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MULTIPLE-LOOP GUARANTEED COST CONTROL FOR UNCERTAIN TIME-DELAYED DESCRIPTOR SYSTEMS

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Abstract. *This paper focuses on the guaranteed cost control issue of quadratic stabilization for a class of time-delayed descriptor systems with norm-bounded uncertainties. By applying the Lyapunov stability theorem, a feedback controller with multiple-loops is developed to ensure quadratic stability of the system and restrict the cost performance within a predefined upper bound. Then, the equivalent conditions of the system stability and the cost function satisfying the requirements are given in terms of linear matrix inequality (LMI) method. Based on the convex optimization techniques, the upper bound of the minimum performance index and the optimal guaranteed cost controller can be determined. Finally, two examples are presented to demonstrate the reasonableness and validness of the proposed scheme.*

Keywords: Time-delayed descriptor system, Norm-bounded uncertainty, Quadratic stabilization, Guaranteed cost control, Linear matrix inequality

1. **Introduction.** Descriptor systems, also known as generalized, singular, differential-algebraic or semi-state systems, have been instrumental in a myriad of applications across different fields, including but not limited to robotics, aerospace control, chemical processes, and multi-sector economic systems [1, 2, 3]. In contrast to the standard state-space models, descriptor system models have garnered significant attention in the control and mathematics communities due to their inherent advantages in describing physical systems with greater convenience and naturalness. One of the key challenges in the process control of descriptor systems is how to ensure reliable stability and obtain target performance [4]. In a network of linear singularly perturbed systems, to address the guaranteed cost control problem, a state feedback controller is designed to realize system synchronization and minimize the prescribed performance cost [5]. In [6], a non-fragile sampled-data controller is developed to meet quadratic stabilization performance standards and handle uncertainties. In [7], the authors propose a control protocol that utilizes local information from neighboring agents to achieve leader-following consensus while preserving the guaranteed cost performance for the agents. Drawing from the insights of these articles, it becomes evident that guaranteed cost control is a critical aspect when discussing descriptor systems. It should be noted that the presence of uncertainties and time delays

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requires the development of novel control strategies to ensure system stability and control performance.

In the realm of real-world systems, uncertainties and time delays are intrinsic factors that significantly impact the performance and behavior of various dynamic processes [8]. From the operation of complex industrial systems to the control of autonomous vehicles, the presence of uncertainties and time delays poses formidable challenges for engineers and researchers. Uncertainties in system dynamics stem from many factors, including inherent variations in manufacturing processes, imprecise knowledge of system parameters and unpredictable environmental conditions [9]. In [10], the main idea of the work focuses on the development of reduced-order dynamic output feedback controllers for linear time-varying systems with norm-bounded parameter uncertainties. In [11], the authors propose a distributed robust optimal control protocol that is applicable for addressing parametric uncertainties. In addition to uncertainties, time delays represent another critical aspect of real-world systems [12]. Time delays emerge when there is an interval in the transmission of information or when physical processes take a certain amount of time to propagate. By constructing a modified Lyapunov-Krasovskii functional, the authors investigate the utilization of both delay-product-type functions and delay-dependent matrices to improve the stability conditions for generalized neural networks with time delays [13]. With the help of switching Lyapunov function approach and free-weighting matrix knowledge, the authors derive delay-dependent criteria for consensus control and fault estimation [14]. In [15], the authors investigate the congestion control problems, where the Ad Hoc networks have restricted bandwidth and exhibit greater susceptibility to time-delays.

With the persistent research and exploration, there have been a lot of achievements on the optimal control problems by using the single-loop feedback controller. However, people are often confronted with various factors that tend to be much more complex in practical applications. Most of the time, designing a simple state feedback [16, 17], output feedback [18] or proportional-derivative feedback controller [19] does not achieve fast and accurate control performance, as well as realize the expected optimal effects. Therefore, to achieve this goal, we need to consider all aspects of the factors and take the advantage of multiple-loop control technology to establish a multiple-loop feedback control system. Multiple-loop control systems are widely adopted in process control, robot control and flight control, which provides many conveniences for industrial production and daily life [20, 21, 22]. By introducing the Lyapunov function and leveraging the advantages of multiple-loop control technology, we propose a proportional-differential and static output feedback controller. This controller ensures quadratic stability of the closed-loop system while constraining the cost performance to a predefined upper bound value. Then, the equivalent conditions of system stability and guaranteed cost function satisfying requirements are given by using LMI method. Next, the upper bound of the minimum performance index and optimal guaranteed cost controller are obtained based on convex optimization technique. Finally, the effectiveness of this method is discussed and analyzed through simulation results. The simulation results demonstrate the superiority of our proposed multiple-loop feedback controller over the controller presented in [4] in terms of steady-state performance and cost value. These findings highlight the efficacy of our approach in attaining the desired control objectives. The results provide compelling evidence supporting the validity and effectiveness of our proposed strategy for guaranteed cost control applications.

The main contributions of this work can be summarized as 1) a multiple-loop feedback control scheme with proportional-differential and static output feedback; 2) LMI-based sufficient conditions for quadratic stability; and 3) convex optimization techniques to determine the upper bound of guaranteed cost performance.

Notations: Throughout this paper, R^n denotes the n-dimensional Euclidean space, and $\mathrm{R}^{m\times n}$ indicates the set of all real matrices. The symbols '$^{\mathrm{T}}$' and '$*$' donate, respectively, the matrix transposition and an ellipsis for the terms induced by symmetry in symmetric block matrices. The matrix I represents an identity matrix with appropriate dimensions, $\mathrm{diag}\{\cdots\}$ for a block-diagonal matrix. Symbol $\|\cdot\|$ refers to the Euclidean norm for vector.

The remainder of this paper is structured as follows. Section 2 presents the formulation of the guaranteed cost control problem and some necessary preliminaries. Section 3 provides the performance analysis and main theoretical results for different guaranteed cost controllers. Section 4 verifies the main results through two numerical examples. Finally, Section 5 concludes the paper.

2. Problem Formulation and Preliminaries.

Consider a class of time-delayed descriptor systems with norm-bounded uncertainties that can be described as

$$\begin{cases} (E+\Delta E)\dot{x}(t) = (A+\Delta A)x(t) + (A_d+\Delta A_d)x(t-d) + (B+\Delta B)u(t), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

$$x(t) = \phi(t), \quad t \in [-d, 0], \tag{2}$$

where $x(t) \in \mathrm{R}^n$, $u(t) \in \mathrm{R}^m$ and $y(t) \in \mathrm{R}^p$ are the state vector, control input vector and output vector, respectively. Symbols $(E, A, A_d) \in \mathrm{R}^{n\times n}$ and $(B, C) \in \mathrm{R}^{m\times n}$ are constant matrices with appropriate dimensions, and $rank(E) = r \le n$. Matrix C is of full row rank, and ΔE, ΔA, ΔA_d, ΔB stand for norm-bounded uncertainties with the following form:

$$\begin{bmatrix} \Delta E & \Delta A & \Delta A_d & \Delta B \end{bmatrix} = GF(t) \begin{bmatrix} N_E & N_A & N_{A_d} & N_B \end{bmatrix}, \tag{3}$$

where G, N_E, N_A, N_{A_d}, N_B are known real constant matrices of appropriate dimensions, and the unknown matrix $F(t)$ satisfies $F^{\mathrm{T}}(t)F(t) \le I$.

For the system (1), a proportional differential and static output feedback controller is designed as follows:

$$u(t) = K_p x(t) - K_d \dot{x}(t) + K_c y(t), \tag{4}$$

where K_p, K_d and K_c are to be determined gain matrices with appropriate dimensions.

Substituting Equation (4) into Equation (1) yields a closed-loop system with three interconnected feedback loops:

$$\bar{E}\dot{x}(t) = \bar{A}x(t) + \bar{A}_d x(t-d), \tag{5}$$

where $\bar{E} = E + \Delta E + (B+\Delta B)K_d$, $\bar{A} = A + \Delta A + (B+\Delta B)K_p + (B+\Delta B)K_c C$, $\bar{A}_d = A_d + \Delta A_d$.

Taking the system (5) into account, and introducing a set of positive definite matrices Q_1, Q_2 and R, we can provide the cost function

$$J = \int_0^{+\infty} \left[x^{\mathrm{T}}(t)Q_1 x(t) + \dot{x}^{\mathrm{T}}(t)Q_2 x(t) + u^{\mathrm{T}}(t)Ru(t)\right] dt. \tag{6}$$

Definition 2.1. *Consider the system (1) associated with the cost function (6). If there exists a control law (4) that makes the system (1) quadratically stable, and the corresponding value of the cost function (6) satisfies the condition $J \le J^*$, then J^* is said to be a guaranteed cost. In this manner, Equation (4) is a quadratically stable guaranteed cost control law (QSGCCL) for the system (1).*

3. Main Results.

Theorem 3.1. *Consider the system (1) associated with the cost function (6). If there exist matrices $P > 0$, $M > 0$, T_1 and T_2 such that*

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ * & \Omega_{22} & \Omega_{23} \\ * & * & \Omega_{33} \end{bmatrix} < 0, \tag{7}$$

where $\Omega_{11} = \bar{A}T_1 + T_1^{\mathrm{T}}\bar{A} + M + Q_1 + \left(K_p^{\mathrm{T}} + C^{\mathrm{T}}K_c^{\mathrm{T}}\right)R(K_p + K_cC)$, $\Omega_{12} = T_1^{\mathrm{T}}\bar{A}_d$, $\Omega_{13} = P - T_1^{\mathrm{T}}\bar{E} + \bar{A}^{\mathrm{T}}T_2 - \left(K_p^{\mathrm{T}} + C^{\mathrm{T}}K_c^{\mathrm{T}}\right)RK_d$, $\Omega_{22} = -M$, $\Omega_{23} = \bar{A}_d^{\mathrm{T}}T_2$, $\Omega_{33} = -T_2^{\mathrm{T}}\bar{E} - \bar{E}^{\mathrm{T}}T_2 + Q_2 + K_d^{\mathrm{T}}RK_d$, *then Equation (4) is a QSGCCL, and the cost function satisfies* $J \le J^* = x_0^{\mathrm{T}}Px_0 + \int_{-d}^{0} x^{\mathrm{T}}(\tau)Mx(\tau)d\tau$.

Proof: Construct the following Lyapunov functional as

$$V(t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-d}^{t} x^{\mathrm{T}}(\tau)Mx(\tau)d\tau, \tag{8}$$

where P and M are real symmetric positive definite matrices. For introducing matrices T_1 and T_2 with appropriate dimensions, we have

$$2\left[-x^{\mathrm{T}}(t)T_1^{\mathrm{T}} - \dot{x}(t)T_2^{\mathrm{T}}\right]\left[\bar{E}\dot{x}(t) - \bar{A}x(t) - \bar{A}_d(t-d)\right] = 0. \tag{9}$$

Differentiating $V(t)$ along the system trajectory with respect to t, and combining Equation (6) with Equation (9), we can obtain

$$\begin{aligned} &\dot{V}(t) + x^{\mathrm{T}}(t)Q_1x(t) + \dot{x}^{\mathrm{T}}(t)Q_2\dot{x}(t) + u^{\mathrm{T}}(t)Ru(t) \\ &= 2x^{\mathrm{T}}(t)P\dot{x}(t) + x^{\mathrm{T}}(t)Mx(t) - x^{\mathrm{T}}(t-d)Mx(t-d) + x^{\mathrm{T}}(t)Q_1x(t) + \dot{x}^{\mathrm{T}}(t)Q_2\dot{x}(t) \\ &\quad + x^{\mathrm{T}}(t)\left(K_p^{\mathrm{T}} + C^{\mathrm{T}}K_c^{\mathrm{T}}\right)R(K_p + K_cC)x(t) \\ &\quad + \dot{x}^{\mathrm{T}}(t)K_d^{\mathrm{T}}RK_d\dot{x}(t) - 2x^{\mathrm{T}}(t)\left(K_p^{\mathrm{T}} + C^{\mathrm{T}}K_c^{\mathrm{T}}\right)RK_d\dot{x}(t) \\ &\quad + 2\left[-x^{\mathrm{T}}(t)T_1^{\mathrm{T}} - \dot{x}(t)T_2^{\mathrm{T}}\right]\left[\bar{E}\dot{x}(t) - \bar{A}x(t) - \bar{A}_d(t-d)\right] \\ &= \xi^{\mathrm{T}}(t)\Omega\xi(t), \end{aligned} \tag{10}$$

where $\xi^{\mathrm{T}}(t) = \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-d) & \dot{x}^{\mathrm{T}}(t) \end{bmatrix}$. If Equation (7) holds, we have

$$\dot{V}(t) + x^{\mathrm{T}}(t)Q_1x(t) + \dot{x}^{\mathrm{T}}(t)Q_2\dot{x}(t) + u^{\mathrm{T}}(t)Ru(t) < 0. \tag{11}$$

Thus, it can be seen that the $\dot{V}(t) < 0$. This proves the stability of a closed-loop system (5). Integrating both sides of Equation (11) from $t = 0$ to $t = +\infty$, we can get

$$\begin{aligned} J &= \int_0^{+\infty}\left[x^{\mathrm{T}}(t)Q_1x(t) + \dot{x}^{\mathrm{T}}(t)Q_2x(t) + u^{\mathrm{T}}(t)Ru(t)\right]dt \le -\int_0^{+\infty}\dot{V}(t)dt \\ &= V(0) - V(+\infty) = x_0^{\mathrm{T}}Px_0 + \int_{-d}^{0} x^{\mathrm{T}}(\tau)Mx(\tau)d\tau. \end{aligned} \tag{12}$$

Therefore, according to the above Definition 2.1, the proof of this theorem is completed.

Based on the aforementioned Theorem 3.1, the method of QSGCCL has been found. However, the norm-bounded uncertainties in $\bar{E}$, $\bar{A}$ and $\bar{A}_d$ make it challenging to obtain feasible LMI conditions. Therefore, we need to perform the following equivalent transformation for Theorem 3.1. Multiply Equation (7) both sides by Δ^{T} and Δ respectively, and let $\Delta = \begin{bmatrix} V_1 & 0 & 0 \\ 0 & V_2 & 0 \\ V_4 & 0 & V_3 \end{bmatrix}$, $V_1 = P^{-1}$, $V_2 = M^{-1}$, $V_3 = T_2^{-1}$, $V_4 = -T_2^{-1}T_1P^{-1}$; then, we can get the following corollary.

Corollary 3.1. *Consider the system (1) associated with the cost function (6). If there exist matrices $V_1 > 0$, $V_2 > 0$, V_3 and V_4 such that*

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ * & \Pi_{22} & \Pi_{23} \\ * & * & \Pi_{33} \end{bmatrix} < 0, \tag{13}$$

where

$$\Pi_{11} = V_4 + V_4^{\mathrm{T}} + V_1^{\mathrm{T}} V_2^{-1} V_1 + V_1^{\mathrm{T}} Q_1 V_1 + V_4^{\mathrm{T}} Q_2 V_4 + \left[V_1^{\mathrm{T}}\left(K_p^{\mathrm{T}} + C^{\mathrm{T}} K_c^{\mathrm{T}}\right) - V_4^{\mathrm{T}} K_d^{\mathrm{T}}\right] R[(K_p + K_c C)V_1 - K_d V_4],$$

$$\Pi_{12} = 0,\ \Pi_{13} = V_3 + V_1^{\mathrm{T}} \bar{A} - V_4^{\mathrm{T}} \bar{E} + V_4^{\mathrm{T}} Q_2 V_3 + \left[V_1^{\mathrm{T}}\left(K_p^{\mathrm{T}} + C^{\mathrm{T}} K_c^{\mathrm{T}}\right) - V_4^{\mathrm{T}} K_d^{\mathrm{T}}\right] R(-K_d V_3),$$

$$\Pi_{22} = -V_2^{\mathrm{T}},\ \Pi_{23} = V_2^{\mathrm{T}} \bar{A}_d^{\mathrm{T}},\ \Pi_{33} = -\bar{E} V_3 - V_3^{\mathrm{T}} \bar{E}^{\mathrm{T}} + V_3^{\mathrm{T}} Q_2 V_3 + \left(-V_3^{\mathrm{T}} K_d^{\mathrm{T}}\right) R(-K_d V_3),$$

then Equation (4) is a QSGCCL, and the cost function satisfies $J \le J^* = x_0^{\mathrm{T}} V_1^{-1} x_0 + \int_{-d}^{0} x^{\mathrm{T}}(\tau) V_2^{-1} x(\tau) d\tau$.

Theorem 3.2. *Consider the system (1) associated with the cost function (6). If there exist a scalar $\lambda > 0$ and matrices $V_1 > 0$, $V_2 > 0$, V_3, V_4, S_1, S_2 and S_3 such that*

$$\begin{bmatrix} V_4 + V_4^{\mathrm{T}} & 0 & \Theta_{13} & \Theta_{14} & V_1^{\mathrm{T}} & 0 & V_4^{\mathrm{T}} & S_1^{\mathrm{T}} + S_2^{\mathrm{T}} \\ * & -2V_2^{\mathrm{T}} & \Theta_{23} & \Theta_{24} & 0 & V_2^{\mathrm{T}} & 0 & 0 \\ * & * & \Theta_{33} & \Theta_{34} & 0 & 0 & V_3^{\mathrm{T}} & S_3^{\mathrm{T}} \\ * & * & * & -\lambda I & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q_1^{-1} - V_2 & 0 & 0 & 0 \\ * & * & * & * & * & -V_2 & 0 & 0 \\ * & * & * & * & * & * & -Q_2^{-1} & 0 \\ * & * & * & * & * & * & * & -R^{-1} \end{bmatrix} < 0, \tag{14}$$

where $\Theta_{13} = V_3 + V_1^{\mathrm{T}} A^{\mathrm{T}} - V_4^{\mathrm{T}} E^{\mathrm{T}} + S_1^{\mathrm{T}} B^{\mathrm{T}} + S_2^{\mathrm{T}} B^{\mathrm{T}}$, $\Theta_{14} = V_1^{\mathrm{T}} N_A^{\mathrm{T}} - V_4^{\mathrm{T}} N_E^{\mathrm{T}} + S_1^{\mathrm{T}} N_B^{\mathrm{T}} + S_2^{\mathrm{T}} N_B^{\mathrm{T}}$, $\Theta_{23} = V_2^{\mathrm{T}} A_d^{\mathrm{T}}$, $\Theta_{24} = V_2^{\mathrm{T}} N_{A_d}^{\mathrm{T}}$, $\Theta_{33} = -EV_3 - V_3^{\mathrm{T}} E^{\mathrm{T}} + BS_3 + S_3^{\mathrm{T}} B^{\mathrm{T}} + \lambda GG^{\mathrm{T}}$, $\Theta_{34} = -V_3^{\mathrm{T}} N_E^{\mathrm{T}} + S_3^{\mathrm{T}} N_B^{\mathrm{T}}$, and choose the guaranteed cost control law as

$$u(t) = \left(S_1 - \frac{1}{2} S_3 V_3^{-1} V_4\right) V_1^{-1} x(t) + S_3 V_3^{-1} \dot{x}(t) + \left(S_2 - \frac{1}{2} S_3 V_3^{-1} V_4\right) V_1^{-1} C^{\mathrm{T}} \left(CC^{\mathrm{T}}\right)^{-\mathrm{T}} y(t), \tag{15}$$

then Equation (15) is a QSGCCL, and the cost function satisfies $J \le J^* = x_0^{\mathrm{T}} V_1^{-1} x_0 + \int_{-d}^{0} x^{\mathrm{T}}(\tau) V_2^{-1} x(\tau) \mathrm{d}\tau$.

Proof: For technique convenience, define some new notations:

$$\eta = \begin{bmatrix} V_4 + V_4^{\mathrm{T}} & 0 & V_3 + V_1^{\mathrm{T}} A^{\mathrm{T}} - V_4^{\mathrm{T}} E^{\mathrm{T}} \\ * & -2V_2^{\mathrm{T}} & V_2^{\mathrm{T}} A_d^{\mathrm{T}} \\ * & * & -EV_3 - V_3^{\mathrm{T}} E^{\mathrm{T}} \end{bmatrix},\ \kappa = \begin{bmatrix} S_1^{\mathrm{T}} + S_2^{\mathrm{T}} \\ 0 \\ S_3^{\mathrm{T}} \end{bmatrix},$$

$$\mu = \begin{bmatrix} 0 & 0 & \left(S_1^{\mathrm{T}} + S_2^{\mathrm{T}}\right) B^{\mathrm{T}} \\ * & 0 & 0 \\ * & * & BS_3 + S_3^{\mathrm{T}} B^{\mathrm{T}} \end{bmatrix},$$

$$\nu = \begin{bmatrix} V_1^{\mathrm{T}} & 0 & V_4^{\mathrm{T}} \\ 0 & V_2^{\mathrm{T}} & 0 \\ 0 & 0 & V_3^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} Q_1 + V_2^{-1} & 0 & 0 \\ 0 & V_2^{-1} & 0 \\ 0 & 0 & Q_2 \end{bmatrix} \begin{bmatrix} V_1 & 0 & 0 \\ 0 & V_2 & 0 \\ V_4 & 0 & V_3 \end{bmatrix}.$$

For a scalar $\lambda > 0$, recalling the previous condition $F^{\mathrm{T}}(t)F(t) \le I$, and using the similar method in [23], the following results can be obtained:

$$\eta+\kappa+\mu+\nu+\begin{bmatrix}0\\0\\G\end{bmatrix}F(t)\begin{bmatrix}\Theta_{14}^{\mathrm{T}} & \Theta_{24}^{\mathrm{T}} & \Theta_{34}^{\mathrm{T}}\end{bmatrix}+\begin{bmatrix}\Theta_{14}\\\Theta_{24}\\\Theta_{34}\end{bmatrix}F^{\mathrm{T}}(t)\begin{bmatrix}0 & 0 & G^{\mathrm{T}}\end{bmatrix}<0, \quad (16)$$

$$\Leftrightarrow \eta+\kappa+\mu+\nu+\lambda\begin{bmatrix}0\\0\\G\end{bmatrix}\begin{bmatrix}0 & 0 & G^{\mathrm{T}}\end{bmatrix}+\lambda^{-1}\begin{bmatrix}\Theta_{14}\\\Theta_{24}\\\Theta_{34}\end{bmatrix}\begin{bmatrix}\Theta_{14}^{\mathrm{T}} & \Theta_{24}^{\mathrm{T}} & \Theta_{34}^{\mathrm{T}}\end{bmatrix}<0. \quad (17)$$

By applying Schur complement, it is easy to obtain that Inequalities (16) and (14) are equivalent. Moreover, we can set

$$\begin{cases} S_1 = K_pV_1 - \dfrac{1}{2}K_dV_4, \\ S_2 = K_cCV_1 - \dfrac{1}{2}K_dV_4, \\ S_3 = -K_dV_3. \end{cases} \quad (18)$$

Then, by associating Equation (4) with Equation (18), we can obtain the guaranteed cost control law (15). Substituting Equation (18) into Equation (16) yields Equation (13). Therefore, according to the above Corollary 3.1, the proof of this theorem is completed.

Theorem 3.2 presents a method of designing the QSGCCL for system (1). However, the guaranteed cost performance in Theorem 3.2 depends on the selection of the guaranteed cost controller. The following theorem provides a solution to obtain the upper bound of the minimum performance index, and the optimal guaranteed cost controller can also be derived using convex optimization techniques.

Theorem 3.3. *Consider the system (1) associated with the cost function (6), and suppose that the initial condition x_0 is known. If the following optimization problem*

$$\min_{\lambda,\rho,V_1,V_2,V_3,V_4,S_1,S_2,S_3,Z}\{\rho+\mathrm{trace}(Z)\}$$
$$\begin{cases} \text{(i) LMI(14)}, \\ \text{(ii)} \begin{bmatrix} -\rho & x_0^{\mathrm{T}} \\ x_0 & -V_1 \end{bmatrix} < 0, \\ \text{(iii)} \begin{bmatrix} -Z & U^{\mathrm{T}} \\ U & -V_2 \end{bmatrix} < 0, \end{cases} \quad (19)$$

has an optimal solution set: $\left(\hat{\lambda}>0,\hat{\rho}>0,\hat{V}_1>0,\hat{V}_2>0,\hat{V}_3,\hat{V}_4,\hat{S}_1,\hat{S}_2,\hat{S}_3,\hat{Z}>0\right)$, *then* $u(t)=\left(\hat{S}_1-\frac{1}{2}\hat{S}_3\hat{V}_3^{-1}\hat{V}_4\right)\hat{V}_1^{-1}x(t)+\hat{S}_3\hat{V}_3^{-1}\dot{x}(t)+\left(\hat{S}_2-\frac{1}{2}\hat{S}_3\hat{V}_3^{-1}\hat{V}_4\right)\hat{V}_1^{-1}C^{\mathrm{T}}\left(CC^{\mathrm{T}}\right)^{-\mathrm{T}}y(t)$ *is the optimal guaranteed cost controller for system (1).*

Proof: It can be seen from Theorem 3.2 that the upper bound of guaranteed cost is obtained as $J^* = x_0^{\mathrm{T}}V_1^{-1}x_0 + \int_{-d}^{0} x^{\mathrm{T}}(\tau)V_2^{-1}x(\tau)d\tau$. By applying the Schur complement to the condition (ii) in Theorem 3.3, it is easy to get $x_0^{\mathrm{T}}V_1^{-1}x_0 < \rho$. Next, by recalling $\mathrm{trace}(AB) = \mathrm{trace}(BA)$, and according to (iii) in Theorem 3.3, we have

$$\begin{aligned}\int_{-d}^{0} x^{\mathrm{T}}(\tau)V_2^{-1}x(\tau)\mathrm{d}\tau &= \int_{-d}^{0}\mathrm{trace}\left(x^{\mathrm{T}}(\tau)V_2^{-1}x(\tau)\right)\mathrm{d}\tau \\ &= \mathrm{trace}\left(UU^{\mathrm{T}}V_2^{-1}\right) = \mathrm{trace}\left(UV_2^{-1}U^{\mathrm{T}}\right) < \mathrm{trace}(Z),\end{aligned}$$

where $\int_{-d}^{0} x(\tau)x^{\mathrm{T}}(\tau)\mathrm{d}\tau = UU^{\mathrm{T}}$. Therefore, we can obtain the following result: $J^* < \rho + \mathrm{trace}(Z)$.

Several special cases can be listed as follows.

(i) When $K_c = 0$, then we have $S_1 = K_pV_1 - \frac{1}{2}K_dV_4$, $S_2 = -\frac{1}{2}K_dV_4$, $S_3 = -K_dV_3$. Next, let $H_1 = S_1 + S_2 = K_pV_1 - K_dV_4$, $H_2 = S_3 = -K_dV_3$, and we can obtain $K_p = \left(H_1 - H_2V_3^{-1}V_4\right)V_1^{-1}$, $K_d = -H_2V_3^{-1}$. In this manner, the designed controller is a proportional differential feedback controller.

(ii) When $K_c = 0$, $K_d = 0$, then we have $S_1 = K_pV_1$, $S_2 = S_3 = 0$, $K_p = S_1V_1^{-1}$. In this case, the designed controller is a state feedback controller.

(iii) When $K_p = 0$, $K_d = 0$, then we have $S_1 = S_3 = 0$, $S_2 = K_cCV_1$, $K_c = S_2V_1^{-1}C^{\mathrm{T}}\left(CC^{\mathrm{T}}\right)^{-\mathrm{T}}$. In this case, the designed controller is a static output feedback controller.

Thus, according to Theorem 3.2, we can provide the following results.

Corollary 3.2. *Consider the system (1) associated with the cost function (6). If there exist a scalar $\lambda > 0$ and matrices $V_1 > 0$, $V_2 > 0$, V_3, V_4, H_1 and H_2 such that*

$$\begin{bmatrix} V_4 + V_4^{\mathrm{T}} & 0 & \Theta_{13} & \Theta_{14} & V_1^{\mathrm{T}} & 0 & V_4^{\mathrm{T}} & H_1^{\mathrm{T}} \\ * & -2V_2^{\mathrm{T}} & \Theta_{23} & \Theta_{24} & 0 & V_2^{\mathrm{T}} & 0 & 0 \\ * & * & \Theta_{33} & \Theta_{34} & 0 & 0 & V_3^{\mathrm{T}} & H_2^{\mathrm{T}} \\ * & * & * & -\lambda I & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q_1^{-1} - V_2 & 0 & 0 & 0 \\ * & * & * & * & * & -V_2 & 0 & 0 \\ * & * & * & * & * & * & -Q_2^{-1} & 0 \\ * & * & * & * & * & * & * & -R^{-1} \end{bmatrix} < 0, \tag{20}$$

where $\Theta_{13} = V_3 + V_1^{\mathrm{T}}A^{\mathrm{T}} - V_4^{\mathrm{T}}E^{\mathrm{T}} + H_1^{\mathrm{T}}B^{\mathrm{T}}$, $\Theta_{14} = V_1^{\mathrm{T}}N_A^{\mathrm{T}} - V_4^{\mathrm{T}}N_E^{\mathrm{T}} + H_1^{\mathrm{T}}N_B^{\mathrm{T}}$, $\Theta_{23} = V_2^{\mathrm{T}}A_d^{\mathrm{T}}$, $\Theta_{24} = V_2^{\mathrm{T}}N_{A_d}^{\mathrm{T}}$, $\Theta_{33} = -EV_3 - V_3^{\mathrm{T}}E^{\mathrm{T}} + BH_2 + H_2^{\mathrm{T}}B^{\mathrm{T}} + \lambda GG^{\mathrm{T}}$, $\Theta_{34} = -V_3^{\mathrm{T}}N_E^{\mathrm{T}} + H_2^{\mathrm{T}}N_B^{\mathrm{T}}$, choose the guaranteed cost control law as

$$u(t) = (H_1 - H_2V_3^{-1}V_4)V_1^{-1}x(t) + H_2V_3^{-1}\dot{x}(t), \tag{21}$$

then Equation (21) is a QSGCCL, and the cost function satisfies $J \leq J^* = x_0^{\mathrm{T}}V_1^{-1}x_0 + \int_{-d}^{0} x^{\mathrm{T}}(\tau)V_2^{-1}x(\tau)\mathrm{d}\tau$.

Corollary 3.3. *Consider the system (1) associated with the cost function (6). If there exist a scalar $\lambda > 0$ and matrices $V_1 > 0$, $V_2 > 0$, V_3, V_4 and S_1 such that*

$$\begin{bmatrix} V_4 + V_4^{\mathrm{T}} & 0 & \Theta_{13} & \Theta_{14} & V_1^{\mathrm{T}} & 0 & V_4^{\mathrm{T}} & S_1^{\mathrm{T}} \\ * & -2V_2^{\mathrm{T}} & \Theta_{23} & \Theta_{24} & 0 & V_2^{\mathrm{T}} & 0 & 0 \\ * & * & \Theta_{33} & \Theta_{34} & 0 & 0 & V_3^{\mathrm{T}} & 0 \\ * & * & * & -\lambda I & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q_1^{-1} - V_2 & 0 & 0 & 0 \\ * & * & * & * & * & -V_2 & 0 & 0 \\ * & * & * & * & * & * & -Q_2^{-1} & 0 \\ * & * & * & * & * & * & * & -R^{-1} \end{bmatrix} < 0, \tag{22}$$

where $\Theta_{13} = V_3 + V_1^{\mathrm{T}}A^{\mathrm{T}} - V_4^{\mathrm{T}}E^{\mathrm{T}} + S_1^{\mathrm{T}}B^{\mathrm{T}}$, $\Theta_{14} = V_1^{\mathrm{T}}N_A^{\mathrm{T}} - V_4^{\mathrm{T}}N_E^{\mathrm{T}} + S_1^{\mathrm{T}}N_B^{\mathrm{T}}$, $\Theta_{23} = V_2^{\mathrm{T}}A_d^{\mathrm{T}}$, $\Theta_{24} = V_2^{\mathrm{T}}N_{A_d}^{\mathrm{T}}$, $\Theta_{33} = -EV_3 - V_3^{\mathrm{T}}E^{\mathrm{T}} + \lambda GG^{\mathrm{T}}$, $\Theta_{34} = -V_3^{\mathrm{T}}N_E^{\mathrm{T}}$, choose the guaranteed cost control law as

$$u(t) = S_1V_1^{-1}x(t), \tag{23}$$

then Equation (23) is a QSGCCL, and the cost function satisfies $J \leq J^* = x_0^{\mathrm{T}} V_1^{-1} x_0 + \int_{-d}^{0} x^{\mathrm{T}}(\tau) V_2^{-1} x(\tau) \mathrm{d}\tau$.

Corollary 3.4. *Consider the system (1) associated with the cost function (6). If there exist a scalar* $\lambda > 0$ *and matrices* $V_1 > 0$, $V_2 > 0$, V_3, V_4 *and* S_2 *such that*

$$\begin{bmatrix} V_4+V_4^{\mathrm{T}} & 0 & \Theta_{13} & \Theta_{14} & V_1^{\mathrm{T}} & 0 & V_4^{\mathrm{T}} & S_2^{\mathrm{T}} \\ * & -2V_2^{\mathrm{T}} & \Theta_{23} & \Theta_{24} & 0 & V_2^{\mathrm{T}} & 0 & 0 \\ * & * & \Theta_{33} & \Theta_{34} & 0 & 0 & V_3^{\mathrm{T}} & 0 \\ * & * & * & -\lambda I & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q_1^{-1}-V_2 & 0 & 0 & 0 \\ * & * & * & * & * & -V_2 & 0 & 0 \\ * & * & * & * & * & * & -Q_2^{-1} & 0 \\ * & * & * & * & * & * & * & -R^{-1} \end{bmatrix} < 0, \tag{24}$$

where $\Theta_{13} = V_3 + V_1^{\mathrm{T}} A^{\mathrm{T}} - V_4^{\mathrm{T}} E^{\mathrm{T}} + S_2^{\mathrm{T}} B^{\mathrm{T}}$, $\Theta_{14} = V_1^{\mathrm{T}} N_A^{\mathrm{T}} - V_4^{\mathrm{T}} N_E^{\mathrm{T}} + S_2^{\mathrm{T}} N_B^{\mathrm{T}}$, $\Theta_{23} = V_2^{\mathrm{T}} A_d^{\mathrm{T}}$, $\Theta_{24} = V_2^{\mathrm{T}} N_{A_d}^{\mathrm{T}}$, $\Theta_{33} = -EV_3 - V_3^{\mathrm{T}} E^{\mathrm{T}} + \lambda G G^{\mathrm{T}}$, $\Theta_{34} = -V_3^{\mathrm{T}} N_E^{\mathrm{T}}$, *choose the guaranteed cost control law as*

$$u(t) = S_2 V_1^{-1} C^{\mathrm{T}} \left(CC^{\mathrm{T}}\right)^{-\mathrm{T}} y(t), \tag{25}$$

then Equation (25) is a QSGCCL, and the cost function satisfies $J \leq J^* = x_0^{\mathrm{T}} V_1^{-1} x_0 + \int_{-d}^{0} x^{\mathrm{T}}(\tau) V_2^{-1} x(\tau) \mathrm{d}\tau$.

4. **Numerical Examples.** In this section, two simulation examples are provided to demonstrate the reasonableness and effectiveness of the theoretical results.

4.1. **Example 1.** Consider the descriptor system (1) and the cost function (6), where the system parameters are given as follows:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A = \begin{bmatrix} 4 & 0 & 1 \\ 3 & 0.5 & 1 \\ 1.5 & 6 & 1 \end{bmatrix},\ A_d = \begin{bmatrix} -1 & 0.5 & 0 \\ 0.3 & -0.5 & 1 \\ 0.8 & 0.5 & 1.5 \end{bmatrix},\ B = \begin{bmatrix} 1.5 \\ 0.8 \\ 0.4 \end{bmatrix},$$

$$C = \begin{bmatrix} -5.1 & 4.2 & 3.5 \\ 4.2 & 2.8 & 7.1 \\ 3.9 & 1.3 & 1.2 \end{bmatrix},\ G = \begin{bmatrix} 0.5 \\ 1 \\ 0.8 \end{bmatrix},\ N_E = \begin{bmatrix} 1 & 0.2 & 0.5 \end{bmatrix},\ N_A = \begin{bmatrix} 1.5 & 1 & 0.2 \end{bmatrix},$$

$$N_{A_d} = \begin{bmatrix} 0.5 & 2 & 0 \end{bmatrix},\ N_B = 0.8,\ Q_1 = \mathrm{diag}\{5,5,5\},\ Q_2 = \mathrm{diag}\{0.2,0.2,0.2\},\ R = 0.2.$$

Suppose that the initial condition is defined as $\phi(t) = \begin{bmatrix} e^t & e^t - e^{-t} & -e^t \end{bmatrix}^{\mathrm{T}}$, $t \in [-1,0]$, and the uncertain matrix is given as $F(t) = \sin(t)$. Since $\det(E) = 0$, it is clear to see that the nominal descriptor system $E\dot{x}(t) = Ax(t)$ is a singular one. Moreover, it can be calculated that $\det(sE - A) = -s^2 - 3s + \frac{19}{4}$, $\deg[\det(sE - A)] = 2 = rank(E)$.

In this simulation, the matrices E, A, and A_d define the time-delayed descriptor system. The matrices B and C represent the input and output matrices of the system, respectively. These matrices determine the relationship between the control inputs, system states, and measured outputs. The matrices G, N_E, N_A, N_{A_d} and N_B represent the system uncertainties. These parameters influence the trade-off between system performance and control effort. The matrices Q_1, Q_2 and R serve as the weighting matrices within the cost function, delineating the relative significance of various states and control inputs in the cost calculation. The initial condition $\phi(t)$ specifies the initial state of the system, affecting its transient behavior.

Based on the above analysis, it is evident that the nominal descriptor system satisfies the regularity and impulsive-free conditions specified in [24, 25]. Consequently, we obtain a finite eigenvalue $s = 1.1458 > 0$, indicating that the nominal descriptor system is unstable.

According to Theorem 3.3, and using Matlab LMI tool box for system (1), we can obtain a set of optimal solutions as follows:

$$\hat{V}_1 = \begin{bmatrix} 0.4563 & 0.0624 & -0.1899 \\ 0.0624 & 0.0404 & -0.1237 \\ -0.1899 & -0.1237 & 0.5889 \end{bmatrix}, \ \hat{V}_2 = \begin{bmatrix} 0.1250 & -0.0287 & -0.1236 \\ -0.0287 & 0.0575 & 0.0285 \\ -0.1236 & 0.0285 & 0.1225 \end{bmatrix},$$

$$\hat{V}_3 = \begin{bmatrix} -0.6783 & -2.1904 & -0.8043 \\ -1.1330 & 4.1599 & -0.1632 \\ -2.816 & -1.0892 & -0.3948 \end{bmatrix}, \ \hat{V}_4 = \begin{bmatrix} -2.0328 & 0.5659 & 1.1350 \\ 0.5854 & -4.8483 & 0.2240 \\ 1.4683 & 0.2826 & -4.3932 \end{bmatrix},$$

$$\hat{S}_1 = \begin{bmatrix} -48.5397 \\ -5.2964 \\ -0.1136 \end{bmatrix}^{\mathrm{T}}, \ \hat{S}_2 = \begin{bmatrix} 46.1767 \\ 4.8317 \\ -0.7939 \end{bmatrix}^{\mathrm{T}}, \ \hat{S}_3 = \begin{bmatrix} -2.9465 \\ -2.2218 \\ -1.3518 \end{bmatrix}^{\mathrm{T}},$$

$$\hat{Z} = \begin{bmatrix} 2.1392 & -5.0065 & -2.1140 \\ -5.0065 & 26.6714 & 5.0065 \\ -2.1140 & 5.0065 & 2.1392 \end{bmatrix}, \ \hat{\lambda} = 0.2039, \ \hat{\rho} = 7.5747.$$

Then, the gain matrices of the optimal guaranteed cost controller can be given by

$$K_p^* = \begin{bmatrix} -110.7510 & -170.0478 & -70.6795 \end{bmatrix}, \ K_d^* = \begin{bmatrix} -1.3214 & -0.2957 & -0.6096 \end{bmatrix},$$
$$K_c^* = \begin{bmatrix} 28.7399 & -18.3691 & 85.1763 \end{bmatrix}.$$

Further, the optimal cost value can be calculated as $J^* = 38.4742$. With the above designed controller, the state trajectories of the open-loop system and the closed-loop system are displayed in Figure 1 and Figure 2, respectively.

Figures 1 and 2 depict the simulation results, which clearly demonstrate the rapid convergence of the system to zero and the minimization of the upper bound of the cost

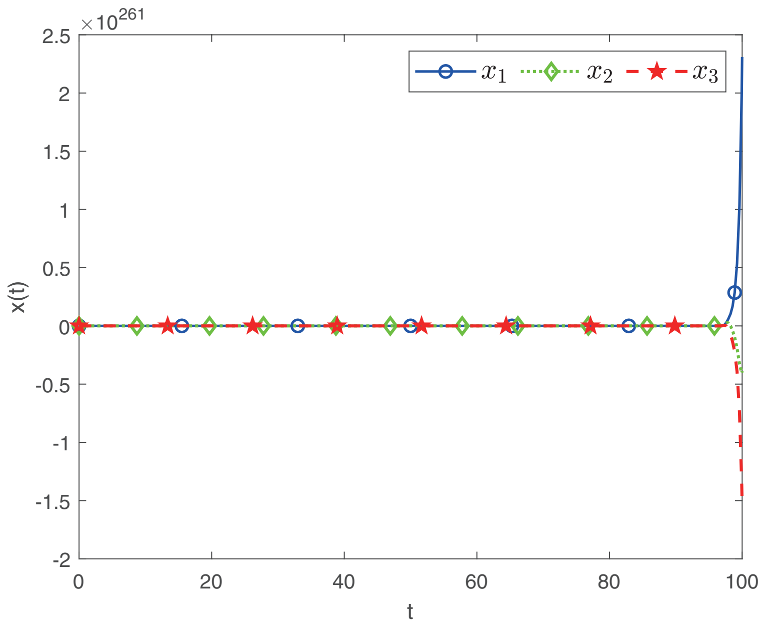

FIGURE 1. The state trajectories of the open-loop system

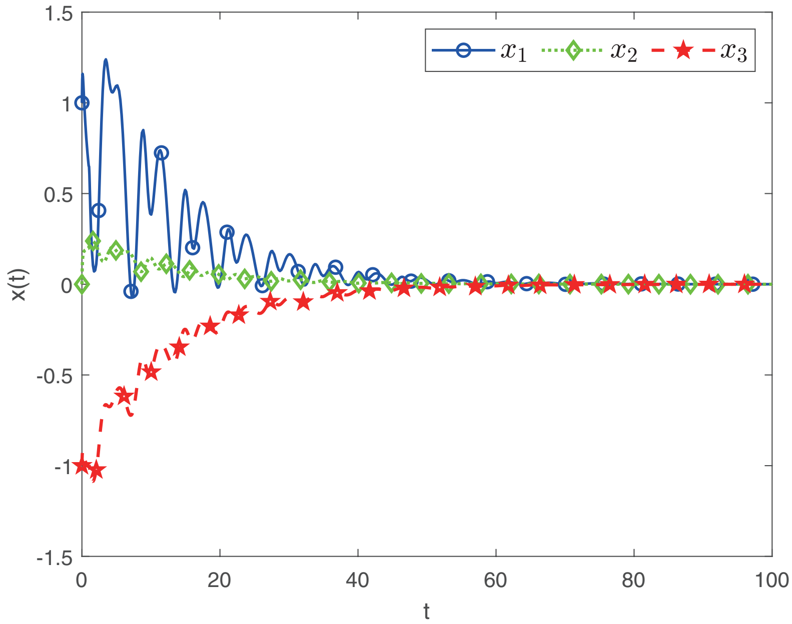

FIGURE 2. The state trajectories of the closed-loop system

achieved by the designed controller. These results illustrate the significance of the proposed multiple-loop control scheme with an optimal controller in addressing the guaranteed cost problem. Notably, the state curves presented in Figure 1 exhibit rapid divergence, indicating the instability of the original system. Conversely, the state trajectories displayed in Figure 2 showcase rapid convergence to zero, thereby affirming the efficiency of the designed controller in stabilizing the system. Additionally, Figure 2 serves as evidence that the designed controller successfully minimizes the upper bound of the cost functional.

In summary, the simulation results provide compelling evidence supporting the efficacy of the proposed multiple-loop control scheme with an optimal controller in resolving the guaranteed cost problem. The fast convergence, minimal cost, and robustness exhibited by the controller highlight its potential for successful implementation in control applications.

4.2. **Example 2.** To further verify the effectiveness of this method, consider an example in [4] with the following parameters:

$$E = \begin{bmatrix} 10.5 & -3.5 & -3.5 \\ 27.6 & 19.3 & -18.7 \\ -1.2 & -8.6 & 3.4 \end{bmatrix}, \ A = \begin{bmatrix} 0.45 & 8.05 & 50.5 \\ 28.1 & -112.11 & 77.1 \\ 66.22 & -1.22 & 0.18 \end{bmatrix}, \ A_d = \begin{bmatrix} -1 & 0 & -1 \\ 1 & -1 & 0.5 \\ 31 & -5 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}, \ G = \begin{bmatrix} -4 \\ 3 \\ -2 \end{bmatrix}, \ N_A = \begin{bmatrix} 2 & -1 & -6 \end{bmatrix}, \ N_{A_d} = \begin{bmatrix} -1 & -0.1 & 0.1 \end{bmatrix}, \ N_B = -0.1,$$

$$Q_1 = \mathrm{diag}\{10, 10, 10\}, \ R = 0.1, \ \phi(t) = \begin{bmatrix} 0 & 0.5e^{-(t+1)} & 0.5e^{t+1} \end{bmatrix}, \ t \in [-1, 0].$$

Since the existing work [4] does not consider perturbation of the derivative term and system output, the following assumptions can be made: $N_E = 0$, $Q_2 = 0$, $C = I$. By applying the Matlab LMI toolbox and solving the LMI of Theorem 3.3, we can obtain the following results: $K_p^* = \begin{bmatrix} -13.6182 & 6.7898 & -0.4354 \end{bmatrix}$, $K_d^* = \begin{bmatrix} 0.0953 & 0.0714 & 0.2147 \end{bmatrix}$, $K_c^* = \begin{bmatrix} -17.3461 & -8.3300 & 0.7068 \end{bmatrix}$, $J^* = 0.5245$. However, the conventional state feedback controller used in [4] yields a cost value of 12.8255. Figures 3 and 4 provide

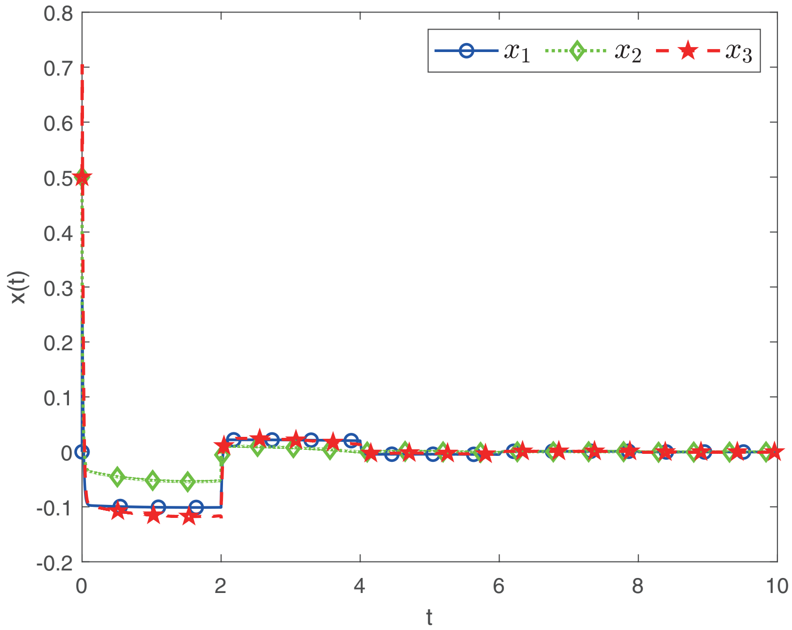

FIGURE 3. The state trajectories with the method in this paper

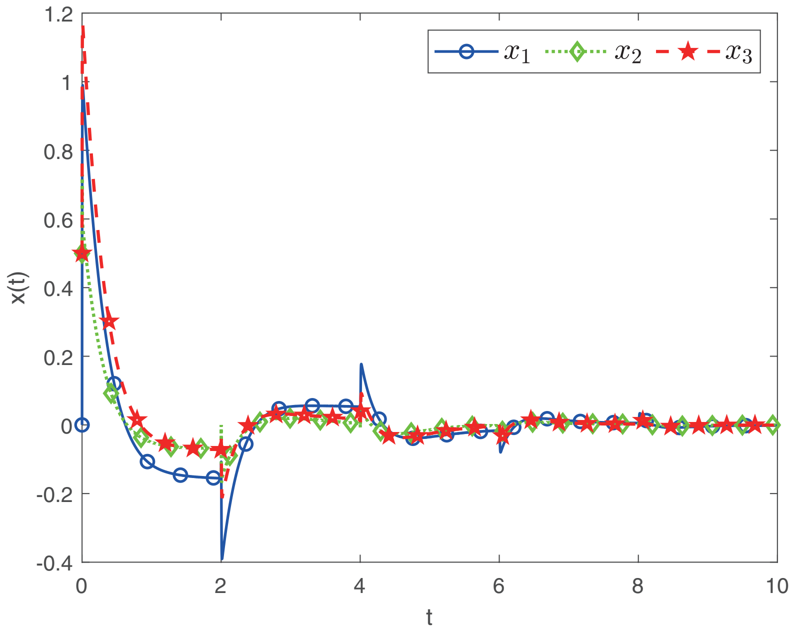

FIGURE 4. The state trajectories with state feedback controller in [4]

a comparison simulation to demonstrate the improved convergence performance and robustness of the proposed controller compared to the existing results in [4]. As shown in Figure 4, the first state component with the conventional controller exhibits significant oscillation in the initial stage, and the convergence time is longer than that with the multiple-loop feedback controller. These results indicate the advantages of our proposed method in addressing the guaranteed cost control problem.

Furthermore, we will conduct a detailed analysis of the aforementioned simulation results. The results show that our multiple-loop feedback controller not only outperforms the controller presented in [4] in terms of steady-state performance, but also achieves a lower cost value. This also illustrates the superiority of our approach in achieving the desired control objectives. The obtained results provide strong evidence to support the validity and effectiveness of our proposed strategy in guaranteed cost control applications.

5. **Conclusion.** In this paper, the guaranteed cost control problem of quadratic stabilization is investigated for a class of norm-bounded uncertain time-delayed descriptor systems. According to the Lyapunov stability theory, and introducing multiple-loop control techniques, a feedback controller with multiple-loops is designed, aiming to ensure quadratic stability of the system and that the cost performance does not surpass the predetermined value. Then, the equivalent conditions of system stability and guaranteed cost function satisfying requirements are given in the form of LMIs. Next, the upper bound of the minimum performance index and optimal guaranteed cost controller are derived using convex optimization techniques. Finally, two numerical examples are provided to illustrate the reasonableness and effectiveness of the proposed method. The results of this paper may facilitate the study of guaranteed cost control for saturated/Lipschitz nonlinear descriptor systems and discrete descriptor systems with mixed-delays. These topics will be the focus of our future research.

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