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OPTIMAL AREA FOR RECTANGULAR FOUNDATION SLABS IN PLAN SUPPORTED ON SOIL

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Abstract. *Foundation slabs are also called foundation raft. This paper shows the optimal or minimum area for rectangular foundation slabs in plan that rest on the soil to support the building structures. The normal practice in structural engineering is to use the trial and error procedure to estimate the sides and area of the foundation under biaxial bending supported on elastic soils. The model is developed using optimization techniques to obtain the minimum area in plan and the sides of the foundation slab. The soil pressure is considered linear, where the maximum pressure is the available allowable bearing capacity of the soil and the minimum pressure is zero. Current model is obtained by trial and error, assuming a uniform pressure, which is the maximum pressure. Three numerical examples are presented. Example 1: Unlimited sides. Example 2: Limited sides in positive and negative X direction. Example 3: Limited sides in positive and negative Y direction. The main advantage of this document over other documents is that the model presented in this work shows the minimum area and sides of the foundation slab and can be used for rectangular isolated footings and rectangular combined footings.*
Keywords: Optimal area, Minimum area, Rectangular foundation slabs, Linear distribution of soil pressure

1. **Introduction.** A foundation slab is a combined foundation slab that covers the entire area under a structure and supports all walls and columns. A foundation slab usually rests directly on soil or rocks: however, it may also sit on piles, if hard stratum is not available at a reasonably small depth.

Common types of foundation slabs are (see Figure 1): (a) Flat slabs: It corresponds to a flat concrete slab of uniform thickness throughout the area; this type is suitable for widely spaced columns supporting small loads; (b) Flat slabs with reinforcement under the columns: This type provides sufficient strength for relatively large column loads; (c) Bidirectional and flat: It is a slab with thickened bands provided along the column lines in both directions; this type provides sufficient strength, when the space between columns is large and the loads between columns are unequal; (d) Flat with pedestal: It represents a slab in which pedestals are added under each column; this alternative has the same purpose as the slabs with reinforcement under the columns; (e) With cell design: It represents a

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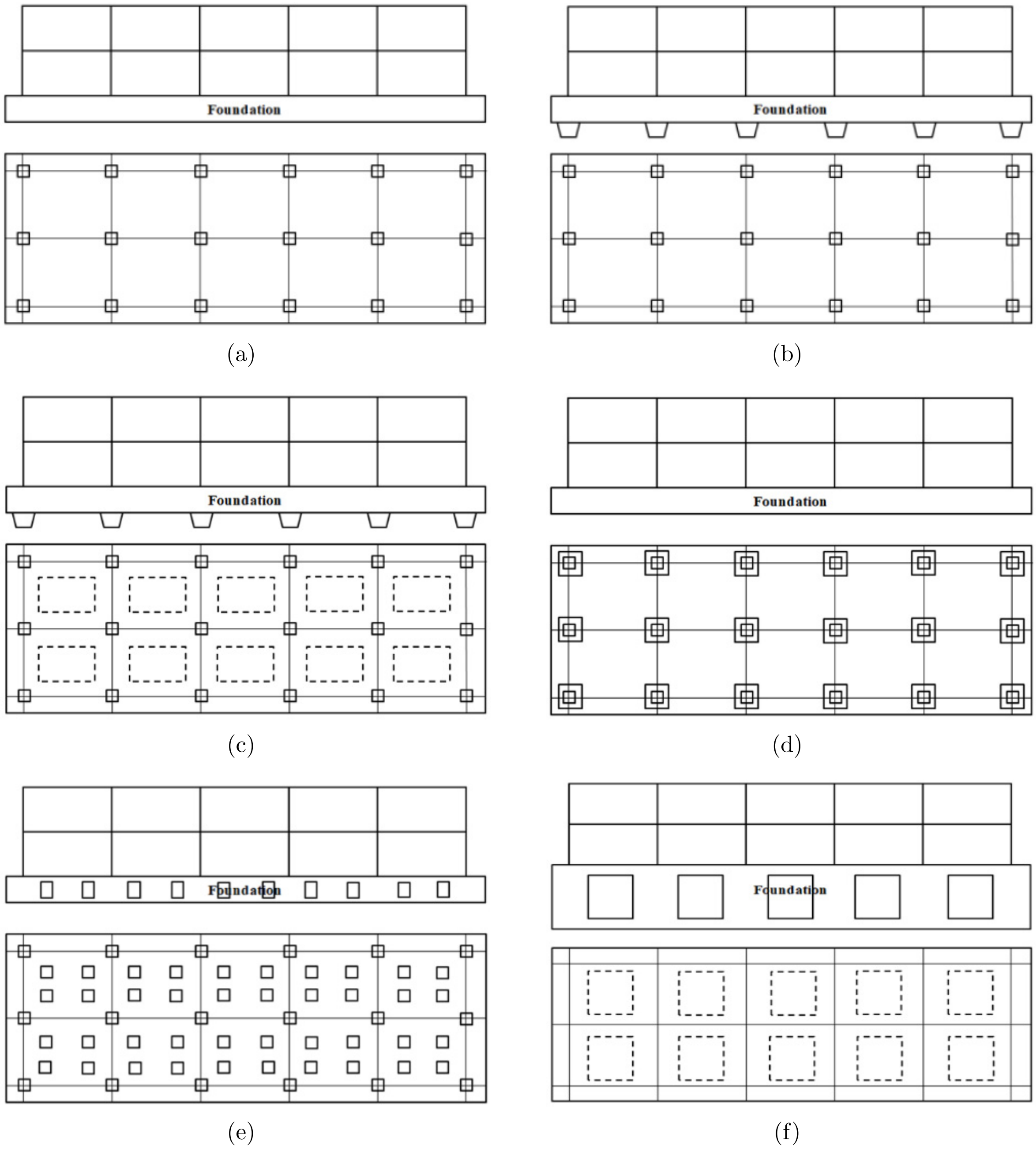

Figure 1. Types of foundation slabs

two-way grid structure made of cellular construction and intersecting structural steel construction; (f) Base walls as a rigid frame (In drawer): It represents a slab in which the walls or basement walls have been used as ribs or deep beams.

The ground pressure beneath a foundation depends on the type of terrain, the relative stiffness of the soil and the foundation, and the depth of foundation at level of contact between the foundation and the soil.

Figure 2 shows the distribution of ground pressure under the foundation depending on the type of ground and the stiffness of the foundation [1]. Figure 2(a) shows a rigid base on a sandy ground. Figure 2(b) presents a rigid base on a clay ground. Figure 2(c) shows a flexible base on a sandy ground. Figure 2(d) presents a flexible base on a clay ground. Figure 2(e) shows the uniform distribution used in design.

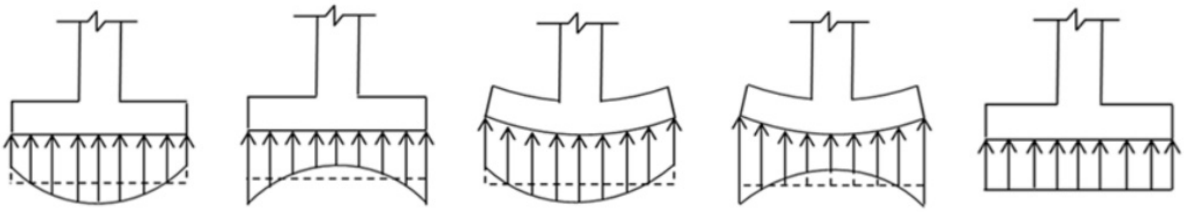

FIGURE 2. Distribution of ground pressure under the footing

The mathematical models have been investigated by many mathematicians on the topic of optimal area of reinforced concrete foundations.

The models for the smallest contact area with the soil have been presented for circular [2-4], square [5,6] and rectangular [7-11] isolated footings under biaxial bending. Likewise, models have been presented for combined footings for the smallest contact area with the ground under biaxial bending in each column of trapezoidal shape [12], rectangular [13,14], L or corner [15], strap [16,17] and T [18].

Several researchers have designed the raft or slab foundations supported on piles for high-rise buildings supported on soils for different studies: Allievi et al. [19] investigated on granular soils; Sundaram et al. [20] studied on alluvial deposits; Pechorskaya [21] used charts; Shakir et al. [22] used software; Jamil et al. [23], Poulos [24], Nguyen et al. [25] considered the interaction effects; Sharma et al. [26] presented a comparison of raft foundation and beam with respect to slab raft foundation; Ojha and Srivastava [27] developed a case study in Lucknow; Punekar et al. [28], Poulos [29] showed an application; Azhar et al. [30] developed a parametric study for high rise buildings. Deb and Pal [31] estimated the load-settlement and load-sharing behavior. Deb and Pal [32] presented structural and geotechnical aspects through numerical analysis. Deb and Pal [33] showed the interaction behavior and load sharing pattern using nonlinear regression and Levenberg-Marquardt algorithm-based artificial neural network. Kannaujiya and Srivastava [34] investigated the behavior of different configuration for a high-rise building using the finite element method.

According to the bibliographic review, the works closest to the subject of rectangular foundation slabs in plan are: the papers for rectangular isolated footings (footings or slabs that support a column) have been investigated by Luévanos-Rojas [7], Algin [8], Aydogdu [9], Highter and Anders [10] and Vela-Moreno et al. [11]; the works for rectangular combined footings (footings or slabs that support two columns) have been developed by Luévanos-Rojas [13] and Montes-Paramo et al. [14]. Therefore, there is no work with the current level of knowledge about the optimal or minimum area for rectangular foundation slabs.

This paper shows the minimum area for rectangular foundation slabs in plan that rest on the soil to support the entire building using optimization techniques, and the soil pressure is linear, where the soil pressure must be between the available allowable bearing capacity of the soil and zero. The normal practice in structural engineering is to use the trial and error procedure to estimate the sides and area of the foundation under biaxial bending supported on elastic soils, assuming a uniform pressure, which is the maximum pressure. Three numerical examples are presented to obtain the minimum area for rectangular foundation slabs. Example 1: Unlimited sides. Example 2: Limited sides in positive and negative X direction. Example 3: Limited sides in positive and negative Y direction. This document is verified with other documents presented by other authors for rectangular isolated footings [35] and rectangular combined footings [36]. The main advantage of this document over other documents is that it can be used for rectangular isolated footings and rectangular combined footings.

The paper is organized as follows. Section 2 shows the formulation of the equations to obtain minimum area of rectangular foundation slabs. Section 3 describes the application of the proposed model through three numerical examples. Section 4 presents the results of the proposed model applied for rectangular isolated footings and rectangular combined footings. Section 5 presents the conclusions.

2. **Formulation of the Model.** Figure 3 presents a rectangular foundation slab subjected to biaxial bending in each column.

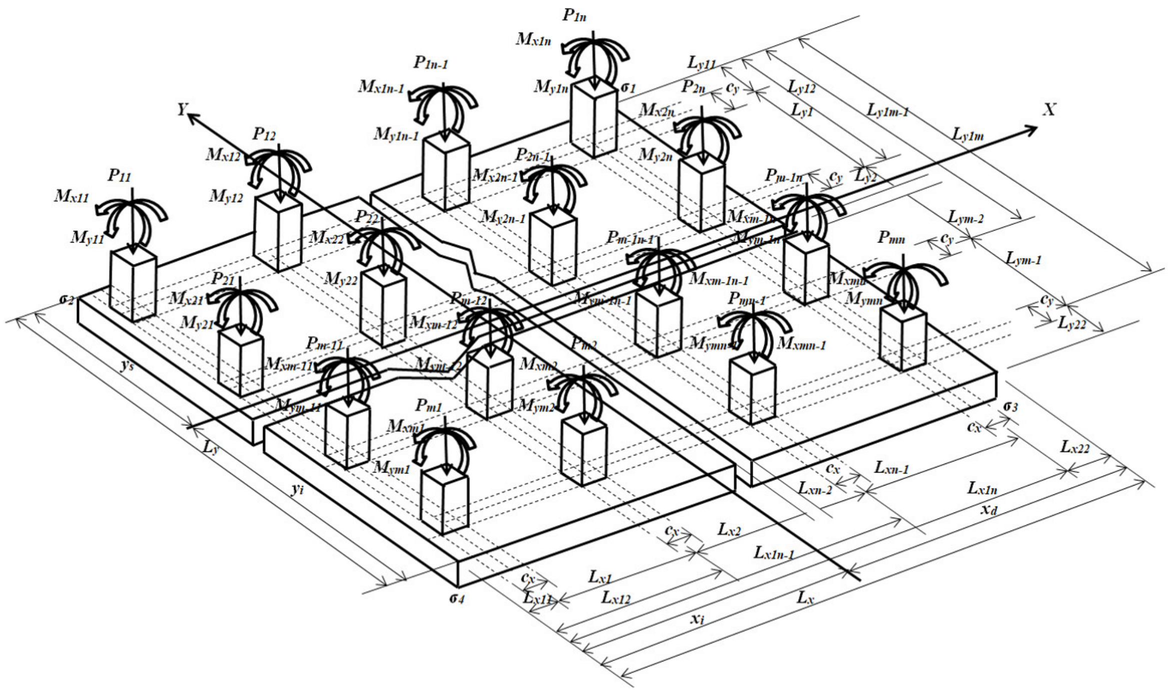

FIGURE 3. Rectangular foundation slab

General equation at any foundation point subjected to biaxial bending is

$$\sigma = \frac{R}{A} + \frac{M_{xT}y}{I_x} + \frac{M_{yT}x}{I_y} \tag{1}$$

where σ is the soil pressure on the foundation, A is the contact surface of the foundation with the soil, R is the resultant force from all the forces applied to the foundation, M_{xT} is the resultant moment applied on the X axis, M_{yT} is the resultant moment applied on the Y axis, x is the coordinate on the X axis to the fiber under study, y is the coordinate on the Y axis to the fiber under study, I_x is the moment of inertia on the X axis and I_y is the moment of inertia on the Y axis.

The pressure in each corner of the rectangular foundation slab by Equation (1) is obtained:

$$\sigma_1 = \frac{R}{A} + \frac{M_{xT}y_s}{I_x} + \frac{M_{yT}x_d}{I_y} \tag{2}$$

$$\sigma_2 = \frac{R}{A} + \frac{M_{xT}y_s}{I_x} + \frac{M_{yT}\left(x_d - L_x\right)}{I_y} \tag{3}$$

$$\sigma_3 = \frac{R}{A} + \frac{M_{xT}\left(y_s - L_y\right)}{I_x} + \frac{M_{yT}x_d}{I_y} \tag{4}$$

$$\sigma_4 = \frac{R}{A} + \frac{M_{xT}\left(y_s - L_y\right)}{I_x} + \frac{M_{yT}\left(x_d - L_x\right)}{I_y} \tag{5}$$

where R, M_{xT} and M_{yT} are obtained as follows:

$$R = \sum_{i=1}^{n}\sum_{j=1}^{m} P_{(i,j)} \tag{6}$$

$$M_{xT} = \sum_{i=1}^{n}\sum_{j=1}^{m} M_{x(i,j)} + \sum_{i=1}^{n}\sum_{j=1}^{m} P_{(i,j)}\left(y_s - L_{y(i,j)}\right) \tag{7}$$

$$M_{yT} = \sum_{i=1}^{n}\sum_{j=1}^{m} M_{y(i,j)} + \sum_{i=1}^{n}\sum_{j=1}^{m} P_{(i,j)}\left(x_i - L_{x(i,j)}\right) \tag{8}$$

The geometrical properties of the rectangular section are $A = L_xL_y$, $y_s = L_y/2$, $x_d = L_x/2$, $I_x = L_xL_y{}^3/12$, $I_y = L_x{}^3L_y/12$.

Now, the geometric conditions depending on the limited sides can be $L_{x11} \geq c_x/2$, $L_{x11} = c_x/2$, $L_{y11} \geq c_y/2$, $L_{y11} = c_y/2$, $L_{x22} \geq c_x/2$, $L_{x22} = c_x/2$, $L_{y22} \geq c_y/2$, $L_{y22} = c_y/2$.

Objective function of the minimum area "$A_{\min}$" is

$$A_{\min} = L_xL_y \tag{9}$$

Constraint functions are Equations (2) to (8), the geometrical properties of the rectangular section, the geometric conditions depending on the limited sides and $0 \leq \sigma_1, \sigma_2, \sigma_3, \sigma_4 \leq \sigma_{\max}$ (Available allowable capacity of the soil).

Figure 4 presents the flowchart for the minimum area procedure of a foundation slab.

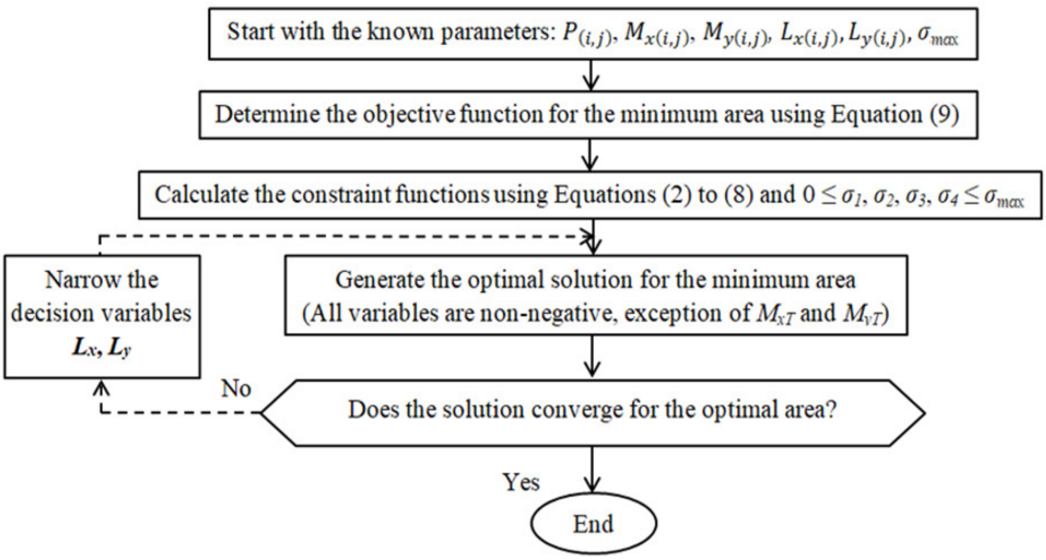

FIGURE 4. Flowchart for the minimum area procedure of a rectangular foundation slab

Figure 5 shows the flowchart for using Maple software for the minimum area of a foundation slab.

3. **Numerical Examples.** Figure 6 presents the distribution of the building columns to obtain the minimum area of the foundation slab under the same loads and moments. This example is shown for the three cases. Case I: Not limited at the ends ($L_{x11} \geq c_x/2$, $L_{y11} \geq$

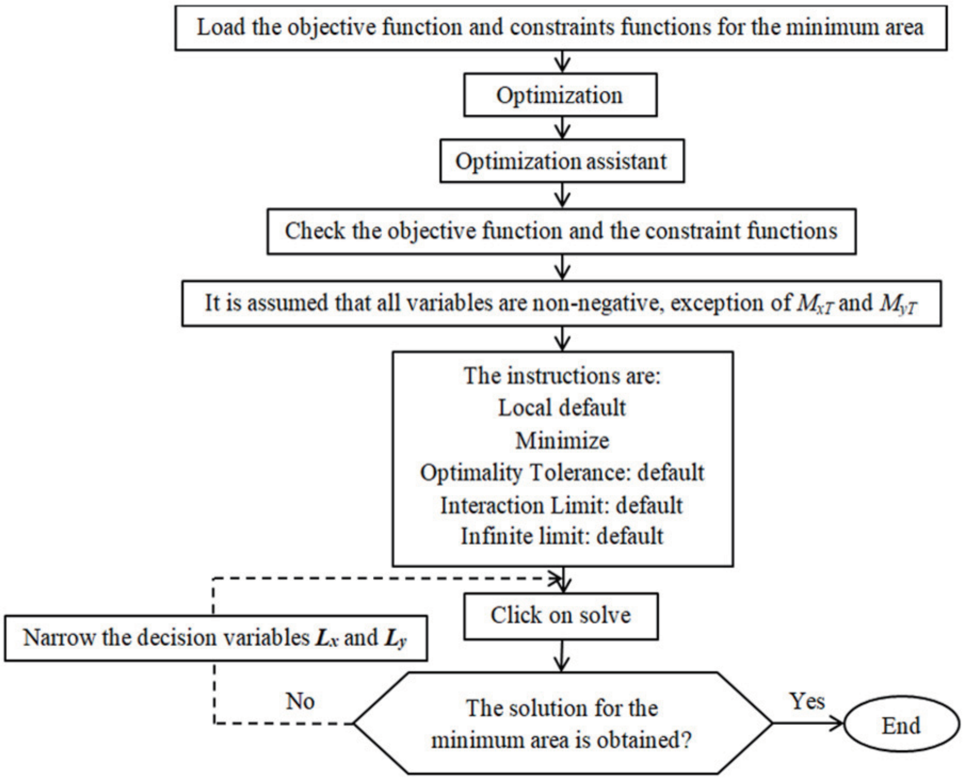

Figure 5. Flowchart for using Maple software on a rectangular foundation slab

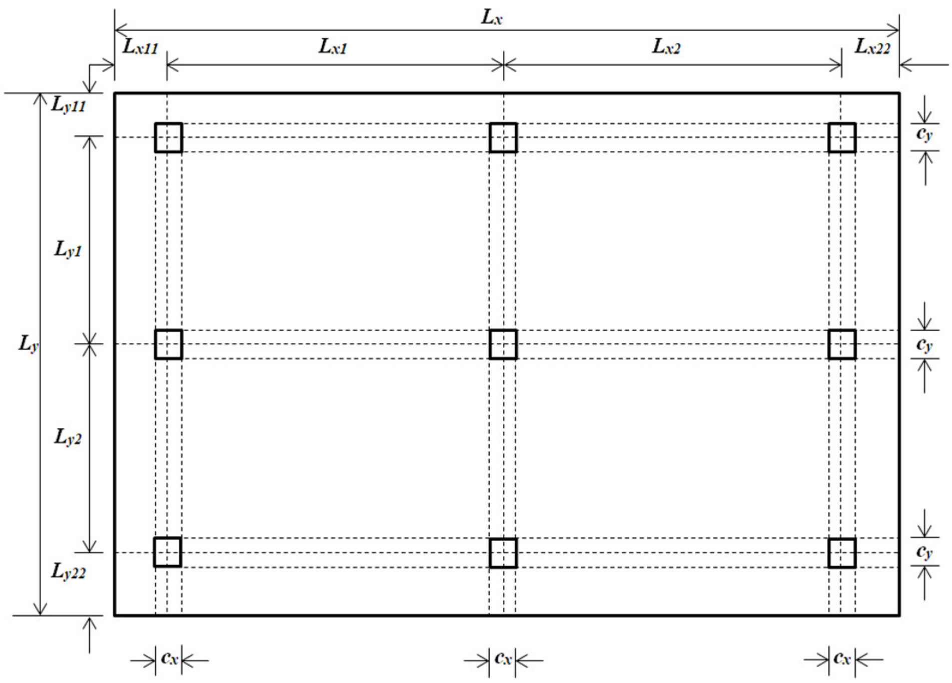

Figure 6. Distribution of the building columns

$c_y/2$, $L_{x22} \geq c_x/2$, $L_{y22} \geq c_y/2$). Case II: Limited in negative and positive X direction and not limited in Y direction ($L_{x11} = c_x/2$, $L_{y11} \geq c_y/2$, $L_{x22} = c_x/2$, $L_{y22} \geq c_y/2$). Case III: Limited in negative and positive Y direction and not limited in X direction ($L_{x11} \geq c_x/2$, $L_{y11} = c_y/2$, $L_{x22} \geq c_x/2$, $L_{y22} = c_y/2$). The data for the three cases are $P_{11} = 1000$ kN, $P_{12} = 2000$ kN, $P_{13} = 1000$ kN, $P_{21} = 2000$ kN, $P_{22} = 4000$ kN, $P_{23} = 2000$ kN,

$P_{31} = 1000$ kN, $P_{32} = 2000$ kN, $P_{33} = 1000$ kN, $M_{x11} = 100$ kN-m, $M_{x12} = 200$ kN-m, $M_{x13} = 100$ kN-m, $M_{x21} = 200$ kN-m, $M_{x22} = 0$ kN-m, $M_{x23} = 200$ kN-m, $M_{x31} = 100$ kN-m, $M_{x32} = 200$ kN-m, $M_{x33} = 100$ kN-m, $M_{y11} = 100$ kN-m, $M_{y12} = 200$ kN-m, $M_{y13} = 100$ kN-m, $M_{y21} = 200$ kN-m, $M_{y22} = 0$ kN-m, $M_{y23} = 200$ kN-m, $M_{y31} = 100$ kN-m, $M_{y32} = 0$ kN-m, $M_{y33} = 100$ kN-m, $c_x = 0.50$ m, $c_y = 0.50$ m, $L_{x1} = 6.00$ m, $L_{x2} = 6.00$ m, $L_{y1} = 6.00$ m, $L_{y2} = 6.00$ m, $\sigma_{\max} = 200$, 150, 100, 50 kN/m^2.

The equations for this example are:

Objective function "$A_{\min}$" is Equation (9).

Constraint functions are

$$y_s = \frac{L_y}{2} \tag{10}$$

$$x_d = \frac{L_x}{2} \tag{11}$$

$$I_x = \frac{L_x {L_y}^3}{12} \tag{12}$$

$$I_y = \frac{{L_x}^3 L_y}{12} \tag{13}$$

$$\sigma_1 = \frac{R}{A} + \frac{M_{xT} y_s}{I_x} + \frac{M_{yT} x_d}{I_y} \tag{14}$$

$$\sigma_2 = \frac{R}{A} + \frac{M_{xT} y_s}{I_x} + \frac{M_{yT} (x_d - L_x)}{I_y} \tag{15}$$

$$\sigma_3 = \frac{R}{A} + \frac{M_{xT} (y_s - L_y)}{I_x} + \frac{M_{yT} x_d}{I_y} \tag{16}$$

$$\sigma_4 = \frac{R}{A} + \frac{M_{xT} (y_s - L_y)}{I_x} + \frac{M_{yT} (x_d - L_x)}{I_y} \tag{17}$$

$$L_{x12} = L_{x11} + L_{x1} \tag{18}$$

$$L_{x13} = L_{x11} + L_{x1} + L_{x2} \tag{19}$$

$$L_x = L_{x13} + L_{x22} \tag{20}$$

$$L_{y12} = L_{y11} + L_{y1} \tag{21}$$

$$L_{y13} = L_{y11} + L_{y1} + L_{y2} \tag{22}$$

$$L_y = L_{y13} + L_{y22} \tag{23}$$

$$R = P_{11} + P_{12} + P_{13} + P_{21} + P_{22} + P_{23} + P_{31} + P_{32} + P_{33} \tag{24}$$

$$\begin{aligned} M_{xT} = {} & M_{x11} + M_{x12} + M_{x13} + M_{x21} + M_{x22} + M_{x23} + M_{x31} + M_{x32} + M_{x33} \\ & + P_{11} (y_s - L_{y11}) + P_{12} (y_s - L_{y11}) + P_{13} (y_s - L_{y11}) \\ & + P_{21} (y_s - L_{y12}) + P_{22} (y_s - L_{y12}) + P_{23} (y_s - L_{y12}) \\ & + P_{31} (y_s - L_{y13}) + P_{32} (y_s - L_{y13}) + P_{33} (y_s - L_{y13}) \end{aligned} \tag{25}$$

$$\begin{aligned} M_{yT} = {} & M_{y11} + M_{y12} + M_{y13} + M_{y21} + M_{y22} + M_{y23} + M_{y31} + M_{y32} + M_{y33} \\ & + P_{11} (x_d - L_{x11}) + P_{12} (x_d - L_{x12}) + P_{13} (x_d - L_{x13}) \\ & + P_{21} (x_d - L_{x11}) + P_{22} (x_d - L_{x12}) + P_{23} (x_d - L_{x13}) \\ & + P_{31} (x_d - L_{x11}) + P_{32} (x_d - L_{x12}) + P_{33} (x_d - L_{x13}) \end{aligned} \tag{26}$$

$$\left\{ \begin{matrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{matrix} \right\} \leq \sigma_{\max} \tag{27}$$

In addition, the geometric conditions must be added based on the limited sides. For case I: $L_{x11} \geq c_x/2$, $L_{y11} \geq c_y/2$, $L_{x22} \geq c_x/2$, $L_{y22} \geq c_y/2$. For case II: $L_{x11} = c_x/2$, $L_{y11} \geq c_y/2$, $L_{x22} = c_x/2$, $L_{y22} \geq c_y/2$. For case III: $L_{x11} \geq c_x/2$, $L_{y11} = c_y/2$, $L_{x22} \geq c_x/2$, $L_{y22} = c_y/2$.

Assume that all variables are non-negative (exception of M_{xT} and M_{yT}).

Table 1 presents the results of the three cases.

4. **Results.** Table 1 presents the minimum area for the three cases. For $\sigma_{\max} = 200$ and 150 kN/m^2, it is observed that they have the same values in three cases, because it does not reach the admissible available capacity of the soil. For the cases I, II and III $\sigma_{\max} = 100$ and 50 kN/m^2, it is observed that when the admissible available capacity of the soil decreases, the minimum area "$A_{\min}$" increases. For the case I, it is observed that when the admissible available capacity of the soil ($\sigma_{\max} = 100$ and 50 kN/m^2) decreases, the "L_x" and "L_y" increase. For the case II, it is observed that when the admissible available capacity of the soil ($\sigma_{\max} = 100$ and 50 kN/m^2) decreases, the "L_y" increases and "L_x" is the same (because L_x is limited in positive and negative X direction). For the case III, it is observed that when the admissible available capacity of the soil ($\sigma_{\max} = 100$ and 50 kN/m^2) decreases, the "L_x" increases and "L_y" is the same (because L_y is limited in positive and negative Y direction).

This model is applied for rectangular isolated footings with the column located in any part of the footing and for rectangular combined footings.

Table 2 shows the minimum area for rectangular isolated footings with the column located in anywhere on the footing. Case 1: Column located at the center of the footing ($e_x = 0$ and $e_y = 0$). Case 2: Column located at the boundary of the footing in the Y direction ($e_x = 0$ and $e_y = L_y/2 - c_y/2$). Case 3: Column located at the boundary of the footing in the X direction ($e_x = L_x/2 - c_x/2$ and $e_y = 0$). Case 4: Column located at the corner of the footing ($e_x = L_x/2 - c_x/2$, and $e_y = L_y/2 - c_y/2$). The general data for all footings are $c_x = c_y = 0.40$ m, $\sigma_{\max} = 180$ kN/m^2.

The equations for rectangular isolated footings are:

Objective function "$A_{\min}$" is Equation (9).

Constraint functions are Equations (10) to (17), (27) and the following equations:

$$L_x = L_{x11} + L_{x22} \tag{28}$$

$$L_y = L_{y11} + L_{y22} \tag{29}$$

$$R = P_{11} \tag{30}$$

$$M_{xT} = M_{x11} + P_{11}(y_s - L_{y11}) \tag{31}$$

$$M_{yT} = M_{y11} + P_{11}(x_d - L_{x11}) \tag{32}$$

where L_{x11} is the distance from the center of the column to the free end of the footing in the positive X direction, L_{x22} is the distance from the center of the column to the free end of the footing in the negative X direction, L_{y11} is the distance from the center of the column to the free end of the footing in the positive Y direction, L_{y22} is the distance from the center of the column to the free end of the footing in the negative Y direction.

Table 3 shows the minimum area for rectangular combined footings with two and three columns. Case 1: Rectangular combined footing that supports two columns and is not limited in the X direction. Case 2: Rectangular combined footing that supports two columns and is limited in the positive and negative X direction. Case 3: Rectangular combined footing that supports three columns and is not limited in the X direction. Case 4: Rectangular combined footing that supports three columns and is limited in the positive and

TABLE 1. Example for the three cases to obtain the minimum area of the foundation slab

Concept	**Case I**				**Case II**				**Case III**			
$\sigma_{\max}$	200	150	100	50	200	150	100	50	200	150	100	50
I_x	2034.51	2034.51	2184.94	8651.20	2034.51	2034.51	2423.13	19275.51	2034.51	2034.51	2172.85	4329.43
I_y	2034.51	2034.51	2150.80	8651.20	2034.51	2034.51	2156.58	4305.01	2034.51	2034.51	2478.41	19605.31
M_{xT}	1200	1200	0	0	1200	1200	0	0	1200	1200	1200	1200
M_{yT}	1000	1000	−200	−200	1000	1000	1000	1000	1000	1000	−200	200
R	16000	16000	16000	16000	16000	16000	16000	16000	16000	16000	16000	16000
L_x	12.50	12.50	12.65	17.89	12.50	12.50	12.50	12.50	12.50	12.50	13.35	26.60
L_y	12.50	12.50	12.75	17.89	12.50	12.50	13.25	26.45	12.50	12.50	12.50	12.50
y_s	6.25	6.25	6.38	8.98	6.25	6.25	6.62	13.22	6.25	6.25	6.25	6.25
x_d	6.25	6.25	6.32	8.98	6.25	6.25	6.25	6.25	6.25	6.25	6.68	13.30
L_{x11}	0.25	0.25	0.40	3.05	0.25	0.25	0.25	0.25	0.25	0.25	0.75	7.35
L_{x12}	6.25	6.25	6.40	9.05	6.25	6.25	6.25	6.25	6.25	6.25	6.75	13.35
L_{x13}	12.25	12.25	12.40	15.05	12.25	12.25	12.25	12.25	12.25	12.25	12.75	19.35
L_{x22}	0.25	0.25	0.25	2.90	0.25	0.25	0.25	0.25	0.25	0.25	0.60	7.25
L_{y11}	0.25	0.25	0.45	3.05	0.25	0.25	0.70	7.30	0.25	0.25	0.25	0.25
L_{y12}	6.25	6.25	6.45	9.05	6.25	6.25	6.70	13.30	6.25	6.25	6.25	6.25
L_{y13}	12.25	12.25	12.45	15.05	12.25	12.25	12.70	19.30	12.25	12.25	12.25	12.25
L_{y22}	0.25	0.25	0.30	2.90	0.25	0.25	0.55	7.15	0.25	0.25	0.25	0.25
σ_1	109.16	109.16	98.61	49.45	109.16	109.16	99.50	49.84	109.16	109.16	98.79	49.99
σ_2	103.01	103.01	99.79	49.87	103.01	103.01	93.71	46.94	103.01	103.01	98.87	49.72
σ_3	101.79	101.79	98.61	49.45	101.79	101.79	99.50	49.84	101.79	101.79	91.89	46.52
σ_4	95.64	95.64	99.79	49.87	95.64	95.64	93.71	46.94	95.64	95.64	92.97	46.25
$A_{\min}$	156.25	156.25	161.29	322.20	156.25	156.25	165.62	330.62	156.25	156.25	166.88	332.50

TABLE 2. Rectangular isolated footings with different locations of the column on the footing

Concept	Case I				Case II				Case III				Case IV			
Input data																
P	1000	850	750	600	1025	880	750	600	1025	880	750	600	750	605	450	300
M_x	225	225	225	225	−225	−225	−225	−225	150	150	150	150	−750	−750	−750	−750
M_y	150	150	150	150	150	150	150	150	−225	−225	−225	−225	−600	−600	−600	−600
e_x	0				0				$L_x/2 - c_x/2$				$L_x/2 - c_x/2$			
e_y	0				$L_y/2 - c_y/2$				0				$L_y/2 - c_y/2$			
$\sigma_{\max}$	180															
Output data																
I_x	11.66	9.93	9.33	22.78	0.75	0.58	0.43	0.51	61.77	27.98	11.38	6.13	2.16	3.15	7.24	31.84
I_y	5.25	4.47	4.15	10.12	61.77	27.98	11.38	6.13	0.75	0.58	0.43	0.51	1.57	2.15	4.96	21.26
M_{xT}	225	225	225	225	82.50	39.00	0	0	150	150	150	150	−18.75	−84.50	−120.00	−112.50
M_{yT}	150	150	150	150	150	150	150	150	82.50	39.00	0	0	9.99	−70.62	−93.75	−90.00
R	1000	850	750	600	1025	880	750	600	1025	880	750	600	750	605	450	300
L_x	2.55	2.45	2.40	3.00	9.05	6.95	5.15	4.00	1.00	1.00	1.00	1.15	2.00	2.15	2.65	3.80
L_y	3.80	3.65	3.60	4.50	1.00	1.00	1.00	1.15	9.05	6.95	5.15	4.00	2.35	2.60	3.20	4.65
y_s	1.90	1.82	1.80	2.25	0.50	0.50	0.50	0.58	4.52	3.48	2.58	2.00	1.18	1.30	1.60	2.32
x_d	1.275	1.22	1.20	1.50	4.52	3.48	2.58	2.00	0.50	0.50	0.50	0.58	1.00	1.08	1.32	1.90
L_{x11}	1.275	1.22	1.20	1.50	4.52	3.48	2.58	2.00	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
L_{x22}	1.275	1.22	1.20	1.50	4.52	3.48	2.58	2.00	0.80	0.80	0.80	0.95	1.80	1.95	2.45	3.60
L_{y11}	1.90	1.82	1.80	2.25	0.20	0.20	0.20	0.20	4.52	3.48	2.58	2.00	0.20	0.20	0.20	0.20
L_{y22}	1.90	1.82	1.80	2.25	0.80	0.80	0.80	0.95	4.52	3.48	2.58	2.00	2.15	2.40	3.00	4.45
σ_1	176.29	177.49	173.61	88.89	178.94	178.92	179.56	179.35	178.94	178.92	179.56	179.35	149.39	38.09	1.50	0.72
σ_2	103.44	95.33	86.81	44.44	156.97	141.66	111.70	81.52	69.55	111.58	179.56	179.35	149.39	108.60	51.56	16.80
σ_3	102.96	94.77	86.81	44.44	69.55	111.58	179.56	179.35	156.97	141.66	111.70	81.52	169.76	107.85	54.56	17.15
σ_4	30.11	12.61	0	0	47.57	74.32	111.70	81.52	47.57	74.32	111.70	81.52	169.76	178.37	104.63	33.24
$A_{\min}$	9.69	8.94	8.64	13.50	9.05	6.95	5.15	4.60	9.05	6.95	5.15	4.60	4.70	5.59	8.48	17.67

where e_x is the distance from the center of the footing to the center of the column in the X direction, and e_y is the distance from the center of the footing to the center of the column in the Y direction.

TABLE 3. Rectangular combined footings for two and three columns on the footing

Concept	Case I				Case II				Case III				Case IV			
Input data																
P_1	600	600	600	600	600	600	600	600	600	600	600	600	600	600	600	600
M_{x1}	300	200	100	0	300	200	100	0	300	200	100	0	300	200	100	0
M_{y1}	150	100	50	0	150	100	50	0	150	100	50	0	150	100	50	0
P_2	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200	1200
M_{x2}	600	400	200	0	600	400	200	0	600	400	200	0	600	400	200	0
M_{y2}	300	200	100	0	300	200	100	0	300	200	100	0	300	200	100	0
P_3									1200	1200	1200	1200	1200	1200	1200	1200
M_{x3}									600	400	200	0	600	400	200	0
M_{y3}									300	200	100	0	300	200	100	0
L_{x1}	7.00	6.00	5.00	4.00	7.00	6.00	5.00	4.00	7.00	6.00	5.00	4.00	7.00	6.00	5.00	4.00
L_{x2}									7.00	6.00	5.00	4.00	7.00	6.00	5.00	4.00
$\sigma_{\max}$	250															
Output data																
I_x	20.78	5.38	2.67	0.74	983.67	407.61	97.20	11.18	37.58	9.64	4.14	1.32	229.32	66.13	10.16	7.92
I_y	196.80	87.48	45.68	25.18	394.56	199.73	78.73	22.18	1164.37	504.61	243.44	108.96	1433.27	635.54	212.97	110.88
M_{xT}	900	600	300	0	900	600	300	0	1500	1000	500	0	1500	1000	500	0
M_{yT}	0	0	0	0	−1650	−1500	−1350	−1200	0	0	0	0	−3450	−3100	−2750	−2400
R	1800	1800	1800	1800	1800	1800	1800	1800	3000	3000	3000	3000	3000	3000	3000	3000
L_x	9.23	8.07	6.90	6.48	7.40	6.40	5.40	4.40	16.70	14.47	12.23	10.44	14.40	12.40	10.40	8.40
L_y	3.00	2.00	1.67	1.11	11.68	9.14	6.00	3.12	3.00	2.00	1.60	1.15	5.76	4.00	2.27	2.24
y_s	1.50	1.00	0.83	0.56	5.84	4.57	3.00	1.56	1.50	1.00	0.80	0.57	2.88	2.00	1.14	1.12
x_d	4.62	4.03	3.45	3.24	3.70	3.20	2.70	2.20	8.35	7.23	6.12	5.22	7.20	6.20	5.20	4.20
L_{x11}	0.20	0.20	0.20	0.57	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.42	0.20	0.20	0.20	0.20
L_{x12}	7.20	6.20	5.20	4.57	7.20	6.20	5.20	4.20	7.20	6.20	5.20	4.42	7.20	6.20	5.20	4.20
L_{x13}									14.20	12.20	10.20	8.42	14.20	12.20	10.20	8.20
L_{x22}	2.33	1.87	1.70	1.91	0.20	0.20	0.20	0.20	2.50	2.27	2.03	2.02	0.20	0.20	0.20	0.20
L_{y11}	1.50	1.00	0.83	0.56	5.84	4.57	3.00	1.56	1.50	1.00	0.80	0.57	2.88	2.00	1.14	1.12
L_{y22}	1.50	1.00	0.83	0.56	5.84	4.57	3.00	1.56	1.50	1.00	0.80	0.57	2.88	2.00	1.14	1.12
σ_1	129.96	223.14	250.00	250.00	10.69	13.46	18.52	11.90	119.76	207.37	250.00	250.00	37.68	60.48	115.71	68.18
σ_2	129.96	223.14	250.00	250.00	41.64	61.52	111.11	250.00	119.76	207.37	250.00	250.00	72.34	120.97	250.00	250.00
σ_3	0	0	62.65	250.00	0	0	0	11.90	0	0	57.37	250.00	0	0	3.94	68.18
σ_4	0	0	62.65	250.00	30.95	48.07	92.59	250.00	0	0	57.37	250.00	34.66	60.48	138.23	250.00
$A_{\min}$	27.00	16.13	11.51	7.20	86.46	58.51	32.40	13.75	50.10	28.93	19.52	12.00	82.94	49.60	23.63	18.86

negative X direction. The general data for all footings are $c_x = c_y = 0.40$ m, $\sigma_{\max} = 250$ kN/m^2.

The equations for rectangular combined footings with two columns are:

Objective function "$A_{\min}$" is Equation (9).

Constraint functions are Equations (10) to (17), (27) and the following equations:

$$L_x = L_{x12} + L_{x22} \quad (33)$$

$$L_y = L_{y11} + L_{y22} \quad (34)$$

$$R = P_{11} + P_{12} \quad (35)$$

$$M_{xT} = M_{x11} + M_{x12} \quad (36)$$

$$M_{yT} = M_{y11} + M_{y12} + P_{11}(x_d - L_{x11}) + P_{12}(x_d - L_{x12}) \quad (37)$$

The equations for rectangular combined footings with three columns are:

Objective function "$A_{\min}$" is Equation (9).

Constraint functions are Equations (10) to (17), (27) and the following equations:

$$L_x = L_{x13} + L_{x22} \quad (38)$$

$$L_y = L_{y11} + L_{y22} \quad (39)$$

$$R = P_{11} + P_{12} + P_{13} \quad (40)$$

$$M_{xT} = M_{x11} + M_{x12} \quad (41)$$

$$M_{yT} = M_{y11} + M_{y12} + M_{y13} + P_{11}(x_d - L_{x11}) + P_{12}(x_d - L_{x12}) + P_{13}(x_d - L_{x13}) \quad (42)$$

Table 2 shows the results for rectangular isolated footings with different column locations, and these results can be verified in the document presented by Luévanos-Rojas [35]. When the axial load P decreases: Case I; $A_{\min}$ decreases until $P = 750$ kN and then increases. Case II; $A_{\min}$ decreases. Case III; $A_{\min}$ decreases. Case IV; $A_{\min}$ increases.

Table 3 presents the results for rectangular combined footings for two and three columns on the footing; these results can be verified in the paper shown by García-Galván et al. [36]. When the moments (M_{x1}, M_{y1}, M_{x2} and M_{y2}) and the separation between the two columns L_{x1} decrease: Case I; $A_{\min}$ decreases. Case II; $A_{\min}$ decreases. When the moments (M_{x1}, M_{y1}, M_{x2}, M_{y2}, M_{x3} and M_{y3}) and the separation between the three columns L_{x1}, L_{x2} decrease: Case III; $A_{\min}$ decreases. Case IV; $A_{\min}$ decreases.

5. **Conclusions.** This work presents a model to find the minimum area of rectangular foundation slabs subjected to biaxial bending in each column supported on elastic soils, and assuming a linear pressure distribution of the soil.

The optimal model is developed from a mathematical approach based on minimum area criterion. The independent variables or known parameters are all the loads and moments that act on the foundation slab, the available allowable load capacity of the soil and the limitations of the terrain where the building will be located. The dependent or unknown variables are area and sides of the foundation slab, and the pressure acting on each corner of the foundation slab.

The main findings are

1) The normal practice in structural engineering is to use the trial and error procedure to estimate the sides and area of the foundation under biaxial bending supported on elastic soils.

2) Some authors present the complete design of foundation slabs based on the known ground contact areas.

3) In this work, the simplified and precise equations of minimum area for foundation slabs are presented.

4) The proposed model can be used to verify the available allowable load capacity of the soil considering the objective function to maximize "$\sigma_{\max}$", and the same constraint functions and the known parameters "L_x" and "L_y".

5) The proposed model can be applied to rectangular isolated footings with a column located anywhere of the footing (see Table 2).

6) The proposed model can be applied to rectangular combined footings that support two and three columns on the footing (see Table 3).

Suggestions for future research are

1) Minimum area for T-shaped foundation slabs.

2) Complete design for rectangular foundation slabs (Thickness and reinforcing steel areas of the slab).

3) Minimum area for rectangular foundation slabs considering that contact surface works partially to compression.

REFERENCES

[1] J. E. Bowles, *Foundation Analysis and Design*, McGraw-Hill, New York, USA, 2001.

[2] A. Luévanos-Rojas, A mathematical model for the dimensioning of circular footings, *Far East Journal of Mathematical Sciences (FJMS)*, vol.71, no.2, pp.357-367, 2012.

[3] A. Luévanos-Rojas, A comparative study for dimensioning of footings with respect to the contact surface on soil, *International Journal of Innovative Computing, Information and Control*, vol.10, no.4, pp.1313-1326, 2014.

[4] S. Soto-García, A. Luévanos-Rojas, J. D. Barquero-Cabrero, S. López-Chavarría, M. Medina-Elizondo, O. M. Farias-Montemayor and C. Martínez-Aguilar, A new model for the contact surface with soil of circular isolated footings considering that the contact surface works partially under compression, *International Journal of Innovative Computing, Information and Control*, vol.18, no.4, pp.1103-1116, 2022.

[5] A. Luévanos-Rojas, A mathematical model for dimensioning of footings square, *International Review of Civil Engineering (IRECE)*, vol.3, no.4, pp.346-350, 2012.

[6] S. Lopez-Chavarria, A. Luévanos-Rojas and M. Medina-Elizondo, A mathematical model for dimensioning of square isolated footings using optimization techniques: General case, *International Journal of Innovative Computing, Information and Control*, vol.13, no.1, pp.67-74, 2017.

[7] A. Luévanos-Rojas, A mathematical model for dimensioning of footings rectangular, *ICIC Express Letters, Part B: Applications*, vol.4, no.2, pp.269-274, 2013.

[8] H. M. Algin, Practical formula for dimensioning a rectangular footing, *Engineering Structures*, vol.29, no.6, pp.1128-1134, 2007.

[9] I. Aydogdu, New iterative method to calculate base stress of footings under biaxial bending, *International Journal of Engineering Applied Sciences*, vol.8, no.4, pp.40-48, 2016.

[10] W. H. Highter and J. C. Anders, Dimensioning footings subjected to eccentric loads, *Journal of Geotechnical Engineering*, vol.111, no.5, pp.659-665, 1985.

[11] V. B. Vela-Moreno, A. Luévanos-Rojas, S. López-Chavarría, M. Medina-Elizondo, R. Sandoval-Rivas and C. Martinez-Aguilar, Optimal area for rectangular isolated footings considering that contact surface works partially to compression, *Structural Engineering and Mechanics*, vol.84, no.4, pp.561-573, 2022.

[12] A. Luévanos-Rojas, A new mathematical model for dimensioning of the boundary trapezoidal combined footings, *International Journal of Innovative Computing, Information and Control*, vol.11, no.4, pp.1269-1279, 2015.

[13] A. Luévanos-Rojas, A mathematical model for the dimensioning of combined footings of rectangular shape, *Revista Técnica de la Facultad de Ingeniería Universidad del Zulia*, vol.39, no.1, pp.3-9, 2016.

[14] P. Montes-Paramo, A. Luévanos-Rojas, S. López-Chavarría, M. Medina-Elizondo and R. Sandoval-Rivas, Optimal area for rectangular combined footings assuming that the contact surface with the soil works partially to compression, *Ingeniería Investigación y Tecnología*, vol.24, no.2, pp.1-15, 2023.

[15] S. López-Chavarría, A. Luévanos-Rojas and M. Medina-Elizondo, Optimal dimensioning for the corner combined footings, *Advances in Computational Design*, vol.2, no.2, pp.169-183, 2017.

[16] G. Aguilera-Mancilla, A. Luévanos-Rojas, S. López-Chavarría and M. Medina-Elizondo, Modeling for the strap combined footings part I: Optimal dimensioning, *Steel and Composite Structures*, vol.30, no.2, pp.97-108, 2019.

[17] M. A. Moreno-Hernandez, A. Luévanos-Rojas, S. López-Chavarría and M. Medina-Elizondo, Mathematical modeling for corner strap combined footings resting on the ground: Part 1, *Computación y Sistemas*, vol.26, no.3, pp.1259-1272, 2022.

[18] A. Luévanos-Rojas, S. López-Chavarría and M. Medina-Elizondo, A new model for T-shaped combined footings part I: Optimal dimensioning, *Geomechanics Engineering*, vol.14, no.1, pp.51-60, 2018.

[19] L. Allievi, S. Ferrero, A. Mussi, R. Persio and F. Petrella, Structural and geotechnical design of a piled raft for a tall building founded on granular soil, *Proc. of the 18th Int. Conf. on Soil Mechanics and Geotechnical Engineering*, Paris, France, pp.2659-2661, 2013.

[20] R. Sundaram, S. Gupta and S. Gupa, Foundations for tall buildings on alluvial deposits-geotechnical aspects, in *Frontiers in Geotechnical Engineering. Developments in Geotechnical Engineering*, M. G. Latha (ed.), Singapore, Springer, 2019.

[21] S. A. Pechorskaya, Raft foundations design charts: Development and applications, *International Scientific and Practical Conference Engineering Systems, IOP Conf. Series: Materials Science and Engineering*, vol.675, Article ID 012014, 2019.

[22] S. Shakir, S. Mudassir, S. Abdullah, Q. Najesh, M. Pathan, S. Sufyan and O. Kharodia, Analysis of raft & pile raft foundation using safe software, *International Journal of Engineering Research & Technology (IJERT)*, vol.9, no.7, pp.57-61, 2020.

[23] I. Jamil, I. Ahmad, S. A. Khan, W. Ullah, M. Amjad, B. J. Khan and H. Nasir, Analysis and design of piled raft foundation taking into account interaction factors, *Advances in Civil Engineering*, vol.2022, Article ID 1334136, 2022.

[24] H. G. Poulos, An approximate numerical analysis of pile-raft interaction, *International Journal for Numerical and Analytical Methods in Geomechanics*, vol.18, no.2, pp.73-92, 1994.

[25] D. D. C. Nguyen, S.-B. Jo and D.-S. Kim, Design method of piled-raft foundations under vertical load considering interaction effects, *Computers and Geotechnics*, vol.47, pp.16-27, 2013.

[26] S. M. Sharma, M. G. Vanza and D. D. Mehta, Comparison of raft foundation and beam & slab raft foundation for high rise building, *International Journal of Engineering Development and Research (IJEDR)*, vol.2, no.1, pp.571-575, 2014.

[27] D. Ojha and R. K. Srivastava, Structural design of raft foundation for 30 story high rise building – A case study in Lucknow, Uttar Pradesh region, India, *Journal of University of Shanghai for Science and Technology*, vol.24, no.8, pp.130-151, 2022.

[28] Z. D. S. Punekar, M. H. Kolhar, A. Algur and M. K. Kushappa, Analysis and design of raft foundation, *International Journal of Research in Engineering and Technology (IJRET)*, vol.6, no.10, pp.14-19, 2017.

[29] H. G. Poulos, Piled raft foundations: Design and applications, *Géotechnique*, vol.51, no.2, pp.95-113, 2001.

[30] S. Azhar, A. Patidar and S. Jaurker, Parametric study of piled raft foundation for high rise buildings, *International Journal of Engineering Research & Technology (IJERT)*, vol.9, no.12, pp.548-555, 2020.

[31] P. Deb and S. K. Pal, Load-settlement and load-sharing behavior of a piled raft foundation resting on layered soils, *Acta Geotechnica Slovenica*, vol.17, no.1, pp.71-86, 2020.

[32] P. Deb and S. K. Pal, Structural and geotechnical aspects of piled raft foundation through numerical analysis, *Marine Georesources & Geotechnology*, vol.40, no.7, pp.823-846, 2021.

[33] P. Deb and S. K. Pal, Interaction behavior and load sharing pattern of piled raft using nonlinear regression and LM algorithm-based artificial neural network, *Frontiers of Structural and Civil Engineering*, vol.14, pp.1181-1198, 2021.

[34] P. Kannaujiya and V. K. Srivastava, Behavior of different configuration of piled raft foundation for a high-rise building by using FEM, *IOP Conference Series: Materials Science and Engineering*, vol.1236, Article ID 012006, 2021.

[35] A. Luévanos-Rojas, Minimum cost design for rectangular isolated footings taking into account that the column is located in any part of the footing, *Buildings*, vol.13, no.9, pp.1-16, 2023.

[36] M. García-Galván, A. Luévanos-Rojas, S. López-Chavarría, M. Medina-Elizondo and J. B. Rivera-Mendoza, A general model for rectangular footings part I: Optimal surface, *Dyna*, vol.89, no.221, pp.132-141, 2022.

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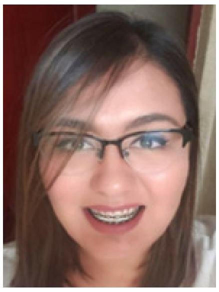

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