International Journal of Innovative Computing, Information and Control
Volume **20**, Number **4**, August **2024**

ICIC International ©2024 ISSN 1349-4198
pp. 1215–1227

SYNTHESIS ANALYSIS FOR OPTIMAL CONTROLLER DESIGN FOR UNKNOWN CLOSED LOOP SYSTEM

Ruchun Wen and Jianhong Wang

School of Electronic Engineering and Automation
Jiangxi University of Science and Technology
No. 86, Hongqi Road, Ganzhou 341000, P. R. China
{ 9119940012; 9120180002 }@jxust.edu.cn

Received October 2023; revised February 2024

Abstract. *This new paper formulates our previous contributions on model reference control and direct data driven control and proposes new aspects for latter studying. Specifically, consider one closed loop system with unknown plant and unknown controller simultaneously, our mission is to design an optimal controller, while achieving the perfect tracking and disturbance rejection. Two different control strategies are applied to design this unknown controller, i.e., classical model reference control and direct data driven control, respectively. For classical model reference control, the detailed process and statistical analysis are given with other two subjects, for example, condition as one convex optimization problem and regularization. On the other hand, for direct data driven control, the controller design process and its statistical property are also reviewed, while introducing adaptive idea to be adaptive direct data control with one explicit recursive form. Finally, above two different control strategies are applied into aircraft flight system to guarantee aircraft fly along the desired trajectory.*

Keywords: Model reference control, Direct data driven control, Statistical property, Regularization, Adaptation

1. **Introduction.** During many engineering applications, an accurate mathematical description of the actual physical plant is needed for latter analysis and controller design. The behavior of the considered plant of interest is modeled from the first principle laws of physical, chemistry, Newton law, etc., or system identification, meaning a detailed knowledge of the interesting plant is essential to derive a mathematical model from these physical laws. However, many physical plants have complex system structure, so it is very hard to model them using the first principle laws. Instead, a system identification approach can be used to describe the intrinsic behavior of the considered plant. Whatever the physical modeling or system identification approach is used to construct one mathematical model for our considered unknown plant, then the obtained model is deemed as one model basis for latter task mission, for example, controller design, anomaly detection, and fault isolation, generally, above description means two steps, i.e., the first system modeling and second controller design. Although controller design is our terminated goal, system model is necessary through spending lots of time and energy resource, being called as model based control.

During our new and convenient lives in new era, information showing in data science blows up with time increases, meaning all our features are contained in the related data, such as figure, face, gene, and voice, so the useful feature can be extracted from these data when it is needed, thus making our lives more convenient. By the way, the main essence of above system identification is similar to extract the useful information for that

DOI: 10.24507/ijicic.20.04.1215

unknown plant. More specifically, after collecting the left and right input-output data with respect to the considered plant, system identification is to plot one continuous or discrete curve, fitting the collected input-output data, i.e., curve fitting problem, that is denoted as one mathematical equation or model. This idea on constructing mathematical model for unknown plant from the input-output data is extended to design controller only through the input-output data directly, being called direct data driven control. Generally, direct data driven control designs the unknown controller to get one original controller from data science directly, while avoiding the first system modeling step. As the final goals for system identification and direct data driven control are the same as each other, i.e., one for system modeling and the other for controller, all related methods about how to extract the useful information from our collected data for system identification are benefit for our studied direct data driven control in this new paper. The reason why direct data driven control is more popular than classical model based control in academy is that more subjects appear to support it, for example, machine learning, deep learning, reinforcement learning, and adaptation.

Due to the widely studied fact about direct data driven control in academy and engineering respectively, lots of researches on it are ongoing. [1] calls it as model free control, while combining the adaptation to analyze the stability property. Data driven dissipativity analysis and its computation approach are done in [2,3]. The dual combinations with direct data driven and model predictive control are proposed together in [4], and other innovative statistical properties are analyzed continuously, such as stabilization and optimality [6], and data informativity [5]. Data informativity guarantees the enough rich for the measured data, so that the intrinsic principle of the considered plant is excited persistently [7], being applied into one algebraic regulator problem [8]. In recent years, behavior theory is introduced into direct data driven control to yield one new control strategy, i.e., data enable predictive control in [9], where the future outputs are expressed as one linear combination form with respect to the finite inputs and outputs within one finite time interval [10,11]. Due to the similar relations for system identification and direct data driven control, many identification methods are suited for direct data driven control, for example, maximum likelihood estimation in data driven control [12], and robust parameter estimation for robust data driven controller [13], meaning the designed data driven controller from the noisy environment. Other than above mentioned recent references on direct data driven control strategy, we also study it from different aspects in these years, and achieve some new contributions, being formulated as follows: 1) optimization algorithm, 2) statistical analysis, 3) optimal input signal design, 4) construct one regularized cost function, 5) combination with other classical control strategies. Specifically, [14] gives a complete statistical analysis for one kind of direct data driven control, i.e., virtual reference feedback tuning control, and adaptive idea is combined with direct data driven control together to form adaptive iterative correlation tuning control [15].

Based on our previous contributions about direct data driven control, this new paper continues to complete our ongoing research, and formulates some interesting issues. For the sake of completeness, the considered closed loop system structure is given with one unknown plant, and unknown controller simultaneously, and then our main mission is to design this controller to guarantee the real output track the expected or desired output, i.e., the goal of perfect tracking. Firstly, classical model reference control strategy is applied to be our reviewed model based control, that designs one controller through solving one optimization problem, related with the unknown plant and expected performance. Considering this optimization problem with the unknown controller as the decision variable, one explicit form about controller and its statistical analysis are all analyzed completely. To improve the computational speed, i.e., applying the existing convex optimization algorithm, we derive one condition to guarantee the constructed optimization problem be one feasible convex optimization problem. Secondly, after describing the dependence on that unknown plant and to avoid the system modeling process, our considered direct data driven control is proposed from different points, such as the explicit form, statistical analysis, and optimal input signal. Furthermore, direct data driven control is combined with the adaptive idea, i.e., forming adaptive direct data driven control. Thirdly, these two different control strategies, i.e., classical model reference control and direct data driven control are used in aircraft flight system, while letting aircraft fly according to the orders from the ground station. After comparing the simulation results, the merit of direct data driven control is proven in case of large data set.

Generally, the main contributions in this continuous paper are listed as follows. 1) The detailed controller design process are explained for classical model reference control and direct data driven control together. 2) To improve the readability, some developments are given to obtain one accurate controller estimate. Then this paper generalizes and formulates our previous contributions. 3) The detailed application of direct data driven control into aircraft flight system is given to combine the theoretic analysis and engineering application.

This paper is organized as follows. In Section 2, the considered system is reviewed and some existing relations are shown. Our main work is around during Section 3 and Section 4. Specifically, Section 3 gives classical model reference control strategy, such as controller design, statistical analysis, algorithm and regularization. Section 4 proposes our considered direct data driven control from algorithm, statistical analysis and adaptation. Section 5 gives an example to prove our theoretical results. Finally, Section 6 formulates the main conclusion and points out our next work.

2. **System Structure.** Consider the following closed loop system structure with one unknown plant and unknown controller, plotted in Figure 1. It is similar to one aircraft flight control system, being described in later Section 5 in more details.

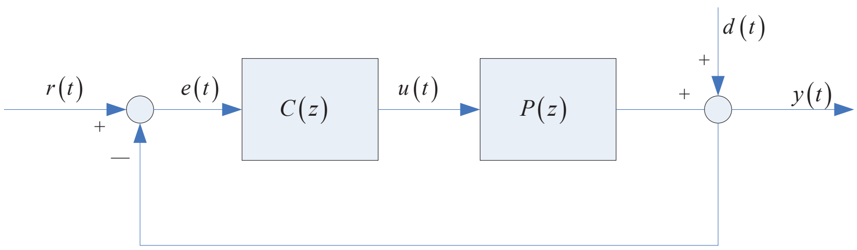


FIGURE 1. Closed loop system structure

In above Figure 1, $P(z)$ is the unknown plant, and z is the shift operator. $C(z)$ is one unknown feed forward controller, i.e., $\{P(z), C(z)\}$ are unknown. $r(t)$ is one external excitation signal, being used to excite the whole closed loop system. $d(t)$ is also one external disturbance or noise, not being neglected in academy and practice. $\{u(t), y(t)\}$ correspond to the input-output signal for that unknown plant $P(z)$. Error signal $e(t)$ means the deviation between external input $r(t)$ and feedback output $y(t)$, i.e., $e(t) = r(t) - y(t)$.

Observing Figure 1, after simple computations, the following obvious equations are yielded.

$$\begin{cases} y(t) = P(z)u(t) + d(t) \\ u(t) = C(z)e(t) = C(z)[r(t) - y(t)] \end{cases} \tag{1}$$

i.e.,

$$y(t) = P(z)C(z)[r(t) - y(t)] + d(t)$$
$$y(t) = \frac{P(z)C(z)}{1 + P(z)C(z)}r(t) + \frac{1}{1 + P(z)C(z)}d(t) \tag{2}$$

The problem of controller design for Figure 1 is to design that unknown feed forward controller $P(z)$ from different points, i.e., perfect tracking, robustness and adaptation, etc. Observing Figure 1, our mission is to design that unknown controller $C(z)$ in case of the unknown plant $P(z)$, i.e., closed loop controller design.

3. Classical Model Reference Control.

The goal of model reference control guarantees the closed loop output response track one expected or desired reference model $M(z)$, while satisfying other control performances.

3.1. Controller design.

Considering the main goal of model reference control, more specifically, we want that closed loop transfer function from external input $P(z)$ to closed loop output $y(t)$ is the same as reference model $M(z)$, given in priori, i.e.,

$$\frac{P(z)C(z)}{1 + P(z)C(z)} \to M(z) \tag{3}$$

Equation (3) is shown in Figure 2.

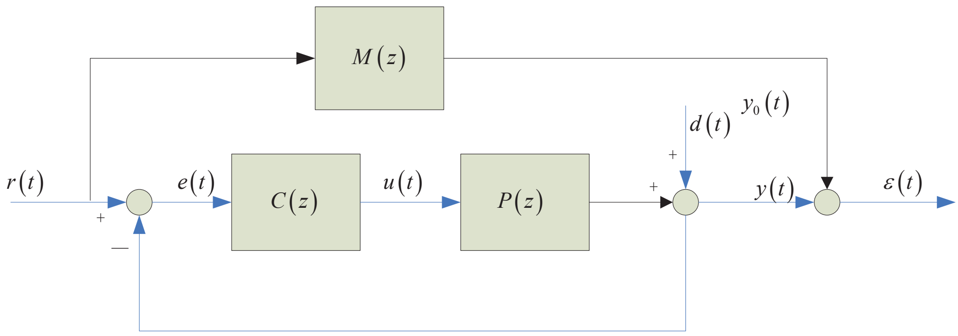


FIGURE 2. Model reference control scheme

From Figure 2, we have

$$\begin{aligned}\varepsilon(t) = y(t) - y_0(t) &= \frac{P(z)C(z)}{1 + P(z)C(z)}r(t) + \frac{1}{1 + P(z)C(z)}d(t) - M(z)r(t) \\ &= \left[\frac{P(z)C(z)}{1 + P(z)C(z)} - M(z)\right] r(t) + \frac{1}{1 + P(z)C(z)}d(t)\end{aligned} \tag{4}$$

Combining Equations (3), (4) and the main goal of model reference control, the optimal feed forward controller $C(z)$ is designed from one optimization problem, i.e.,

$$C(z) = \arg\min_{C(z)} \left\| \frac{P(z)C(z)}{1 + P(z)C(z)} - M(z) \right\|_2^2 \tag{5}$$

where $\|.\|$ is the commonly used Euclidean norm.

To get one explicit form for controller $C(z)$, we take the partial derivative with respect to controller $C(z)$ and set the derivative equal to zero, then we have

$$\left[\frac{P(z)C(z)}{1 + P(z)C(z)} - M(z)\right] \frac{P(z)}{[1 + P(z)C(z)]^2} = 0 \tag{6}$$

i.e.,

$$1 - \frac{1}{1 + P(z)C(z)} - M(z) = 0; \quad P(z)C(z) = \frac{1}{1 - M(z)} - 1 = \frac{M(z)}{1 - M(z)} \tag{7}$$

so the final controller is that

$$C(z) = \frac{1}{P(z)} \frac{M(z)}{1 - M(z)} \tag{8}$$

where from above Equation (8), we see after given that reference model $M(z)$, and identified that considering plant $P(z)$, the optimal controller $C(z)$ is yielded to satisfy the control goal, i.e., Equation (3).

Furthermore, due to the dependence of controller $C(z)$ on plant $P(z)$, classical model reference control is also model based control strategy, as plant $P(z)$ appears in that explicit form of controller $C(z)$.

3.2. **Statistical analysis.** For the sake of completeness, the quality of that final controller $C(z)$ is needed to evaluate. Assume one ideal controller $C_0(z)$ exists to satisfy that

$$\frac{P(z)C_0(z)}{1 + P(z)C_0(z)} = M(z) \tag{9}$$

i.e.,

$$1 - M(z) = 1 - \frac{P(z)C_0(z)}{1 + P(z)C_0(z)} = \frac{1}{1 + P(z)C_0(z)} \tag{10}$$

Substituting Equations (9) and (10) into Equation (8), it holds that

$$C(z) = \frac{P(z)C_0(z)}{1 + P(z)C_0(z)} \frac{1 + P(z)C_0(z)}{1} \frac{1}{P(z)} = C_0(z) \tag{11}$$

Equation (11) means our final controller, showing in Equation (8), is equal to that assumed ideal controller, while guaranteeing the condition (3), i.e., perfect tracking or matching.

3.3. **Algorithm.** When to solve that optimization problem (5), lots of classical optimization algorithms can be applied directly, such as Newton algorithm, gradient algorithm, and conjugate algorithm. To let the final controller be one optimal controller, i.e., $C(z) = C_0(z)$, we can resort to the nice convex optimization algorithm, in case of a convex cost function.

As the second derivative of one convex cost function is positive, we continuously derive that cost function twice to get the following inequity.

$$\frac{1}{[1 + P(z)C(z)]^3} - 3\frac{P(z)C(z)}{[1 + P(z)C(z)]^4} + \frac{2M(z)}{[1 + P(z)C(z)]^3} \geq 0 \tag{12}$$

Multiplying $[1 + P(z)C(z)]^4$ on both sides of above Equation (12) to get one simplified form.

Making use of the optimality necessary condition to differentiate with respect to $P(z)$ and set the derivative equal to zero, we have

$$\begin{aligned} &1 + P(z)C(z) - 3P(z)C(z) + 2M(z)(1 + P(z)C(z)) \geq 0 \\ &C(z)[2P(z) - 2M(z)P(z)] \leq 1 + 2M(z) \\ &C(z) \leq \frac{1 + 2M(z)}{2P(z) - 2M(z)P(z)} \end{aligned} \tag{13}$$

Equation (13) gives a condition about guaranteeing the constructed cost function (5) be convex, so the final controller, solving by each optimization in form (8), is one optimal controller.

3.4. **Regularization.** Above description about optimal controller design is around that desired condition (3), but disturbance or noise $e(t)$ exists really, so it is necessary to reject the bad effect coming from disturbance $e(t)$.

Combining Equation (3) and the bad effect from disturbance $e(t)$, here we consider them together, i.e., designing one optimal controller while satisfying the following two conditions.

$$\frac{P(z)C(z)}{1+P(z)C(z)} \to M(z); \quad \frac{1}{1+P(z)C(z)} \to 0 \tag{14}$$

Then one improved optimization problem is constructed to design the optimal controller within the case of above two conditions.

$$\begin{aligned} C(z) &= \arg\min_{C(z)} \left\| \frac{P(z)C(z)}{1+P(z)C(z)} - M(z) \right\|_2^2 + \lambda \left\| \frac{1}{1+P(z)C(z)} \right\|_2^2 \\ &= \arg\min_{C(z)} \left[\frac{P(z)C(z)}{1+P(z)C(z)} - M(z) \right]^2 + \lambda \left[\frac{1}{1+P(z)C(z)} \right]^2 \end{aligned} \tag{15}$$

In Equation (15), the second term is called the regularization term, being to restrict the bad effect from disturbance. Regularization parameter λ is chosen by designers.

After simple but tedious calculation, the optimal controller is given by

$$C(z) = \frac{1}{P(z)} \frac{M(z)+\lambda}{1-M(z)} \tag{16}$$

Substituting Equation (9) into above Equation (16), we have

$$C(z) = \frac{1}{P(z)} \frac{\frac{P(z)C_0(z)}{1+P(z)C_0(z)} + \lambda}{1 - \frac{P(z)C_0(z)}{1+P(z)C_0(z)}} = C_0(z) + \lambda C_0(z) + \frac{\lambda}{P(z)} \tag{17}$$

Continue to substitute Equation (17) into that first condition, i.e.,

$$\frac{P(z)C(z)}{1+P(z)C(z)} - M(z) = \frac{P(z)\left[\lambda C_0(z) + \frac{\lambda}{P(z)}\right]}{[1+P(z)C(z)][1+P(z)C_0(z)]} \tag{18}$$

Equation (18) tells us the goal of perfect tracking is not impossible in case of two conditions, but one minimum value about that improved cost function (15) is obtained, i.e., one tradeoff between perfect tracking and disturbance rejection.

4. **Direct Data Driven Control.** Observing Figure 1 or cost functions (5), (15), plant $P(z)$ exists in all above mathematical derivation. Moreover, those two final controllers (8) and (16) are dependent on unknown plant $P(z)$, meaning we must identify that unknown plant $P(z)$ firstly, and substitute its explicit or implicit forms in the final controller.

4.1. **Algorithm.** To avoid the identification process for the unknown plant $P(z)$ and achieve our terminate goal of designing unknown controller $C(z)$, here this section proposes our considered direct data driven control, whose idea is plotted in Figure 3.

To show the main essence of direct data driven control and avoid the identification process for the unknown plant, only observed input-output, corresponding to the unknown controller is applied to design the controller directly, i.e., extracting some useful information from the observed input-output data. Two sensors are placed on the left and right side of controller $C(z)$, whose output is $u(t)$ and input is $e(t) = r(t) - y(t)$.

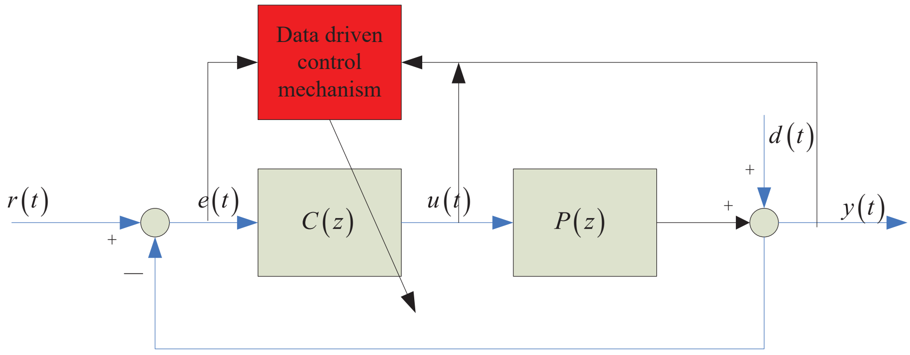


FIGURE 3. Direct data driven control scheme

Furthermore, reference model $M(z)$ means $y(t) = M(z)r(t)$, then the input signal $r(t)$ satisfies $r(t) = M^{-1}(z)y(t)$, so the input-output data with respect to controller $C(z)$ are formulated as follows:

$$\begin{aligned}&\{e(t), u(t)\} \to \{r(t) - y(t), u(t)\} \to \left\{M^{-1}(z)y(t) - y(t), u(t)\right\}\\&\to \left\{\left(M^{-1}(z) - 1\right) y(t), u(t)\right\}\end{aligned} \tag{19}$$

Based on above input-output data, direct data driven control works to design one optimal controller only through data $\{u(t), y(t)\}$, while considering the property of perfect tracking, i.e.,

$$C(z) = \arg\min_{C(z)} \frac{1}{N} \sum_{t=1}^{N} [u(t) - C(z)y_1(t)]^2; \quad y_1(t) = \left(M^{-1}(z) - 1\right) y(t) \tag{20}$$

where N is the total number of observed data.

Observing Equation (20), $\{u(t), y(t)\}_{t=1}^{N}$ are collected by some physical sensors, $M(z)$ is the given reference model, only $C(z)$ is unknown, and unknown plant $P(z)$ does not exist. When to obtain one explicit form for optimal controller, by differentiating with respect to $C(z)$ and setting the derivative equal to zero, we have

$$\frac{2}{N} \sum_{t=1}^{N} [u(t) - C(z)y_1(t)]y_1(t) = 0; \quad y_1(t) = \left(M^{-1}(z) - 1\right) y(t) \tag{21}$$

i.e.,

$$\begin{aligned}&\sum_{t=1}^{N} u(t)y_1(t) = C(z) \sum_{t=1}^{N} y_1(t)y_1(t);\\&C(z) = \left[\sum_{t=1}^{N} y_1(t)y_1(t)\right]^{-1} \left[\sum_{t=1}^{N} u(t)y_1(t)\right] = \frac{\phi_{uy_1}(w)}{\phi_{y_1}(w)} = \frac{\phi_{uy}(w)}{\left(M^{-1}(z) - 1\right) \phi_y(w)}\end{aligned} \tag{22}$$

where $\phi_y(w)$, $\phi_{uy}(w)$ are auto spectrum and cross spectrum between input-output $\{u(t), y(t)\}_{t=1}^{N}$, i.e.,

$$\phi_y(w) = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} y^T(t)y(t); \quad \phi_{uy}(w) = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} y^T(t)u(t)$$

Without loss of generality, the detailed algorithm for direct data driven control is listed as follows.

Algorithm 1

Step 1: Given one reference model $M(z)$ in priori.

Step 2: Collect the input-output data $\{u(t), y(t)\}_{t=1}^{N}$ with respect to that unknown controller $C(z)$.

Step 3: Compute two kinds of power spectral $\{\phi_y(w), \phi_{uy}(w)\}$.

Step 4: Set the designed controller to be

$$C(z) = \frac{\phi_{uy}(w)}{(M^{-1}(z) - 1)\,\phi_y(w)}$$

Above algorithm gives a rough controller by using the idea of direct data driven control, that only depends on input-output data $\{u(t), y(t)\}_{t=1}^{N}$ and reference model $M(z)$.

4.2. **Statistical analysis.** To testify the statistical property about optimal controller $C(z)$ in Equation (22), we take the expectation operator on both sides of Equation (22) to get

$$E[C(z)] = \frac{E[\phi_{uy}(w)]}{(M^{-1}(z) - 1)E[\phi_y(w)]} \tag{23}$$

where the following equations are used in deriving Equation (23) and assume the covariance of disturbance $d(t)$ to be σ^2.

$$\begin{aligned}
y(t) &= \frac{P(z)C(z)}{1+P(z)C(z)}r(t) + \frac{1}{1+P(z)C(z)}d(t); \\
u(t) &= \frac{C(z)}{1+P(z)C(z)}r(t) - \frac{C(z)}{1+P(z)C(z)}d(t); \\
\phi_{uy}(w) &= \frac{P(z)C^2(z)}{[1+P(z)C(z)]^2}\phi_r(w) - \frac{C(z)}{[1+P(z)C(z)]^2}\sigma^2; \\
\phi_y(w) &= \frac{P^2(z)C^2(z)}{[1+P(z)C(z)]^2}\phi_r(w) + \frac{1}{[1+P(z)C(z)]^2}\sigma^2; \\
E[r(t)d(t)] &= 0; \\
M(z) &= \frac{P(z)C_0(z)}{1+P(z)C_0(z)}; \quad M^{-1}(z) - 1 = \frac{1}{1+P(z)C_0(z)}
\end{aligned}$$

Then after simple computations, Equation (23) is simplified as that

$$E[C(z)] = C_0(z) - \frac{[P(z)C_0(z) + 1]\sigma^2}{P^2(z)C_0^2(z)\phi_r(w) + \sigma^2} \tag{24}$$

Here $\phi_r(w)$ denotes the input power spectral. The second term is the bias term, meaning that optimal controller is a biased controller. By the way, external input $r(t)$ and disturbance $e(t)$ are uncorrelated.

An interesting problem about reducing this bias term is solved through choosing an approximate input power spectral $\phi_r(w)$ so that our designed controller $C(z)$ is one unbiased controller, i.e., $E[C(z)] = C_0(z)$. The process of choosing an approximate input power spectral corresponds to the subject of optimal input design, i.e.,

$$\begin{aligned}
&\arg\min_{\phi_r(w)} \frac{1}{\phi_r(w)} \\
&\text{subject to } \phi_1(w) \le \phi_r(w) \le \phi_2(w)
\end{aligned} \tag{25}$$

or its equivalent form as

$$\arg\min_{\phi_r(w)} \frac{[P(z)C_0(z)+1]\sigma^2}{P^2(z)C_0^2(z)\phi_r(w)+\sigma^2}$$
$$\text{subject to } \phi_1(w) \leq \phi_r(w) \leq \phi_2(w) \tag{26}$$

where $\{\phi_1(w), \phi_2(w)\}$ are upper and lower bound for input power spectral. Based on our previous contribution, one explicit form for input power spectral $\phi_r(w)$ is derived through our own derivations.

4.3. **Adaptation.** Here this last section proposes to combine the adaptive idea, i.e., adaptation, into solving that optimal controller $C(z)$ recursively in real-time way.

From Equation (23), we have

$$C(z) = \left[\sum_{t=1}^{N} y_1(t)y_1(t)\right]^{-1}\left[\sum_{t=1}^{N} u(t)y_1(t)\right]; \quad y_1(t) = \left(M^{-1}(z)-1\right)y(t) \tag{27}$$

As the total number of observed input-output data is t, then we define regressor variable and unknown parameter vector as follows.

$$C_t(z) = \left[\sum_{i=1}^{t} y_1(i)y_1(i)\right]^{-1}\left[\sum_{t=1}^{t} u(i)y_1(i)\right] = F(t)\left[\sum_{i=1}^{t} u(i)y_1(i)\right]$$
$$F^{-1}(t) = \sum_{i=1}^{t} y_1(i)y_1(i) \tag{28}$$

$C_t(z)$ means the designed controller $C(z)$ is related with the total number or observed input-output data being t, using one recursive form as that

$$C_{t+1}(z) = F(t+1)\left[\sum_{i=1}^{t+1} u(i)y_1(i)\right];$$
$$F^{-1}(t+1) = \sum_{i=1}^{t+1} y_1(i)y_1(i) = \sum_{i=1}^{t} y_1(i)y_1(i) + y_1(t+1)y_1(t+1)$$
$$= F^{-1}(t) + y_1(t+1)y_1(t+1) \tag{29}$$

Express $C_{t+1}(z)$ as a function of $C_t(z)$ to get

$$C_{t+1}(z) = C_t(z) + \Delta C_{t+1}(z) \tag{30}$$

i.e.,

$$C_{t+1}(z) = F(t+1)\left[\sum_{i=1}^{t} u(i)y_1(i) + u(t+1)y_1(t+1)\right]$$
$$= F(t+1)\left[F^{-1}(t)C_t(z) + u(t+1)y_1(t+1)\right];$$
$$F^{-1}(t+1)C_t(z) = F^{-1}(t)C_t(z) + y_1(t+1)y_1(t+1)C_t(z);$$
$$F^{-1}(t)C_t(z) = F^{-1}(t+1)C_t(z) - y_1(t+1)y_1(t+1)C_t(z) \tag{31}$$

Then

$$C_{t+1}(z) = F(t+1)[F^{-1}(t+1)C_t(z) - y_1(t+1)y_1(t+1)C_t(z) + u(t+1)y_1(t+1)]$$
$$= C_t(z) + F(t+1)y_1(t+1)(u(t+1) - C_t(z)y_1(t+1)) \tag{32}$$

This result is that

$$C_{t+1}(z) = C_t(z) + F(t+1)y_1(t+1)\varepsilon(t+1);$$

$$\varepsilon(t+1) = u(t+1) - C_t(z)y_1(t+1);$$
$$F(t+1) = F(t) - \frac{F(t)y_1(t+1)y_1(t+1)F(t)}{1 + y_1(t+1)F(t)y_1(t+1)} \tag{33}$$

where Equation (33) shows one recursive expression in deriving that optimal controller with time increases. The merit of recursive form is to implement the recursive algorithm in real-time way.

Generally, adaptive direct data driven control scheme is plotted in Figure 4.

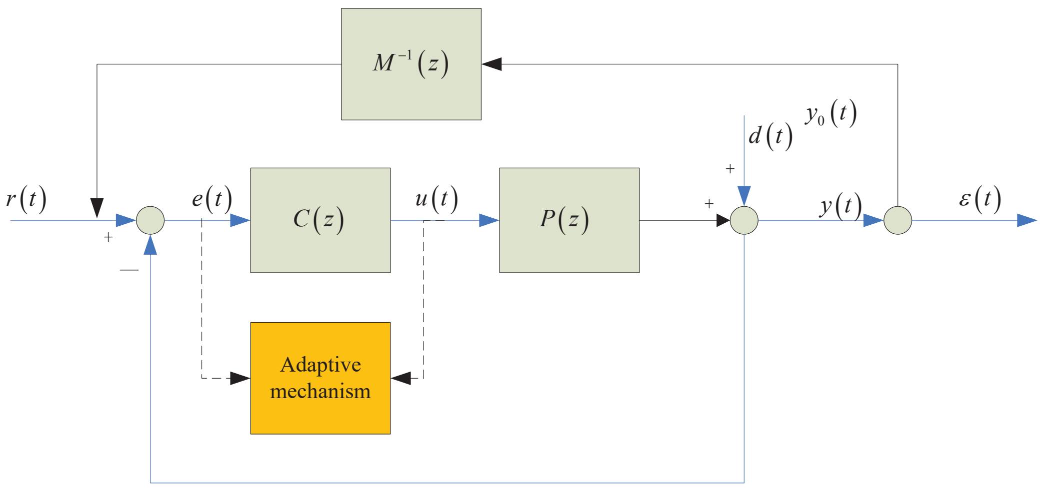


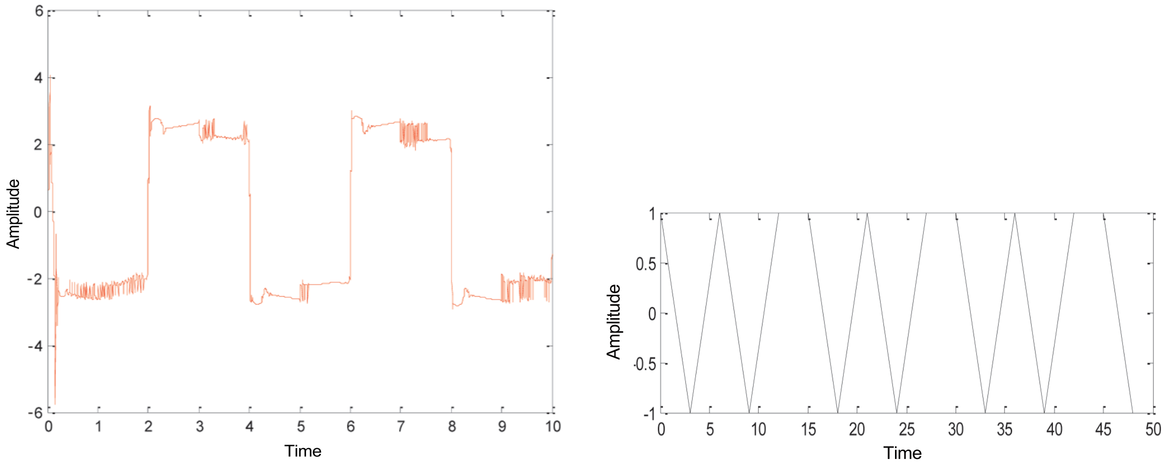
FIGURE 4. Adaptive direct data driven control scheme

5. **Aircraft Flight Control System.** This section gives some simulations to prove our proposed theories. During the aircraft formation simulation, three aircrafts exist, i.e., one leader and two followers, and the simulation time interval is 160 second. The original position, horizontal velocity, and heading angle are denoted as $(0, 100, 30)$, respectively. The whole flight stages are divided into two stages. The first stage is in $(0, 60)$, and the second stage is in $(60, 140)$. More specifically, within the first stage, the leader flies in a straight and level flight with a constant velocity, and its heading angle is 0. Similarly, in the second stage, the leader will rotate around at a constant angular velocity, but its flight velocity remains the same.

To avoid the identification process for that follower, two sides of data are collected around the formation controller, after one order from leader is sent or excite the nonlinear closed loop system. This order from leader is chosen a constant or one special signal, i.e., square wave, shown in Figure 5. The corresponding output signal is measured and plotted in Figure 6. Based on the input signal and output signal in Figure 5 and Figure 6, our mission is to design one nonlinear form for that formation controller. Through using our proposed direct data driven control strategy, one approximated linear controller is applied to replace that nonlinear formation controller.

Specifically, the commonly used linear affine form is used here, i.e., $u(t) = a_0 + \sum_{i=1}^{4} a_i e(t)$. Then our mission is to change to design these four above unknown parameters $\{a_1, a_2, a_3, a_4\}$ from the input-output measured data. It corresponds to one data fitting problem, i.e., designing three parameters to guarantee the real output be the same as the output in Figure 6.

In practice, leader sends one order to those two followers, i.e., telling two followers to fly around leader. The flight trajectory of leader is the desired or expected flight path.



(a) Actual input signal (b) Approximated input signal

FIGURE 5. The applied input signal

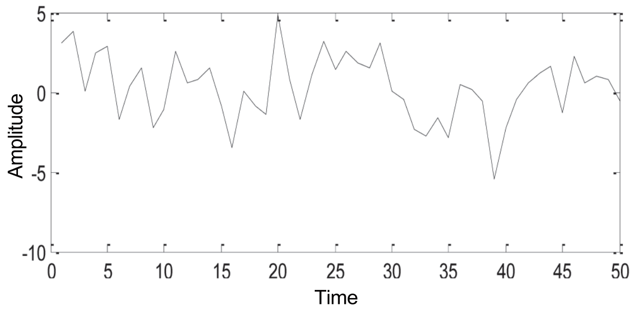


FIGURE 6. The observed output signal

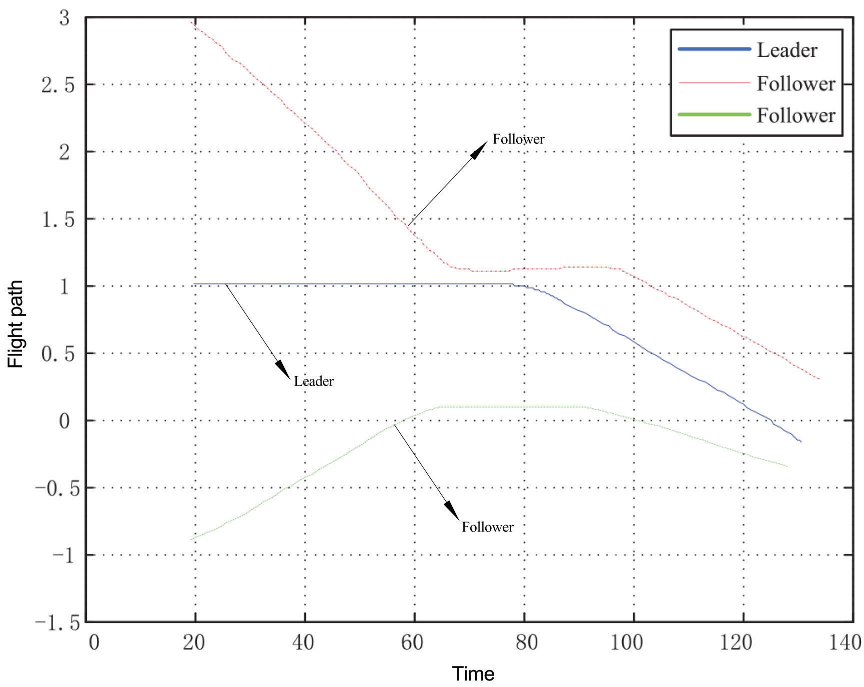


FIGURE 7. The whole flight trajectories for UAV group

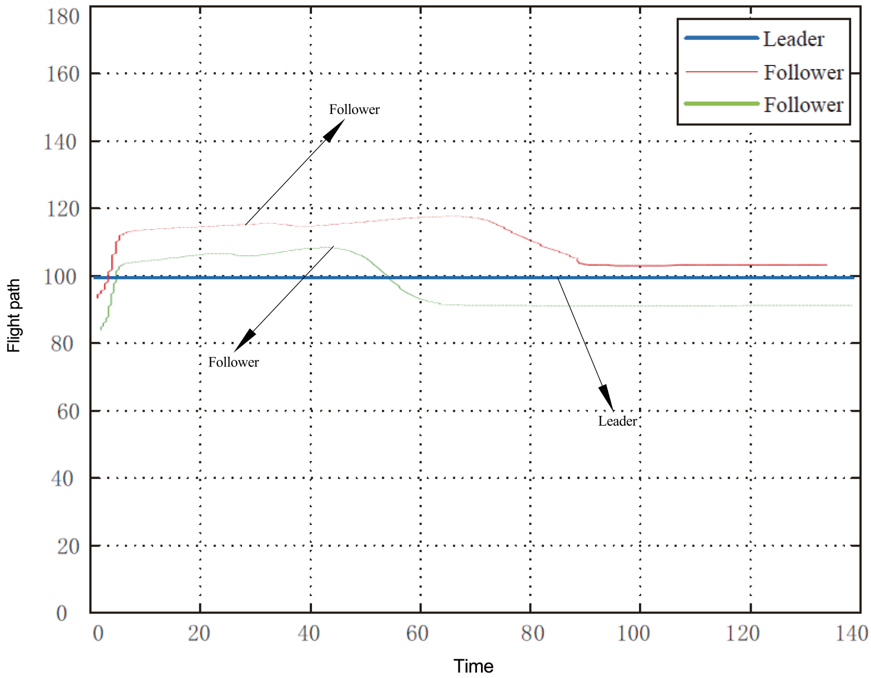


FIGURE 8. The horizontal velocity varying curve

After two followers receive the order from the leader, then two followers will modify their flight situations, and track the leader as soon as possible. According to the leader's flight trajectory, the formation controllers will design the above four parameters, so that the two followers will fly near to the leader. Figure 7 shows the whole flight trajectories for the leader and two followers, and the horizontal velocity varying curve is plotted in Figure 8. From these three figures, we see after two followers receive the order from the leader, they will fly near to the leader, then achieving the tracking goal perfectly. This tracking performance can be proven during the later 40 second, where three flight trajectories are close to each other.

6. **Conclusion.** To formulate our previous contributions and propose new interesting topics on controller design within closed loop structure, we analyze two different control strategies through whether the whole controller design depends on the unknown plant. Consider classical model reference control and direct data driven control respectively, their own special properties are all described in detail through our own mathematical derivations. To combine theoretical analysis and engineering application, our considered control strategies are applied to design one optimal aircraft flight controller. Generally, this new paper reviews our previous contributions and covers adaptive direct data driven control. Later, nonlinear direct data driven control is deemed as our future work.

Acknowledgment. This work is partially supported by Jiangxi Provinical National Science Foundation (No. 20232BAB201015).

REFERENCES

[1] D. Piga and S. Formentin, Direct data driven control of costrained systems, *IEEE Transactions on Control Systems Technology*, vol.25, no.2, pp.331-351, 2018.
[2] M. Tanaskovic, L. Fagiano and C. Novara, Data-driven control of nonlinear systems: An on-line direct approach, *Automatica*, vol.75, no.1, pp.1-10, 2017.
[3] M. Tanaskovic, L. Fagiano and V. Gligorovski, Adaptive model predictive control for linear time varying MIMO systems, *Automatica*, vol.105, no.2, pp.237-245, 2019.

[4] E. Terzi, L. Fagiano, M. Farina and R. Scattolini, Learning based predictive control for linear systems: A unitary approach, *Automatica*, vol.108, no.1, pp.2697-2705, 2019.
[5] A. Bemporad, M. Morari and V. Dua, The explicit linear quadratic regulator for constrained systems, *Automatica*, vol.38, no.3, pp.3-20, 2002.
[6] A. Bemporad, A. Casavola and E. Mosca, Control of constrained linear systems via predictive reference management, *IEEE Transactions on Automatic Control*, vol.42, no.2, pp.340-349, 1997.
[7] M. Gevers, Identification for control: From the early achievements to the revial of experiment design, *European Journal of Control*, vol.11, no.12, pp.1-10, 2017.
[8] P. Falugi and D. Q. Mayne, Getting robustness against unstructured uncertainty: A tube based MPC approach, *IEEE Transactions on Automatic Control*, vol.59, no.10, pp.1290-1295, 2014.
[9] U. Rosolia and F. Borrelli, Learning model predictive control for iterative tasks: A data driven control framework, *IEEE Transactions on Automatic Control*, vol.63, no.11, pp.1883-1896, 2018.
[10] E. Garone, S. Di Cairano and I. Kolmanovsky, Reference and command governors for systems with constraints: A survey on theory and applications, *Automatica*, vol.75, no.4, pp.306-328, 2017.
[11] M. Palanikumar, K. Arulmozhi, A. Iampan and K. Rangarajan, Multiple attribute decision-making based on sine trigonometric Fermatean normal fuzzy aggregation operator, *International Journal of Innovative Computing, Information and Control*, vol.18, no.5, pp.1431-1444, 2022.
[12] J. Wang, Closed loop identification for aircraft flutter model parameters, *Aircraft Engineering and Aerospace Technology*, vol.94, no.7, pp.1117-1127, 2022.
[13] J. Wang and R. A. Ramirez-Mendoza, Direct data driven strategy for closed loop aircraft flutter text, *Aircraft Engineering and Aerospace Technology*, vol.95, no.6, pp.1-10, 2023.
[14] J. Wang and R. A. Ramirez-Mendoza, Stealth identification strategy for closed loop system structure, *International Journal of Systems Science*, vol.51, no.6, pp.1084-1101, 2020.
[15] M. Yahara, K. Fujimoto, D. Nishiguchi, Y. Harada and M. Fukuhara, Digital frequency-locked loop with wide lock-in range and low frequency error based on multi-phase clock, *International Journal of Innovative Computing, Information and Control*, vol.18, no.6, pp.1979-1988, 2022.

Author Biography

Ruchun Wen received bachelor degree from Jiangxi University of Science and Technology, China, in 1994. In 1998, she received master degree in the same university. She is currently an associate professor with Jiangxi University of Science and Technology, China. Her current research interests include machine leaning, direct data driven control, convex optimizaiton and nonlinear control theory.

Jianhong Wang received the diploma in Engineering Cybernetics from Yunnan University, China, in 2007. In 2011, he received the Dr.Sc. degree in College of Automation Engineering from Nanjing University of Aeronautics and Astronautics, China. From 2013 to 2015, he was a postdoctoral fellow in Informazione Politecnico di Milano, Italy. From 2016 to 2018, he was a professor in University of Seville, Spain. From 2019 until to now, he is a professor in Jiangxi University of Science and Technology, China. His current research interests include real-time distributed control, nonlinear control and differential geometry control.