International Journal of Innovative
Computing, Information and Control
Volume **21**, Number **1**, February **2025**

ICIC International ©2025 ISSN 1349-4198
pp. **197–208**

# FAILURE DETECTION USING AN EXTENDED SEMI-STRONGLY STABILIZING CONTROLLER WITH A POLE AT THE ORIGIN AND TWO PAIRS OF POLES ON THE IMAGINARY AXIS

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Received February 2024; revised June 2024

Abstract. *To make control systems stable using a stable controller is called the strongly stabilization. The strongly stabilizing controller, which is a stable controller that makes the system internally stable, is often used to make the control system stable even if a sensor or actuator fails for stable plants. However, the strongly stabilizing controller has some problems: not make the output follow the step reference input or sinusoidal reference input, and not make the step disturbance or sinusoidal disturbance be attenuate. This is because the strongly stabilizing controller is not able to have poles at the origin and on the imaginary axis. Since sinusoidal inputs are often used to detect the faulty in some reliable control system design, it is important to overcome problems that the strongly stabilizing controller has. To overcome these problems, the extended strongly stabilizing controller, which has a pole at the origin and two pairs of poles on the imaginary axis, has been proposed. Using the extended strongly stabilizing controller, we can construct a reliable control system that has failure detection and follows the output to the reference input. In this paper, we propose a control system with failure detection without an error between the output and the reference input by using the extended semi-strongly stabilizing controller that has a pole at the origin and two pairs of poles on the imaginary axis.*
**Keywords:** Resonance, Extended semi-strongly stabilization, Parameterization

1. **Introduction.** This paper concerns the control system with failure detection using the parameterization of all extended semi-strongly stabilizing controllers with a pole at the origin and two pairs of poles on the imaginary axis. The parameterization is a method of finding all stabilizing controllers for a given plant [1, 2]. By using the parametrization, the stability of the control system is guaranteed [3]. Various papers have been published

DOI: 10.24507/ijicic.21.01.197

on the parameterization problems, such as PID control [4], two-degree-of-freedom stabilizing controller [5], disturbance observer [6], modified Smith predictor [7], and internally stabilizing controller [8]. However, the stability of the controller obtained in this parametrization is not taken into account. If the controller is unstable, the control system will be highly sensitive when parameters under control change [3, 9, 10]. Thus, to be lowly sensitive to a parameter change, we had better use stable controllers. In addition, if the feedback-loop of the control system is cut, that is, the control system breaks down to a feedforward control system, the unstable poles of the stabilizing controller produce the unstable poles of the control system [11]. Thus, the control system becomes unstable even if the plant is stable. Based on the presented reasons, it is desirable in practice that the control system is stabilized by a stable stabilizing controller [10, 11]. To design stable stabilizing controllers, there exists a control method called strongly stabilization, which is a method to stabilize the control system by using stable controllers. By using this method, there is no need to consider problems such as high sensitivity to disturbances and degradation of target tracking performance that occurs when unstable controllers are used [12, 13]. However, for any plant, there does not necessarily exist strongly stabilizing controllers. The condition that there exist strongly stabilizing controllers is known as the parity interlacing property (p.i.p.) condition [9, 14], that is, the plant needs to satisfy the p.i.p. condition. Wakaiki et al. examined the sensitivity reduction problem with stable controllers for the linear time-invariant multi-input/multi-output distributed parameter system [15, 16]. However, they do not clarify the class of strongly stabilizable plants. If the class of strongly stabilizable plants is clarified, we can obtain the parameterization of all stable stabilizing controllers. In addition, we can clarify the characteristics of strongly stabilizable plants. From this viewpoint, Hoshikawa et al. clarified the class of all strongly stabilizable plants [17]. [18] clarified the parameterization of all two-degree-of-freedom strongly stabilizing controllers.

The strongly stabilizing controller has a problem when the output from the control system is not followed to the step reference without steady-state error for plants with uncertainty or a step disturbance. To overcome the problem of what the strongly stabilizing controller has, the controller must have a pole at the origin. Hoshikawa et al. expanded the concept of the strongly stabilization and proposed a new concept of semi-strongly stabilization. The semi-strongly stabilization is to make the control system stable using the controller that has the pole at the origin and the others in the open left half-plane [19, 20]. According to [20], a class of semi-strongly stabilizable plants is clarified and a design method of the control system using the strongly stabilizing controller for the class of semi-strongly stabilizable plants is proposed. On the other hand, the method by [20] fails to place the pole at the origin and poles on the imaginary axis. The controller is often required to have poles on the imaginary axis for several purposes such as sinusoidal reference tracking or fault detection. Sinusoidal reference tracking is required in important practical applications [21]. The sinusoidal reference tracking is applied for Robotics [22, 23], and so on. In addition, from the internal model principle, the control system using the semi-strongly stabilizing controller cannot attenuate the sinusoidal disturbances. This is an important problem because, in several nonlinear industrial processes, disturbances can be sinusoidal signals [24]. Thus, it is important to solve the stabilization problem using a controller that has a pole on the imaginary axis and the others in the open left half-plane. Kimura et al. defined an extended semi-strongly stabilizing controller that has a pair of poles on the imaginary axis and no other unstable poles [26]. According to [26], the parameterization of all extended semi-strongly stabilizing controllers is clarified. Using the result of [26], Niiyama et al. proposed a control system with failure detection and faulty sensors using an unbounded resonance of a linear detection filter. This control system proposed by [27] is based on the idea of [25] that the self-repairing control uses the method that exploits an unbounded resonance of a linear detection filter to find failure.

However, the control system proposed by [27] cannot eliminate the steady-state error between the reference input and the output. This is because the extended semi-strongly stabilizing controller defined by [26] has no pole at the origin. According to [29, 30], the extended semi-strongly stabilizing controller is redefined having no unstable poles except for a pole at the origin and a pair of poles on the imaginary axis. The parameterization of all extended semi-strongly stabilizable plants and the parameterization of all extended semi-strongly stabilizing controllers were also clarified [29, 30]. If we can construct the control system with failure detection using the extended semi-strongly stabilizing controller defined by [29, 30] as with the results of [27], the control system is suitable for the high-performance reliable control system design. However, in [29, 30], it is not considered a design method of the control system with failure detection using the extended semi-strongly controller.

In this paper, we consider a design method for the control system with fault detection using the extended semi-strongly controller which has a pole at the origin and two pairs of poles on the imaginary axis without a steady-state error. This paper is organized as follows. In Section 2, the problem considered in this paper is explained. In Section 3, the structure of the control system with failure detection using resonance is presented. In Section 4, steady-state analysis is explained. In Section 5, we describe a method for detecting sensor failure using resonance. In Section 6, we present a design method for the control system with failure detection. In Section 7, a numerical example is illustrated to show the effectiveness of the proposed method. Section 8 gives concluding remarks.

## 2. **Problem Formulation.**

Consider the control system written by

$$\begin{cases} y(s) = G(s)u(s) + d(s) \\ u(s) = C(s)(r(s) - y(s)) \end{cases}, \tag{1}$$

where $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$ is the controller, $y(s)$ is the output, $u(s)$ is the control input, $d(s)$ is the disturbance and $r(s)$ is the step references input, that is,

$$r(t) = r_0 \in R. \tag{2}$$

$G(s)$ is assumed to be semi-strongly stabilizable. That is, from [26], $G(s)$ is written by the form

$$G(s) = \frac{N_b(s) + N_c(s)Q_2(s)}{\frac{1-N_b(s)Q_1(s)}{N_c(s)} - Q_1(s)Q_2(s)}. \tag{3}$$

Here, $N_c(s) \in RH_\infty$ is written as

$$N_c(s) = \frac{s\left(s^2 + \omega^2\right)^2}{n_{cd}(s)}, \tag{4}$$

$\omega \in R$ is any constant positive real number, $n_{cd}(s)$ is any Hurwitz polynomial of 5 degree, $Q_1(s) \in RH_\infty$ is any function satisfying

$$Q_1(s)|_{s=0,\pm j\omega} \neq 0, \tag{5}$$

$N_b(s) \in RH_\infty$ is any function satisfying

$$\frac{1}{s^2 + \omega^2}\{1 - N_b(s)Q_1(s)\}\Big|_{s=0,\pm j\omega} = 0 \tag{6}$$

and

$$N_b(s)|_{s=0,\pm j\omega} \neq 0, \tag{7}$$

and $Q_2(s) \in RH_\infty$ is any function. Equation (3) shows the parameterization of all semi-strongly stabilizable plant $G(s)$. Thus, all semi-strongly stabilizable plant can be written by the form in (3).

The sensor that senses the value of the output $y(t)$ has a backup sensor in case of failure. Initially, the original sensor is used. When sensor failure has been detected, the sensor is switched from the original sensor to the backup sensor. Even if $C(s)$ is designed to stabilize the control system in (1), the sensor failures sometimes make the control system in (1) unstable. If we can detect failures of the control system in (1), the control system in (1) becomes a highly reliable control system.

In this paper, we extend the control system with failure detection proposed by Niiyama et al. [27] and propose the control system with failure detection that eliminates an error between the output and the reference input by using the extended semi-strongly stabilizing controller with a pole at the origin and two pairs of poles on the imaginary axis.

3. **Structure for Control System with Failure Detection.** In this section, using the parameterization of all extended semi-strongly stabilizing controllers with a pole at the origin and two pairs of poles on the imaginary axis, we clarify the control structure of the control system with failure detection.

According to [30], for the extended semi-strongly stabilizable plant $G(s)$ in (3), the parameterization of all extended semi-strongly stabilizing controllers $C(s)$ can be expressed by

$$C(s) = \frac{Q_1(s) + \left\{\frac{1-N_b(s)Q_1(s)}{N_c(s)} - Q_1(s)Q_2(s)\right\} P(s)}{N_c(s) - \{N_b(s) + N_c(s)Q_2(s)\} P(s)}, \tag{8}$$

where $P(s) \in RH_\infty$ and $Q(s) \in RH_\infty$ are functions written as

$$P(s) = N_c(s)Q(s) \tag{9}$$

and

$$Q(s) = \frac{1 - \hat{Q}(s)}{N_b(s) + N_c(s)Q_2(s)}, \tag{10}$$

respectively, $\hat{Q}(s) \in \mathcal{U}$ is a unimodular function that makes $Q(s)$ proper and satisfies

$$\left.\frac{1}{(s - s_i)^{m_i - 1}} \left\{1 - \hat{Q}(s)\right\}\right|_{s=s_i} = 0 \ \forall i, \tag{11}$$

$s_i \in R$ is unstable zeros of $N_b(s) + N_c(s)Q_2(s)$ and $m_i$ is its multiplicity [30].

When $N_c(s)$ in (4) is settled by

$$N_c(s) = \frac{S_N(s)S_D(s)^2}{S_D(s)K + Z(s)S_N(s)} = \frac{s\left(s^2 + \omega^2\right)^2}{\left(Ks^2 + \zeta s + K\omega^2\right)(s + \sigma)^3}, \tag{12}$$

$C(s)$ in (8) can be designed as the controller having the structure with failure detection for sensor shown in Figure 1. In the control system in Figure 1, the entire system surrounded by the dotted lines is the controller $C(s)$ with failure detection. The control system shown in Figure 1 can detect failure by using the resonance of the auxiliary input $\tau(s)$. The auxiliary input $\tau(s) \in R(s)$ is denoted by $\tau(s) = \mathcal{L}\{\tau(t)\}$, where $\tau(t)$ is written by

$$\tau(t) = \delta \sin(\omega t), \tag{13}$$

$\delta$ is $\delta > 0$, and $v(s)$ is the detector output denoted by $v(s) = \mathcal{L}\{v(t)\}$. The detector output $v(t)$ determines whether the control system has failed or not. If $v(t) \neq 0$, the control system

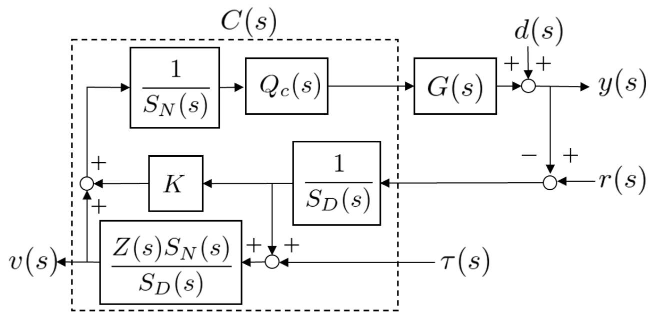


FIGURE 1. Feedback control system with failure detection

has failed. $Z(s)S_N(s)/S_D(s)$ is called by a detector. $Q_c(s) \in RH_\infty$, $S_N(s) \in RH_\infty(s)$, $S_D(s) \in RH_\infty(s)$ and $Z(s) \in RH_\infty(s)$ are written by

$$Q_c(s) = Q_1(s) + \frac{Q(s)}{1 - (N(s) + N_c(s)Q_2(s))\,Q(s)}, \tag{14}$$

$$S_N(s) = \frac{s}{s+\sigma}, \tag{15}$$

$$S_D(s) = \frac{s^2+\omega^2}{(s+\sigma)^2} \tag{16}$$

and

$$Z(s) = \frac{\zeta}{s+\sigma}, \tag{17}$$

respectively, where $\sigma > 0$, $\zeta > 0$ and $K > 0$.

Note that the controller $C(s)$ in Figure 1 is equivalent to (8). Therefore, the controller $C(s)$ in Figure 1 can represent all stabilizing controllers for the pant $G(s)$ in (3) and has a pole at the origin and two pair of poles on the imaginary axis.

4. **Steady-State Analysis.** This section describes the steady-state analysis.

From (3) and (8), transfer functions from the reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ and from the disturbance $d(s)$ to the error $e(s)$ are given by

$$\frac{e(s)}{r(s)} = \frac{1}{1+G(s)C(s)}r(s) = (1 - N_b(s)Q_1(s) - N_c(s)Q_1(s)Q_2(s))\,\hat{Q}(s) \tag{18}$$

and

$$\frac{e(s)}{d(s)} = \frac{-1}{1+G(s)C(s)}d(s) = -\,(1 - N_b(s)Q_1(s) - N_c(s)Q_1(s)Q_2(s))\,\hat{Q}(s), \tag{19}$$

respectively. From (6) and (4), the transfer functions in (18) and (19) have a zero at the origin. Therefore, the output $y(t)$ in Figure 1 follows the step reference input $r(t)$ without steady state error. In addition, the step disturbance $d(t)$ is attenuated.

Next, we consider the effect of the auxiliary input $\tau(t)$ on the output $y(t)$. From Figure 1, the transfer function from $\tau(s)$ to $y(s)$ is

$$\begin{aligned}\frac{y(s)}{\tau(s)} &= \frac{G(s)C(s)}{1+G(s)C(s)}\frac{N_c(s)Z(s)}{S_D(s)}\\ &= (N_c(s)Q_2(s) + N_b(s))\left(Q_1(s)\hat{Q}(s) + Q(s)\right)\frac{s\left(s^2+\omega^2\right)\zeta}{\left(Ks^2+\zeta s+K\omega^2\right)(s+\sigma)^2}.\end{aligned} \tag{20}$$

From this equation and (13), it is obvious that

$$\lim_{t\to\infty} y(t) = 0 \tag{21}$$

is satisfied.

In this way, we find following expressions hold true.

1) The output $y(t)$ follows the step reference input $r(t)$ without steady state error.
2) The step disturbance $d(t)$ is attenuated.
3) The auxiliary input $\tau(t)$ does not appear on the output $y(t)$.

5. **Failure Detection.** In this section, we describe a method for detecting sensor failure using the detector.

Consider a situation, where a sensor fails and the sensor value is stuck as

$$y(t) = y_f. \tag{22}$$

Here, $y_f \in R$ is the value of $y(t)$ after the sensor failure. In this case, the correct output $y(t)$ is not feedback, that is, the control system in Figure 1 changes to a kind of feed-forward control system. When the sensor failure occurs, that is, (22) is satisfied. From (2) and (22), $v(s)$ in Figure 1 is written by

$$v(s) = \frac{\zeta s}{s^2+\omega^2}\left(\frac{r_0 - y_f}{s} + \frac{\delta\omega}{s^2+\omega^2}\right). \tag{23}$$

This yields

$$\begin{aligned} v(t) &= \mathcal{L}^{-1}[v(s)] \\ &= \frac{1}{2}\delta t\sin(\omega t) + \zeta(r(t)-y_f)\left(\frac{1}{2\omega}t\sin(\omega t) + \frac{\sigma}{2\omega^3}t\sin(\omega t) - \frac{\sigma}{2\omega^2}t\cos(\omega t)\right) \end{aligned} \tag{24}$$

and

$$\lim_{t\to\infty} v(t) = \infty. \tag{25}$$

This implies that when the sensor failure occurs, a resonance phenomenon occurs. Therefore, we can detect sensor failure if

$$|v(t)| > \Gamma \in R, \tag{26}$$

where $\Gamma$ is threshold.

When $|v(t)|$ satisfies (26), that is, a failure is detected, the sensor switches from the original failed sensor to a normal backup sensor. After switching to the normal backup sensor, the normal output value is feedback and the system returns to the normal state. Since the original feedback system is stable as shown previously, the system returns to stable. Failure detection is performed in this way.

6. **Design Method for Control System with Failure Detection.** In this section, we present a design method for a control system with failure detection in Figure 1.

A design method is summarized as the following procedure.

Step 1) $K > 0$, $\omega > 0$, $\zeta > 0$ and $\sigma > 0$ are determined.
Step 2) $S_N(s)$ and $S_D(s)$ are settled as (15) and (16), respectively.
Step 3) $Z(s)$ is set by (17).
Step 4) $N_c(s)$ is given by (12).
Step 5) Extended semi-strongly stabilizable plant $G(s)$ can be factorized by (3).
Step 6) Threshold $\Gamma$ is set, so that if $v(s)$ satisfies (26), we find that the sensor failure occurs and change the original sensor to the backup sensor.
Step 7) $\hat{Q}(s)$ is designed so that $\hat{Q}(s) \in \mathcal{U}$ makes $Q(s)$ in (10) proper and satisfy (11).

Step 8) $Q_c(s)$ is given by (14). We can design the control system in Figure 1.
Step 9) $\tau(s)$ is set by (13). The threshold $\Gamma$ for failure detection in (26) is settled.

## 7. **Numerical Example.**

In this section, a numerical example is shown to illustrate the effectiveness of the proposed method.

Consider the problem to design the control system with failure detection in Figure 1 for the plant $G(s)$ written by

$$G(s) = \frac{90.9 \times 10^3}{(s+0.117)\left(s^2+3.97s+2.02 \cdot 10^3\right)}, \tag{27}$$

which is considered in [26]. $K$, $\omega$, $\zeta$ and $\sigma$ are settled by

$$K = 1.01, \tag{28}$$

$$\omega = 1, \tag{29}$$

$$\zeta = 2 \tag{30}$$

and

$$\sigma = 0.1, \tag{31}$$

respectivey. From (15), (16) and (17), we have

$$S_N(s) = \frac{s}{s^2+0.1}, \tag{32}$$

$$S_D(s) = \frac{s^2+1}{(s+0.1)^2} \tag{33}$$

and

$$Z(s) = \frac{2}{s+0.1}, \tag{34}$$

respectively.

From (12), $N_c(s)$ is given by

$$N_c(s) = \frac{s(s^2+1)^2}{(1.01s^2+2s+1.01)(s+0.1)^3}. \tag{35}$$

Since $G(s)$ in (27) is factorized by (3), where

$$Q_1(s) = 0.01, \tag{36}$$

$$Q_2(s) = \frac{-(s+z_1)(s+z_2)(s+z_3)(s+z_4)^3(s+z_5)(s+z_6)}{(s+p_1)(s+p_2)(s+p_3)(s+p_4)(s+p_5)(s+p_6)(s+p_7)(s+p_8)}, \tag{37}$$

$$\begin{cases} z_1 = 197.7 & z_2 = -0.7893 & z_3 = 0.3697 \\ z_4 = 0.1 & z_5 = 3.383 + 39.69j & z_6 = 3.383 - 39.69j \\ p_1 = 0.117 & p_2 = 0.003568 & p_3 = 1.758 + 44.88j \\ p_4 = 1.758 - 44.88j & p_5 = 0.1239 + 1.576j & p_6 = 0.1239 - 1.576j \\ p_7 = 0.1012 + 0.6528j & p_8 = 0.1012 - 0.6528j & \end{cases},$$

and

$$N_b(s) = \frac{s^2+200s+1}{1.01s^2+2s+1.01}. \tag{38}$$

$\hat{Q}(s)$ is set as

$$\hat{Q}(s) = \frac{(s+0.003345)\left(s^2+2.997s+2.99\right)}{(s+1)^3}. \tag{39}$$

From (14), we have

$$Q_c(s) = \frac{0.010011(s+z_{c1})(s+z_{c2})(s+z_{c3})(s+z_{c4})(s+z_{c5})(s+z_{c6})(s+z_{c7})(s+z_{c8})}{(s+p_{c1})^3(s+p_{c2})(s+p_{c3})(s+p_{c4})(s+p_{c5})(s+p_{c6})}, \quad (40)$$

where

$$\begin{cases} z_{c1} = 0.117 & z_{c2} = 0.003565 & z_{c3} = -0.06335 + 0.5050j \\ z_{c4} = -0.06335 - 0.5050j & z_{c5} = 0.4262 + 0.9478j & z_{c6} = 0.4262 - 0.9478j \\ z_{c7} = 2.217 + 1.910j & z_{c8} = 2.217 - 1.910j & \\ p_{c1} = 0.1 & p_{c2} = 0.003345 & p_{c3} = 0.99 + 0.1410j \\ p_{c4} = 0.99 - 0.1410j & p_{c5} = 1.4985 + 0.862843j & p_{c6} = 1.4985 - 0.862843j \end{cases}.$$

The auxiliary signal $\tau(s)$ is set by (13), where $\delta = 0.1$ and the threshold for failure detection in (26) is

$$\Gamma = 5. \quad (41)$$

When $t = t_F = 15$[sec], we suppose that the sensor failure occurs. That is, if $t \geq t_F$, the output $y(t)$ is assumed to be

$$y(t) = y(t_F). \quad (42)$$

If

$$t \geq t_D = \min\{t|\, |v(t)| \geq \Gamma\}, \quad (43)$$

we find that the sensor failure occurs and change the failed sensor to the backup sensors.

When $r(s) = 1/s$, the response of the output $y(t)$, the sensor value $y_s(t)$ and $|v(t)|$ of the control system in Figure 1 are shown in Figure 2. Here, in the upper figure, the solid line shows the response of the output $y(t)$, dashed line shows that of the sensor value $y_s(t)$ and the dash-dot line shows that of the reference input $r(t) = 1$. In the lower figure, the solid line shows the response of $|v(t)|$ and the dash-dot line shows threshold $\Gamma$ in

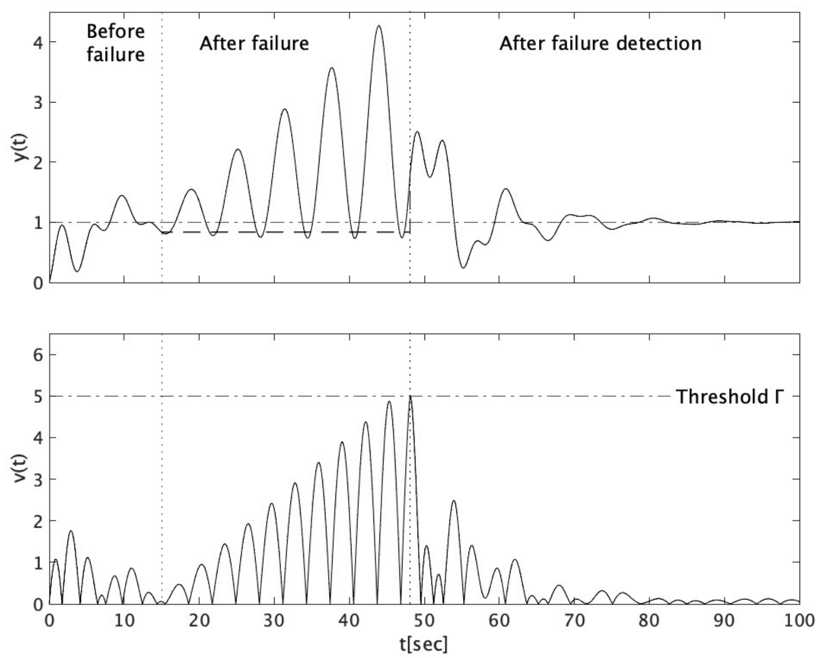


FIGURE 2. Response of the output $y(t)$ and the sensor value (top), the response of absolute value of detector output $v(t)$ (bottom)

(41). As seen in Figure 2, the output $y(t)$ follows the reference input $r(t) = 1$ without a steady-state error. When $t_F = 15$[sec] the sensor signal sticks, $|v(t)|$ increases gradually. Around 48[sec], $|v(t)|$ exceeds the threshold $\Gamma$ in (41), and the failed sensor is replaced by a backup sensor. Since then, from Figure 2, the stability of the control system has been regained again. In addition, from Figure 2, the output $y(t)$ follows the reference input $r(t)$ without steady state error.

8. **Conclusion.** In this paper, we have expanded the control system with failure detection proposed by Niiyama et al. [27] and have proposed a design method for the control system to eliminate the steady-state error between the output and the reference input using the extended semi-strongly stabilizing controller with a pole at the origin and two pairs of poles on the imaginary axis [29, 30]. Using the extended semi-strong stabilizing controllers with a pole at the origin and two pairs of poles on the imaginary axis, the reference input follows the output without the steady-state error. We present a design procedure the proposed control system with failure detection. Finally, a numerical example has been shown to illustrate the effectiveness of the proposed method. The results of this paper are not considered for the multiple-input/multiple-output systems. In the future, we will expand the result of this paper to the multiple-input/multiple-output. In addition, we do not consider the application of the control system with failure detection in this paper. These problems are our future studies.

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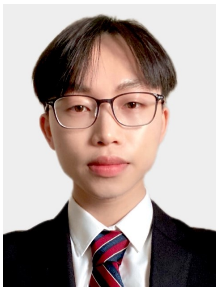

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