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CONTROL SYSTEM TO ATTENUATE PERIODIC DISTURBANCE WITHOUT USING REPETITIVE CONTROL

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Abstract. *In this paper, we propose a control system to attenuate periodic disturbance using disturbance observer. It is well known that repetitive control is effective for tracking periodic reference inputs and suppressing periodic disturbances. However, when the reference input is non-periodic input, repetitive control is not necessary. For the output to follow the non-periodic reference input, a robust higher-order repetitive control is proposed by Pipeleers et al. However, this remains the difficulty to reduce order of controller. In this paper, in order to overcome this problem, we propose a new control system to attenuate periodic disturbance using the disturbance observer for periodic disturbances.*
Keywords: Disturbance observer, Non-periodic reference input, Periodic disturbance

1. **Introduction.** In this paper, we propose a design method of control system to attenuate the periodic disturbance and to follow the non-periodic reference input using disturbance observer without using repetitive control. Repetitive control is a well known method for the output to track a periodic reference input and to attenuate periodic disturbances [1]. However, repetitive control is not necessary when the input is a non-periodic input [2]; however, when the disturbance is a periodic disturbance, repetitive control is used to suppress the disturbance. For the output to follow the non-periodic reference input, a robust higher-order repetitive control [3] is proposed. However, this remains the difficulty to reduce order of controller.

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In order to design control systems to follow non-periodic reference input and to attenuate periodic disturbance without using repetitive control, we have a possibility to use disturbance observer that can estimate the disturbance, since disturbance observers can estimate the disturbance [5, 6, 7, 8]. A disturbance observer is used to estimate the disturbances in the factory plant [9], and various studies are examined [5, 6, 7, 8]. In addition, many studies are studying on disturbance observer control systems that utilize these studies [10, 11, 12, 13, 14]. Currently, the applications of disturbance observers have been used in many control systems such as a motion-control field [15, 16, 17]. A disturbance observer is used in motion control to cancel the disturbance or to make the closed-loop system robustly stable [18, 19, 20]. Typically a disturbance observer includes a disturbance signal generator and an observer, and the disturbance that normally considered a step disturbances is estimated by the observer. Since the disturbance observer is simple to understand the structure, it is used in many cases [18, 19, 21].

Mita et al. pointed out that disturbance observers are not the only alternative design of complete controllers [20]; extended H_∞ control in [20] has therefore been proposed as an effective motion control method that cancels disturbances. From another point of view, Kobayashi et al. considered an observer design method for obtaining phase compensation based on disturbance observers [21]. Another important control problem is the parameterization problem which is the problem of finding all stable controllers for the plant [22]. Since if the parameterization of all disturbance observers for any disturbances could be obtained, we could express results from previous studies of disturbance observers in a uniform manner, in addition, disturbance observers for any disturbances could be designed systematically, Yamada et al. examined the parameterization of all disturbance observers [23]. There exists another study that motion control realization bases on the disturbance observer and the Kalman filter [15]. This study realizes high robustness against disturbance, parameter variations, effective noise suppression and wideband force sensing by using disturbance observer and Kalman filter. In this way, the research on disturbance observer has been progressed.

Resently, the parameterization of all disturbance observer for the periodic disturbance was clarified [4]. Using this parameterization in [4], we have a possibility to design a control system to follow non-periodic reference input and to attenuate periodic disturbance without using repetitive control. However, no paper tackles this problem.

In this paper, in order to overcome this problem, we propose a design method for control system to attenuate the periodic disturbance effectively and to follow the non-periodic reference input without steady state error using disturbance observer. We show that the disturbance can be sufficiently attenuated by using a disturbance observer and the output follows the non-periodic reference input. This paper is organized as follows. In Section 2, we propose a control system and explain the problem considered in this paper. In Section 3, we clarify the condition that the transfer function from the disturbance to the output has a finite number of poles. In Section 4, the stability condition for the control system is presented. In Section 5, it is shown control characteristics of the control system. In Section 6, we present a design method for the control system. In Section 7, we provide a numerical example to illustrate the effectiveness of the proposed method. Section 8 gives concluding remarks.

2. Problem Formulation.

Consider the plant described by

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + d(t) \end{array} \right. , \tag{1}$$

where $x \in R^n$ is the state variable, $u \in R$ is the control input, $y \in R$ is the output to follow the non-periodic reference input $r \in R$, $d \in R$ is the periodic output disturbance with period $T > 0$ satisfying

$$d(t+T) = d(t) \quad (\forall t \geq 0), \tag{2}$$

$A \in R^{n\times n}$, $B \in R^n$ and $C \in R^{1\times n}$. It is assumed that (A, B) is stabilizable, (C, A) is detectable, A has no eigenvalue on the imaginary axis,

$$\det \begin{bmatrix} A - sI & B \\ C & 0 \end{bmatrix} = 0 \tag{3}$$

has no root in the closed right half plane, and $u(t)$ and $y(t)$ are available, but $d(t)$ is unavailable. The transfer function in (1) is denoted by

$$y(s) = G(s)u(s) + d(s), \tag{4}$$

where

$$G(s) = C(sI - A)^{-1}B \in R(s), \tag{5}$$

$y(s) = \mathcal{L}y(t)$, $u(s) = \mathcal{L}u(t)$ and $d(s) = \mathcal{L}d(t)$. Note that the assumption in (3) implies that $G(s)$ in (5) is assumed to have no zero in the closed right half plane, that is, $G(s)$ assumed to be of minimum phase.

In order to make a control system such that the output $y(t)$ follows the non-periodic reference input $r(t)$ and the periodic disturbance $d(t)$ is attenuated, we adopt the disturbance observer for the periodic disturbance [4]. Using the disturbance observer for the periodic disturbance, we propose a control system in Figure 1. Here, $\tilde{d}(s)$ is a disturbance observer for periodic disturbance [4] written by

$$\tilde{d}(s) = F_1(s)e^{-sT}y(s) + F_2(s)e^{-sT}u(s), \tag{6}$$

$C_1(s) \in R(s)$ is the feedback controller, $C_2(s) \in R(s)$ is the controller for disturbance observer, $F_1(s) \in RH_\infty$, $F_2(s) \in RH_\infty$ and $r(s) = \mathcal{L}r(t)$.

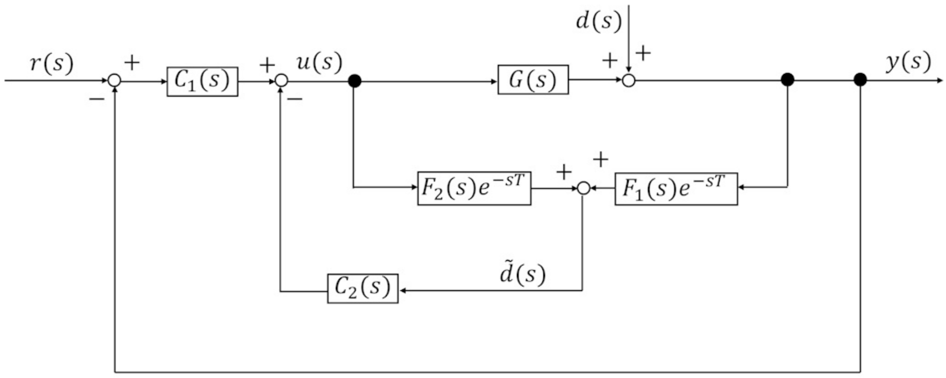

Figure 1. Structure of a disturbance observer system

According to [4], the parameterization of all $F_1(s)$ and $F_2(s)$ satisfying

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \left(d(t) - \tilde{d}(t)\right) = 0 \tag{7}$$

for any initial state $x(0)$, control input $u(t)$ and periodic output disturbance $d(t)$ is given by

$$F_1(s) = D(s) + Q(s)D(s) \in RH_\infty \tag{8}$$

and

$$F_2(s) = -N(s) - Q(s)N(s) \in RH_\infty, \tag{9}$$

where $D(s) \in RH_\infty$ and $N(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)}, \tag{10}$$

respectively, and $Q(s) \in RH_\infty$ is any function satisfying

$$D(s_i) + Q(s_i)D(s_i) = 1 \quad \forall s_i \ (i = 0, 1, \dots), \tag{11}$$

$$s_i = j\omega_i, \tag{12}$$

$$\omega_i = \frac{2\pi i}{T} \ (i = 0, 1, \dots) \tag{13}$$

and j is the imaginary unit.

Here, substituting (4) to (6), we have the following equation,

$$\tilde{d}(s) = F_1(s)e^{-sT}(G(s)u(s) + d(s)) + F_2(s)e^{-sT}u(s). \tag{14}$$

Substituting (8), (9) and (10) to (14), we have

$$\tilde{d}(s) = (D(s) + Q(s)D(s))e^{-sT}\left(\frac{N(s)}{D(s)}u(s) + d(s)\right) - (N(s) + Q(s)N(s))e^{-sT}u(s), \tag{15}$$

and summarizing (15), we have the following equation

$$\begin{aligned}\tilde{d}(s) = &(N(s) + Q(s)N(s))e^{-sT}u(s) + (D(s) + Q(s)D(s))e^{-sT}d(s) \\ &- (N(s) + Q(s)N(s))e^{-sT}u(s).\end{aligned} \tag{16}$$

Therefore, we have the following equation

$$\tilde{d}(s) = (D(s) + Q(s)D(s))e^{-sT}d(s). \tag{17}$$

From (11), the following equation

$$\tilde{d}(s_i) = d(s_i) \quad \forall s_i \ (i = 0, 1, \dots) \tag{18}$$

holds true. Thus, it can be seen that (6) estimates the disturbance $d(s)$ and (7) holds true from (18).

Using the disturbance observer for periodic disturbances, in general, the transfer function from the disturbance $d(s)$ to the output $y(s)$ has infinite number of poles. In order to make the transfer function from $d(s)$ to $y(s)$, $C_2(s)$ is set as the form in [24]

$$C_2(s) = \frac{C_n(s)}{1 + C_d(s)e^{-sT}}, \tag{19}$$

where $C_n(s) \in RH_\infty$ and $C_d(s) \in RH_\infty$.

The problem considered in this paper is to propose a design method for control system in Figure 1 to attenuate the periodic disturbance $d(s)$, for $y(s)$ to follow the reference input $r(s)$ and the transfer function from $d(s)$ to $y(s)$ has finite number of poles.

3. **Condition for Finite Number of Poles.** In this section, we clarify the condition such that the transfer function from $d(s)$ to $y(s)$ has finite number of poles.

Transfer function from $d(s)$ to $y(s)$ in Figure 1 is given by

$$\frac{y(s)}{d(s)} = \frac{1 + (C_d(s) + F_2(s)C_n(s))e^{-sT}}{1 + C_1(s)G(s) + \{(1 + C_1(s)G(s))C_d(s) + (F_1(s)G(s) + F_2(s))C_n(s)\}e^{-sT}}. \tag{20}$$

From this equation, we have the following theorem.

Theorem 3.1. *The transfer function from $d(s)$ to $y(s)$ has finite number of poles if and only if*

$$(1 + C_1(s)G(s))C_d(s) + (F_1(s)G(s) + F_2(s))C_n(s) = 0 \tag{21}$$

holds true.

Proof: It is obvious from (20). □

From Theorem 3.1, $C_2(s)$ in (19) is settled satisfying (21). A design method for $C_2(s)$ satisfying (21) will be described in Section 6.

4. Stability Condition.

In this section, we clarify the stability condition for the control system in Figure 1.

From the definition of internal stability [25], when all transfer functions $V_i(s)$ ($i = 1, 2, 3, 4$) written by

$$\begin{bmatrix} u(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} V_1(s) & V_2(s) \\ V_3(s) & V_4(s) \end{bmatrix} \begin{bmatrix} r(s) \\ d(s) \end{bmatrix} \tag{22}$$

are stable, the control system in Figure 1 is stable.

On the stability of the control system in Figure 1, we have the following theorem.

Theorem 4.1. *The control system in Figure 1 is stable if and only if the following conditions are satisfied.*

1) $C_1(s)$ stabilizes $G(s)$. That is, all transfer functions $C_1(s)G(s)/(1 + C_1(s)G(s))$, $G(s)/(1 + C_1(s)G(s))$, $C_1(s)/(1 + C_1(s)G(s))$ and $1/(1 + C_1(s)G(s))$ are stable.
2) $C_d(s) \in RH_\infty$ in (19).
3) $C_n(s) \in RH_\infty$ in (19).

Proof: From Figure 1, all transfer functions $V_i(s)$ ($i = 1, 2, 3, 4$) in (22) are written by

$$V_1(s) = \frac{C_1(s)}{1 + C_1(s)G(s)}, \tag{23}$$

$$V_2(s) = \frac{-C_1(s) + (-C_1(s)C_d(s) - F_1(s)C_n(s))\, e^{-sT}}{1 + C_1(s)G(s) + ((1 + C_1(s)G(s))\, C_d(s) + (F_1(s)G(s) + F_2(s))\, C_n(s))\, e^{-sT}}, \tag{24}$$

$$V_3(s) = \frac{C_1(s)G(s)}{1 + C_1(s)G(s)} \tag{25}$$

and

$$V_4(s) = \frac{1 + (C_d(s) + F_2(s)C_n(s))\, e^{-sT}}{1 + C_1(s)G(s) + ((1 + C_1(s)G(s))\, C_d(s) + (F_1(s)G(s) + F_2(s))\, C_n(s))\, e^{-sT}}. \tag{26}$$

Substitution of (21) for (24) and (26) gives

$$V_2(s) = \frac{-C_1(s) + (-C_1(s)C_d(s) - F_1(s)C_n(s))e^{-sT}}{1 + C_1(s)G(s)}, \tag{27}$$

and

$$V_4(s) = \frac{1 + (C_d(s) + F_2(s)C_n(s))\, e^{-sT}}{1 + C_1(s)G(s)}. \tag{28}$$

We will prove all transfer functions in (23), (25), (27) and (28) are stable if and only if $C_1(s)$ stabilizes $G(s)$, $C_n(s) \in RH_\infty$ and $C_d(s) \in RH_\infty$. First necessity is shown. That is if the control system in Figure 1 is stable, $C_1(s)$ stabilizes $G(s)$, $C_n(s) \in RH_\infty$ and $C_d(s) \in RH_\infty$. From (23), (25), (27) and (28), if $C_1(s)G(s)/(1+C_1(s)G(s))$, $G(s)/(1+C_1(s)G(s))$, $C_1(s)/(1 + C_1(s)G(s))$, $1/(1 + C_1(s)G(s))$, $C_n(s)$ and $C_d(s)$ are unstable, then transfer function in (23), (25), (27) and (28) are unstable. Thus, the necessity is shown.

Conversely if all transfer functions $C_1(s)G(s)/(1+C_1(s)G(s))$, $G(s)/(1+C_1(s)G(s))$, $C_1(s)/(1+C_1(s)G(s))$, $1/(1+C_1(s)G(s))$, $C_n(s)$ and $C_d(s)$ are stable, it is obvious that transfer functions in (23), (25), (27) and (28) are stable. The sufficiency is shown.

We have thus proved Theorem 4.1. □

In this section, the stability condition has been clarified.

5. **Control Characteristics.** In this section, we explain control characteristics of the control system in Figure 1.

First, the input output characteristic of control system in Figure 1 is shown. The transfer function from the reference input $r(s)$ to the output $y(s)$ and that from the reference input $r(s)$ to the error $e_r(s) = r(s) - y(s)$ are written by

$$y(s) = \frac{C_1(s)G(s)}{1+C_1(s)G(s)} r \tag{29}$$

and

$$e_r(s) = r(s) - y(s) = \frac{1}{1+C_1(s)G(s)} r(s), \tag{30}$$

respectively. In order for the output $y(s)$ to follow the non-periodic reference input $r(s)$ without steady-state error, from the internal model principle [26], $C_1(s)$ is written by the form

$$C_1(s) = C_r(s)\bar{C}_1(s), \tag{31}$$

where $C_r(s)$ is model of the reference input $r(s)$ and $\bar{C}_1(s) \in R(s)$. Therefore, $C_1(s)$ needs to be written by (31) and stabilizes $G(s)$.

Next, the disturbance attenuation characteristic is shown. Transfer functions from the periodic disturbance $d(s)$ to the output $y(s)$ is written by

$$y(s) = \frac{1 + (C_d(s) + F_2(s)C_n(s))\, e^{-sT}}{1+C_1(s)G(s)} d(s). \tag{32}$$

Therefore, in order to attenuate periodic disturbances $d(s)$, it is necessary to satisfy

$$1 - (C_d(s) + F_2(s)C_n(s))\big|_{s_i = j\frac{2\pi i}{T}} = 0 \quad (i = 0, 1, 2, \ldots). \tag{33}$$

From (33), the disturbance characteristic is specified using controllers $C_d(s)$ and $C_n(s)$ of $C_2(s)$ in (19).

Therefore, the purpose of $C_1(s)$ is to specify the input/output characteristics, and the purpose of $C_2(s)$ is to specify the disturbance attenuation characteristics. This implies that the control system in Figure 1 is a two-degree-of-freedom control system.

6. **Design Method for $C_1(s)$ and $C_2(s)$.** In this section, we present a design method for $C_1(s)$ and $C_2(s)$ to attenuate the periodic disturbance $d(s)$ and for the output $y(s)$ to follow the non-periodic reference input $r(s)$.

From the discussion in previous sections, $C_n(s)$ and $C_d(s)$ in $C_2(s)$ in (19) and $C_1(s)$ need to satisfy (21) and (33). From (33), if $C_n(s)$ and $C_d(s)$ satisfy

$$\frac{C_n(s)}{C_d(s)} = -\frac{1}{F_2(s)}, \tag{34}$$

then $C_n(s)$ and $C_d(s)$ satisfy (33). By substituting (34) to (21) and solve, then the solution is $C_n(s)$. Thus, we cannot design $C_n(s)$ and $C_d(s)$ satisfying (34).

In order to overcome this problem, we change the problem to satisfy (33) by

$$C_d(s) + F_2(s)C_n(s) = -q(s), \tag{35}$$

where $q(s) \in RH_\infty$ is a strictly proper low-pass filter written by

$$q(s) = \frac{1}{(1+\tau s)^{n_q}}, \tag{36}$$

$n_q > 0$ is positive integer and $\tau > 0$ is small real number. In the frequency range satisfying

$$q(s_i) \simeq 1 \quad (i = 0, 1, \ldots, k_{\max}), \tag{37}$$

frequency component of the periodic disturbance $d(s)$ is attenuated, where $k_{\max}$ is maximum integer.

Equation (35) is rewritten by

$$C_n(s) = -\frac{q(s) + C_d(s)}{F_2(s)}. \tag{38}$$

Substituting (38) to (21), we have

$$C_d(s) = \frac{(F_1(s)G(s) + F_2(s))q(s)}{(F_2(s)C_1(s) - F_1(s))G(s)}. \tag{39}$$

From Theorem 4.1, in order to make the control system in Figure 1 stable, the stability $C_n(s) \in RH_\infty$ and $C_d(s) \in RH_\infty$ must be satisfied. First, we will clarify that the condition to hold $C_d(s) \in RH_\infty$ in (39). Since (39), (8), (9), (36), the assumption that $G(s)$ is of minimum phase, $F_1(s) \in RH_\infty$, $F_2(s) \in RH_\infty$, $q(s) \in RH_\infty$, $1/G(s)$ has no pole in the closed right half plane. Therefore, $C_d(s)$ in (39) is stable if $F_2(s)C_1(s) - F_1(s) \in \mathcal{U}$, where $\mathcal{U}$ is unimodular, that is, $F_2(s)C_1(s) - F_1(s) \in \mathcal{U}$ implies $F_2(s)C_1(s) - F_1(s) \in RH_\infty$ and $1/(F_2(s)C_1(s) - F_1(s)) \in RH_\infty$. In addition, $C_d(s)$ is proper if relative degree of $(F_1(s)G(s) + F_2(s))\,q(s)$ is greater than or equal to that of $G(s)$. From (8) and (9), $F_1(s)$ is bi-proper and $F_2(s)$ is strictly proper. Therefore, $F_1(s)G(s) + F_2(s)$ is bi-proper. This implies that if $q(s)/G(s)$ is proper, then $C_d(s)$ is proper. In this way, we find that if the relative degree of $q(s)$ is greater than or equal to $G(s)$ and $F_2(s)C_1(s) - F_1(s) \in \mathcal{U}$, then $C_d(s) \in RH_\infty$ in (39). Second, we will clarify that the condition of $C_n(s) \in RH_\infty$ in (38). From (38), if $F_2(s)$ is of minimum phase and relative degree of $q(s) + C_d(s)$ is greater than or equal to $F_2(s)$, $C_n(s) \in RH_\infty$ is satisfied. Since $F_1(s)$ in (8) is bi-proper, $G(s)$ is strictly proper, and $F_2(s)$ in (9) is strictly proper, then $F_1(s)G(s) + F_2(s)$ is strictly proper. Since $F_1(s)G(s) + F_2(s)$ is strictly proper, the relative degree of $C_d(s)$ is greater than that of $q(s)$. That is, the relative degree of $q(s) + C_d(s)$ is equal to that of $q(s)$. Thus if $q(s)/F_2(s)$ is proper, then $C_d(s)$ is proper. Thus, if $F_2(s)$ is of minimum phase and $q(s)/F_2(s)$ is proper, then $C_n(s) \in RH_\infty$.

Using above expressions, a design procedure of the control system in Figure 1 is summarized as follows.

Step 1) Obtain the coprime factors $N(s)$ and $D(s)$ satisfying (10).

Step 2) Design $F_1(s) \in RH_\infty$ and $F_2(s) \in \mathcal{U}$ using the method in [11]. That is, $Q(s)$ in (8) and (9) is settled to satisfy (11).

Step 3) According to [25], all stabilizing controllers for $G(s)$ are given by

$$C_1(s) = \frac{N_c(s)}{D_c(s)} = \frac{X(s) + D(s)\bar{Q}(s)}{Y(s) - N(s)\bar{Q}(s)}, \tag{40}$$

where

$$N_c(s) = X(s) + D(s)\bar{Q}(s), \tag{41}$$

$$D_c(s) = Y(s) - N(s)\bar{Q}(s), \tag{42}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are a pair of solutions of

$$N(s)X(s) + D(s)Y(s) = 1, \tag{43}$$

and $\bar{Q}(s) \in RH_\infty$ is any function satisfying (31). Substitution of (10) and (40) to (39) gives

$$C_d(s) = \frac{(F_1(s)N(s) + F_2(s)D(s))\, q(s)D_c(s)}{(F_2(s)N_c(s) - F_1(s)D_c(s))\, N(s)}. \tag{44}$$

Using $\bar{Q}(s)$ in (41) and (42), $N_c(s)$ and $D_c(s)$ are designed to make $F_2(s)N_c(s) - F_1(s)D_c(s) \in \mathcal{U}$ and to written by the form $D_c(s) = \hat{D}_c(s)/C_r(s)$, where $\hat{D}_c(s) \in RH_\infty$ is to make $D_c(s)$ proper. To find $N_c(s)$ and $D_c(s)$ to make $F_2(s)N_c(s) - F_1(s)D_c(s) \in \mathcal{U}$ is equivalent to find a pair of solution to

$$F_2(s)N_c(s) - F_1(s)D_c(s) = 1. \tag{45}$$

Obtain $N_c(s)$ and $D_c(s)$ satisfying (45).

$q(s)$ is settled by (36), where the maximum frequency range $k_{\max}$ in (37) to estimate the periodic disturbance $d(s)$ is settled. $\tau > 0$ is settled to small real number that $q(s_i) = 1$ in the frequency range from 0 to $k_{\max}$. n_q is settled to make $C_d(s)$ and $C_n(s)$ proper.

Step 4) $C_d(s)$ and $C_n(s)$ are given by (44) and (38).

7. **Numerical Example.** In this section, a numerical example is illustrated to show the effectiveness of the proposed method.

Consider the problem to design the control system in Figure 1 to attenuate periodic disturbances $d(t)$ with period $T = \pi$ [sec] and to follow the reference input $r(t) = 1$ for the minimum-phase plant $G(s)$ written by

$$G(s) = \frac{s+1}{s^2 - 43s - 350}. \tag{46}$$

In this paper, in order to compare with the results of [4], we adopt plant $G(s)$ of [4].

Coprime factors $N(s)$ and $D(s)$ of $G(s)$ in (46) satifying (10) is written

$$N(s) = \frac{-2s - 2}{s^2 + 1007s + 7000} \tag{47}$$

and

$$D(s) = \frac{-2s + 100}{s + 1000}. \tag{48}$$

$F_1(s)$ and $F_2(s)$ are given by (8) and (9). We have

$$F_1(s) = \frac{-5s^2 - 750s + 50000}{s^2 + 1050s + 50000} \tag{49}$$

and

$$F_2(s) = \frac{5s^2 + 1005s + 1000}{s^3 + 1057s^2 + 57350s + 350000}, \tag{50}$$

where $Q(s)$ in (8) and (9) is settled by

$$Q(s) = \frac{1.5s + 450}{s + 50}. \tag{51}$$

In this paper, $G(s)$ from [4] is adopted. Therefore, $F_1(s)$, $F_2(s)$, and $Q(s)$ have the same values as [4].

From (36), to cover a wide frequency range, $q(s)$ is given by

$$q(s) = \frac{1}{0.001s + 1}. \tag{52}$$

Using Step 4) in the preceding section, solution to (45) is obtained by

$$N_c(s) = \frac{27890s^3 + 1.17 \cdot 10^7 s^2 + 1.789 \cdot 10^9 s + 3.5 \cdot 10^9}{s^4 + 1251s^3 + 2.612 \cdot 10^5 s^2 + 1.026 \cdot 10^7 s + 10^7} \tag{53}$$

and

$$D_c(s) = \frac{0.2s^4 + 431.4s^3 + 2.371 \cdot 10^5 s^2 + 2.284 \cdot 10^7 s}{s^4 + 1257s^3 + 2.688 \cdot 10^5 s^2 + 1.182 \cdot 10^7 s + 7 \cdot 10^7}. \tag{54}$$

Using above parameters, $C_d(s)$ and $C_n(s)$ are given by (44) and (38), respectively.

Using designed control system in Figure 1, the response of the output $y(t)$ for the step reference input $r(t) = 1$ is shown Figure 2. Figure 2 shows that the control system in Figure 1 is stable and the output $y(t)$ follows the step reference input $r(t) = 1$ without steady state error.

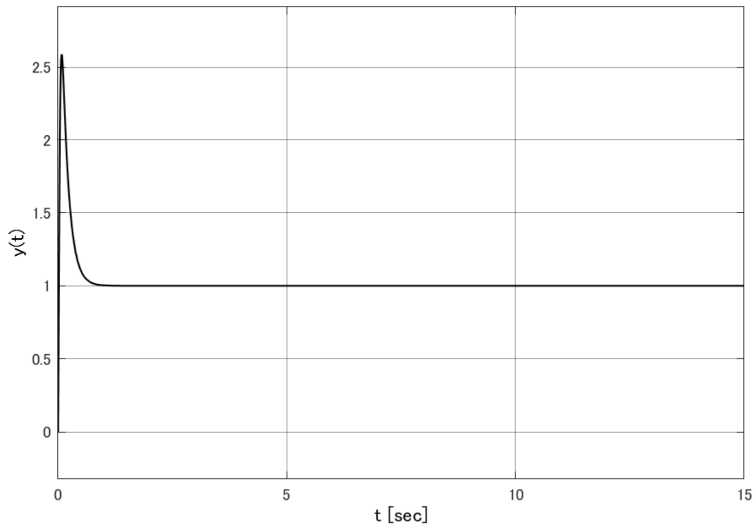

Figure 2. The response of the output $y(t)$ for the step reference input $r(t) = 1$

Next, the disturbance attenuation characteristic is shown. When the periodic output disturbance $d(t)$ is given by

$$d(t) = \begin{cases} \frac{2}{\pi}t & 0 \le t - kt < \frac{\pi}{2} \\ \frac{2}{\pi}(\pi - t) & \frac{\pi}{2} \le t - kt < \pi \end{cases}, \tag{55}$$

where k is the maximum integer which will not exceed t/T, the disturbance $d(t)$ in (55) is a triangular wave with the period T as shown in Figure 3. The response of the output $y(t)$ for the disturbance $d(t)$ in (55) is shown in Figure 4. Figure 4 shows that the periodic disturbance $d(t)$ in (55) is attenuated effectively.

In this way, we can easily design a control system to attenuate the periodic disturbance and to follow the reference input without steady state error using Figure 1.

Here, in this result, we consider the advantage of the control system using disturbance observer. If the reference input is the step input, we consider a repetitive control system as shown in Figure 5, where $\bar{q}(s) \in RH_\infty$ is a strictly proper low-pass filter written by

$$\bar{q}(s) = \frac{1}{(1 + \bar{\tau}s)^{\bar{n}_q}}, \tag{56}$$

$\bar{n}_q > 0$ is positive integer and $\bar{\tau} > 0$ is small real number, $\tilde{C}_1(s) \in R(s)$ is a controller and $G_2(s) \in RH_\infty$ is a plant. Here $\bar{q}(s)$, $\tilde{C}_1(s)$ and $G_2(s)$ are satisfying

$$\left|1 - \tilde{C}_1(s)G_2(s)\right| < \frac{1}{|\bar{q}(s)|} \quad \forall s = j\omega,\ \omega \in R. \tag{57}$$

Repetitive control can reduce the steady-state error of step input to 0 as shown in Figure 6. However, if the reference input is the ramp input, repetitive control system has the steady-state error as shown in Figure 7.

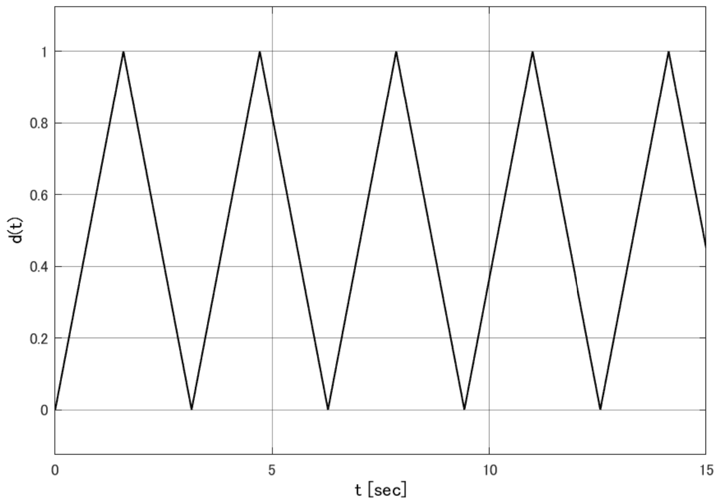

FIGURE 3. Triangular wave disturbance

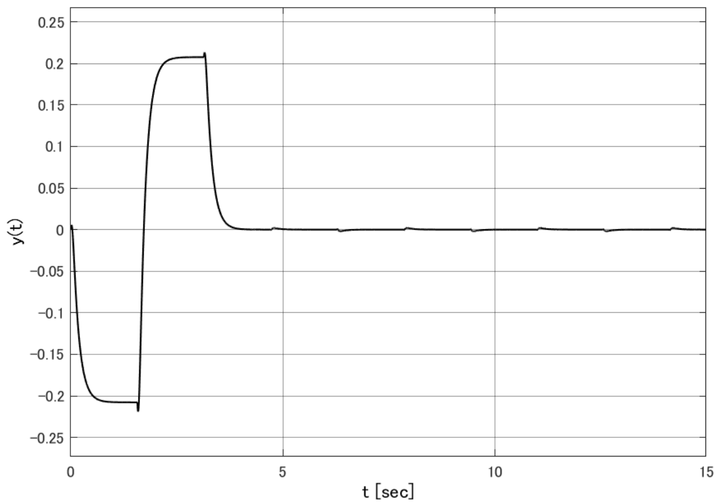

FIGURE 4. The response of the output $y(t)$ for the disturbance $d(t)$ in (55)

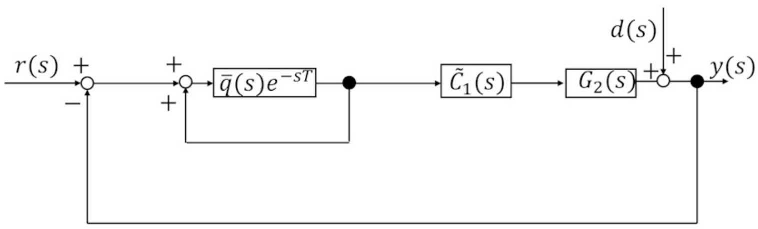

FIGURE 5. Structure of repetitive control system

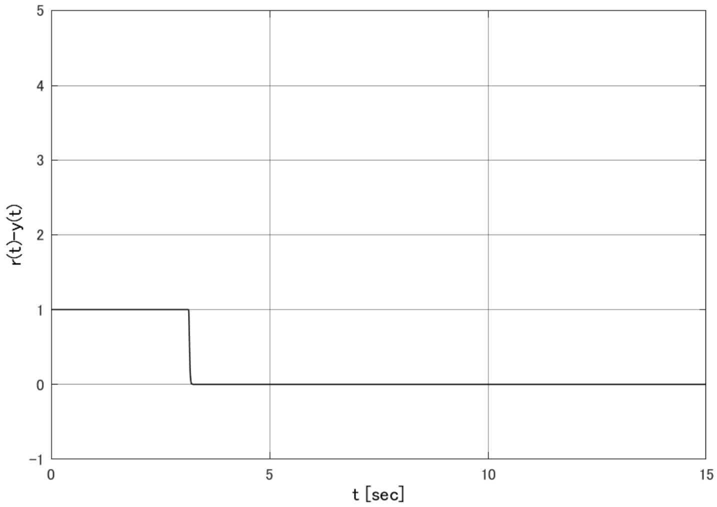

FIGURE 6. Steady-state error of repetitive control with step input

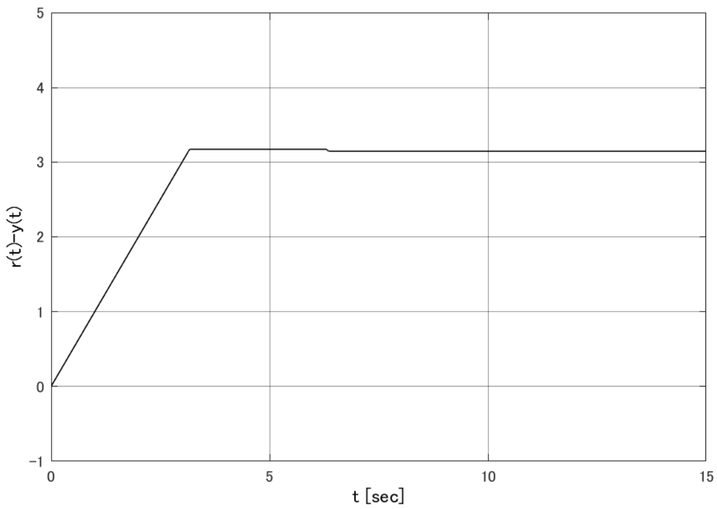

FIGURE 7. Steady-state error of repetitive control with ramp input

In addition, the reference input of repetitive control is repetitive input, if the reference input is not repetitive input, repetitive control has extra data. Therefore, a simple control system that corresponds to non-periodic inputs is required to eliminate redundancy.

Here, the control system using disturbance observer for the periodic disturbance can reduce the steady-state error of ramp input to 0 as shown in Figure 8. This is the advantage of a disturbance observer that suppresses periodic disturbances.

We consider the PID control system in Figure 9. Here, K_p is proportional gain, K_i is integral gain, K_d is differential gain, $G_3(s) \in RH_\infty$ is a plant, $f(s) \in RH_\infty$ is a strictly proper low-pass filter written by

$$f(s) = \frac{1}{(1 + \tilde{\tau} s)^{\tilde{n}_q}}, \tag{58}$$

$\tilde{n}_q > 0$ is positive integer and $\tilde{\tau} > 0$ is small real number.

PID control system can reduce steady-state error of step input to 0 as Figure 10. However, if reference input is ramp input, two integrators are needed and PID control system is steady-state error as Figure 11. In addition, if the disturbance is periodic disturbance written by

$$d(t) = \begin{cases} \dfrac{2}{\pi} t & 0 \le t - kt < \dfrac{\pi}{2} \\ \dfrac{2}{\pi}(\pi - t) & \dfrac{\pi}{2} \le t - kt < \pi \end{cases}, \tag{59}$$

PID control system has disturbance as Figure 12.

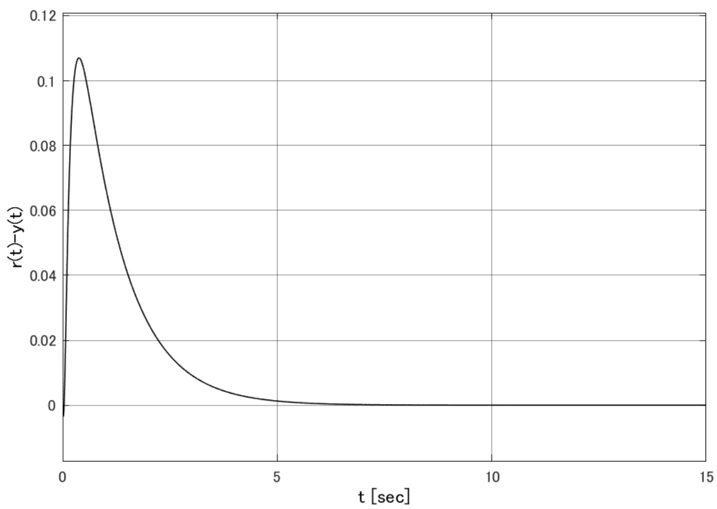

FIGURE 8. Steady-state error of the control system using disturbance observer for the periodic reference input for ramp input

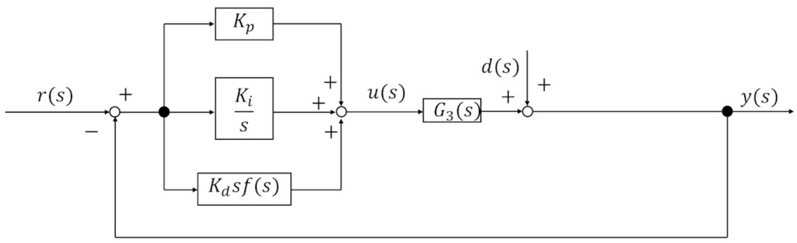

FIGURE 9. Structure of PID control system

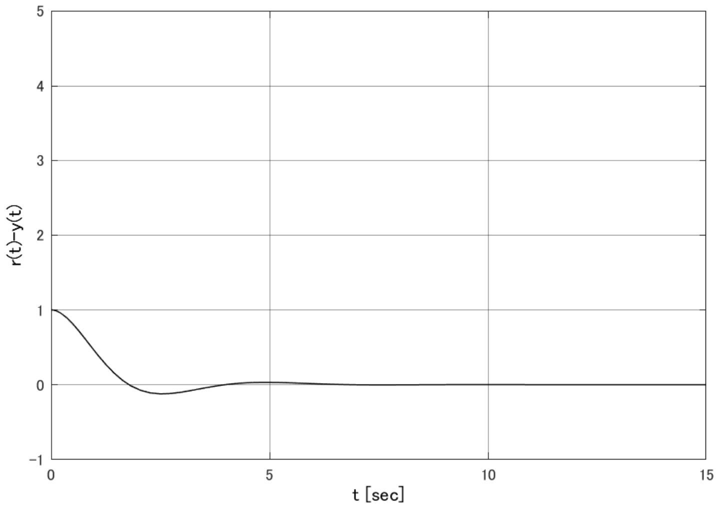

FIGURE 10. Steady-state error of PID control system with step input

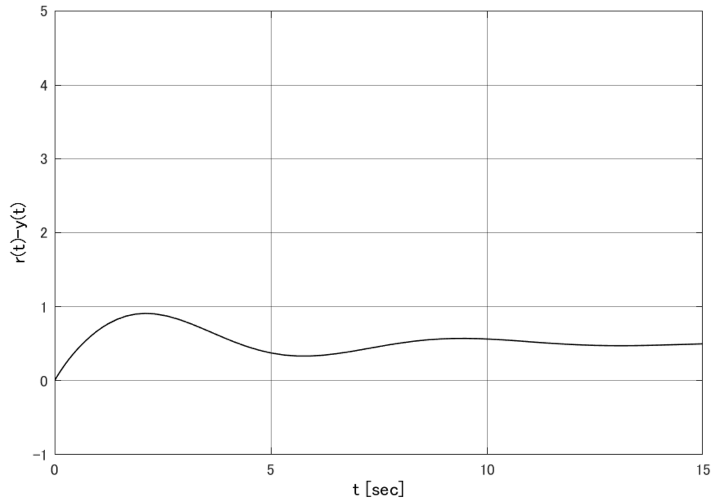

FIGURE 11. Steady-state error of PID control system with ramp input

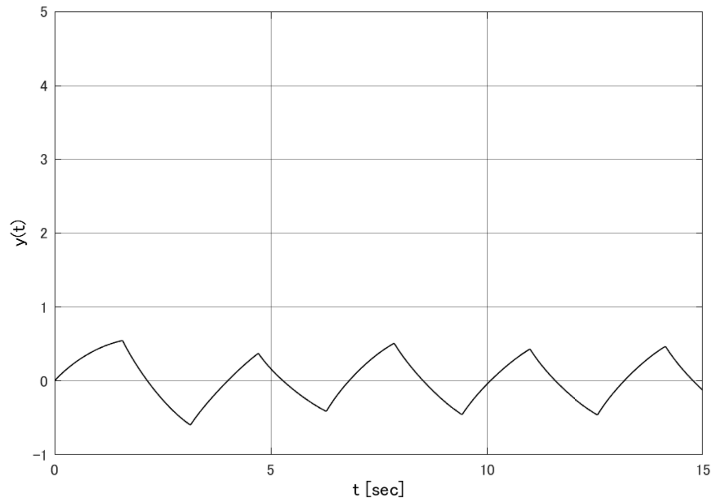

FIGURE 12. The response of the output $y(t)$ for the disturbance $d(t)$ of PID control system

Therefore, the disturbance observer has advantage for ramp input and periodic disturbance. However, the disturbance observer in this paper is the dead time system; therefore, the disturbance observer of in this paper cannot suppress the disturbance of first cycle as Figure 4.

8. **Conclusion.** In this paper, we have proposed a design method for control system to attenuate the periodic disturbances and to follow non-periodic reference input without steady state error using the disturbance observer for periodic disturbance. First, we proposed a control system using disturbance observer for the periodic disturbance. We clarify the condition that the transfer function from the disturbance $d(s)$ to the output $y(s)$ has finite number of poles and that the proposed control system is stable. The control characteristics of the proposed system and a design procedure of the proposed system are shown. We show features of the proposed design method through a numerical example. Finally, in this paper, the plant $G(s)$ is assumed to be of minimum-phase; however, if the plant is not minimum-phase plant, the results of this paper cannot be applied. Therefore, we will examine a design method for disturbance observers control system for non-minimum phase systems.

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