1]$, the sets $U(\mu_F; t)$ and $L(\gamma_F; s)$ are either empty or strong IUP-ideals of X.*

Proof: It is straightforward by Theorem 3.1. □

The relationship between upper t-level subsets, lower t-level subsets, and their IFSs is demonstrated in Theorems 3.18, 3.19, 3.20, and 3.21. Additionally, we establish the relationship between upper t-strong level subsets, lower t-strong level subsets, and their IFSs, as depicted in Theorems 3.22, 3.23, 3.24, and 3.25.

Theorem 3.22. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X if and only if for all $t, s \in [0, 1]$, the sets $U^+(\mu_F; t)$ and $L^-(\gamma_F; s)$ are either empty or IUP-subalgebras of X.*

Proof: Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X. Let $t \in [0, 1]$ be such that $U^+(\mu_F; t) \neq \emptyset$. Let $x, y \in U^+(\mu_F; t)$. Then $\mu_F(x) > t$ and $\mu_F(y) > t$. Thus, $\min\{\mu_F(x), \mu_F(y)\} > t$. By (24), we have $\mu_F(x\cdot y) \geq \min\{\mu_F(x), \mu_F(y)\} > t$. Thus, $x \cdot y \in U^+(\mu_F; t)$. Hence, $U^+(\mu_F; t)$ is an IUP-subalgebra of X.

Let $s \in [0, 1]$ be such that $L^-(\gamma_F; s) \neq \emptyset$. Let $x, y \in L^-(\gamma_F; s)$. Then $\gamma_F(x) < s$ and $\gamma_F(y) < s$. Thus, $\max\{\gamma_F(x), \gamma_F(y)\} < s$. By (25), we have $\gamma_F(x\cdot y) \leq \max\{\gamma_F(x), \gamma_F(y)\} < s$. Thus, $x \cdot y \in L^-(\gamma_F; s)$. Hence, $L^-(\gamma_F; s)$ is an IUP-subalgebra of X.

Conversely, assume that for all $t, s \in [0, 1]$, the sets $U^+(\mu_F; t)$ and $L^-(\gamma_F; s)$ are either empty or IUP-subalgebras of X. Let $x, y \in X$. Assume that $\mu_F(x\cdot y) < \min\{\mu_F(x), \mu_F(y)\}$. Let $t = \mu_F(x \cdot y)$. Then $\mu_F(x) > t$ and $\mu_F(y) > t$. Thus, $x, y \in U^+(\mu_F; t) \neq \emptyset$. By the assumption, we have $U^+(\mu_F; t)$ is an IUP-subalgebra of X. By (17), we have $x \cdot y \in U^+(\mu_F; t)$. So $\mu_F(x \cdot y) > t = \mu_F(x \cdot y)$, which is a contradiction. Thus, $\mu_F(x \cdot y) \geq \min\{\mu_F(x), \mu_F(y)\}$.

Let $x, y \in X$. Assume that $\gamma_F(x \cdot y) > \max\{\gamma_F(x), \gamma_F(y)\}$. Let $s = \gamma_F(x \cdot y)$. Then $\gamma_F(x) < s$ and $\gamma_F(y) < s$. Thus, $x, y \in L^-(\gamma_F; s) \neq \emptyset$. By the assumption, we have $L^-(\gamma_F; s)$ is an IUP-subalgebra of X. By (17), we have $x \cdot y \in L^-(\gamma_F; s)$. So $\gamma_F(x \cdot y) < s = \gamma_F(x \cdot y)$, which is a contradiction. Thus, $\gamma_F(x \cdot y) \leq \max\{\gamma_F(x), \gamma_F(y)\}$.

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-subalgebra of X. □

Theorem 3.23. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-filter of X if and only if for all $t, s \in [0, 1]$, the sets $U^+(\mu_F; t)$ and $L^-(\gamma_F; s)$ are either empty or IUP-filters of X.*

Proof: Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-filter of X. Let $t \in [0, 1]$ be such that $U^+(\mu_F; t) \neq \emptyset$. Let $a \in U^+(\mu_F; t)$. Then $\mu_F(a) > t$. By (26), we have $\mu_F(0) \geq \mu_F(a) > t$. Thus, $0 \in U^+(\mu_F; t)$. Let $x, y \in X$ be such that $x \cdot y \in U^+(\mu_F; t)$ and $x \in U^+(\mu_F; t)$. Then $\mu_F(x \cdot y) > t$ and $\mu_F(x) > t$. Thus, $\min\{\mu_F(x \cdot y), \mu_F(x)\} > t$. By (28), we have $\mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\} > t$. Thus, $y \in U^+(\mu_F; t)$. Hence, $U^+(\mu_F; t)$ is an IUP-filter of X.

Let $s \in [0, 1]$ be such that $L^-(\gamma_F; s) \neq \emptyset$. Let $b \in L^-(\gamma_F; s)$. Then $\gamma_F(b) < s$. By (27), we have $\gamma_F(0) \leq \gamma_F(b) < s$. Thus, $0 \in L^-(\gamma_F; s)$. Let $x, y \in X$ be such that $x \cdot y \in L^-(\gamma_F; s)$ and $x \in L^-(\gamma_F; s)$. Then $\gamma_F(x\cdot y) < s$ and $\gamma_F(x) < s$. Thus, $\max\{\gamma_F(x\cdot y), \gamma_F(x)\} < s$. By (29), we have $\gamma_F(y) \leq \max\{\gamma_F(x\cdot y), \gamma_F(y)\} < s$. Thus, $x\cdot z \in L^-(\gamma_F; s)$. Hence, $L^-(\gamma_F; s)$ is an IUP-filter of X.

Conversely, assume that for all $t, s \in [0, 1]$, the sets $U^+(\mu_F; t)$ and $L^-(\gamma_F; s)$ are either empty or IUP-filters of X. Let $x \in X$. Assume that $\mu_F(0) < \mu_F(x)$. Let $t = \mu_F(0)$. Then $x \in U^+(\mu_F; t) \neq \emptyset$. By the assumption, we have $U^+(\mu_F; t)$ is an IUP-filter of X. By (18), we have $0 \in U^+(\mu_F; t)$. So $\mu_F(0) > t = \mu_F(0)$, which is a contradiction. Thus, $\mu_F(0) \geq \mu_F(x)$. Let $x, y \in X$. Assume that $\mu_F(y) < \min\{\mu_F(x \cdot y), \mu_F(x)\}$. Let $t = \mu_F(y)$. Then $t < \mu_F(x \cdot y)$ and $t < \mu_F(x)$. Thus, $x \cdot y, x \in U^+(\mu_F; t) \neq \emptyset$. By the assumption, we have $U^+(\mu_F; t)$ is an IUP-filter of X. By (19), we have $y \in U^+(\mu_F; t)$. So $\mu_F(y) > t = \mu_F(y)$, which is a contradiction. Thus, $\mu_F(y) \geq \min\{\mu_F(x \cdot y), \mu_F(x)\}$.

Let $x \in X$. Assume that $\gamma_F(0) > \gamma_F(x)$. Let $s = \gamma_F(0)$. Then $x \in L^-(\gamma_F; s) \neq \emptyset$. By the assumption, we have $L^-(\gamma_F; s)$ is an IUP-filter of X. By (18), we have $0 \in L^-(\gamma_F; s)$. So $\gamma_F(0) < s = \gamma_F(0)$, which is a contradiction. Thus, $\gamma_F(0) \leq \gamma_F(x)$. Let $x, y \in X$.

Assume that $\gamma_F(y) > \max\{\gamma_F(x \cdot y), \gamma_F(x)\}$. Let $s = \gamma_F(y)$. Then $s > \gamma_F(x \cdot y)$ and $s > \gamma_F(x)$. Thus, $x \cdot y, x \in L^-(\gamma_F; s) \neq \emptyset$. By the assumption, we have $L^-(\gamma_F; s)$ is an IUP-filter of X. By (19), we have $y \in L^-(\gamma_F; s)$. So $\gamma_F(y) < s = \gamma_F(y)$, which is a contradiction. Thus, $\gamma_F(y) \leq \max\{\gamma_F(x \cdot y), \gamma_F(x)\}$.

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-filter of X. □

Theorem 3.24. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-ideal of X if and only if for all $t, s \in [0, 1]$, the sets $U^+(\mu_F; t)$ and $L^-(\gamma_F; s)$ are either empty or IUP-ideals of X.*

Proof: Assume that $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-ideal of X. Let $t \in [0, 1]$ be such that $U^+(\mu_F; t) \neq \emptyset$. Let $a \in U^+(\mu_F; t)$. Then $\mu_F(a) > t$. By (26), we have $\mu_F(0) \geq \mu_F(a) > t$. Thus, $0 \in U^+(\mu_F; t)$. Let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in U^+(\mu_F; t)$ and $y \in U^+(\mu_F; t)$. Then $\mu_F(x \cdot (y \cdot z)) > t$ and $\mu_F(y) > t$. Thus, $\min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\} > t$. By (30), we have $\mu_F(x \cdot z) \geq \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\} > t$. Thus, $x \cdot z \in U^+(\mu_F; t)$. Hence, $U^+(\mu_F; t)$ is an IUP-ideal of X.

Let $s \in [0, 1]$ be such that $L^-(\gamma_F; s) \neq \emptyset$. Let $b \in L^-(\gamma_F; s)$. Then $\gamma_F(b) < s$. By (27), we have $\gamma_F(0) \leq \gamma_F(b) < s$. Thus, $0 \in L^-(\gamma_F; s)$. Let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in L^-(\gamma_F; s)$ and $y \in L^-(\gamma_F; s)$. Then $\gamma_F(x \cdot (y \cdot z)) < s$ and $\gamma_F(y) < s$. Thus, $\max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\} < s$. By (31), we have $\gamma_F(x \cdot z) \leq \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\} < s$. Thus, $x \cdot z \in L^-(\gamma_F; s)$. Hence, $L^-(\gamma_F; s)$ is an IUP-ideal of X.

Conversely, assume that for all $t, s \in [0, 1]$, the sets $U^+(\mu_F; t)$ and $L^-(\gamma_F; s)$ are either empty or IUP-ideals of X. Let $x \in X$. Assume that $\mu_F(0) < \mu_F(x)$. Let $t = \mu_F(0)$. Then $x \in U^+(\mu_F; t) \neq \emptyset$. By the assumption, we have $U^+(\mu_F; t)$ is an IUP-ideal of X. By (18), we have $0 \in U^+(\mu_F; t)$. So $\mu_F(0) > t = \mu_F(0)$, which is a contradiction. Thus, $\mu_F(0) \geq \mu_F(x)$. Let $x, y, z \in X$. Assume that $\mu_F(x \cdot z) < \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\}$. Let $t = \mu_F(x \cdot z)$. Then $t < \mu_F(x \cdot (y \cdot z))$ and $t < \mu_F(y)$. Thus, $x \cdot (y \cdot z), y \in U^+(\mu_F; t) \neq \emptyset$. By the assumption, we have $U^+(\mu_F; t)$ is an IUP-ideal of X. By (20), we have $x \cdot z \in U^+(\mu_F; t)$. So $\mu_F(x \cdot z) > t = \mu_F(x \cdot z)$, which is a contradiction. Thus, $\mu_F(x \cdot z) \geq \min\{\mu_F(x \cdot (y \cdot z)), \mu_F(y)\}$.

Let $x \in X$. Assume that $\gamma_F(0) > \gamma_F(x)$. Let $s = \gamma_F(0)$. Then $x \in L^-(\gamma_F; s) \neq \emptyset$. By the assumption, we have $L^-(\gamma_F; s)$ is an IUP-ideal of X. By (18), we have $0 \in L^-(\gamma_F; s)$. So $\gamma_F(0) < s = \gamma_F(0)$, which is a contradiction. Thus, $\gamma_F(0) \leq \gamma_F(x)$. Let $x, y, z \in X$. Assume that $\gamma_F(x \cdot z) > \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\}$. Let $s = \gamma_F(x \cdot z)$. Then $s > \gamma_F(x \cdot (y \cdot z))$ and $s > \gamma_F(y)$. Thus, $x \cdot (y \cdot z), y \in L^-(\gamma_F; s) \neq \emptyset$. By the assumption, we have $L^-(\gamma_F; s)$ is an IUP-ideal of X. By (20), we have $x \cdot z \in L^-(\gamma_F; s)$. So $\gamma_F(x \cdot z) < s = \gamma_F(x \cdot z)$, which is a contradiction. Thus, $\gamma_F(x \cdot z) \leq \max\{\gamma_F(x \cdot (y \cdot z)), \gamma_F(y)\}$.

Hence, $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy IUP-ideal of X. □

Theorem 3.25. *An IFS $F = (\mu_F, \gamma_F)$ is an intuitionistic fuzzy strong IUP-ideal of X if and only if for all $t, s \in [0, 1]$, the sets $U^+(\mu_F; t)$ and $L^-(\gamma_F; s)$ are either empty or strong IUP-ideals of X.*

Proof: It is straightforward by Theorem 3.1. □

The results presented in this paper regarding the study of IFSs in IUP-algebras are also connected to prior research examining IFSs in UP-algebras [5], BG-algebras [8], and Hilbert algebras [21].

4. **Conclusions.** In this paper, we introduce the concepts of intuitionistic fuzzy IUP-subalgebras, intuitionistic fuzzy IUP-filters, intuitionistic fuzzy IUP-ideals, and intuitionistic fuzzy strong IUP-ideals of IUP-algebras and investigate important properties. Our research found that these four concepts relate to characteristic functions and level sets.

In the near future, our research team will also study the concept of neutrosophic sets as defined by Smarandache [32].

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