

## CONTROLLER DESIGN FOR UNMANNED BICYCLES BASED ON TENSOR PRODUCT MODEL TRANSFORMATION AND VARIABLE UNIVERSE FUZZY CONTROL

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**ABSTRACT.** *In this paper, a tensor product variable universe fuzzy (TPVUF) controller is designed for stabilizing the balance of unmanned bicycles. Firstly, the nonlinear exact model of the unmanned bicycles is obtained using Kane's method, and then the tensor product (TP) model transformation technique is used to derive the tensor product model of the unmanned bicycles. Subsequently, the TPVUF controller utilizes the gain calculated by the parallel distributed compensation (PDC) method as fusion coefficients of the error and its rate of change in the variable universe fuzzy (VUF) method. The VUF control method has the ability to quickly converge the error and its rate of change, which improves the response speed of the TPVUF controller. Finally, simulation experiments validate the effectiveness of the designed controller.*

**Keywords:** Unmanned bicycles, Quasi-linear parametric model, Tensor product model transformation, Parallel distributed compensation, Variable universe fuzzy control

1. **Introduction.** As a widely used means of transportation in daily life, bicycles are typically under-actuated systems. The study of stabilization control of unmanned bicycles is of great significance in verifying the accuracy of modeling under-actuated system and evaluating the performance of the controller. Therefore, the research in related fields has attracted extensive attention from scholars.

In recent years, various classical control methods have been employed to solve the balance control problem of unmanned bicycles. To maintain the balance control of the unmanned bicycles, a proportional integral derivative (PID) controller is used in [1] to track the inclination of the bicycle body by adjusting the handlebar rotation angle. The variable gain linear quadratic regulator (VGLQR) method [2] is proposed for real-time balance control of variable-speed unmanned bicycles. To further improve the stability of the system, the active disturbance rejection controller (ADRC) [3] is applied to the balance control of the unmanned bicycles. In [4], sliding mode control (SMC) is used for the stabilization control of unmanned bicycles to improve the robustness of the system. Moreover, some control methods have been applied to unmanned bicycles control, such as

fuzzy control methods [5,6] and neural network methods [7,8], which have achieved good results in the field of unmanned bicycles balance control.

The unmanned bicycles are non-linear models, and the accurate models can restore the properties of the system more completely and improve the stability and robustness of the systems. Therefore, achieving equilibrium control for the nonlinear model is an interesting problem. The TP model transformation [9,10] is a kind of numerical computation method for quasi-linear parametric varying (qLPV) model controller design. This method transforms a complex qLPV model into a tensor product model via high-order singular value decomposition (HOSVD). The TP model is a multicellular system composed of nonlinear weight functions and linear time-invariant matrices, which can approximate the original nonlinear model as closely as possible, and facilitates controller design in conjunction with linear matrix inequality (LMI). Therefore, the TP model transformation is widely used in the controller design for under-actuated systems, such as self-balancing robots [11] and quadrotor unmanned aerial vehicle systems [12]. In addition, path planning [13] for unmanned vehicles is an important part of autonomous driving.

Motivated by these facts, this paper aims to provide a robust controller for balancing the unmanned bicycle. Compared to the traditional unmanned bicycle model after linearization of the equilibrium point, the proposed model can describe the nonlinear features of the unmanned bicycle. The motivation and contributions of this paper are as follows.

1) Compared with the common equilibrium point linearized model, the TP model of the unmanned bicycle system based on the TP model transformation is able to achieve a good balance between model accuracy and computational complexity, and improve the performance of the controller in terms of model accuracy.

2) Based on the high efficiency of the VUF method for error convergence, this paper proposes the TPVUF controller combined with the PDC method. It can achieve fast and precise control of the error by dynamically adjusting the thesis domain, thus enhancing the adaptability and robustness in complex environments.

This paper is organized as follows. In Section 2, the TP model of unmanned bicycles is introduced. Section 3 presents the design of the TPVUF controller based on the TP model and analyzes the stability of the controller. In Section 4, simulation experiments are conducted to verify the effectiveness of the designed controller. Finally, the conclusion of the paper is given.

**2. Unmanned Bicycle Model.** The unmanned bicycle consists of four main parts: the body, the front fork, the front and rear wheels. The model of the unmanned bicycles [14] is shown below:

$$\dot{\mathbf{x}} = [\mathbf{J}(\mathbf{p}(t))^{-1} \mathbf{A}_0(\mathbf{p}(t)) \mathbf{J}(\mathbf{p}(t))^{-1} \mathbf{B}_0(\mathbf{p}(t))] \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} = [\mathbf{A}(\mathbf{p}(t)) \quad \mathbf{B}(\mathbf{p}(t))] \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} \quad (1)$$

where  $\mathbf{A}$  and  $\mathbf{B}$  are parameter matrices dependent on the variable  $\mathbf{p}(t)$ , and their specific expressions are provided in [14],  $\mathbf{x}$  is the state variable and  $\mathbf{u}$  is the input quantity.

The system matrix of the qLPV model (1) for unmanned bicycles can be expressed as

$$\mathbf{S}(\mathbf{p}(t)) = [\mathbf{A}(\mathbf{p}(t)) \quad \mathbf{B}(\mathbf{p}(t))] \quad (2)$$

The system matrix  $\mathbf{S}(\mathbf{p}(t))$  is related to the partial states of  $\mathbf{x} = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [\varphi, \theta, \psi, \dot{\varphi}, \dot{\theta}, \dot{\psi}]^T$ , where  $\varphi$  and  $\theta$  are the body lean angle and handlebar turn angle respectively, and their rates of change are  $\dot{\varphi}$  and  $\dot{\theta}$ , and  $\dot{\psi}$  is the change rate of the angular displacement of the rear wheel  $\psi$ .

The parameter vector of the system matrix is  $\mathbf{p}(t) = [p_1, p_2, p_3, p_4, p_5]^T = [x_1, x_2, x_3, x_4, x_5, x_6]^T$ , and divides the range of values  $[-\pi/7, \pi/7] \times [-\pi/6, \pi/6] \times [-\pi/5, \pi/5] \times [-\pi/5, \pi/5] \times [-\pi/5, \pi/5]$  equally spaced into  $67 \times 67 \times 17 \times 17 \times 17$ , and the number of rules is retained as  $3 \times 3 \times 2 \times 2 \times 2 = 72$ . The system matrix can be converted into a combination of nonlinear weight functions and linear time-invariant matrices:

$$\mathbf{S}(\mathbf{p}(t)) = \sum_{i=1}^{72} w_i(\mathbf{p}(t)) \mathbf{S}_i \tag{3}$$

where  $\mathbf{S}_i$  is a linear time-invariant vertex matrix, and  $w_i(\mathbf{p}(t))$  is the weighting function.

Substituting (3) into (1), the state space equations of the unmanned bicycles can be written in the form of TP model:

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{72} w_i(\mathbf{p}(t)) \mathbf{S}_i \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} = \sum_{i=1}^{72} w_i(\mathbf{p}(t)) (\mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i \mathbf{u}(t)) \tag{4}$$

**3. Variable Universe Fuzzy Controller Design Based on TP Model Transformation.** Since the unmanned bicycle is a complex nonlinear system, this chapter combines the VUF control method and the PDC method, and proposes the TPVUF controller to solve the control problem of the complex system. The control structure of the controller is shown in Figure 1.

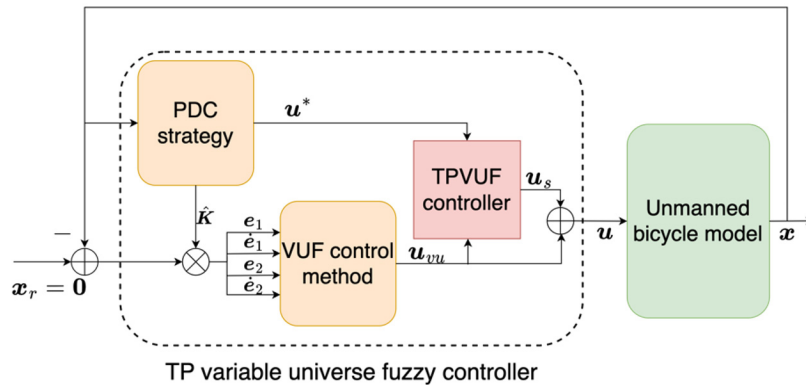


FIGURE 1. Schematic structure of TP variable universe fuzzy controller

As shown in Figure 1, the feedback gains  $\mathbf{K}$  obtained by the PDC method can be normalized to obtain  $\hat{\mathbf{K}}$ , which serves as the fusion coefficient for the error and its rate of change in the VUF controller. This integration allows the error and its rate of change to be adjusted without the need for precise tuning. Finally, the unmanned bicycle system is stabilized by designing the TPVUF controller.

First, the PDC control strategy is presented. The control output of the PDC method can usually be written as

$$\mathbf{u}^*(t) = - \sum_{i=1}^l w_i(\mathbf{p}(t)) \mathbf{K}_i \mathbf{x}(t) \triangleq - \sum_{i=1}^l \mathbf{K} \mathbf{x}(t) \tag{5}$$

where  $\mathbf{K}$  is the feedback gain matrix.

Substituting (5) into the state space (4) of the unmanned bicycle system, the closed-loop state space equation of the system is obtained:

$$\dot{\mathbf{x}}(t) = \sum_{i,j=1}^l w_i(\mathbf{p}(t)) w_j(\mathbf{p}(t)) [(\mathbf{A}_i - \mathbf{B}_i \mathbf{K}_j) \mathbf{x}(t)] \tag{6}$$

For the unmanned bicycle closed-loop system model (6), the PDC controller is stabilized when the following lemmas are satisfied.

**Lemma 3.1.** [15] *For the closed-loop state-space model (6) of the unmanned bicycles, if there exist a common positive definite matrix  $\mathbf{X}$ ,  $\mathbf{M}_i$ , and decay rate  $\alpha > 0$  satisfying the following LMI stability condition:*

$$-\mathbf{X}\mathbf{A}_i^T - \mathbf{A}_i\mathbf{X} + \mathbf{M}_i^T\mathbf{B}_i^T + \mathbf{B}_i\mathbf{M}_i - 2\alpha\mathbf{X} > 0 \quad (7)$$

and

$$-\mathbf{X}\mathbf{A}_i^T - \mathbf{A}_i\mathbf{X} - \mathbf{X}\mathbf{A}_j^T - \mathbf{A}_j\mathbf{X} + \mathbf{M}_j^T\mathbf{B}_i^T + \mathbf{B}_i\mathbf{M}_j + \mathbf{M}_i^T\mathbf{B}_j^T + \mathbf{B}_j\mathbf{M}_i - 4\alpha\mathbf{X} \geq 0 \quad (8)$$

for  $i < j < l$  holds (In addition to the index pairs  $(i, j)$  that makes  $w_i(\mathbf{p}(t))w_j(\mathbf{p}(t)) = 0$ ), then the system is asymptotically stable over the entire state space.

**Lemma 3.2.** [15] *Suppose  $\|\mathbf{x}(0)\|_2 \leq \phi$ , where the upper limit value  $\phi$  of the initial state  $\mathbf{x}(0)$  is known, but  $\mathbf{x}(0)$  is unknown. If the following condition is satisfied:*

$$\phi^2\mathbf{I} \leq \mathbf{X} \quad (9)$$

and

$$\begin{pmatrix} \mathbf{X} & \mathbf{M}_i^T \\ \mathbf{M}_i & u_{\max}^2\mathbf{I} \end{pmatrix} \geq \mathbf{0} \quad (10)$$

for any moment  $t > 0$ , the controller outputs of the system satisfy  $\|\mathbf{u}^*(t)\|_2 \leq u_{\max}$ .

By solving the LMI stability conditions in Lemmas 3.1 and 3.2, the matrices  $\mathbf{X}$  and  $\mathbf{M}_i$  can be obtained, which in turn allow to calculate the gain matrix of the system:

$$\mathbf{K}_i = \mathbf{M}_i\mathbf{X}^{-1} \quad (11)$$

By solving the feasible solution of the LMI stability condition, the gain matrix  $\mathbf{K}$  of the PDC controller and the control output  $\mathbf{u}^*$  can be obtained. Subsequently, combining with the output  $\mathbf{u}_{vu}$  of the VUF controller can be carried out the design of the stability condition of the TPVUF controller, and the stability theorem and proof of the system are shown below.

**Theorem 3.1.** *For the TP model (4) of the unmanned bicycles, the input of the TPVUF controller is designed as  $\mathbf{u} = \mathbf{u}_{vu} + \mathbf{u}_s$ . If the control quantity  $\mathbf{u}_s$  is designed as*

$$\mathbf{u}_s = \begin{cases} \mathbf{0}_{2*1}, & \bar{\mathbf{u}} \leq \alpha_1\mathbf{x}^T\mathbf{P}\mathbf{x}/2 \\ -(\|\mathbf{u}_{vu}\|_2 + \|\mathbf{K}\mathbf{x}\|_2) [\text{sgn}(\mathbf{x}^T\mathbf{P}\mathbf{B}(1)), \text{sgn}(\mathbf{x}^T\mathbf{P}\mathbf{B}(2))]^T, & \bar{\mathbf{u}} > \alpha_1\mathbf{x}^T\mathbf{P}\mathbf{x}/2 \end{cases} \quad (12)$$

where  $\bar{\mathbf{u}} = \sum_{i=1}^l \|w_i\mathbf{x}^T\mathbf{P}\mathbf{B}_i\|_2 (\|\mathbf{u}_{vu}\|_2 + \|\mathbf{K}\mathbf{x}\|_2)$ , then the corresponding unmanned bicycle system is globally asymptotically stable.

**Proof:** Choosing the Lyapunov candidate function as  $V(\mathbf{x}) = \mathbf{x}^T\mathbf{P}\mathbf{x}$ , to prove the stability of the above system, we differentiate the equation:

$$\begin{aligned} \dot{V}(\mathbf{x}) &= \frac{1}{2}\dot{\mathbf{x}}^T\mathbf{P}\mathbf{x} + \frac{1}{2}\mathbf{x}^T\mathbf{P}\dot{\mathbf{x}} \\ &= \frac{1}{2}\sum_{i=1}^l w_i (\mathbf{x}^T\mathbf{A}_i^T + \mathbf{u}^T\mathbf{B}_i^T) \mathbf{P}\mathbf{x} + \frac{1}{2}\sum_{i=1}^l w_i\mathbf{x}^T\mathbf{P}(\mathbf{A}_i\mathbf{x} + \mathbf{B}_i\mathbf{u}) \end{aligned} \quad (13)$$

Based on the PDC controller, the intermediate control input  $\mathbf{u}^* = -\sum_{i=1}^l w_i\mathbf{K}_i\mathbf{x}(t)$  is calculated. Introducing it into (13), we can obtain

$$\dot{V}(\mathbf{x}) = \frac{1}{2}\sum_{i=1}^l w_i (\mathbf{x}^T\mathbf{A}_i^T + \mathbf{u}^T\mathbf{B}_i^T + \mathbf{u}^{*T}\mathbf{B}_i^T - \mathbf{u}^{*T}\mathbf{B}_i^T) \mathbf{P}\mathbf{x}$$

$$\begin{aligned}
 & + \frac{1}{2} \sum_{i=1}^l w_i \mathbf{x}^T \mathbf{P} (\mathbf{A}_i \mathbf{x} + \mathbf{B}_i \mathbf{u}^* - \mathbf{B}_i \mathbf{u}^* + \mathbf{B}_i \mathbf{u}) \\
 & = \sum_{i=1}^l \sum_{j=1}^l w_i w_j \left\{ \frac{1}{2} \mathbf{x}^T [(\mathbf{A}_i - \mathbf{B}_i \mathbf{K}_j)^T \mathbf{P} + \mathbf{P}(\mathbf{A}_i - \mathbf{B}_i \mathbf{K}_j)] \mathbf{x} \right. \\
 & \quad \left. + \mathbf{x}^T \mathbf{P} \mathbf{B}_i (\mathbf{u} - \mathbf{u}^*) \right\} \tag{14}
 \end{aligned}$$

where  $\mathbf{P} = \mathbf{X}^{-1}$  and  $\mathbf{K}_j$  can be obtained through Lemmas 3.1 and 3.2.

Let  $\mathbf{G}_{ij} = \mathbf{A}_i - \mathbf{B}_i \mathbf{K}_j$ , and substitute it into (14):

$$\dot{V}(\mathbf{x}) = \sum_{i=1}^l \sum_{j=1}^l w_i w_j \left[ \frac{1}{2} \mathbf{x}^T (\mathbf{G}_{ij}^T \mathbf{P} + \mathbf{P} \mathbf{G}_{ij}) \mathbf{x} + \mathbf{x}^T \mathbf{P} \mathbf{B}_i (\mathbf{u} - \mathbf{u}^*) \right] \tag{15}$$

To adjust the response speed of the system, a decay rate  $\alpha_1 > 0$  is introduced into the stability condition of the system. Thus, the continuous system satisfying the Lyapunov stability condition  $\dot{V}(\mathbf{x}) \leq -2\alpha_1 V(\mathbf{x})$  is equivalent to [15]

$$\mathbf{G}_{ii}^T \mathbf{P} + \mathbf{P} \mathbf{G}_{ii} + 2\alpha_1 \mathbf{P} < 0 \tag{16}$$

for all  $i$ , and

$$\left( \frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2} \right)^T \mathbf{P} + \mathbf{P} \left( \frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2} \right) + 2\alpha_1 \mathbf{P} \leq 0 \tag{17}$$

Using (16) and (17) to scale (15), and substituting the output of the PDC controller  $\mathbf{u}^* = -\sum_{i=1}^l w_i \mathbf{K}_i \mathbf{x}(t)$  into (15), we have

$$\dot{V}(\mathbf{x}) \leq \sum_{i=1}^l \sum_{j=1}^l w_i w_j \left( -\frac{1}{2} \alpha \mathbf{x}^T \mathbf{P} \mathbf{x} \right) + \sum_{i=1}^l w_i \left[ \mathbf{x}^T \mathbf{P} \mathbf{B}_i \left( \mathbf{u} + \sum_{j=1}^l \mathbf{K}_j \mathbf{x} \right) \right] \tag{18}$$

Furthermore, substitute the input  $\mathbf{u} = \mathbf{u}_{vu} + \mathbf{u}_s$  into (18) and rearrange

$$\begin{aligned}
 \dot{V}(\mathbf{x}) & \leq \sum_{i=1}^l \sum_{j=1}^l w_i w_j \left( -\frac{1}{2} \alpha \mathbf{x}^T \mathbf{P} \mathbf{x} \right) + \sum_{i=1}^l w_i \left[ \mathbf{x}^T \mathbf{P} \mathbf{B}_i \left( \mathbf{u}_{vu} + \mathbf{u}_s + \sum_{j=1}^l \mathbf{K}_j \mathbf{x} \right) \right] \\
 & \leq -\frac{1}{2} \alpha \mathbf{x}^T \mathbf{P} \mathbf{x} + \sum_{i=1}^l \|w_i \mathbf{x}^T \mathbf{P} \mathbf{B}_i\|_2 (\|\mathbf{u}_{vu}\|_2 + \|\mathbf{K} \mathbf{x}\|_2) + \sum_{i=1}^l w_i \mathbf{x}^T \mathbf{P} \mathbf{B}_i \mathbf{u}_s \tag{19}
 \end{aligned}$$

Let  $\bar{\mathbf{u}} = \sum_{i=1}^l \|w_i \mathbf{x}^T \mathbf{P} \mathbf{B}_i\|_2 (\|\mathbf{u}_{vu}\|_2 + \|\mathbf{K} \mathbf{x}\|_2)$ , and (19) can be written as

$$\dot{V}(\mathbf{x}) \leq -\frac{1}{2} \alpha \mathbf{x}^T \mathbf{P} \mathbf{x} + \bar{\mathbf{u}} + \sum_{i=1}^l w_i \mathbf{x}^T \mathbf{P} \mathbf{B}_i \mathbf{u}_s \tag{20}$$

where  $\alpha \mathbf{x}^T \mathbf{P} \mathbf{x} / 2 \geq 0$  always holds. When  $\bar{\mathbf{u}} \leq \alpha \mathbf{x}^T \mathbf{P} \mathbf{x} / 2$ , let  $\mathbf{u}_s = \mathbf{0}$ ,  $\dot{V}(\mathbf{x}) \leq 0$  holds.

When  $\bar{\mathbf{u}} > \alpha \mathbf{x}^T \mathbf{P} \mathbf{x} / 2$ , the design of  $\mathbf{u}_s$  is required. Further scale the (20):

$$\begin{aligned}
 \dot{V}(\mathbf{x}) & \leq -\frac{1}{2} \alpha \mathbf{x}^T \mathbf{P} \mathbf{x} + \|\mathbf{x}^T \mathbf{P} \mathbf{B}\|_2 (\|\mathbf{u}_{vu}\|_2 + \|\mathbf{K} \mathbf{x}\|_2) + \mathbf{x}^T \mathbf{P} \mathbf{B} \mathbf{u}_s \\
 & \leq -\frac{1}{2} \alpha \mathbf{x}^T \mathbf{P} \mathbf{x} + \|\mathbf{x}^T \mathbf{P} \mathbf{B}\|_1 (\|\mathbf{u}_{vu}\|_2 + \|\mathbf{K} \mathbf{x}\|_2) + \mathbf{x}^T \mathbf{P} \mathbf{B} \mathbf{u}_s \tag{21}
 \end{aligned}$$

Substitute the input  $\mathbf{u}_s = -(\|\mathbf{u}_{vu}\|_2 + \|\mathbf{K}\mathbf{x}\|_2) [\text{sgn}(\mathbf{x}^\top \mathbf{P}\mathbf{B}(1)), \text{sgn}(\mathbf{x}^\top \mathbf{P}\mathbf{B}(2))]^\top$  into (21):

$$\begin{aligned} \dot{V}(\mathbf{x}) &\leq -\frac{1}{2}\alpha_1 \mathbf{x}^\top \mathbf{P}\mathbf{x} + (\|\mathbf{u}_{vu}\|_2 + \|\mathbf{K}\mathbf{x}\|_2) \|\mathbf{x}^\top \mathbf{P}\mathbf{B}\|_1 \\ &\quad - (\|\mathbf{u}_{vu}\|_2 + \|\mathbf{K}\mathbf{x}\|_2) \mathbf{x}^\top \mathbf{P}\mathbf{B} \cdot [\text{sgn}(\mathbf{x}^\top \mathbf{P}\mathbf{B}(1)), \text{sgn}(\mathbf{x}^\top \mathbf{P}\mathbf{B}(2))]^\top \\ &\leq -\frac{1}{2}\alpha_1 \mathbf{x}^\top \mathbf{P}\mathbf{x} + (\|\mathbf{u}_{vu}\|_2 + \|\mathbf{K}\mathbf{x}\|_2) [|\mathbf{x}^\top \mathbf{P}\mathbf{B}(1)| - \mathbf{x}^\top \mathbf{P}\mathbf{B}(1) \cdot \text{sgn}(\mathbf{x}^\top \mathbf{P}\mathbf{B}(1)) \\ &\quad + |\mathbf{x}^\top \mathbf{P}\mathbf{B}(2)| - \mathbf{x}^\top \mathbf{P}\mathbf{B}(2) \cdot \text{sgn}(\mathbf{x}^\top \mathbf{P}\mathbf{B}(2))] \end{aligned} \quad (22)$$

Since  $|\mathbf{x}^\top \mathbf{P}\mathbf{B}(1)| - \mathbf{x}^\top \mathbf{P}\mathbf{B}(1) \cdot \text{sgn}(\mathbf{x}^\top \mathbf{P}\mathbf{B}(1)) = 0$  holds,  $\dot{V}(\mathbf{x}) \leq -1/2\alpha_1 \mathbf{x}^\top \mathbf{P}\mathbf{x} \leq 0$ . Therefore, when the control input of the TPVUF is designed as (12), the Lyapunov stability condition  $\dot{V}(\mathbf{x}) \leq 0$  holds, indicating that the unmanned bicycle system remains in a globally asymptotically stable state.

The sign function  $\text{sgn}(x)$  in (12) exhibits discontinuities near the zero point due to step jumps, which can lead to chattering of the system in the steady state.  $\text{sgn}(x)$  is expressed as

$$\text{sgn}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases} \quad (23)$$

In order to further minimize the chattering phenomenon and thus improve the performance of the controller, we replace the sign function  $\text{sgn}(x)$  in (12) with function  $\text{sat}(\cdot)$ . The expression for the saturation function  $\text{sat}(\cdot)$  is given by

$$\text{sat}(\Psi) = \begin{cases} \text{sgn}(\Psi), & |\Psi| \geq 1 \\ \Psi, & |\Psi| < 1 \end{cases} \quad (24)$$

where  $\Psi = \mathbf{x}^\top \mathbf{P}\mathbf{B}(i)/\delta$  ( $i = 1, 2$ ) and  $\delta$  is the width of the boundary layer.

Based on the TP fuzzy model of the unmanned bicycles, the TPVUF controller utilizes the PDC control method for solving the feasible solution of the system and combines it with the VUF control method for regulating the system states. The unmanned bicycle is a nonlinear system, which can be transformed into the TP model by TP model transformation, thus transforming the design of the PDC controller into a problem of solving the LMI stability condition. Meanwhile, the VUF control method has the advantage of fast convergence, which can effectively improve the response speed of the controller.

**4. Simulation Results.** In this section, the control performance of the TPVUF controller is verified by experimental simulation. Parameters of unmanned bicycles are selected from [14] and the TPVUF controller parameters are set as  $\alpha = 0.25$ ,  $\phi = 0.656$ ,  $u_{\max} = 2200$ ,  $\alpha_1 = 0.01$ ,  $\delta = 0.01$ .

This experiment conducts numerical simulations on the unmanned bicycle system under different controllers. LQR and VUF are two classical control methods that are widely applied in the balance control of underactuated systems. Therefore, this paper adopts these two methods as the comparative algorithms. Furthermore, to verify the effectiveness of the VUF method in the TPVUF controller, a TPDC controller based on TP model transformation and PDC method is used for comparative experiments. The initial state of the unmanned bicycles is set as  $\mathbf{x}_0 = [0.1, -0.2, 0, 0, 0, 0]^\top$ , and the simulation runtime is set to 20 s. The variation of the system states of the unmanned bicycles is shown in Figure 2.

From Figure 2, it can be observed that under the LQR controller, the body angle and handlebar angle of the unmanned bicycles continuously fluctuate around the equilibrium

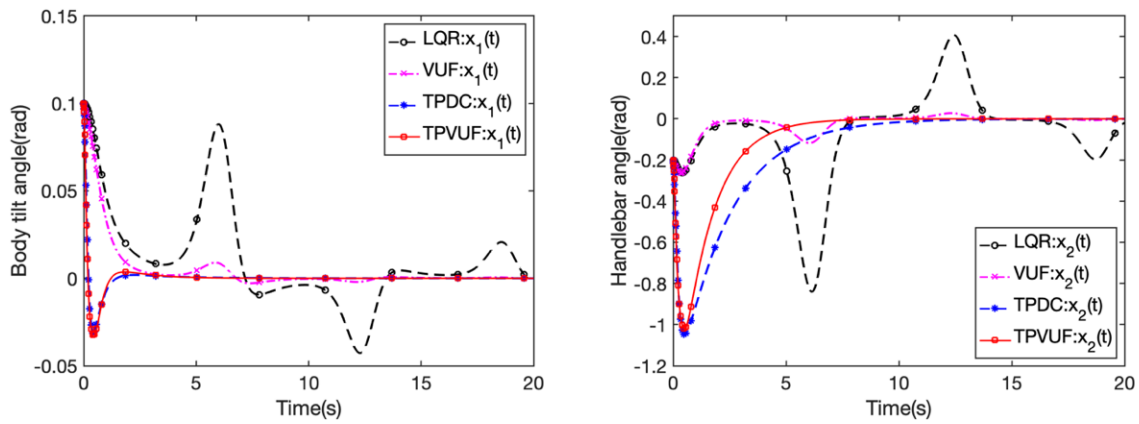


FIGURE 2. State change graph of unmanned bicycles under sedation conditions

point without reaching a stable state. However, under the VUF, TPDC and TPVUF controllers, the bicycle system can be balanced within a finite time. The simulation experiments show that the TPVUF controllers have a significant advantage over the LQR, VUF and TPDC controllers in adjusting the speed of the unmanned bicycle’s body angle and handlebar angle states, which shows a faster response speed.

To verify the anti-interference performance of the designed controller, the state of the unmanned bicycles is set to  $\mathbf{x}_0 = [0.1, -0.2, 0, 0, 0, 0]^T$ . A disturbance of 0.1 rad lasting for 1 s is added to the body inclination at the 10th second of traveling, and the simulation duration is set to 30 s. The evolution of the system states is shown in Figure 3.

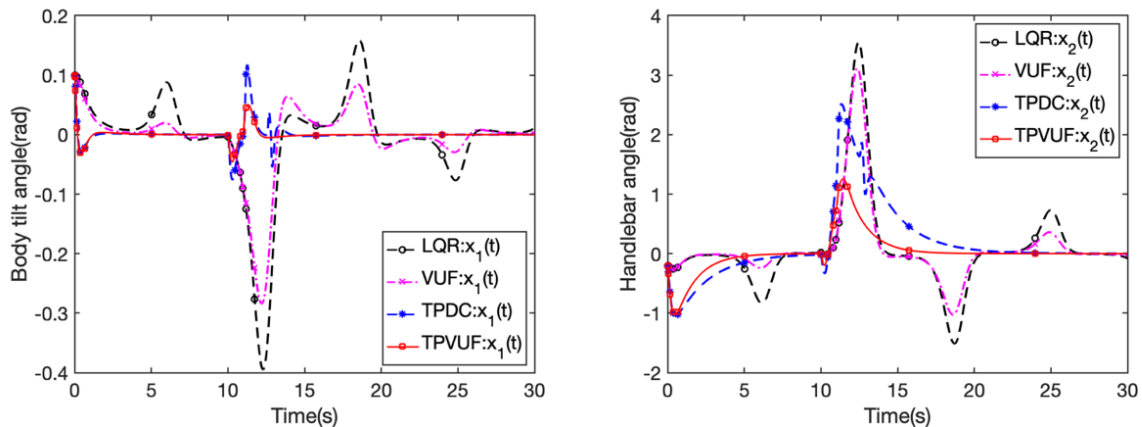


FIGURE 3. State change graph of unmanned bicycles under collision-disturbance conditions

From the state curves in Figure 3, it can be observed that after the unmanned bicycle system experiences disturbance at 10 s, the LQR and VUF controllers fail to adjust the system state. Instead, their states continue to fluctuate and fail to stabilize the system. The experimental results demonstrate that compared to the LQR, VUF, and TPDC controllers, the controller designed in this paper can rapidly adjust the bicycle’s state when subjected to external collision disturbances, showing faster response speed and stronger robustness.

**5. Conclusions.** This paper employs TP model transformation to obtain the TP model of the unmanned bicycle system. Based on this model, the TPVUF controller is designed

by integrating PDC method and VUF method for balance control of the unmanned bicycles. Experimental simulations indicate that the TPVUF controller exhibits good disturbance rejection and robustness. In future research, aiming at the complex outdoor bumpy road conditions, we will further enhance the anti-interference and robustness of the controller.

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