International Journal of Innovative Computing, Information and Control
Volume **21**, Number **2**, April **2025**

ICIC International ©2025 ISSN 1349-4198
pp. **373–387**

ADAPTIVE DIRECT DATA DRIVEN CONTROL FOR UNKNOWN CLOSED LOOP SYSTEM

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Received August 2024; revised November 2024

Abstract. *During this era of data science, vast data are collected to infer the unknown plant and unknown controller, existing in a closed loop system structure, which corresponds to direct data driven modelling and direct data driven control. This new paper studies the latter direct data driven control, i.e., designing that feedback controller based on measured input-output data sequence directly. Firstly, after introducing the basic essence of direct data driven control, nonparametric controller and parameterized control are all derived through our own derivations. Secondly, due to the widely application of adaptation into other aircraft control, adaptive idea is applied to identifying the unknown controller parameters, and its related algorithm is formulated in detail while analyzing the termination condition. Thirdly, to guarantee the dual goals, i.e., perfect tracking and asymptotic unbias estimation, Lyapunov function is constructed to derive the accurate nonparametric controller and parameterized controller. Fourthly, application of our considered adaptive direct data driven control is proved in a practical example of aircraft control. Generally, the mission of this paper is to apply adaptive idea into direct data driven control although research only on direct data driven control is very mature. Furthermore, perfect tracking and asymptotic unbias controller are guaranteed through Lyapunov function analysis.*

Keywords: Adaptation, Direct data driven control, Perfect tracking, Lyapunov analysis, Asymptotic unbias, Closed loop system

1. **Introduction.** The ultimate goal of direct data driven modeling and control is to construct a mathematical model or a detailed controller from noisy data for either prediction or physical interpretation, control, computer aided design, fault detection, monitoring, classification, denoising of medical images. During this new information era, other innovative ideas are introduced into control theory to establish a new research direction – direct data driven strategy, i.e., extracting the useful or important knowledge for unknown plant or unknown controller. Above innovative ideas are about machine learning, deep learning and reinforcement learning. For example, within the terminology of machine learning, the case we handle is supervise learning, meaning that the data (input-output) is labeled. Starting from one or more noisy data sets, a model is estimated or a controller is designed only through the measured data. Then this model is used to predict the response to new inputs, or to detect a fault operation in new events, or to classify new medical images.

It is well known that two methods exist for controller design, i.e., model based control and direct data driven control. Firstly, considering model based control method, physical principle or system identification is applied to getting one approximated mathematical

DOI: 10.24507/ijicic.21.02.373

model, corresponding to the unknown plant, then this obtained mathematical model is deemed as a basis for latter controller design or other interesting missions, so the identification accuracy about that unknown plant will influence greatly the latter control performance. In our opinion, the identification accuracy can be compensated, while determining the time in designing controller. More specifically, it means if the mathematical model for that unknown plant is not good, then latter we need spend more time in devising controller. To testify whether the identified mathematical model is good or bad, model validation process is studied during the research field. As the first step for model based control is to identify that unknown plant through system identification strategy, i.e., getting one detailed mathematical equation to fit the measured input-output data, we also think we can get one mathematical form or equation for that unknown controller directly from the measured input-output data, thus causing our studied direct data driven control strategy in this paper. The idea about extracting some useful knowledge from data is feasible during our big data era, corresponding to direct data driven modelling and direct data driven control. Their common difference is the different part, i.e., one is for unknown plant, and the other is for unknown controller. However, we think the main essence between direct data driven modelling and direct data driven control is the same as each other. As the research about system identification or direct data driven modelling is very mature, the earlier research about direct data driven control concerns on how to extend the existing results from direct data driven modelling or system identification to direct data driven control. Now the ongoing research on direct data driven control strategy is concentrated on other new concepts from different fields, for example, behavior property, differential geometry, topology, and game theory. These above new research directions complete the whole study about control theory. As our previous research is to study the application about advanced control theory into flight platform and so obtain some existing contributions for control theory and engineering, during this year, we turn to control the aircraft or other vehicle, and find it possible to implement the adaptive control. It means adaptive idea is widely applied to forming adaptive model reference control, adaptive pole place control, adaptive tuning control, etc. Based on above adaptive idea and our existing contributions on direct data driven control, this new paper combines them together to form adaptive direct data driven control from two points of theoretical analysis and engineering application.

Some references are listed as follows about direct data driven control strategy. [1] studies data driven control through behavior property, describing as Petersen's lemma. One nonconservative design via a matrix S-lemma is used to design feedback controllers from noisy data [2]. Some other statistical properties about data driven control are reviewed in [3], such as stabilization optimality and robustness. Learning strategy is introduced to approximate nonlinear cancellation, being to cancel the nonlinear effect in [4]. Furthermore, model based control and data driven control are all proposed for event and self-triggered discrete time linear system, and their own performances are all compared in [5], where the event is described as a piecewise function. [6] proposes a data driven approach to robust control of multivariable system, and the controller design problem is changed into data driven control within the era of big data, meaning data includes lots of important information for unknown plant and unknown controller [7]. Recently, researchers turn to study nonlinear data driven control by virtue of Koopman operator as tool [8,9], and then apply Koopman operator into model predictive control. [10] proposes a proximally stabilized semismooth algorithm to design a stabilized controller, and convex quadratic programming is similar to show the optimal controller. During these recent years, we apply direct data driven scheme for UAV flight control [11], and also propose direct data driven safety control for aircraft flight system [12].

After showing above our understanding about direct data driven control, and our previous research plants, such as aircraft and vehicle, we think how to apply direct data driven control into aircraft control. We want to introduce adaptive idea into it to form adaptive direct data driven control strategy, and then its theoretical analysis and engineering application are studied deeply. Consider one commonly used closed loop system structure with unknown plant and unknown controller simultaneously, the final goal is to design that controller, while guaranteeing the closed loop system output track one expected or desired output [13]. To avoid the complex modelling process for that unknown plant, direct data driven control is described in detail to achieve our control goal. Then adaptive idea is introduced to complete the existing direct data driven control through our own mathematical derivations. More specifically, from the point of adaptive idea, the nonparametric controller and parameterized controller are yielded adaptively. It means if no any information about unknown controller is known in priori, then its nonparametric controller can be regarded as a rough estimation, constructed only by the measured input-output data, i.e., ratio of two power spectrums. By the way, the commonly used PID controller is a special parameterized controller with three unknown parameters [14], and then parameter estimations are identified only through the measured input-output data sequence, being similar to the system identification process, i.e., parameter identification. Furthermore, to analyze the stability of the proposed adaptive algorithm, Lyapunov function is constructed to do some deep analysis, being our new contributions in this paper. To prove our above theoretical results on adaptive direct data driven control, finally, aircraft control is used as examples to show the theoretical results, i.e., making an achievement of theory and application.

Generally, the main contributions of this paper are formulated as follows.

1) After showing the basic result, the detailed mathematical derivations are shown to give the nonparametric controller and parameterized controller adaptively.

2) Adaptative idea is combined with direct data driven control strategy to form adaptive direct data driven control, while the detailed algorithm is given and the terminal condition is designed.

3) To guarantee both perfect tracking and asymptotic unbias controller, Lyapunov function is established to derive the accurate adaptive controller.

Furthermore, methodologies of this paper are given as follows. Firstly, after introducing the basic essence of direct data driven control, nonparametric controller and parameterized control are all derived through our own derivations. Secondly, due to the wide application of adaptive idea into other advanced control theory, adaptive idea is applied to identifying the unknown controller parameters, and its related algorithm is formulated in detail while analyzing the termination condition. Thirdly, to guarantee the dual goals, i.e., perfect tracking and asymptotic unbias estimation, Lyapunov function is constructed to derive the accurate nonparametric controller and parameterized controller. Fourthly, application of our considered adaptive direct data driven control is proved in a practical example of aircraft control. Generally, the mission of this paper is to apply adaptive idea into direct data driven control although research only on direct data driven control is very mature.

2. **Considered System.** Consider one common closed loop system structure, plotting in Figure 1.

In Figure 1, $r(t)$ is the chosen external input, $e(t)$ is the unavoided external noise or disturbance, $\{u(t), y(t)\}$ corresponds to input-output signal for plant $P(z)$, $H(z)$ is the filter for noise $e(t)$, and $C(z)$ is the feedback controller. Variable z denotes the delay operator, i.e., $zu(t) = u(t-1)$. All three parts $\{P(z), H(z), C(z)\}$ are unknown.

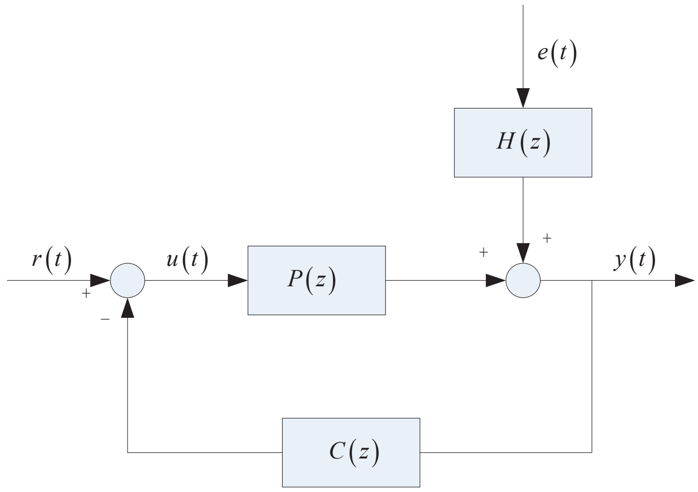


FIGURE 1. Closed loop system structure

Observing Figure 1 again, some existing results are easily obtained, i.e.,

$$y(t) = P(z)[r(t) - C(z)y(t)] + H(z)e(t) \tag{1}$$

i.e.,

$$y(t) = \frac{P(z)}{1 + P(z)C(z)}r(t) + \frac{H(z)}{1 + P(z)C(z)}e(t) \tag{2}$$

As three parts $\{P(z), H(z), C(z)\}$ are all unknown, here we say $\{P(z), H(z)\}$ are plant models, and $C(z)$ is the feedback controller, being designed by researcher. Classical model based control is firstly to construct the detailed mathematical equations for plant models $\{P(z), H(z)\}$, and then secondly feedback controller $C(z)$ is sequently designed based on these two mathematical equations $\{P(z), H(z)\}$. To avoid the modelling process for two plant models $\{P(z), H(z)\}$, direct data driven control strategy appears to design that unknown controller $C(z)$ directly on the basis of the measured input-output data sequence $\{r(t), u(t), y(t)\}_{t=1}^{N}$, where N is the total number of measured data.

Then during this paper, our main mission is to design that unknown feedback controller $C(z)$ from the measured data without any information about that unknown plant models $\{P(z), H(z)\}$.

3. **Adaptive Direct Data Driven Control.** To describe the main idea of direct data driven control and combine it with adaptive idea, we show our main contributions about theoretical results in Section 3.

3.1. **Basis result.** Whatever in theoretical research or engineering application, controller design must achieve one certain of goal, i.e., prefect tracking, minimum energy, robustness, pole placement, etc. Here perfect tracking is used as our controller design goal, i.e., guaranteeing closed loop output $y(t)$ track one expected or desired output $y_0(t)$ perfectly. And this desired output $y_0(t)$ is deemed as follows:

$$y_0(t) = M(z)r(t) \tag{3}$$

where above $M(z)$ is called the desired transfer function from external input $r(t)$ to closed loop output $y_0(t)$.

Comparing Equations (2) and (3), the goal of controller controller design is to satisfy the following two conditions, i.e.,

$$\frac{P(z)}{1+P(z)C(z)} \to M(z), \quad \frac{H(z)}{1+P(z)C(z)} \to 0 \tag{4}$$

where the first condition means perfect tracking, and the second condition is disturbance rejection.

Above explanation is plotted in Figure 2 to understand well.

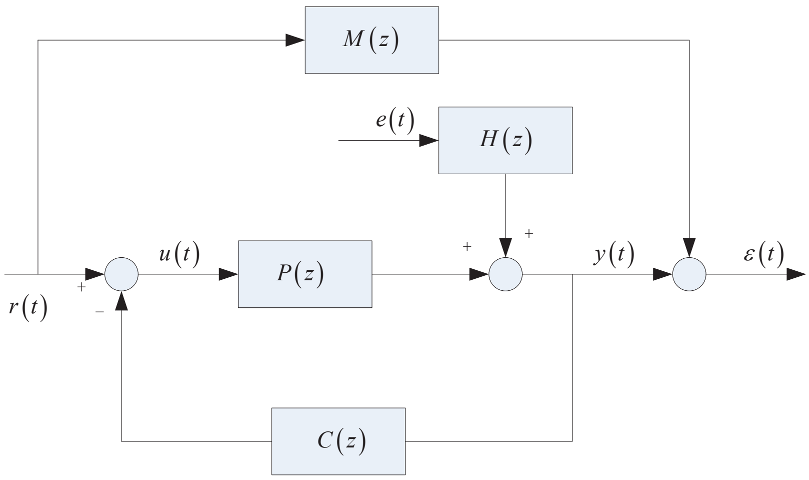


FIGURE 2. Perfect tracking

In Figure 2, Equation (4) means the output error $\varepsilon(t) = y(t) - y_0(t)$ approaches to zero with time increases.

To apply direct data driven control strategy for designing that unknown feedback controller $C(z)$ from the measured input-output data sequence $\{r(t), u(t), y(t)\}_{t=1}^{N}$, we observe that feedback controller $C(z)$ must satisfy the following relation, i.e.,

$$\begin{aligned} &u(t) = r(t) - C(z)y(t) = M^{-1}(z)y(t) - C(z)y(t) = \left[M^{-1}(z) - C(z)\right] y(t); \\ &r(t) = M^{-1}(z)y_0(t) = M^{-1}(z)y(t) \end{aligned} \tag{5}$$

where above relation uses the perfect tracking, i.e., $y(t) = y_0(t) = M(z)r(t)$. If Equation (5) holds, it means $y(t) = y_0(t)$, being an ideal case, i.e., $u(t) - \left[M^{-1}(z) - C(z)\right] y(t) = 0$, then no any input is imposed on plant $P(z)$, so the whole closed loop system stops to work.

Equation (5) is an ideal case, i.e., that ideal controller $C(z)$ must satisfy it, so we can design that feedback controller $C(z)$ in a reversed way, i.e., solving the following optimization problem.

$$C(z) = \arg\min_{C(z)} \frac{1}{N} \sum_{t=1}^{N} \left[u(t) - \left[M^{-1}(z) - C(z)\right] y(t)\right]^2 \tag{6}$$

Equation (6) corresponds to a data fitting problem, whose variables $\{u(t), y(t)\}_{t=1}^{N}$, $M(z)$ are all known, but only expect that unknown controller $C(z)$.

An easy way to solve that feedback controller $C(z)$ is to differentiate with respect to $C(z)$, and then it holds that

$$\frac{2}{N} \sum_{t=1}^{N} \left[u(t) - \left[M^{-1}(z) - C(z)\right] y(t)\right] y^T(t) = 0 \tag{7}$$

i.e.,

$$\sum_{t=1}^{N} u(t)y^T(t) = \left[M^{-1}(z) - C(z)\right] \sum_{t=1}^{N} y(t)y^T(t);$$

$$\phi_{uy}(w) = \left[M^{-1}(z) - C(z)\right] \phi_y(w); \quad M^{-1}(z) - C(z) = \frac{\phi_{uy}(w)}{\phi_y(w)} \tag{8}$$

Then the feedback controller is

$$C(z) = \frac{1}{M(z)} - \frac{\phi_{uy}(w)}{\phi_y(w)} \tag{9}$$

where $\phi_{uy}(w)$ and $\phi_y(w)$ are two kinds of power spectrums.

Observing Equation (9), we see after collecting input-output data sequence $\{u(t), y(t)\}_{t=1}^{N}$ and giving the expected transfer function $M(z)$, the feedback controller $C(z)$ is estimated to be a rough value, corresponding to a nonparametric estimation.

3.2. **Parameterized controller.** As Equation (9) is a rough estimation, and parameterized controller is always used in engineering, such as PID controller, we start to describe how to design the parameterized controller through combining adaptive idea.

The parameterized controller is denoted by $C(z, \theta) = \alpha(z)\theta$, where parameter vector θ includes all controller parameters, and $\alpha(z)$ is one priori chosen basis function vector, for example, PID controller.

$$C(z, \theta) = \begin{pmatrix} e(t) & \dot{e(t)} & \int e(t) \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} \tag{10}$$

Substituting above chosen parameterized form $C(z, \theta) = \alpha(z)\theta$ into Equation (6), then the whole controller design problem is transformed into one parameter estimation problem, i.e.,

$$\theta = \arg\min_{\theta} \frac{1}{N} \sum_{t=1}^{N} \left[u(t) - \left[M^{-1}(z) - \alpha(z)\theta\right] y(t)\right]^2 \tag{11}$$

Differentiating with respect to unknown parameter vector θ, we have

$$\frac{2}{N} \sum_{t=1}^{N} \left[u(t) - \left[M^{-1}(z) - \alpha(z)\theta\right] y(t)\right] \alpha(z)y^T(t) = 0 \tag{12}$$

After complex derivation, it holds that

$$\theta = \left[\sum_{t=1}^{N} \alpha^T(z)y^T(t)y(t)\alpha(z)\right]^{-1} \left[\sum_{t=1}^{N} \alpha^T(z)y(t)\left[u(t) - M^{-1}(z)y(t)\right]\right] \tag{13}$$

where matrix $\sum_{t=1}^{N} \alpha^T(z)y^T(t)y(t)\alpha(z)$ is called as regression matrix, appearing in its inverse matrix, so this regression matrix must be full rank, i.e., corresponding to the persistent excitation condition.

Definition 3.1. [Persistent excitation condition]. *To guarantee the efficiency of above parameter estimation (13), the measured output sequence must be full rank, i.e., regression matrix* $\sum_{t=1}^{N} \alpha^T(z)y^T(t)y(t)\alpha(z)$ *has its reverse form.*

To let readers understand well, an example is given here. Set that parameterized controller $C(z, \theta) = \alpha(z)\theta$ be

$$C(z,\theta)=\alpha(z)\theta=\begin{pmatrix}1 & z & z^2 & \cdots & z^n\end{pmatrix}\begin{pmatrix}\theta_1\\ \theta_2\\ \vdots\\ \theta_n\end{pmatrix} \tag{14}$$

where n is the order. Then

$$\alpha(z)y(t)=\begin{pmatrix}1 & z & z^2 & \cdots & z^n\end{pmatrix}y(t)=\begin{pmatrix}y(t) & y(t+1) & y(t+2) & \cdots & y(t+n)\end{pmatrix} \tag{15}$$

Substitute above Equation (15) into regression matrix to get

$$\begin{aligned}\sum_{t=1}^{N}\alpha^T(z)y^T(t)y(t)\alpha(z) &= \sum_{t=1}^{N}\begin{pmatrix}y(t)\\ y(t+1)\\ y(t+2)\\ \vdots\\ y(t+n)\end{pmatrix}\begin{pmatrix}y(t) & y(t+1) & y(t+2) & \cdots & y(t+n)\end{pmatrix}\\ &= \begin{pmatrix}\phi_y(0) & \phi_y(1) & \cdots & \phi_y(n)\\ \phi_y(1) & \phi_y(0) & \cdots & \phi_y(n-1)\\ \vdots & \vdots & \cdots & \vdots\\ \phi_y(n) & \phi_y(n-1) & \cdots & \phi_y(0)\end{pmatrix}\end{aligned} \tag{16}$$

so that about regression matrix $\sum_{t=1}^{N}\alpha^T(z)y^T(t)y(t)\alpha(z)$ is full rank, corresponding to nonzero output power spectrum, i.e., $\phi_y(0)\neq 0$.

3.3. **Adaptive idea.** From that optimization problem (6), no any priori knowledge about that unknown plant $P(z)$ is needed, so it is the meaning of direct data driven control strategy. Also for that controller parameter estimation problem (11), we want that parameter vector θ will be yielded iteratively or adaptively. Here adaptive idea is introduced to complete above described direct data driven control strategy.

For that parameter estimation problem (11), adaptive iteration is applied to generating a sequence of parameter estimations at different iterative steps, i.e.,

$$\theta_{k+1}=\theta_k-\gamma\bigtriangledown J(\theta_k) \tag{17}$$

where γ is a scale value, for example, $\gamma=0.5$, θ_k and θ_{k+1} denote the iterative values at k step and continuous $k+1$ step. $\bigtriangledown J(\theta_k)$ means the gradient of cost function, i.e.,

$$\bigtriangledown J(\theta_k)=\frac{2}{N}\sum_{t=1}^{N}\left[u(t)-\left[M^{-1}(z)-\alpha(z)\theta_k\right]y(t)\right]\alpha(z)y^T(t) \tag{18}$$

To analyze the estimation error for $\|\theta_{k+1}-\theta_k\|$, one sufficient small value ξ is chosen in priori. Substituting above gradient computation $\bigtriangledown J(\theta_k)$ into that iterative value, we get

$$\begin{aligned}\|\theta_{k+1}-\theta_k\| &= \|\gamma\bigtriangledown J(\theta_k)\|=\|0.5\bigtriangledown J(\theta_k)\|\\ &= \frac{1}{N}\left\|\alpha(z)\phi_{uy}(w)-\alpha(z)\left[M^{-1}(z)-\alpha(z)\theta_k\right]\phi_y(w)\right\|\le\xi\end{aligned} \tag{19}$$

i.e.,

$$\frac{1}{N}\left\|\alpha(z)\phi_{uy}(w)-\alpha(z)M^{-1}(z)\phi_y(w)+\alpha(z)\alpha^T(z)\theta_k\phi_y(w)\right\|\le\xi \tag{20}$$

Applying triangle inequality on the left side of Equation (20), it yields

$$\frac{1}{N}\left\|\alpha(z)\phi_{uy}(w)-\alpha(z)M^{-1}(z)\phi_y(w)+\alpha(z)\alpha^T(z)\theta_k\phi_y(w)\right\|$$

$$\leq \frac{1}{N}\|\alpha(z)\phi_{uy}(w)\| + \left\|\alpha(z)M^{-1}(z)\phi_y(w)\right\| + \left\|\alpha(z)\alpha^T(z)\theta_k\phi_y(w)\right\| \leq \xi \tag{21}$$

Choose $\alpha(z)\alpha^T(z) = 1$, the above inequality is simplified as

$$\|\theta_k\phi_y(w)\| \leq N\xi + \phi_{uy}(w) + M^{-1}(z)\phi_y(w);$$
$$\|\theta_k\| \leq \frac{N\xi + \phi_{uy}(w) + M^{-1}(z)\phi_y(w)}{\phi_y(w)} = \frac{N\xi}{\phi_y(w)} + \frac{\phi_{uy}(w)}{\phi_y(w)} + M^{-1}(z) \tag{22}$$

Equation (22) tells us that the continuous estimation error $\|\theta_{k+1} - \theta_k\|$ is less than one given sufficient small value ξ, satisfies above Inequality (22). Generally, adaptive direct data driven control is formulated as the following algorithm.

Algorithm 1

Step 1: Collect input-output data sequence $\{u(t), y(t)\}_{t=1}^N$.
Step 2: Given one expected or desired reference model $M(z)$.
Step 3: Compute two kinds of power spectral $\{\phi_y(w), \phi_{uy}(w)\}$.
Step 4: For the parameterized controller $C(z, \theta) = \alpha(z)\theta$, use that iterative form (17) to compute parameter vector θ, and an initial parameter vector is set as $\theta_0 = 0.5I$, then list the generated sequence as that

$$\theta_0, \theta_1, \theta_2, \ldots, \theta_k, \theta_{k+1}, \ldots \tag{23}$$

Step 5: Testify if

$$\|\theta_k\| \leq \frac{N\xi}{\phi_y(w)} + \frac{\phi_{uy}(w)}{\phi_y(w)} + M^{-1}(z), \quad \xi = 0.1 \tag{24}$$

holds, then θ_k is deemed as the final controller parameter vector, or turn to step 4 again, until Equation (24) is satisfied.

4. **Lyapunov Analysis.** Based on our detailed description about direct data driven control, the complex modelling process about plant $P(z)$ is avoided, meaning that knowledge of unknown plant $P(z)$ is included implicitly in measured input-output data $\{u(t), y(t)\}_{t=1}^N$. Then our mission is to extract some information for feedback controller $C(z)$. To implement it more widely in engineering application, adaptive idea is proposed to identify the unknown controller parameters iteratively, as the parameterized controller exists in industry truly. After combining above ideas, we want to guarantee the closed loop output $y(t)$ tracks the desired output $y_0(t)$, and the parameter estimation θ approaches to its true value θ^*, i.e.,

$$u(t) = r(t) - C(z, \theta)y(t) = \left[M^{-1}(z) - C(z, \theta)\right] y(t); \quad \theta \to \theta^* \tag{25}$$

Above dual goals are considered simultaneously, i.e., the parameter estimation θ is unbiased so that the perfect tracking is achieved. To analyze these two aspects in Equation (25), Lyapunov analysis is worth. For the sake of completeness, two errors are defined as tracking error $e(t)$ and parameter estimation error $\tilde{\theta}$, i.e.,

$$e(t) = u(t) - \left[M^{-1}(z) - C(z, \theta)\right] y(t); \quad \tilde{\theta} = \theta - \theta^* \tag{26}$$

Then we have

$$e(t+1) - e(t) = [u(t+1) - u(t)] - \left[M^{-1}(z) - C(z, \theta)\right] [y(t) - y(t+1)];$$
$$\tilde{\theta}(t) = \theta(t) - \theta^*; \quad \tilde{\theta}(t+1) = \theta(t+1) - \theta^*; \quad \tilde{\theta}(t+1) - \tilde{\theta}(t) = \theta(t+1) - \theta(t) \tag{27}$$

To simplify the following mathematical notation, we rewrite above Equation (27) as

$$e(t+1) = e(t) + \Delta u(t) + \Delta y(t);$$
$$\tilde{\theta}(t+1) = \tilde{\theta}(t) + \Delta\theta(t); \quad \Delta u(t) = u(t+1) - u(t);$$
$$\Delta y(t) = \left[M^{-1}(z) - C(z,\theta)\right][y(t) - y(t+1)]; \quad \Delta\theta(t) = \theta(t+1) - \theta(t) \tag{28}$$

where t means the iterative step. Construct one Lyapunov function as

$$V\left(e, \tilde{\theta}, t\right) = \frac{e^2(t)}{2} + \frac{\tilde{\theta}^2(t)}{2\gamma} = V(t) > 0 \tag{29}$$

where above $V\left(e, \tilde{\theta}, t\right)$ is obviously positive. Let us compute the following tedious mathematical computations.

$$V(t+1) = \frac{e^2(t+1)}{2} + \frac{\tilde{\theta}^2(t+1)}{2\gamma} > 0;$$
$$V(t+1) - V(t) = \frac{1}{2}[e(t) + \Delta u(t) + \Delta y(t)][e(t) + \Delta u(t) + \Delta y(t)]^T - \frac{1}{2}e(t)e^T(t)$$
$$+ \frac{1}{2\gamma}\left[\tilde{\theta}(t) + \Delta\theta(t)\right]\left[\tilde{\theta}(t) + \Delta\theta(t)\right]^T - \tilde{\theta}(t)\tilde{\theta}^T(t)$$
$$= e(t)[\Delta u(t) + \Delta y(t)] + [\Delta u(t) + \Delta y(t)]^2 + \tilde{\theta}(t)\Delta\theta(t) + \Delta\theta^2(t) \tag{30}$$

To guarantee $V(t+1) - V(t)$ be negative, we choose

$$e(t) = -2[\Delta u(t) + \Delta y(t)]; \quad \tilde{\theta}(t) = -2\Delta\theta(t) \tag{31}$$

so

$$V(t+1) - V(t) = -[\Delta u(t) + \Delta y(t)]^2 - \Delta\theta^2(t) < 0 \tag{32}$$

Substituting Equation (31) into (26), we have

$$e(t) = u(t) - \left[M^{-1}(z) - C(z,\theta)\right] y(t)$$
$$= -2\left[u(t+1) - u(t) - \left[M^{-1}(z) - C(z,\theta)\right][y(t+1) - y(t)]\right]$$
$$= -2u(t+1) + 2u(t) + 2\left[M^{-1}(z) - C(z,\theta)\right][y(t+1) - y(t)];$$
$$2u(t+1) - u(t) = \left[M^{-1}(z) - C(z,\theta)\right][2y(t+1) - y(t)] \tag{33}$$

and

$$\tilde{\theta}(t) = \theta(t) - \theta^* = -2(\theta(t+1) - \theta(t));$$
$$\theta(t+1) - \theta(t) = -\frac{1}{2}\theta(t) + \frac{1}{2}\theta^* \tag{34}$$

Multiplying $u(t)$ and $y(t)$ on both sides of Equation (33), it holds that

$$2\phi_{uy}(1) - \phi_{uy}(0) = \left[M^{-1}(z) - C(z,\theta)\right](2\phi_y(1) - \phi_y(0)) \tag{35}$$

which means

$$M^{-1}(z) - C(z,\theta) = \frac{2\phi_u(1) - \phi_u(0)}{2\phi_{uy}(1) - \phi_{uy}(0)}; \quad C(z,\theta) = M^{-1}(z) - \frac{2\phi_u(1) - \phi_u(0)}{2\phi_{uy}(1) - \phi_{uy}(0)} \tag{36}$$

and

$$M^{-1}(z) - C(z,\theta) = \frac{2\phi_{uy}(1) - \phi_{uy}(0)}{2\phi_y(1) - \phi_y(0)}; \quad C(z,\theta) = M^{-1}(z) - \frac{2\phi_{uy}(1) - \phi_{uy}(0)}{2\phi_y(1) - \phi_y(0)} \tag{37}$$

Equations (36) and (37) give two nonparametric controllers, being similar to our previous result in Equation (9). Similarly, Equation (34) means

$$\theta(t+1) = \theta(t) + \frac{1}{2}\theta^* \tag{38}$$

However, in Equation (38), the true parameter vector θ^* is needed. From both Equations (37) and (38), Lyapunov analysis is done to achieve our dual goals, i.e., perfect tracking and asymptotic unbiased estimation. The whole Lyapunov analysis for adaptive direct data driven control theory, is plotted in Figure 3, where the modelling process of plant $P(z)$ is avoided.

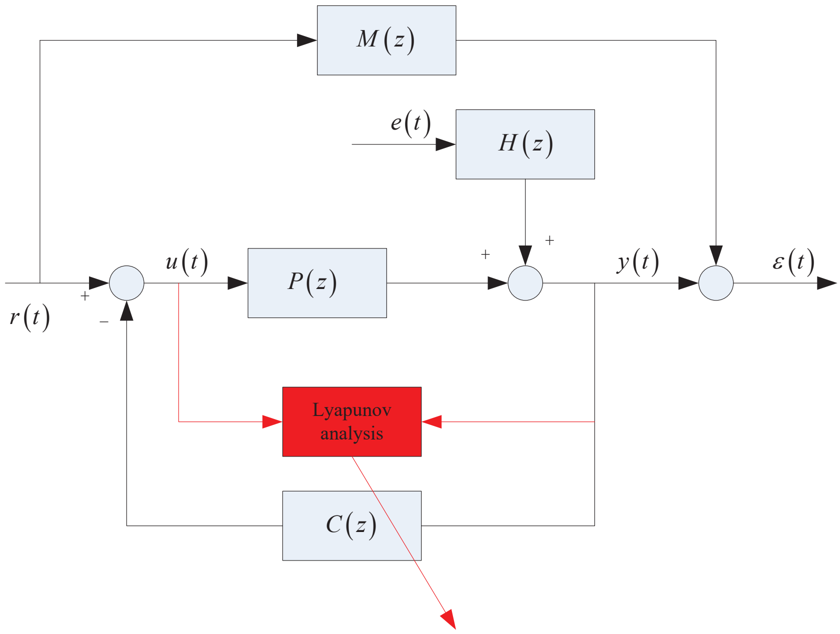


FIGURE 3. Lyapunov analysis

5. **Simulation.** In simulation example, aircraft is shown in Figure 4. The ground coordinate system $E(Oxyz)$ and the body coordinate system $B(Oxyz)$ are established to mathematically describe the position and state of aircraft. With the widespread application of aircraft in civilian and military domains, UAV warm technology has transitioned from concept to reality. This paper focuses on the study of fixed-wing aircraft swarms, addressing key issues related to formation control and collaborative strategies in dynamic

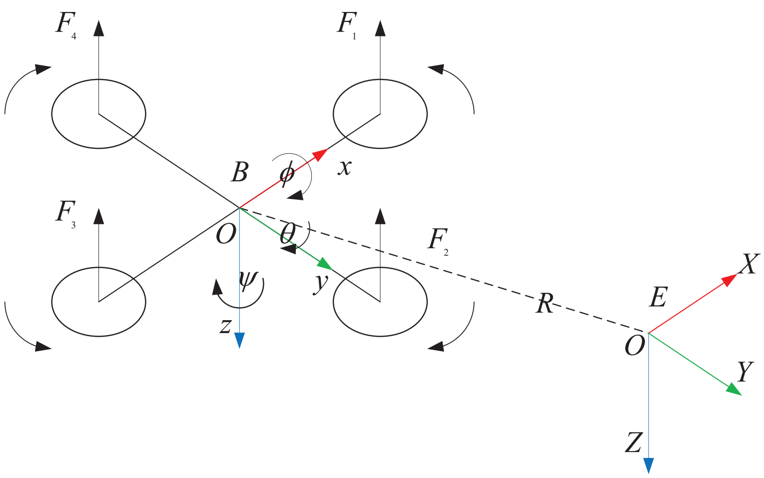


FIGURE 4. Body coordinate system and ground coordinate system

environments. The research explores theoretical aspects and technological implementations confronting challenges such as limited models, control constraints, and scale limitations from the perspectives of model control, data-driven control, and swarm intelligence algorithms. Simulation experiments have been conducted to achieve formation aggregation, maintenance, and reconstruction control of fixed-wing aircraft swarms, as well as collaborative obstacle avoidance and multi-target allocation strategies.

The structure of the four-rotor flight control system is shown in Figure 5, where a dual-loop control structure is adopted, in which the inner loop controls the attitude and the outer loop controls the position.

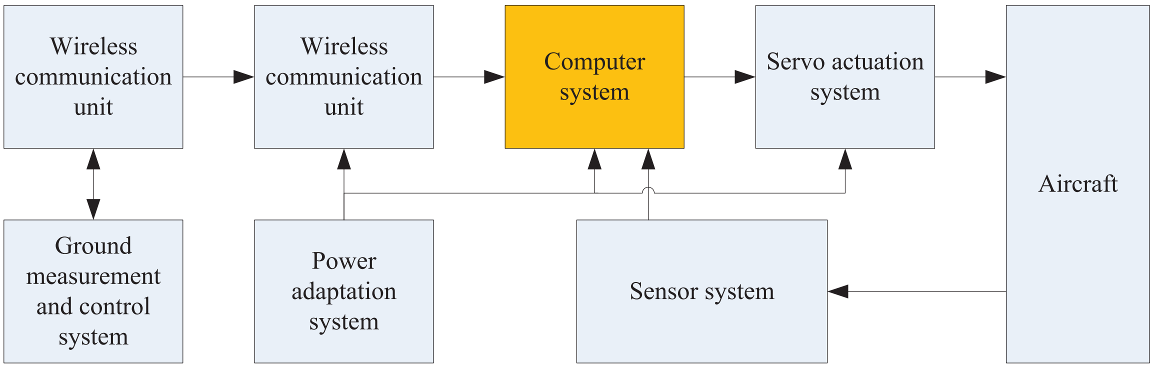


FIGURE 5. Aircraft control system structure

Because the design of the four-rotor controller is the same, the control effect of its position and attitude is similar. The following only takes the position control as an example and compares the control effects for different disturbances. Within the closed loop system structure of Figure 5, there are two parts, i.e., aircraft model and feedback controller. Our considered adaptive direct data driven control strategy is to design that unknown feedback control only through the measured input-output signal, while avoiding the modelling process for that unknown plant. For clarity of simulation, plant model and ideal controller are assumed to be

$$P(z) = \frac{0.7z + 1.2}{z^2 - 0.7z + 0.5}; \quad C(z) = \frac{0.6z^2 - 0.2z}{z^2 - 1.3z + 0.6} \tag{39}$$

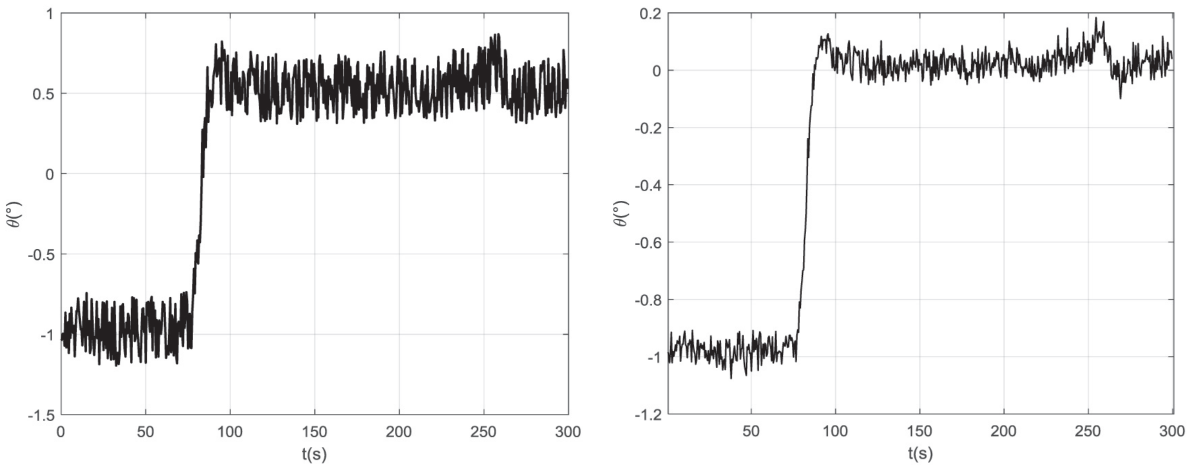
To testify the asymptotic unbiased parameter estimations and perfect tracking performance, the desired or expected transfer function is set at

$$M(z) = \frac{0.6z^2 + 0.8z + 1.2}{z^2 - 0.75z + 0.7} \tag{40}$$

Remember that direct data driven control uses the measured input-output signal to extract some important information about that unknown feedback controller, so it is urgent to collect the measured input-output signal after the aircraft starts to fly. Figure 6 gives the measured input-output signal sequence through some physical devices. From Figure 6, we see some certainty of disturbance of noise exists in collecting, so to filter these disturbance and noise, one filter device is used to do it.

As adaptive idea is combined to identify the unknown controller parameters iteratively, here the unknown controller parameters are combined together to form one unknown parameter vector.

$$\theta = [0.6 \ \ -0.2 \ \ -1.3 \ \ 0.6]^T \tag{41}$$



(a) Measured input signal (b) Measured output signal

FIGURE 6. Measured input-output signal

Then adaptive direct data driven control strategy is applied to identifying that unknown controller parameter vector in an iterative form. Furthermore, our proposed parameter identification method is used to identify the controller parameter, and also that unknown plant parameter. Figure 7 and Figure 8 give our identification results, which correspond to their true values. It means the controller parameter estimations approach to their real or true values, and then the unbiased property of parameter estimations is achieved.

Furthermore, to show the perfect tracking property of our adaptive idea, adaptive direct data driven control strategy transforms the parameterized controller design problem into one parameter estimation problem. Based on the given transfer function $M(z)$, the desired flight trajectory is potted in Figure 9, which shows aircraft fly according to this given trajectory. Latter, our designed parameterized controller needs to guarantee the considered aircraft fly with the given one in Figure 9. From above description, after the desired

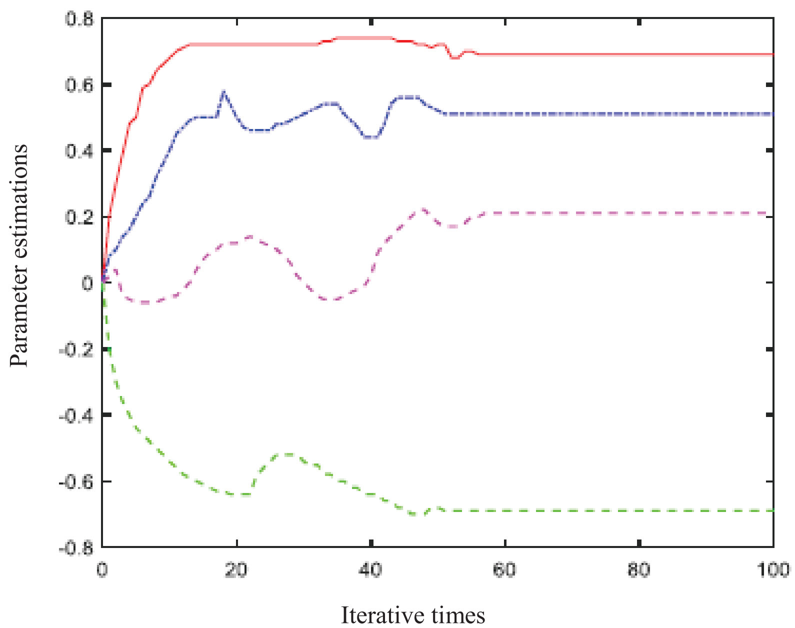


FIGURE 7. Parameter estimation result for plant

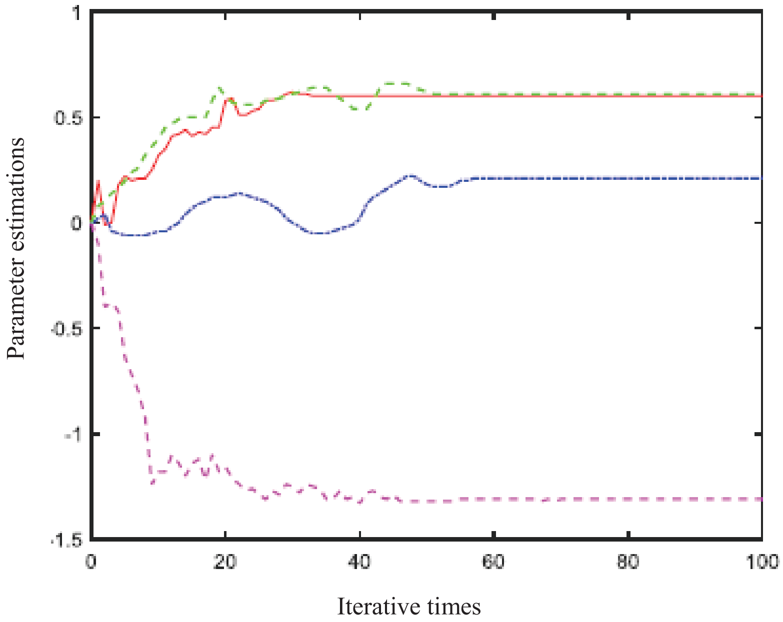


FIGURE 8. Parameter estimation result for controller

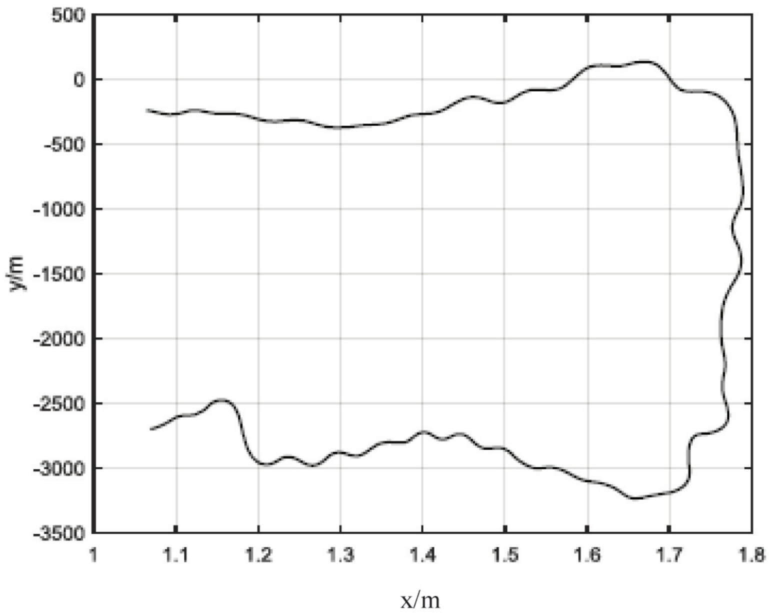


FIGURE 9. Tracking trajectory

or expected transfer function $M(z)$, and the parameterized controller $C(z, \theta)$, then our proposed adaptive direct data driven control changes the controller design problem into one parameter estimation problem, i.e., identifying or estimating that unknown controller parameters θ.

6. **Conclusion.** Based on our previous contributions about direct data driven control, and our new plant-aircraft, we find it is impossible and urgent to introduce adaptive idea into direct data driven control. After showing the basic result, the detailed mathematical derivations are shown to give the nonparametric controller and parameterized controller adaptively. Moreover, to guarantee both perfect tracking and asymptotic unbias controller, Lyapunov function is established to derive the accurate adaptive controller. Due to space

limitation, only one paper cannot cover all aspects, so next we will concentrate on robust analysis for adaptive direct data driven control in case on bounded uncertainty.

Acknowledgment. This work is partially supported by Jiangxi Provinical National Science Foundation (No. 20232BAB201015).

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