International Journal of Innovative
Computing, Information and Control
Volume **20**, Number **6**, December **2024**

ICIC International ©2024 ISSN 1349-4198
pp. 1819–1835

CONTROL SYSTEM TO ATTENUATE PERIODIC DISTURBANCE WITHOUT USING REPETITIVE CONTROL FOR NON-MINIMUM PHASE PLANT

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Received February 2024; revised June 2024

Abstract. *In this paper, we propose a control system to attenuate periodic disturbance using a disturbance observer without using repetitive control for non-minimum phase plants. It is well known that repetitive control is effective for tracking periodic reference inputs and suppressing periodic disturbances. However, repetitive control is not necessary for a non-periodic reference input. In addition, repetitive control has a steady-state error when the reference input is ramp input. Therefore, Yamada et al. proposed a design method of a disturbance observer control system for non-periodic reference. However, the method of Yamada et al. cannot be applied to the non-minimum phase plant. In this paper, in order to overcome this problem, for non-minimum phase plants, we propose a new design method for control system to attenuate periodic disturbance using the disturbance observer for the periodic disturbances. From the result of this paper, we can design a control system to attenuate the periodic disturbance effectively and for the output to follow the non-periodic reference input without steady-state error and the transfer function from the reference input to the output has a finite number of poles.*

Keywords: Disturbance observer, Non-periodic reference input, Periodic disturbance, Non-minimum phase plant

1. **Introduction.** In this paper, we propose a design method for the control system to attenuate the periodic disturbance and for the output to follow the non-periodic reference

DOI: 10.24507/ijicic.20.06.1819

input using a disturbance observer without using repetitive control for non-minimum phase plant.

In order to attenuate the periodic disturbance, the repetitive control is a well-known method [1]. When the reference input is the periodic input, using the repetitive control, the output follows the reference input without steady-state error. When the disturbance is a periodic disturbance, repetitive control is used to suppress the disturbance. However, when the reference input is non-periodic input, repetitive control is not necessary [2]. In addition, even if the reference input is a non-periodic input, the output follows the reference input without steady-state error only when the reference input is a step input. If the reference input is a ramp input, repetitive control has a steady-state error. In order to overcome this problem, a robust higher-order repetitive control [3] is proposed. However, this remains the difficulty to reduce the order of controller.

There is a possibility to overcome the problem to design control systems to follow non-periodic reference input and to attenuate periodic disturbance without using repetitive control, if we adopt the disturbance observer, which is effective to attenuate periodic disturbance effectively.

A disturbance observer is used to estimate the disturbances in the factory plant [8], and various papers are proposed [4, 5, 6, 7]. In addition, many papers are proposing disturbance observer that utilize these papers [9, 10, 11, 12, 13], and the applications of disturbance observers have been proposed in many control systems such as a motion-control field [14, 15, 16]. A disturbance observer is used in the motion control field to cancel the disturbance or to make the closed-loop system robustly stable [17, 18, 20]. Typically, a disturbance observer includes a disturbance signal generator and an observer, and the disturbance that is normally considered as a step disturbance is estimated by the observer. Since the disturbance observer is simple to understand the structure, it is used in many cases [17, 18, 19].

Mita et al. pointed out that disturbance observers are not the only alternative design of complete controllers [20]. Extended H_∞ control in [20] has therefore been proposed as an effective motion control method that cancels disturbances. From another point of view, Kobayashi et al. considered an observer design method for obtaining phase compensation based on disturbance observers [21].

Another important control problem is the parameterization problem which is the problem of finding all stable controllers for the plant [22]. If the parameterization of all disturbance observers for any disturbances could be obtained, we could express results from previous studies of disturbance observers in a uniform manner, in addition, disturbance observers for any disturbances could be designed systematically, and Yamada et al. examined the parameterization of all disturbance observers [23]. There exists another study of motion control realization based on the disturbance observer and the Kalman filter [14]. This study realizes high robustness against disturbance, parameter variations, effective noise suppression, and wide band force sensing by using a disturbance observer and Kalman filter. In this way, the research on disturbance observers has been progressed.

Recently, the parameterization of all disturbance observers for the periodic disturbance was clarified [24]. Using this parameterization in [24], we have the possibility to design a control system to follow non-periodic reference input and to attenuate periodic disturbance without using repetitive control. A design method for non-periodic reference input is proposed by Yamada et al. [25]. Yamada et al. proposed a design method that uses a disturbance observer for non-periodic input without using repetitive control [25]. However, the method in [25] cannot be applied to non-minimum phase plants. There exists a non-minimum phase plant. In addition, the problem of designing a control system to attenuate the periodic disturbance and for the output to follow the non-periodic reference input is important.

In this paper, in order to overcome this problem, we propose a design method for the control system to attenuate the periodic disturbance effectively and for the output to follow the non-periodic reference input without steady-state error using a disturbance observer for a non-minimum phase plant. This paper is organized as follows. In Section 2, we show a control system to attenuate the periodic disturbance effectively and for the output to follow the non-periodic reference input without steady-state error using disturbance observer for non-minimum phase plant and explain the problem considered in this paper. In Section 3, we summarize the control characteristic of the control system and clarify the problem in detail. In Section 4, we propose a design method for the control system. In Section 5, a design procedure of the control system is presented. In Section 6, we provide a numerical example to illustrate the effectiveness of the proposed method. Section 7 gives concluding remarks.

2. **Problem Formulation.** Consider the plant written by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + d(t) \end{cases}, \tag{1}$$

where $x(t) \in R^n$ is the state variable, $u(t) \in R$ is the control input, $y(t) \in R$ is the output, $r(t) \in R$ is the non-periodic reference input, $d(t) \in R$ is the periodic output disturbance with period $T > 0$ satisfying

$$d(t+T) = d(t) \quad (\forall t \geq 0), \tag{2}$$

$A \in R^{n\times n}$, $B \in R^n$ and $C \in R^{1\times n}$. It is assumed that (A, B) is stabilizable, (C, A) is detectable, A has no eigenvalue on the imaginary axis,

$$\det \begin{bmatrix} A - sI & B \\ C & 0 \end{bmatrix} = 0 \tag{3}$$

has roots in the closed right half plane. In addition, we assume that $u(t)$ and $y(t)$ are available, but $d(t)$ is unavailable. The transfer function in Equation (1) is denoted by

$$y(s) = G(s)u(s) + d(s), \tag{4}$$

where

$$G(s) = C(sI - A)^{-1}B \in R(s), \tag{5}$$

$y(s) = \mathcal{L}\{y(t)\}$, $u(s) = \mathcal{L}\{u(t)\}$ and $d(s) = \mathcal{L}\{d(t)\}$. Note that the assumption in Equation (3) implies that $G(s)$ has zeroes in the closed right half plane, that is, $G(s)$ in Equation (5) is of non-minimum phase.

In order to make a control system such that the output $y(t)$ follows the non-periodic reference input $r(t)$ and the periodic disturbance $d(t)$ is attenuated. Yamada et al. propose a design method for the control system to attenuate the periodic disturbance and to follow the non-periodic reference input for the minimum phase plant [25] is shown in Figure 1. Here, $\tilde{d}(s)$ is the disturbance observer for periodic disturbance [24] written by

$$\tilde{d}(s) = F_1(s)e^{-sT}y(s) + F_2(s)e^{-sT}u(s), \tag{6}$$

$C_1(s) \in R(s)$ is the feedback controller, $C_2(s) \in R(s)$ is the controller for disturbance observer [30] and given by

$$C_2(s) = \frac{C_n(s)}{1 + C_d(s)e^{-sT}}, \tag{7}$$

where $C_n(s) \in RH_\infty$ and $C_d(s) \in RH_\infty$, $F_1(s) \in RH_\infty$, $F_2(s) \in RH_\infty$ and $r(s) = \mathcal{L}\{r(t)\}$. $C_2(s)$ in Equation (7) is used to make the transfer function from $d(s)$ to $y(s)$ have a finite number of poles. According to [24], the parameterization of all $F_1(s)$ and $F_2(s)$ satisfying

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \left(d(t) - \tilde{d}(t)\right) = 0 \tag{8}$$

for any initial state $x(0)$, control input $u(t)$ and periodic output disturbance $d(t)$ is given by

$$F_1(s) = D(s) + Q(s)D(s) \in RH_\infty \tag{9}$$

and

$$F_2(s) = -N(s) - Q(s)N(s) \in RH_\infty, \tag{10}$$

where $D(s) \in RH_\infty$ and $N(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)}, \tag{11}$$

respectively, and $Q(s) \in RH_\infty$ is any function satisfying

$$D(s_i) + Q(s_i)D(s_i) = 1 \quad \forall s_i \ (i = 0, 1, \ldots), \tag{12}$$

$$s_i = j\omega_i, \tag{13}$$

$$\omega_i = \frac{2\pi i}{T} \quad (i = 0, 1, \ldots) \tag{14}$$

and j is the imaginary unit.

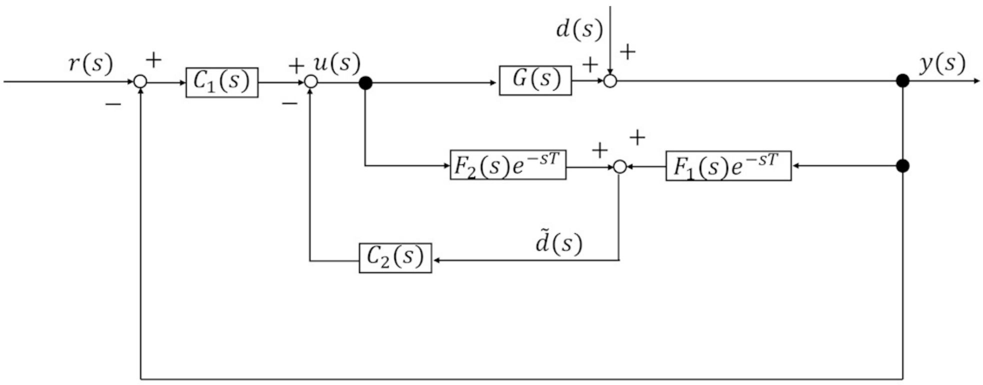

Figure 1. Structure of a disturbance observer system

Using the method proposed in [25], when $G(s)$ is of non-minimum phase, the control system is unstable. The problem considered in this paper is to overcome this problem and propose a design method for the control system in Figure 1 to attenuate the periodic disturbance $d(s)$, for $y(s)$ to follow the reference input $r(s)$ and the transfer function from $d(s)$ to $y(s)$ has the finite number of poles.

3. Control Characteristic.

In this section, when the plant $G(s)$ in Figure 1 is of non-minimum phase, we summarize control characteristics of the control system in Figure 1.

First, we summarize the condition of the transfer function from $d(s)$ to $y(s)$ in Figure 1. According to [25], since the transfer function from $d(s)$ to $y(s)$ in Figure 1 is given by

$$\frac{y(s)}{d(s)} = \frac{1 + (C_d(s) + F_2(s)C_n(s))e^{-sT}}{1 + C_1(s)G(s) + \{(1 + C_1(s)G(s))C_d(s) + (F_1(s)G(s) + F_2(s))C_n(s)\}e^{-sT}}, \tag{15}$$

the transfer function from $d(s)$ to $y(s)$ has finite number of poles if and only if

$$(1 + C_1(s)G(s))C_d(s) + (F_1(s)G(s) + F_2(s))C_n(s) = 0 \tag{16}$$

is satisfied.

Next, the stability of the control system in Figure 1 is summarized. When Equation (16) is satisfied, the control system in Figure 1 is stable if and only if the following expressions are satisfied.

1) $C_1(s)$ stabilizes $G(s)$. That is, all transfer functions $C_1(s)G(s)/(1 + C_1(s)G(s))$, $G(s)/(1 + C_1(s)G(s))$, $C_1(s)/(1 + C_1(s)G(s))$ and $1/(1 + C_1(s)G(s))$ are stable.
2) $C_d(s) \in RH_\infty$ in Equation (7).
3) $C_n(s) \in RH_\infty$ in Equation (7).

From [25], in order for the output $y(s)$ to follow the non-periodic reference input $r(s)$ without steady-state error, from the internal model principle [31], $C_1(s)$ is written by the form in

$$C_1(s) = C_r(s)\bar{C}_1(s), \tag{17}$$

where $C_r(s)$ is the model of the reference input $r(s)$ and $\bar{C}_1(s) \in R(s)$. Therefore, $C_1(s)$ needs to be written by Equation (17) and stabilizes $G(s)$.

Next, the disturbance attenuation characteristic is shown. According to [25], since transfer functions from the periodic disturbance $d(s)$ to the output $y(s)$ is written by

$$y(s) = \frac{1 + (C_d(s) + F_2(s)C_n(s))e^{-sT}}{1 + C_1(s)G(s)} d(s), \tag{18}$$

if

$$1 - (C_d(s) + F_2(s)C_n(s))\big|_{s_i = j\frac{2\pi i}{T}} = 0 \quad (i = 0, 1, 2, \ldots), \tag{19}$$

then the periodic disturbance $d(s)$ is attenuated. From Equation (19), the disturbance characteristic is specified using controllers $C_d(s)$ and $C_n(s)$ of $C_2(s)$ in Equation (7). Therefore, the purpose of $C_1(s)$ is to specify the input/output characteristics, and the purpose of $C_2(s)$ is to specify the disturbance attenuation characteristics.

From the above discussion, the problem in this paper is to design $C_1(s)$, $C_n(s) \in RH_\infty$ and $C_d(s) \in RH_\infty$ satisfying Equation (16) and Equation (19).

According to [25], when the plant $G(s)$ is of non-minimum phase, the control system is unstable. In the next section, we propose a design method for the control system in Figure 1 to attenuate the periodic disturbance $d(s)$, for $y(s)$ to follow the reference input $r(s)$ and the transfer function from $d(s)$ to $y(s)$ has finite number of poles.

4. **Design Method for Control System.** In this section, when $G(s)$ is of non-minimum phase, we propose a design method for the control system in Figure 1 to attenuate the periodic disturbance $d(s)$, for $y(s)$ to follow the reference input $r(s)$ and the transfer function from $d(s)$ to $y(s)$ has finite number of poles.

From Section 3, $C_n(s)$, $C_d(s)$ in Equation (7) and $C_1(s)$ need to satisfy Equation (16), Equation (19), $C_n(s) \in RH_\infty$ and $C_d(s) \in RH_\infty$. Even if the plant is in the non-minimum phase, in order to make the control system stable, we change the problem to satisfy Equation (19) by

$$C_d(s) + F_2(s)C_n(s) = -q(s)N_i(s), \tag{20}$$

where $q(s) \in RH_\infty$ is a strictly proper low-pass filter written by

$$q(s) = \frac{1}{(1 + \tau s)^{n_q}}, \tag{21}$$

$n_q > 0$ is positive integer, $\tau > 0$ is small real number and $N_i(s) \in RH_\infty$ is an inner function of $N(s)$ and satisfy where $N_o(s) \in RH_\infty$ is an outer function of $N(s)$ and satisfy

$$N(s) = N_i(s)N_o(s) \tag{22}$$

and $N_i(0) = 1$, and $N_o(s)$ is an outer function. Then in the frequency range satisfying

$$q(s_i)N_i(s_i) \simeq 1 \quad (i = 0, 1, \dots, k_{\max}), \tag{23}$$

for s_i $(i = 0, 1, \dots, k_{\max})$, Equation (19) is satisfied, that is the frequency component of the periodic disturbance $d(s)$ is attenuated, where $k_{\max}$ is maximum integer.

From Equation (10), when $G(s)$ is of non-minimum phase, $F_2(s)$ is of non-minimum phase and has the same unstable zeroes of $G(s)$, which is equivalent to $N(s)$. Therefore, $F_2(s)$ is factorized by

$$F_2(s) = N_i(s)\bar{F}_2(s), \tag{24}$$

where $\bar{F}_2(s) \in RH_\infty$ is of minimum phase. Substituting Equation (24) to Equation (20), we have

$$C_d(s) + N_i(s)\bar{F}_2(s)C_n(s) = -q(s)N_i(s). \tag{25}$$

From this equation, $C_d(s)$ needs to be written by the form

$$C_d(s) = N_i(s)\bar{C}_d(s). \tag{26}$$

By substituting Equation (26) to Equation (25), we have

$$C_n(s) = -\frac{q(s) + \bar{C}_d(s)}{\bar{F}_2(s)}. \tag{27}$$

Substitution Equation (27) to Equation (16) gives

$$\bar{C}_d(s) = \frac{\left(F_1(s)N_o(s) + \bar{F}_2(s)D(s)\right) q(s)D_c(s)}{\left(\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s)\right) N_o(s)}, \tag{28}$$

where $N_c(s) \in RH_\infty$ and $D_c(s) \in RH_\infty$ are coprime factors of $C_1(s)$ satisfying

$$C_1(s) = \frac{N_c(s)}{D_c(s)}. \tag{29}$$

From the stability condition, in order to make the control system in Figure 1 stable, $C_n(s) \in RH_\infty$ and $C_d(s) \in RH_\infty$ must be satisfied. From Equation (26), this is equivalent to $C_n(s) \in RH_\infty$ and $\bar{C}_d(s) \in RH_\infty$. First, we will clarify the condition to hold $\bar{C}_d(s) \in RH_\infty$ in Equation (28). Since Equation (28), Equation (9), Equation (10), Equation (21), $N_o(s)$ is of minimum phase, $F_1(s) \in RH_\infty$, $\bar{F}_2(s) \in RH_\infty$, $q(s) \in RH_\infty$. Therefore, $\bar{C}_d(s)$ in Equation (28) is stable if $\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s) \in \mathcal{U}$, where $\mathcal{U}$ is unimodular, that is, $\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s) \in \mathcal{U}$ implies $\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s) \in RH_\infty$ and $1/\left(\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s)\right) \in RH_\infty$. In addition, $\bar{C}_d(s)$ is proper if relative degree of $\left(F_1(s)N(s) + \bar{F}_2(s)\right) q(s)$ is greater than or equal to that of $N_o(s)$. Note that the relative degree of $N_o(s)$ is equal to that of $G(s)$. From Equation (9), Equation (10) and Equation (24), $F_1(s)$ is bi-proper and $\bar{F}_2(s)$ is strictly proper. Therefore, $F_1(s)N(s) + \bar{F}_2(s)$ is bi-proper. This implies that if $q(s)/D(s)$ is proper, then $\bar{C}_d(s)$ is proper. In this way, we find that if the relative degree of $q(s)$ is greater than or equal to $N_o(s)$ and $\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s) \in \mathcal{U}$, then $C_d(s) \in RH_\infty$ in Equation (28).

Second, we will clarify the condition of $C_n(s) \in RH_\infty$ in Equation (27). From Equation (27) and $\bar{F}_2(s)$ is of minimum phase if the relative degree of $q(s)N_i(s) + \bar{C}_d(s)$ is greater than or equal to $\bar{F}_2(s)$, $C_n(s) \in RH_\infty$ in Equation (27) is satisfied. Since $F_1(s)$ in Equation (9) is bi-proper, $G(s)$ is strictly proper, and $\bar{F}_2(s)$ in Equation (10) is strictly proper, then

$F_1(s)N_i(s)N_o(s)+\bar{F}_2(s)$ is strictly proper. Since $F_1(s)N_i(s)N_o(s)+\bar{F}_2(s)$ is strictly proper, the relative degree of $C_d(s)$ is greater than that of $q(s)$. That is, the relative degree of $q(s)N_i(s) + C_d(s)$ is equal to that of $q(s)$. Thus, if $q(s)/\bar{F}_2(s)$ is proper, then $\bar{C}_d(s)$ is proper. Thus, if $q(s)/\bar{F}_2(s)$ is proper, then $C_n(s) \in RH_\infty$.

We consider the robustness analysis for the period T of the disturbance $d(t)$, that is, when the period of the disturbance $d(t)$ is different from T, the disturbance attenuation characteristic will be described. Substituting Equation (20) to Equation (18), we have

$$y(s) = \frac{1 - q(s)N_i(s)e^{-sT}}{1 + C_1(s)G(s)} d(s). \tag{30}$$

Equation (30) is rewritten by

$$y(s) = \left(1 - q(s)N_i(s)e^{-sT}\right) T_{yd}(s), \tag{31}$$

where

$$T_{yd}(s) = \frac{1}{1 + C_1(s)G(s)}, \tag{32}$$

which is the transfer function from $d(s)$ to $y(s)$ in the case that the disturbance observer is not used. We can examine the effect of the suppressing periodic disturbances by examining the gain of $1 - q(s)N_i(s)e^{-sT}$.

5. **Design Procedure.** In this section, we propose a design procedure satisfying the preceding section.

A design procedure of the control system in Figure 1 is summarized as follows.

Step 1) Obtain the coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ satisfying Equation (11).

Step 2) Obtained $N(s) \in RH_\infty$ is factorized by Equation (22), that is, the inner function $N_i(s)$ and the outer function $N_o(s)$ are obtained.

Step 3) Design $F_1(s) \in RH_\infty$ and $\bar{F}_2(s) \in \mathcal{U}$ using the method in [10]. That is, $Q(s)$ in Equation (9) and Equation (10) is settled to satisfy Equation (12). $F_2(s)$ is given by Equation (24).

Step 4) According to [32], all stabilizing controllers for $G(s)$ are given by

$$C_1(s) = \frac{X(s) + D(s)\bar{Q}(s)}{Y(s) - N(s)\bar{Q}(s)}, \tag{33}$$

where $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are a pair of solution of

$$N(s)X(s) + D(s)Y(s) = 1, \tag{34}$$

and $\bar{Q}(s) \in RH_\infty$ is any function. From Equation (29) and Equation (33), we have

$$N_c(s) = X(s) + D(s)\bar{Q}(s) \tag{35}$$

and

$$D_c(s) = Y(s) - N(s)\bar{Q}(s). \tag{36}$$

From Equation (28), $N_c(s)$ in Equation (35) and $D_c(s)$ in Equation (36) need to satisfy $\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s) \in \mathcal{U}$. Using $\bar{Q}(s)$ in Equations (32) and (33), $N_c(s)$ and $D_c(s)$ are designed to make $\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s) \in \mathcal{U}$ and to have the form in Equation (17). To find $C_n(s)$ and $C_d(s)$ to make $\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s) \in \mathcal{U}$ is equivalent to finding a pair of solutions to

$$\bar{F}_2(s)N_i(s)N_c(s) - F_1(s)D_c(s) = 1. \tag{37}$$

Obtain $N_c(s)$ and $D_c(s)$ satisfying Equation (37).

$q(s)$ is settled by Equation (21), where the maximum frequency range $k_{\max}$ in Equation (23) to estimate the periodic disturbance $d(s)$ is settled. $\tau > 0$ is settled to small real number that $q(s_i) = 1$ in the frequency range from 0 to $k_{\max}$. n_q is settled to make $C_d(s)$ and $C_n(s)$ proper.

Step 5) $C_d(s)$ and $C_n(s)$ are given by Equation (28) and Equation (27), respectively.

6. **Numerical Example.** In this section, a numerical example is illustrated to show the effectiveness of the proposed method.

Consider the problem of designing the control system in Figure 1 to attenuate periodic disturbances $d(t)$ with period $T = \pi$ [sec] and to follow the reference input $r(t) = 1$ for the non-minimum-phase plant $G(s)$ written by

$$G(s) = \frac{s - 100}{s^2 - 43s - 350}. \tag{38}$$

Following Step 1), coprime factors $N(s)$ and $D(s)$ of $G(s)$ in Equation (38) satisfying Equation (11) are written as

$$N(s) = \frac{-2s + 200}{s^2 + 1007s + 7000} \tag{39}$$

and

$$D(s) = \frac{-2s + 100}{s + 1000}. \tag{40}$$

Following Step 2), the inner-outer decomposition of $N(s)$ satisfies Equation (23) and is given by Equation (22),

$$N_i(s) = \frac{-2s + 200}{2s + 200}, \tag{41}$$

$$N_o(s) = \frac{2s + 200}{s^2 + 1007s + 7000}. \tag{42}$$

Following Step 3), $F_1(s)$ and $F_2(s)$ are given by Equation (9) and Equation (10). We have

$$F_1(s) = \frac{-5s^2 - 750s + 50000}{s^2 + 1050s + 50000} \tag{43}$$

and

$$F_2(s) = \frac{5s^2 + 500s - 100000}{s^3 + 1057s^2 + 57350s + 350000}, \tag{44}$$

where $Q(s)$ in Equation (9) and Equation (10) is settled by

$$Q(s) = \frac{1.5s + 450}{s + 50}. \tag{45}$$

$\bar{F}_2(s)$ is given by Equation (24)

$$\bar{F}_2(s) = \frac{-(5s^2 + 1500s + 100000)}{s^3 + 1057s^2 + 57350s + 350000}. \tag{46}$$

Following Step 4) in the preceding section, solution to Equation (37) is obtained by

$$N_c(s) = \frac{-(14010s^2 + 4257000s + 35000000)}{s^3 + 1250s^2 + 26000s + 10000000} \tag{47}$$

and

$$D_c(s) = \frac{0.2s^4 + 431.4s^3 + 405200s^2 + 7149000s}{s^4 + 1257s^3 + 268800s^2 + 11820000s + 70000000}. \tag{48}$$

From Equation (21), $q(s)$ is given by

$$q(s) = \frac{1}{0.001s + 1}, \tag{49}$$

where $q(s)/G(s)$ and $q(s)/F_2(s)$ are proper as following equations

$$\frac{q(s)}{G(s)} = \frac{(s^2 - 43s - 350)}{(s - 100)(0.001s + 1)}, \tag{50}$$

and

$$\frac{q(s)}{F_2(s)} = \frac{(s^3 + 1057s^2 + 57350s + 350000)}{(5s^2 + 500s - 100000)(0.001s + 1)}. \tag{51}$$

In addition to this result, $q(s)/N_o(s)$ and $q(s)/\bar{F}_2$ in Equation (28) and Equation (27) are proper.

Following Step 5), using above parameters, $C_d(s)$, $\bar{C}_d(s)$ and $C_n(s)$ are given by Equation (26), Equation (28), and Equation (27), respectively.

Using the designed control system in Figure 1, the response of the output $y(t)$ for the step input $r(t) = 1$ is shown in Figure 2. Figure 2 shows that the control system in Figure 1 is stable and the output $y(t)$ follows the step input $r(t) = 1$ without steady-state error.

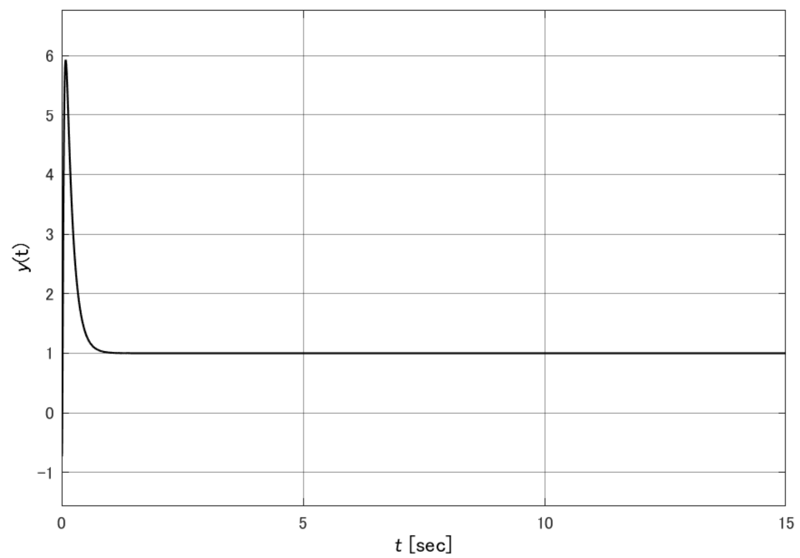

FIGURE 2. The response of the output $y(t)$ for the step input $r(t) = 1$

Next, the disturbance attenuation characteristic is shown. When the periodic output disturbance $d(t)$ is given by

$$d(t) = \begin{cases} \frac{2}{\pi}t & 0 \le t - kt < \frac{\pi}{2} \\ \frac{2}{\pi}(\pi - t) & \frac{\pi}{2} \le t - kt < \pi \end{cases}, \tag{52}$$

where k is the maximum integer which will not exceed t/T, the disturbance $d(t)$ in Equation (52) is a triangular wave with the period T as shown in Figure 3. The response of the output $y(t)$ for the disturbance $d(t)$ in Equation (52) is shown in Figure 4. Figure 4 shows that the periodic disturbance $d(t)$ in Equation (52) is attenuated effectively. Where according to Figure 4, there are disturbances sometimes, however, this is due to the influence of the inner function $N_i(s)$ of Equation (25). Therefore, it is considered that disturbances can be effectively attenuated even in non-minimum phase systems.

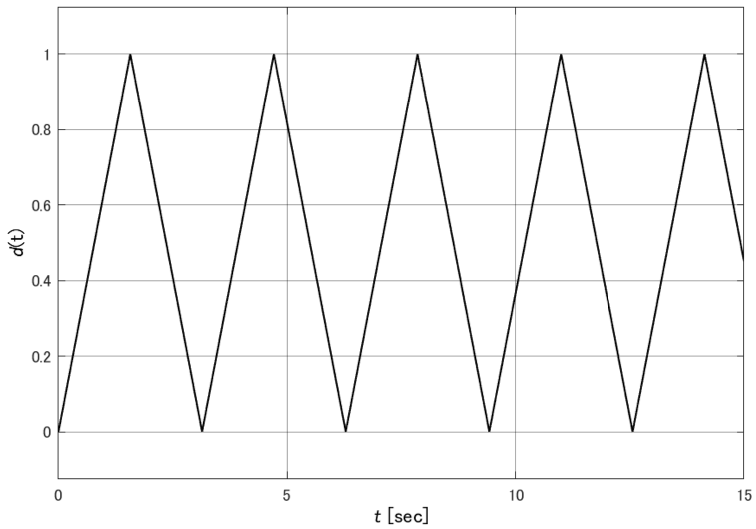

FIGURE 3. Triangular wave disturbance

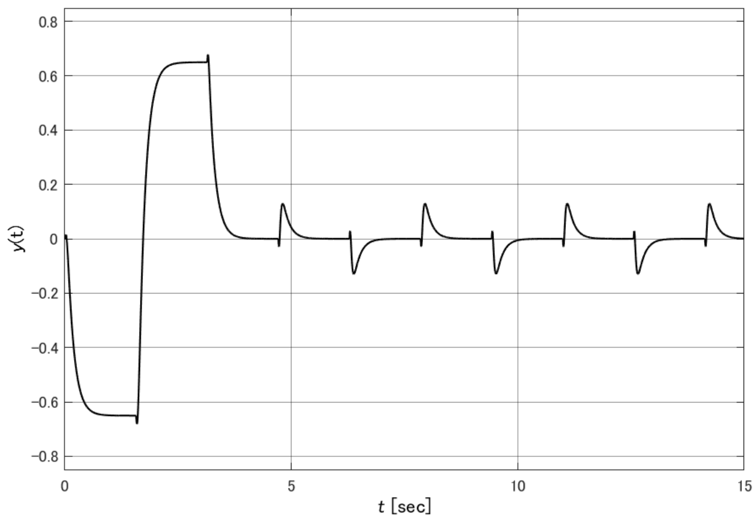

FIGURE 4. The response of the output $y(t)$ for the disturbance $d(t)$ in Equation (52)

In this way, we can easily design a control system to attenuate the periodic disturbance and to follow the reference input without steady-state error using Figure 1.

Yamada et al. proposed a design method that uses a disturbance observer for non-periodic input without using repetitive control [25]. Method by Yamada et al. [25] is designed to satisfy

$$C_d(s) + F_2(s)C_n(s) = -q(s), \tag{53}$$

in order to attenuate the periodic disturbance. Here, if we adopt Equation (53) to non-minimum phase plant, C_n is unstable, since C_n has $N_i^{-1}(s)$ as shown in

$$C_n(s) = -\frac{q(s) + \bar{C}_d(s)N_i(s)}{\bar{F}_2(s)N_i(s)}. \tag{54}$$

Therefore, the response of the output $y(t)$ for the step input is unstable as Figure 5. In this paper, in order to overcome this problem, we adopt Equation (20).

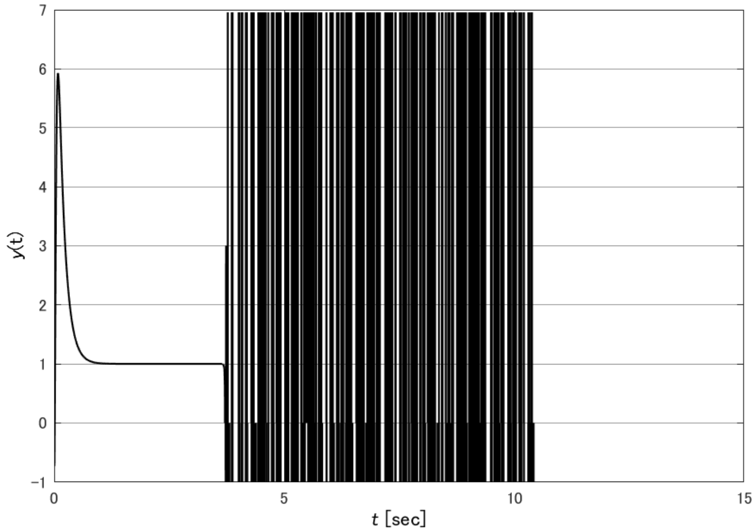

FIGURE 5. The response of the output $y(t)$ for the step input $r(t) = 1$ using a method by Yamada et al. [25]

In addition, the output $y(s)$ of the repetitive control to follow the step input without the steady-state error. However, the repetitive control has the steady-state error when the reference input is ramp input as Figure 6.

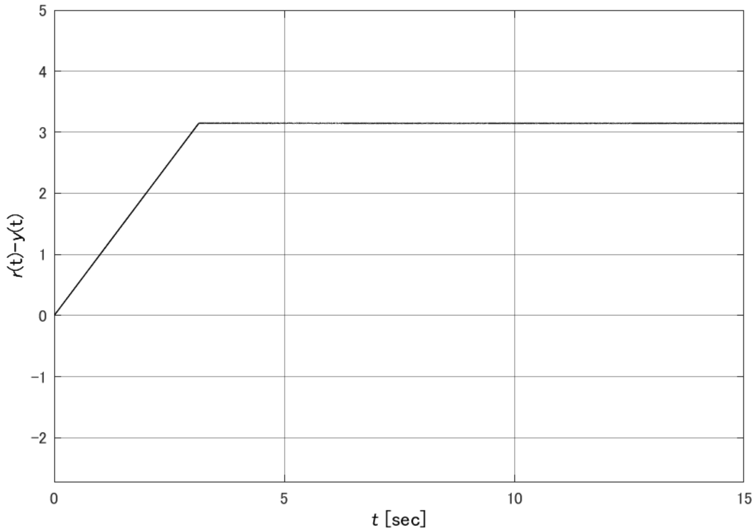

FIGURE 6. The steady-state error of the ramp input of the repetitive control

However, the output $y(s)$ can follow the ramp input without the steady-state error. The steady-state error of the ramp input of the disturbance observer with non-minimum phase is shown as Figure 7. This is the advantage of disturbance observer that suppresses periodic disturbances.

Next, we examine the robustness of the period T of the disturbance $d(t)$. We show the gain diagram of $1 - q(s)N_i(s)e^{-sT}$ in Figure 8. From Figure 8, when the frequency range of the disturbance $d(t)$ between 1.988∼2.013, the disturbance $d(t)$ is attenuated 5% rather than without using the disturbance observer for the periodic disturbance. This implies that the robustness is guaranteed for the uncertainty of period T for ±0.625%. From Figure 8, in order to confirm that the disturbance observer suppresses the periodic disturbance with period T, we will show disturbance attenuation characteristics when the

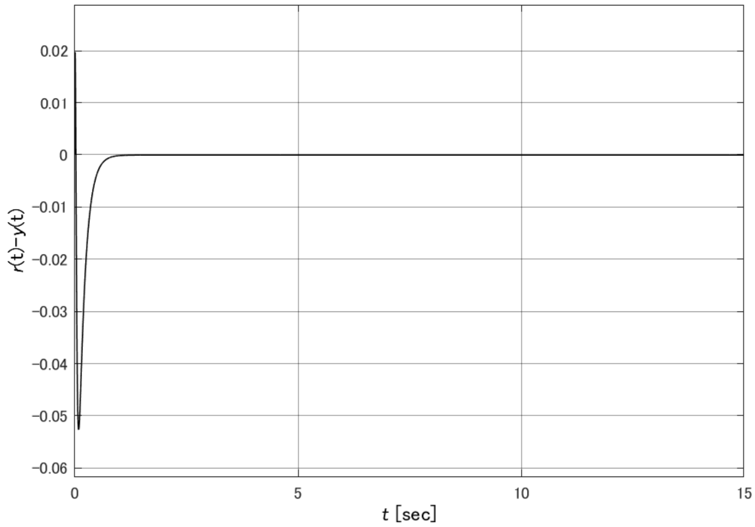

FIGURE 7. The steady-state error of the ramp input of the disturbance observer with non-minimum phase

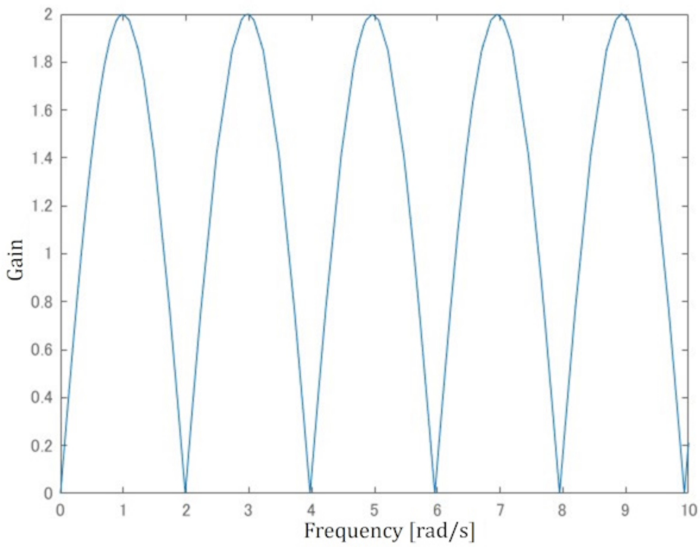

FIGURE 8. The gain diagram of $1 - q(s)N_i(s)e^{-sT}$

period T has uncertainty. We consider that the period T of the periodic disturbance has the shifting of $\pm 0.625\%$. When the period of the disturbance $d(t)$ is equal to 3.120 and 3.160, the responses of the output $y(t)$ for the disturbance $d(t)$ are shown in Figure 9 and Figure 10, respectively. In Figure 9, the solid line shows the response of the output $y(t)$ for the disturbance $d(t)$ and the dotted line shows that of the disturbance $d(t)$ with the period $T = 3.120$ [sec]. In Figure 10, the solid line shows the response of the output $y(t)$ for the disturbance $d(t)$ and the dotted line shows that of the disturbance $d(t)$ with the period $T = 3.160$ [sec]. From Figure 9 and Figure 10, we find that the proposed method has a robustness for the period T of the disturbance $d(t)$.

7. **Conclusion.** In this paper, we have proposed a design method for the control system of a non-minimum phase plant to attenuate the periodic disturbances and to follow non-periodic reference input without steady-state error using the disturbance observer for periodic disturbance. First, we proposed a control system using a disturbance observer for

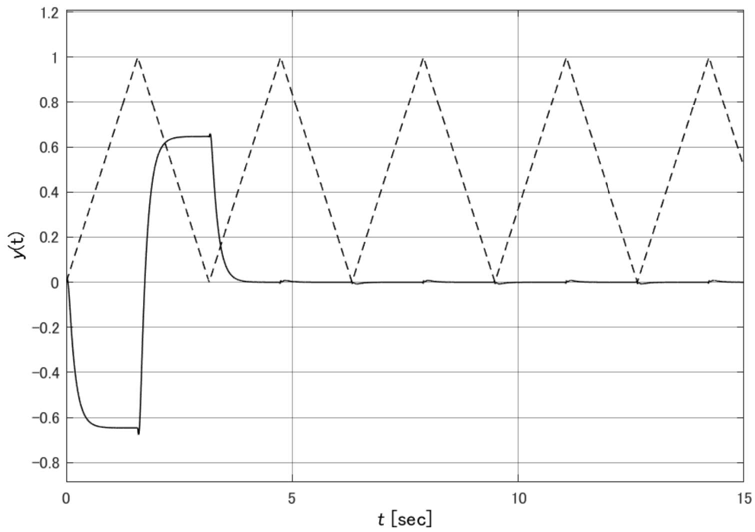

FIGURE 9. The response of the output $y(t)$ for the disturbance $d(t)$ with the period T that has the shifting of $+0.625\%$

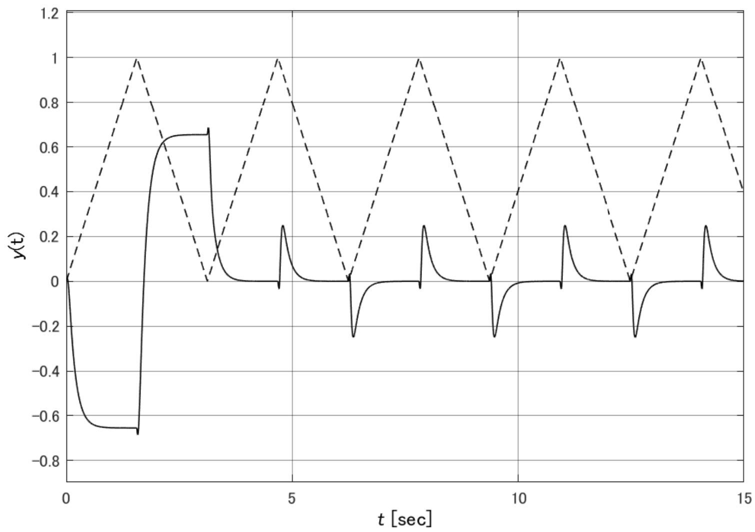

FIGURE 10. The response of the output $y(t)$ for the disturbance $d(t)$ with the period T that has the shifting of -0.625%

the periodic disturbance. We clarify the control characteristic that the transfer function from the disturbance $d(s)$ to the output $y(s)$ has a finite number of poles and that the proposed control system is stable. The design method and the design procedure of the proposed system for a non-minimum phase plant are shown. Finally, we show the features of the proposed design method through a numerical example.

From the result of this paper, we have advantage for non-minimum phase plant with non-periodic input and periodic disturbance. The disturbance observer can control ramp responses that cannot be controlled by repetitive control.

In this paper, the plant $G(s)$ is assumed to be of non-minimum phase. However, the design method for $C_1(s)$ that stabilizes $G(s)$ is insufficient for system stability. $C_1(s)$ must be designed so that the disturbance controllers $C_d(s)$ and $C_n(s)$ are stable. Therefore, we will examine a design method for disturbance observers control system for non-minimum phase systems.

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