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OPTIMAL AREA FOR RECTANGULAR FOUNDATION SLABS IN PLAN SUPPORTED ON A GROUP OF PILES

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Abstract. *One of the few good things about the crisis is that it forces the researcher to generate new methods and paths to do things in the best way and, above all, in the most economical way. For this reason, this document shows the minimum or optimal area for rectangular foundation slabs in plan supported on a group of piles to support the entire building. Some authors present the complete design of foundation slabs supported on a group of piles based on the knowledge of the sides and area of the foundation slab, the distribution and load capacity of the piles. The model is formulated using optimization techniques to obtain the minimum area in plan and the sides of the foundation slab, the distribution and load capacity of the piles. Two numerical examples are presented with the same loads and moments due to each column: Example 1: Foundation slab supported on 9 piles; Example 2: Foundation slab supported on 25 piles. Each example presents five cases: Case I: Free sides; Case II: Sides constrained in the X direction on their opposite sides; Case III: Sides constrained in the Y direction on their opposite sides; Case IV: Sides constrained on all sides; Case V: Free sides and same load capacity of the piles. The main advantage of this study over other works is that the model shows the minimum area and sides of the foundation slab supported on a group of piles and can be used for rectangular isolated footings and rectangular combined footings.*
Keywords: Optimal or minimum area, Rectangular foundation slabs, Group of piles

1. **Introduction.** The foundation is a structural part that is located below the ground surface and that transmits the loads of the structure to the underlying soil or rock. All soils, when subjected to loads, compress and cause settlements in the supported structure. The two main requirements in foundation design are as follows: 1) The total settlement of the structure is restricted to a tolerably small amount; 2) Differential settlement of the different parts of the structure is eliminated.

The loads that come down from the structure to the superficial foundations rest directly on the ground, and in the deep foundations the loads those come from the structure rest on the caps of the piles or slabs and in turn are transmitted to the piles. Piles are column-shaped elements placed deep in the ground to help maintain the structure of any

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construction. They are used in geotechnical projects to ensure the stability of the ground and thus prevent landslides and collapses when constructing any building.

Foundation piles are structural elements that have a small cross section relation to their length, which transmit the loads that descend from the structure and from the foundation itself (piles caps or slabs) to the subsoil, with the purpose of obtaining the stability of the assembly.

Pile caps are thick slabs used to join a group of piles to support and transmit column loads to the piles.

Figure 1 shows the most common types of foundation slabs in plan supported on a group of piles, which are a) flat slabs; b) flat slabs with reinforcement under the columns; c) bidirectional and flat; d) flat with pedestal; e) with cell design; f) base walls as a rigid frame (In drawer).

For foundation slabs resting on the piles, the following requirements must be taken into account [1].

A. To determine the pile spacing, the following must be taken into account:

1) Total cost of the foundation;
2) Nature of the soil;
3) Pile group behavior;
4) Possible lifting or compaction of the soil causing damage to adjacent structures;
5) Cost of the foundation slab;
6) Effective size and length of the soil beam;
7) Type and size of the piles.

B. The piles must be placed in an adequate arrangement, so that the space between the piles oscillates between (2 and 3)D (diameter of the pile) in the case of slabs supported on isolated piles and (2 to 6)D in the case of slabs supported on piles.

C. The center of gravity of pile must be placed as far as possible from the center of gravity of the loads transmitted from the structure to the pile group.

D. In the case of the presence of neighbors, the piles must be away from the property line by a distance not less than D, or as required by the pile installation method.

Many researchers have studied various models on the subject of the optimal or minimum area of reinforced concrete foundations.

The models have been developed to estimate the minimum ground contact area for square, rectangular and circular isolated footings subjected to biaxial bending [2-11]. The models have been investigated to obtain the minimum ground contact area for trapezoidal, rectangular, L or corner, strap and T combined footings subjected to biaxial bending [12-18].

The most relevant studies on deep foundations that have been presented by some researchers are as the following: Kim et al. [19] presented an optimal design of slab-on-pile foundations based on a genetic algorithm with model tests; Leung et al. [20] studied the optimization of pile length in foundation slabs supported on pile groups; Amornfa et al. [21] showed a current practice on the design of pile-supported foundations of high-rise buildings in Bangkok, Thailand; Penteado and de Brito [22] estimated a selection methodology based on expert knowledge to optimize the construction of concrete piles; Letsios et al. [23] developed an optimal design methodology for pile-supported foundations in London; Wang et al. [24] investigated an analysis method to optimize the pile diameter of slab foundations supported on piles based on minimizing differential settlements; Leung et al. [25] studied the optimization of the pile group with multiple objectives considering the superstructure-foundation interaction and the settlement response; Ravichandran et al. [26] presented an optimization procedure for the design of pile-supported foundation slabs to support tall wind turbine in clayey and sandy soils; Shakir et al. [27] analyzed the

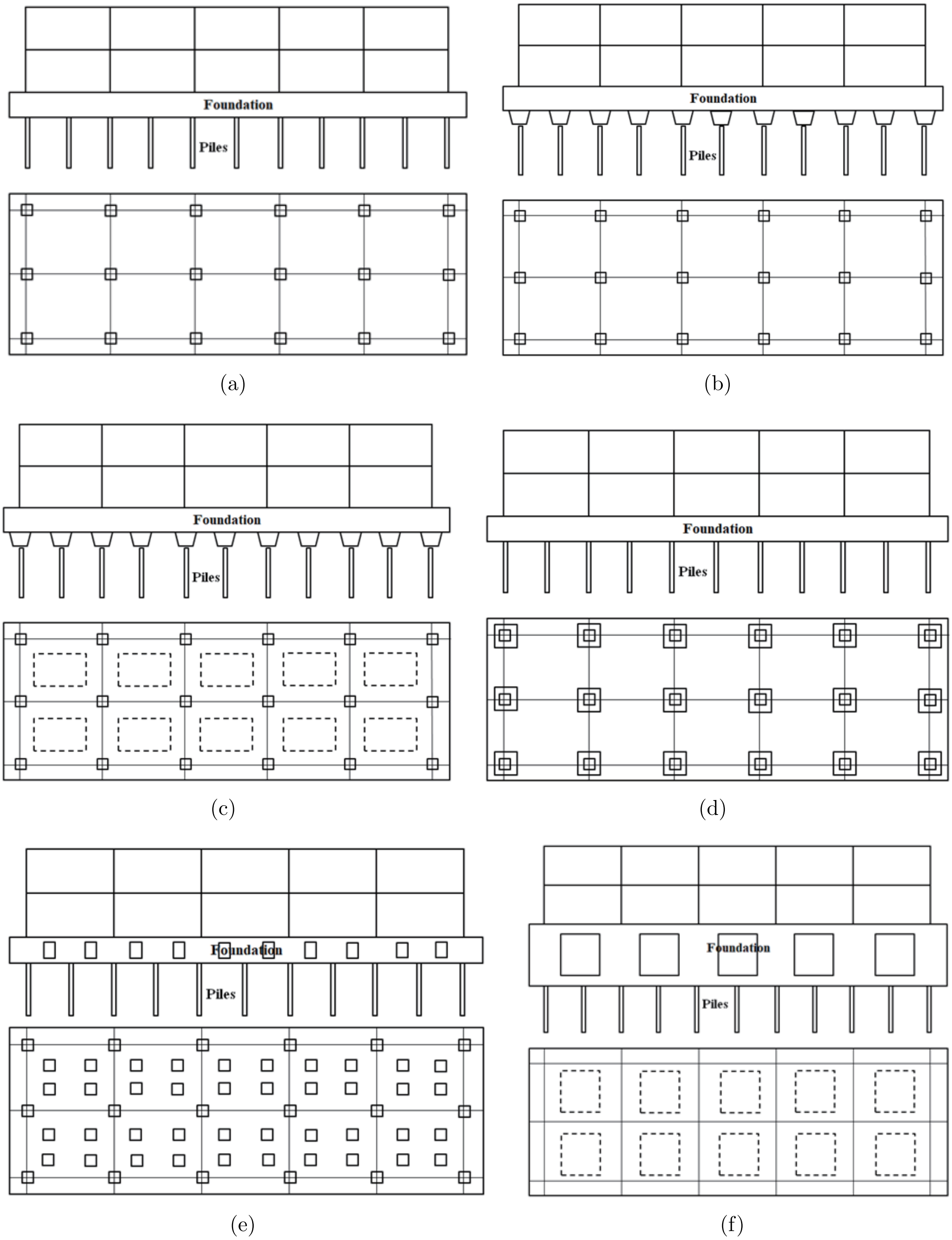

FIGURE 1. Types of foundation slabs supported on a group of piles

foundation slabs supported on pile groups using SAFE software; Jamil et al. [28] analyzed and designed the foundation slabs supported on pile groups considering interaction factors; Ojha and Srivastava [29] designed a foundation slab supported on pile groups for a 30-storey high-rise building in Lucknow, Uttar Pradesh, India; Azhar et al. [30] developed a parametric study for foundation slab supported on pile groups for high rise buildings;

Deb and Pal [31-33] investigated the load-settlement and load-sharing behavior, structural and geotechnical aspects through numerical analysis, the interaction behavior and load sharing pattern using nonlinear regression and Levenberg-Marquardt algorithm-based artificial neural network; Kannaujiya and Srivastava [34] presented the behavior of different configurations for a high-rise building using the finite element method.

According to the bibliographic review, the manuscripts closest to topic of optimal area for rectangular foundation slabs supported on a group of piles are the following: Farías-Montemayor et al. [35,36] developed a model to optimize the area and design for rectangular isolated footings supported on a group of piles; however, these works are presented only for rectangular isolated footings, not for rectangular foundation slabs. Other authors present complete designs for foundation slabs supported on a group of piles based on the knowledge of the area and sides of the foundation slab, the distribution and load capacity of the piles, but first in order to design these concepts must be taken into account. Therefore, there is no work with the current level of knowledge on the topic addressed here.

This manuscript presents the minimum or optimal area for rectangular foundation slabs in plan that rest on a group of piles to support the entire construction. Some authors present the complete design of foundation slabs supported on a group of piles based on the knowledge of the sides and area of the foundation slab, the distribution and load capacity of the piles. This model is formulated using optimization techniques to obtain the minimum area in plan and the sides of the foundation slab, the distribution and load capacity of the piles (This is the main contribution of this manuscript over other authors). Two numerical examples are shown with the same loads and moments due to each column: Example 1: Foundation slab supported on 9 piles; Example 2: Foundation slab supported on 25 piles. Each example presents five cases: Case I: Free sides; Case II: Sides constrained in the X direction on their opposite sides; Case III: Sides constrained in the Y direction on their opposite sides; Case IV: Sides constrained on all sides; Case V: Free sides and same load capacity of the piles. This manuscript is verified with other manuscripts presented by other authors for rectangular isolated footings [35]. The main advantage of this paper over other works is that it can be used for rectangular isolated footings and rectangular combined footings.

The paper is organized as follows. Section 2 shows the formulation of the equations to obtain minimum area of rectangular foundation slabs supported on a group of piles. Section 3 describes the application of the proposed model through two numerical examples. Section 4 presents the results of the proposed model applied for rectangular isolated footings. Section 5 presents the conclusions.

2. Formulation of the Model.

General equation to obtain the pressure at any part of a foundation slab supported on soil under biaxial bending is

$$\sigma_S = \frac{R}{A_S} + \frac{M_{xT}y}{I_{xS}} + \frac{M_{yT}x}{I_{yS}} \tag{1}$$

where σ_S is the pressure generated by the ground on the foundation slab (soil pressure), A_S is the contact surface of the foundation slab with the soil, R is the resultant force from all the forces applied to the foundation slab, M_{xT} and M_{yT} are the resultant moments applied on the X and Y axes, x and y are the coordinates on the X and Y axes of the fiber under study, and I_{xS} and I_{yS} are the moments of inertia in the X and Y axes.

Figure 2 presents a rectangular foundation slab subjected to biaxial bending at each column and the foundation slab is supported on piles.

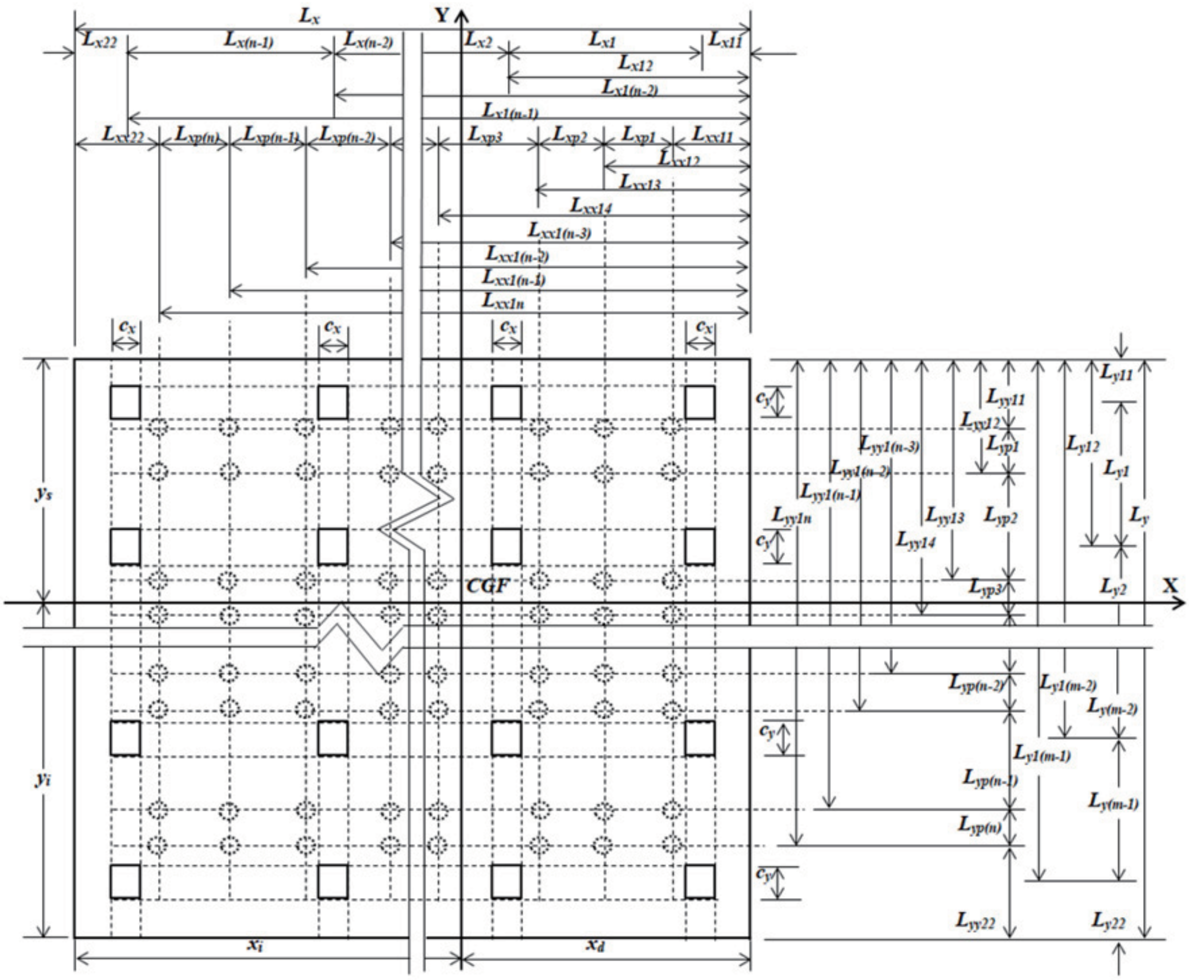

FIGURE 2. Rectangular foundation slab supported on piles

The geometrical properties of the rectangular section are $A = L_xL_y$, $y_s = L_y/2$, $x_d = L_x/2$, and the geometric conditions depending on the limited sides can be $L_{x11} \geq c_x/2$ or $1.5D$, $L_{x11} = c_x/2$ or $1.5D$, $L_{y11} \geq c_y/2$ or $1.5D$, $L_{y11} = c_y/2$ or $1.5D$, $L_{x22} \geq c_x/2$ or $1.5D$, $L_{x22} = c_x/2$ or $1.5D$, $L_{y22} \geq c_y/2$ or $1.5D$, $L_{y22} = c_y/2$ or $1.5D$.

For a simple understanding of the method, let us assume that the following conditions are met.

1) The pile is rigid.
2) The pile is fixed at the top and bottom.
3) Each pile receives the load only vertically, that is, only axial load.
4) The force P acting on the pile is proportional to the displacement due to compression.

The general equation to obtain the pressure generated by each pile in any part of a foundation slab supported by piles (deep foundation) subjected to biaxial bending is

$$\sigma_P = \frac{N_P}{A_P} = \frac{R}{A_{PT}} + \frac{M_{xT}y_P}{\Sigma I_{xP}} + \frac{M_{yT}x_P}{\Sigma I_{yP}} \tag{2}$$

where σ_P is the pressure generated by pile on the foundation slab (pile pressure), N_P is the axial force generated by each pile on the foundation slab, A_P is the contact area of the pile under study, A_{PT} is the contact surface of all piles on the foundation slab, x_P is the distance in the direction of the "X" axis from the "Y" axis to the location of the pile under study, y_P is the distance in the direction of the "Y" axis from the "X" axis to the location of the pile under study, and ΣI_{xP} and ΣI_{yP} are the moments of inertia of all piles on the foundation slab in the X and Y axes.

The values R, M_{xT} and M_{yT} are obtained as follows:

$$R = \sum_{i=1}^{n} \sum_{j=1}^{m} P_{(i,j)} \tag{3}$$

$$M_{xT} = \sum_{i=1}^{n} \sum_{j=1}^{m} M_{x(i,j)} + \sum_{i=1}^{n} \sum_{j=1}^{m} P_{(i,j)} \left(y_s - L_{y(i,j)} \right) \tag{4}$$

$$M_{yT} = \sum_{i=1}^{n} \sum_{j=1}^{m} M_{y(i,j)} + \sum_{i=1}^{n} \sum_{j=1}^{m} P_{(i,j)} \left(x_d - L_{x(i,j)} \right) \tag{5}$$

The moments of inertia of each pile are

$$I_{xP} = I_{xPo} + A_P {y_P}^2 \tag{6}$$

$$I_{yP} = I_{yPo} + A_P {x_P}^2 \tag{7}$$

where I_{xPo} is the moment of inertia of the pile contact surface on the local "X" axis (center of the pile under study), and I_{yPo} is the moment of inertia of the pile contact surface on the local "Y" axis (center of the pile under study).

If all the piles have the same contact area (cross section), then the following is obtained:

$$A_{PT} = nA_P \tag{8}$$

where n is the number of piles.

Now, substituting Equations (6), (7) and (8) into Equation (2) obtains:

$$\frac{N_P}{A_P} = \frac{R}{nA_P} + \frac{M_{xT} y_P}{\Sigma \left(I_{xPo} + A_P {y_P}^2 \right)} + \frac{M_{yT} x_P}{\Sigma \left(I_{yPo} + A_P {x_P}^2 \right)} \tag{9}$$

If the local moment of inertia of all the piles is neglected because it is a very small quantity, the following equation is obtained:

$$N_P = \frac{R}{n} + \frac{M_{xT} y_P}{\Sigma {y_P}^2} + \frac{M_{yT} x_P}{\Sigma {x_P}^2} \tag{10}$$

The general equation for any pile is

$$N_{Pi} = \frac{R}{n} + \frac{M_{xT} y_{Pi}}{\Sigma {y_{Pi}}^2} + \frac{M_{yT} x_{Pi}}{\Sigma {x_{Pi}}^2} \tag{11}$$

where $i = 1, 2, 3, \ldots, n$.

By balance of forces on the foundation slab, the following is obtained:

$$R = \sum_{i=1}^{n} N_{Pi} \tag{12}$$

The objective function to obtain the minimum surface area of the foundation slab "$A_{\min}$" is shown as follows:

$$A_{\min} = L_x L_y \tag{13}$$

The minimum area constraint functions for a foundation slab surface are shown as follows:

$$y_s = \frac{L_y}{2} \tag{14}$$

$$x_d = \frac{L_x}{2} \tag{15}$$

$$\Sigma {x_{Pi}}^2 = k_{c1} \left(y_s - L_{y11} \right)^2 + k_{c2} \left(y_s - L_{y12} \right)^2 + k_{c3} \left(y_s - L_{y13} \right)^2 + \cdots \tag{16}$$

$$\Sigma {y_{Pi}}^2 = k_{f1}\left(x_d - L_{x11}\right)^2 + k_{f2}\left(x_d - L_{x12}\right)^2 + k_{f3}\left(x_d - L_{x13}\right)^2 + \cdots \quad (17)$$

$$N_{Pi} = \frac{R}{n} + \frac{M_{xT} y_{Pi}}{\Sigma {y_{Pi}}^2} + \frac{M_{yT} x_{Pi}}{\Sigma {x_{Pi}}^2} \quad (18)$$

$$R = \sum_{i=1}^{n} N_{Pi} \quad (19)$$

$$L_x \geq \sum_{i=1}^{n-1} L_{xi} + L_{x11} + L_{x22} \quad (20)$$

$$L_x \geq \sum_{i=1}^{n-1} L_{xpi} + L_{xx11} + L_{xx22} \quad (21)$$

$$L_y \geq \sum_{i=1}^{m-1} L_{yi} + L_{y11} + L_{y22} \quad (22)$$

$$L_y \geq \sum_{i=1}^{m-1} L_{ypi} + L_{yy11} + L_{yy22} \quad (23)$$

$$L_{x11} \text{ and } L_{x22} \geq \frac{c_x}{2} \text{ or } 1.5D \quad (24)$$

$$L_{y11} \text{ and } L_{y22} \geq \frac{c_y}{2} \text{ or } 1.5D \quad (25)$$

$$0 \leq N_{Pi} \leq N_{PP} \quad (26)$$

where R, M_{xT} and M_{yT} are given by Equations (3), (4) and (5), N_{PP} is the allowable load capacity of each pile, $k_{c1}, k_{c2}, k_{c3}, \ldots$ are the number of piles found in each horizontal line, and $k_{f1}, k_{f2}, k_{f3}, \ldots$ are the number of piles found in each vertical line.

Furthermore, if it is desired to limit the sides of the foundation slab, the geometric conditions mentioned above must be considered.

Assuming that all variables are non-negative (exception of M_{xT} and M_{yT}).

Figure 3 presents the flowchart for the minimum area procedure for a foundation slab supported on piles. Figure 4 shows the flowchart for using Maple software for the minimum area of a foundation slab supported on piles.

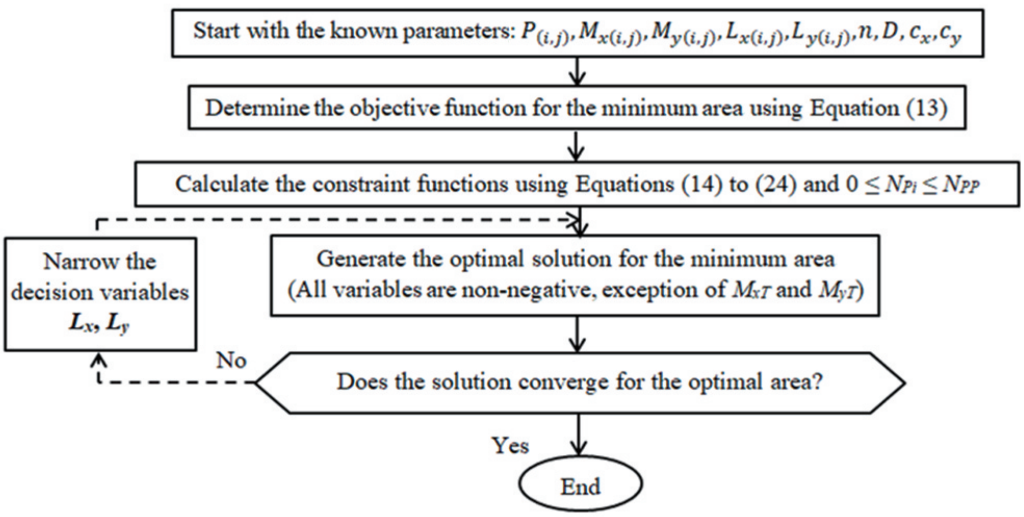

FIGURE 3. Flowchart for the minimum area procedure of a rectangular foundation slab supported on piles

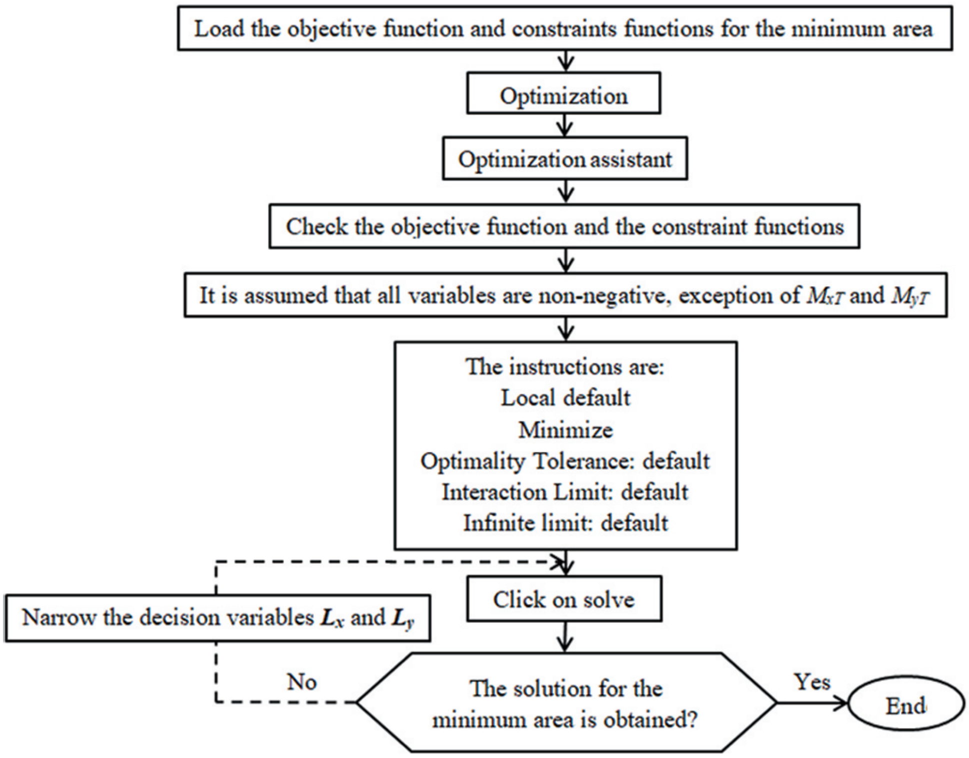

Figure 4. Flowchart for using Maple software on a rectangular foundation slab supported on piles

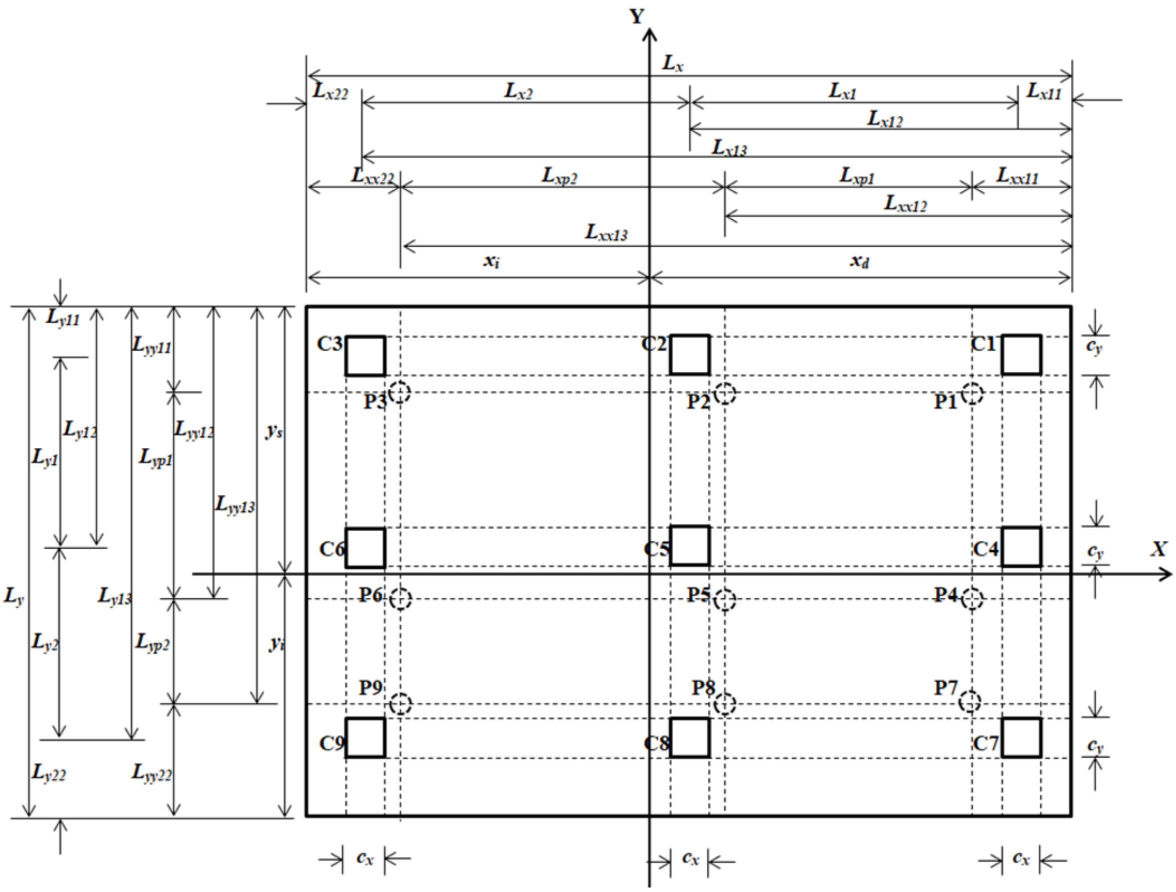

Figure 5. Distribution of 9 piles and 9 columns in a building

3. **Numerical Examples.** Two examples are shown. Example 1 is presented in Figure 5 and Example 2 is shown in Figure 6. Both figures present the distribution of the columns of a building to obtain the minimum area and the distribution of the piles in a foundation slab supported on piles under the same loads and moments due to the columns. These examples are shown for the five cases. Case I: Not limited at the ends, that is, it has no adjacent buildings; therefore, its constraints on the sides of the slab are $L_{x11} \geq c_x/2$, $L_{xx11} \geq 1.5D$, $L_{y11} \geq c_y/2$, $L_{yy11} \geq 1.5D$, $L_{x22} \geq c_x/2$, $L_{xx22} \geq 1.5D$, $L_{y22} \geq c_y/2$, $L_{yy22} \geq 1.5D$. Case II: Limited in negative and positive X direction and not limited in Y direction, that is, it has adjacent buildings in X direction and in Y direction it has no adjacent buildings; therefore, its constraints on the sides of the slab are $L_{x11} \geq c_x/2$, $L_{xx11} = 1.5D$, $L_{y11} \geq c_y/2$, $L_{yy11} \geq 1.5D$, $L_{x22} \geq c_x/2$, $L_{xx22} = 1.5D$, $L_{y22} \geq c_y/2$, $L_{yy22} \geq 1.5D$. Case III: Limited in negative and positive Y direction and not limited in X direction, that is, it has adjacent buildings in Y direction and in X direction it has no adjacent buildings; therefore, its constraints on the sides of the slab are $L_{x11} \geq c_x/2$, $L_{xx11} \geq 1.5D$, $L_{y11} \geq c_y/2$, $L_{yy11} = 1.5D$, $L_{x22} \geq c_x/2$, $L_{xx22} \geq 1.5D$, $L_{y22} \geq c_y/2$, $L_{yy22} = 1.5D$. Case IV: Limited in negative and positive X and Y directions, that is, it has adjacent buildings in both directions; therefore, its constraints on the sides of the slab are $L_{x11} \geq c_x/2$, $L_{xx11} = 1.5D$, $L_{y11} \geq c_y/2$, $L_{yy11} = 1.5D$, $L_{x22} \geq c_x/2$, $L_{xx22} = 1.5D$, $L_{y22} \geq c_y/2$, $L_{yy22} = 1.5D$. Case V: Not limited at the ends and the capacity of the piles is equal; therefore, the constraints on the sides of the slab are the same as in case I. The data for the five cases are $P_1 = 500$ kN, $P_2 = 1000$ kN, $P_3 = 500$ kN, $P_4 = 1000$ kN,

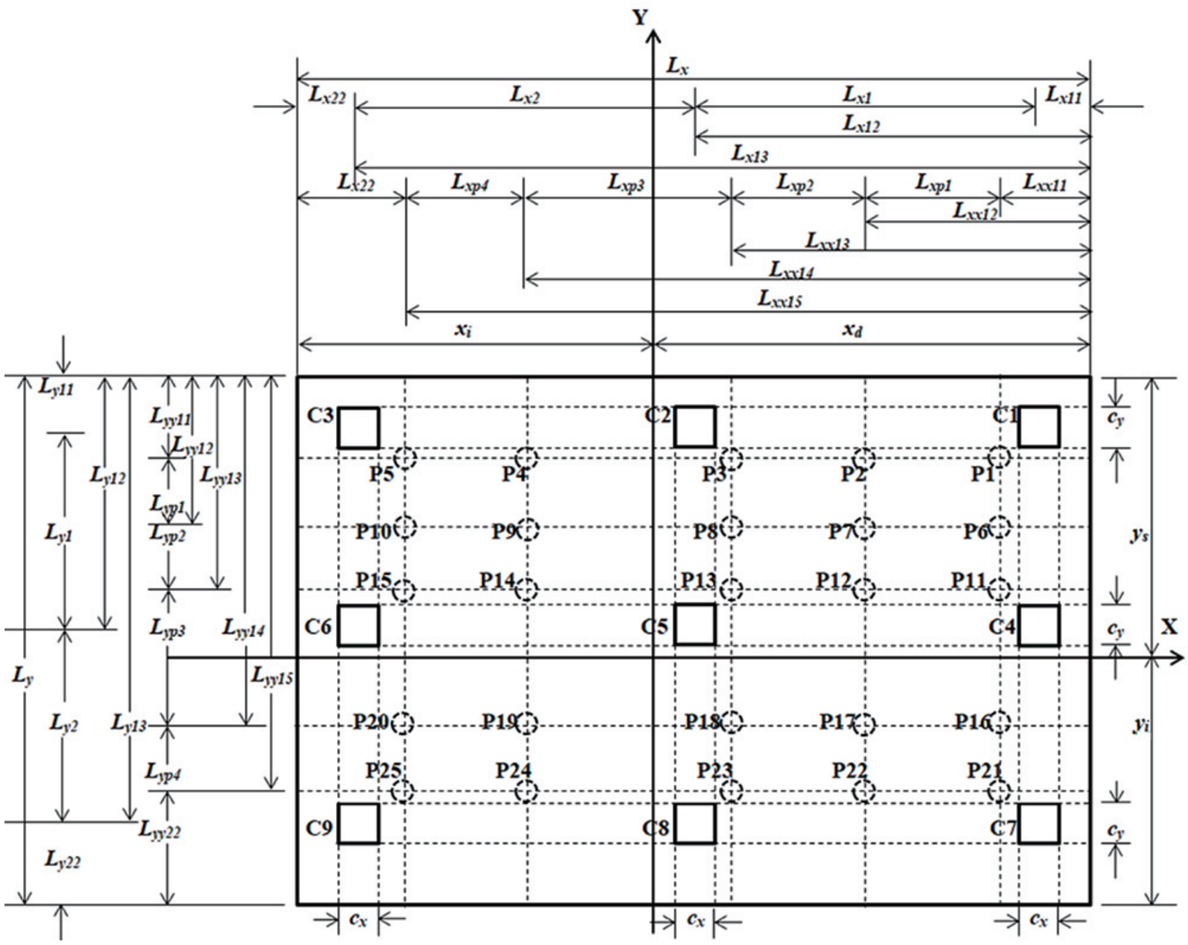

FIGURE 6. Distribution of 25 piles and 9 columns in a building

$P_5 = 2000$ kN, $P_6 = 1000$ kN, $P_7 = 500$ kN, $P_8 = 1000$ kN, $P_9 = 500$ kN, $M_{x1} = 200$ kN-m, $M_{x2} = 400$ kN-m, $M_{x3} = 200$ kN-m, $M_{x4} = 400$ kN-m, $M_{x5} = 800$ kN-m, $M_{x6} = 400$ kN-m, $M_{x7} = 200$ kN-m, $M_{x8} = 400$ kN-m, $M_{x9} = 200$ kN-m, $M_{y1} = 200$ kN-m, $M_{y2} = 400$ kN-m, $M_{y3} = 200$ kN-m, $M_{y4} = 400$ kN-m, $M_{y5} = 800$ kN-m, $M_{y6} = 400$ kN-m, $M_{y7} = 200$ kN-m, $M_{y8} = 400$ kN-m, $M_{y9} = 200$ kN-m, $c_x = 0.50$ m, $c_y = 0.50$ m, $L_{x1} = 6.00$ m, $L_{x2} = 6.00$ m, $L_{y1} = 6.00$ m, $L_{y2} = 6.00$ m, $D = 0.40$ m, $n = 9$ (Example 1), $n = 25$ (Example 2). For all cases the load capacity of the piles "N_{PP}" is not limited, except in case V.

Objective function "$A_{\min}$" is in Equation (13).

The equations of the constraint functions for the two examples are shown in Table 1.

In addition, the geometric conditions must be added based on the limited sides. For case I: $L_{x11} \geq c_x/2$, $L_{xx11} \geq 1.5D$, $L_{y11} \geq c_y/2$, $L_{yy11} \geq 1.5D$, $L_{x22} \geq c_x/2$, $L_{xx22} \geq 1.5D$, $L_{y22} \geq c_y/2$, $L_{yy22} \geq 1.5D$. For case II: $L_{x11} \geq c_x/2$, $L_{xx11} = 1.5D$, $L_{y11} \geq c_y/2$, $L_{yy11} \geq 1.5D$, $L_{x22} \geq c_x/2$, $L_{xx22} = 1.5D$, $L_{y22} \geq c_y/2$, $L_{yy22} \geq 1.5D$. For case III: $L_{x11} \geq c_x/2$, $L_{xx11} \geq 1.5D$, $L_{y11} \geq c_y/2$, $L_{yy11} = 1.5D$, $L_{x22} \geq c_x/2$, $L_{xx22} \geq 1.5D$, $L_{y22} \geq c_y/2$, $L_{yy22} = 1.5D$. For case IV: $L_{x11} \geq c_x/2$, $L_{xx11} = 1.5D$, $L_{y11} \geq c_y/2$, $L_{yy11} = 1.5D$, $L_{x22} \geq c_x/2$, $L_{xx22} = 1.5D$, $L_{y22} \geq c_y/2$, $L_{yy22} = 1.5D$. For case V: $N_{P1} = N_{P2} = N_{P3} = N_{P4} = N_{P5} = N_{P6} = N_{P7} = N_{P8} = N_{P9}$ (Example 1) and $N_{P1} = N_{P2} = N_{P3} = N_{P4} = N_{P5} = N_{P6} = N_{P7} = N_{P8} = N_{P9} = N_{P10} = N_{P11} = N_{P12} = N_{P13} = N_{P14} = N_{P15} = N_{P16} = N_{P17} = N_{P18} = N_{P19} = N_{P20} = N_{P21} = N_{P22} = N_{P23} = N_{P24} = N_{P25}$ (Example 2).

Assuming that all variables are non-negative (exception of M_{xT} and M_{yT}).

Table 2 presents the results of Example 1 for the five cases.

Table 3 presents the results of Example 2 for the five cases.

4. **Results.** Table 2 presents the minimum area and the distribution of the piles in a foundation slab supported on 9 piles for the five cases. The first four cases present the same values for M_{xT}, M_{yT}, R, L_x, L_y, y_s, x_d, L_{x11}, L_{x12}, L_{x13}, L_{x22}, L_{y11}, L_{y12}, L_{y13}, L_{y22}, $A_{\min}$. The largest values in the first four cases appear in L_{xx11}, L_{xx12}, L_{xx22}, L_{yy11}, L_{yy22}, N_{P1}, N_{P2}, N_{P4}, N_{P5}, N_{P7} (case I); L_{xp2}, L_{xx13}, N_{P3} (case II); L_{xp1}, L_{yp2}, L_{yy13} (case III); L_{xx13}, L_{yp1}, L_{yy12}, L_{yy13}, N_{P6}, N_{P8}, N_{P9} (case IV). Case V shows the same values for the capacity that the nine piles must support of 888.89 kN.

Table 3 shows the minimum area and the distribution of the piles in a foundation slab supported on 25 piles for the five cases. The first four cases present the same values for M_{xT}, M_{yT}, R, L_x, L_y, y_s, x_d, L_{x11}, L_{x12}, L_{x13}, L_{x22}, L_{y11}, L_{y12}, L_{y13}, L_{y22}, $A_{\min}$. The largest values in the first four cases appear in L_{xx11}, L_{xx12}, L_{xx13}, L_{xx14}, L_{xx15}, L_{yy22}, N_{P6}, N_{P12}, N_{P17} (case I); L_{xp1}, L_{xp4}, L_{xx15}, L_{yy11}, L_{yy12}, N_{P1}, N_{P2}, N_{P3}, N_{P4}, N_{P5}, N_{P9} (case II); L_{xp2}, L_{xp3}, L_{xx22}, L_{yp1}, L_{yp2}, L_{yp3}, L_{yy13}, L_{yy14}, L_{yy15}, N_{P7}, N_{P8}, N_{P10}, N_{P13}, N_{P18}, N_{P22}, N_{P23}, N_{P24}, N_{P25} (case III); L_{xx15}, L_{yp4}, L_{yy15}, N_{P11}, N_{P14}, N_{P15}, N_{P16}, N_{P19}, N_{P20}, N_{P21} (case IV). Case V shows the same values for the capacity that the nine piles must support of 320.00 kN.

This model can be applied to rectangular isolated footings rested on several piles with a column located at the center of the footing.

Two examples are shown for rectangular isolated footings. The data for the all cases are $P = 1200$ kN, $M_x = 400, 600, 800, 1000$ kN-m, $M_y = 400, 800, 1200$ kN-m, $c_x = 0.40$ m, $c_y = 0.40$ m, $D = 0.30$ m, $n = 4$ (Example 3), $n = 6$ (Example 4). For all cases the load capacity of the piles "N_{PP}" is not limited.

Objective function "$A_{\min}$" is in Equation (13).

The equations of the constraint functions for the two examples are shown in Table 4.

Assume all variables are non-negative.

TABLE 1. Constraint functions for the Examples 1 and 2

Example 1	**Example 2**
$y_s = \frac{L_y}{2}$	
$x_d = \frac{L_x}{2}$	
$\Sigma {x_{Pi}}^2 = 3\left(y_s - L_{yy11}\right)^2 + 3\left(y_s - L_{yy12}\right)^2 + 3\left(y_s - L_{yy13}\right)^2$	$\Sigma {x_{Pi}}^2 = 5\left(y_s - L_{yy11}\right)^2 + 5\left(y_s - L_{yy12}\right)^2 + 5\left(y_s - L_{yy13}\right)^2 + 5\left(y_s - L_{yy14}\right)^2 + 5\left(y_s - L_{yy15}\right)^2$
$\Sigma {y_{Pi}}^2 = 3\left(x_d - L_{xx11}\right)^2 + 3\left(x_d - L_{xx12}\right)^2 + 3\left(x_d - L_{xx13}\right)^2$	$\Sigma {y_{Pi}}^2 = 5\left(x_d - L_{xx11}\right)^2 + 5\left(x_d - L_{xx12}\right)^2 + 5\left(x_d - L_{xx13}\right)^2 + 5\left(x_d - L_{xx14}\right)^2 + 5\left(x_d - L_{xx15}\right)^2$
$N_{Pi} = \frac{R}{n} + \frac{M_{xT} y_{Pi}}{\Sigma {y_{Pi}}^2} + \frac{M_{yT} x_{Pi}}{\Sigma {x_{Pi}}^2}$	
$R = \sum_{i=1}^{n} N_{Pi}$	
$L_{x12} = L_{x11} + L_{x1}$	
$L_{x13} = L_{x11} + L_{x1} + L_{x2}$	
$L_x = L_{x11} + L_{x1} + L_{x2} + L_{x22}$	
$L_{y12} = L_{y11} + L_{y1}$	
$L_{y13} = L_{y11} + L_{y1} + L_{y2}$	
$L_y = L_{y11} + L_{y1} + L_{y2} + L_{y22}$	
$L_{xx12} = L_{xx11} + L_{xp1}$	
$L_{xx13} = L_{xx11} + L_{xp1} + L_{xp2}$	
–	$L_{xx14} = L_{xx11} + L_{xp1} + L_{xp2} + L_{xp3}$
–	$L_{xx15} = L_{xx11} + L_{xp1} + L_{xp2} + L_{xp3} + L_{xp4}$
$L_{yy12} = L_{yy11} + L_{yp1}$	
$L_{yy13} = L_{yy11} + L_{yp1} + L_{yp2}$	
–	$L_{yy14} = L_{yy11} + L_{yp1} + L_{yp2} + L_{yp3}$
–	$L_{yy15} = L_{yy11} + L_{yp1} + L_{yp2} + L_{yp3} + L_{yp4}$
$R = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9$	
$M_{xT} = M_{x1} + M_{x2} + M_{x3} + M_{x4} + M_{x5} + M_{x6} + M_{x7} + M_{x8} + M_{x9} + P_1\left(y_s - L_{y11}\right) + P_2\left(y_s - L_{y11}\right) + P_3\left(y_s - L_{y11}\right) + P_4\left(y_s - L_{y12}\right) + P_5\left(y_s - L_{y12}\right) + P_6\left(y_s - L_{y12}\right) + P_7\left(y_s - L_{y13}\right) + P_8\left(y_s - L_{y13}\right) + P_9\left(y_s - L_{y13}\right)$	
$M_{yT} = M_{y1} + M_{y2} + M_{y3} + M_{y4} + M_{y5} + M_{y6} + M_{y7} + M_{y8} + M_{y9} + P_1\left(x_d - L_{x11}\right) + P_2\left(x_d - L_{x12}\right) + P_3\left(x_d - L_{x13}\right) + P_4\left(x_d - L_{x11}\right) + P_5\left(x_d - L_{x12}\right) + P_6\left(x_d - L_{x13}\right) + P_7\left(x_d - L_{x11}\right) + P_8\left(x_d - L_{x12}\right) + P_9\left(x_d - L_{x13}\right)$	

where N_{Pi} takes the values of $i = 1, 2, 3, \ldots, 9$ (Example 1), and $i = 1, 2, 3, \ldots, 25$ (Example 2).

TABLE 2. Example 1 of the five cases to obtain the minimum area

Concept	**Case I**	**Case II**	**Case III**	**Case IV**	**Case V**
$\Sigma {x_{Pi}}^2$	56.16	158.29	192.56	219.39	221.56
$\Sigma {y_{Pi}}^2$	39.04	222.85	104.94	219.39	143.17
M_{xT}	3200.00	3200.00	3200.00	3200.00	0
M_{yT}	3200.00	3200.00	3200.00	3200.00	0
R	8000.00	8000.00	8000.00	8000.00	8000.00
L_x	12.50	12.50	12.50	12.50	13.30
L_y	12.50	12.50	12.50	12.50	13.30
y_s	6.25	6.25	6.25	6.25	6.65
x_d	6.25	6.25	6.25	6.25	6.65
L_{x11}	0.25	0.25	0.25	0.25	1.05
L_{x12}	6.25	6.25	6.25	6.25	7.05
L_{x13}	12.25	12.25	12.25	12.25	13.05
L_{x22}	0.25	0.25	0.25	0.25	0.25
L_{xx11}	4.39	0.60	2.19	0.60	3.33
L_{xp1}	2.45	2.42	3.54	2.60	3.67
L_{xp2}	2.44	8.88	4.79	8.70	5.70
L_{xx12}	6.84	3.02	5.73	3.20	7.00
L_{xx13}	9.28	11.90	10.52	11.90	12.70
L_{xx22}	3.22	0.60	1.98	0.60	0.60
L_{y11}	0.25	0.25	0.25	0.25	1.05
L_{y12}	6.25	6.25	6.25	6.25	7.05
L_{y13}	12.25	12.25	12.25	12.25	13.05
L_{y22}	0.25	0.25	0.25	0.25	0.25
L_{yy11}	2.60	1.38	0.60	0.60	0.60
L_{yp1}	2.66	7.15	6.24	8.70	6.85
L_{yp2}	3.09	2.60	5.06	3.20	5.25
L_{yy12}	5.26	8.53	6.84	9.30	7.45
L_{yy13}	8.35	11.13	11.90	11.90	12.70
L_{yy22}	4.15	1.37	0.60	0.60	0.60
N_{P1}	1249.37	1068.47	1106.69	1053.71	888.89
N_{P2}	1048.24	1033.74	998.69	1015.75	888.89
N_{P3}	848.28	906.21	852.70	888.89	888.89
N_{P4}	1097.80	923.87	1003.06	926.85	888.89
N_{P5}	896.68	889.13	895.05	888.89	888.89
N_{P6}	696.71	761.61	749.06	762.03	888.89
N_{P7}	921.71	871.32	918.92	888.89	888.89
N_{P8}	720.59	836.58	810.91	850.93	888.89
N_{P9}	520.62	709.06	664.92	724.07	888.89
$A_{\min}$	156.25	156.25	156.25	156.25	176.89

Table 3. Example 2 of the five cases to obtain the minimum area

Concept	Case I	Case II	Case III	Case IV	Case V
$\Sigma x_{Pi}{}^2$	327.19	291.40	416.28	391.89	460.82
$\Sigma y_{Pi}{}^2$	327.95	377.13	418.88	385.33	393.37
M_{xT}	3200.00	3200.00	3200.00	3200.00	0
M_{yT}	3200.00	3200.00	3200.00	3200.00	0
R	8000	8000	8000	8000	8000
L_x	12.50	12.50	12.50	12.50	13.30
L_y	12.50	12.50	12.50	12.50	13.30
y_s	6.25	6.25	6.25	6.25	6.65
x_d	6.25	6.25	6.25	6.25	6.65
L_{x11}	0.25	0.25	0.25	0.25	1.05
L_{x12}	6.25	6.25	6.25	6.25	7.05
L_{x13}	12.25	12.25	12.25	12.25	13.05
L_{x22}	0.25	0.25	0.25	0.25	0.25
L_{xx11}	1.78	0.60	0.60	0.60	0.68
L_{xp1}	2.91	3.30	2.52	4.00	2.52
L_{xp2}	2.41	2.40	2.54	2.40	3.04
L_{xp3}	2.40	2.41	3.78	2.40	2.78
L_{xp4}	2.40	3.19	2.42	2.50	2.68
L_{xx12}	4.69	3.90	3.12	4.60	3.20
L_{xx13}	7.10	6.30	5.66	7.00	6.24
L_{xx14}	9.50	8.71	9.45	9.40	9.02
L_{xx15}	11.90	11.90	11.87	11.90	11.70
L_{xx22}	0.60	0.60	0.63	0.60	1.60
L_{y11}	0.25	0.25	0.25	0.25	1.05
L_{y12}	6.25	6.25	6.25	6.25	7.05
L_{y13}	12.25	12.25	12.25	12.25	13.05
L_{y22}	0.25	0.25	0.25	0.25	0.25
L_{yy11}	0.61	1.40	0.60	0.60	0.60
L_{yp1}	2.40	2.41	2.61	2.40	3.95
L_{yp2}	2.40	2.43	3.47	2.40	3.21
L_{yp3}	2.40	2.40	2.73	2.65	2.54
L_{yp4}	2.90	2.41	2.49	3.85	2.40
L_{yy12}	3.01	3.81	3.21	3.00	4.55
L_{yy13}	5.41	6.24	6.68	5.40	7.76
L_{yy14}	7.81	8.64	9.41	8.05	10.30
L_{yy15}	10.71	11.05	11.90	11.90	12.70
L_{yy22}	1.79	1.45	0.60	0.60	0.60
N_{P1}	418.80	421.25	406.56	413.06	320.00
N_{P2}	390.40	393.26	387.30	379.81	320.00
N_{P3}	366.91	372.90	367.92	359.88	320.00
N_{P4}	343.50	352.44	339.01	339.95	320.00
N_{P5}	320.08	325.37	320.51	319.21	320.00
N_{P6}	395.33	394.74	386.53	393.46	320.00
N_{P7}	366.93	366.76	367.27	360.22	320.00
N_{P8}	343.44	346.39	347.89	340.28	320.00
N_{P9}	320.02	325.93	318.97	320.35	320.00
N_{P10}	296.60	298.86	300.47	299.62	320.00
N_{P11}	371.85	368.10	359.86	373.86	320.00
N_{P12}	343.46	340.12	340.60	340.62	320.00
N_{P13}	319.97	319.75	321.22	320.69	320.00
N_{P14}	296.55	299.30	292.30	300.76	320.00
N_{P15}	273.13	272.22	273.81	280.02	320.00
N_{P16}	348.38	341.73	338.87	352.20	320.00
N_{P17}	319.98	313.74	319.60	318.96	320.00
N_{P18}	296.50	293.38	300.22	299.03	320.00
N_{P19}	273.08	272.92	271.31	279.10	320.00
N_{P20}	249.66	245.84	252.81	258.36	320.00
N_{P21}	319.95	315.21	319.70	320.79	320.00
N_{P22}	291.55	287.22	300.44	287.54	320.00
N_{P23}	268.06	266.86	281.06	267.61	320.00
N_{P24}	244.64	246.40	252.14	247.68	320.00
N_{P25}	221.23	219.32	233.64	226.94	320.00
$A_{\min}$	156.25	156.25	156.25	156.25	176.89

Table 4. Constraint functions for the Examples 3 and 4

Example 3	**Example 4**
$y_s = \frac{L_y}{2}$	
$x_d = \frac{L_x}{2}$	
$\Sigma {x_{Pi}}^2 = 2\left(y_s - L_{yy11}\right)^2 + 2\left(y_s - L_{yy12}\right)^2$	$\Sigma {x_{Pi}}^2 = 2\left(y_s - L_{yy11}\right)^2 + 2\left(y_s - L_{yy12}\right)^2 + 2\left(y_s - L_{yy13}\right)^2$
$\Sigma {y_{Pi}}^2 = 2\left(x_d - L_{xx11}\right)^2 + 2\left(x_d - L_{xx12}\right)^2$	$\Sigma {y_{Pi}}^2 = 3\left(x_d - L_{xx11}\right)^2 + 3\left(x_d - L_{xx12}\right)^2$
$N_{Pi} = \frac{R}{n} + \frac{M_{xT} y_{Pi}}{\Sigma {y_{Pi}}^2} + \frac{M_{yT} x_{Pi}}{\Sigma {x_{Pi}}^2}$	
$R = \sum_{i=1}^{n} N_{Pi}$	
$L_{xx11} = D$	$L_{xx11} = D$
$L_{xp1} \geq 3D$	$L_{xp1} \geq 3D$
$L_{xx12} = L_{xx11} + L_{xp1}$	$L_{xx12} = L_{xx11} + L_{xp1}$
$L_{xp1} \geq 3D$	$L_{xp1} \geq 3D$
$L_{xx22} = D$	$L_{xx22} = D$
$L_x = L_{xx12} + L_{xx22}$	$L_x = L_{xx12} + L_{xx22}$
$L_{yy11} = D$	$L_{yy11} = D$
$L_{yp1} \geq 3D$	$L_{yp1} \geq 3D$
$L_{yy12} = L_{yy11} + L_{yp1}$	$L_{yy12} = L_{yy11} + L_{yp1}$
–	$L_{yp2} \geq 3D$
–	$L_{yy13} = L_{yy11} + L_{yp1} + L_{yp2}$
$L_{yy22} = D$	$L_{yy22} = D$
$L_y = L_{yy12} + L_{y22}$	$L_x = L_{yy13} + L_{y22}$
$R = P$; $M_{xT} = M_x$; $M_{yT} = M_y$	

where N_{Pi} takes the values of $i = 1, 2, 3, 4$ (Example 3), and $i = 1, 2, 3, 4, 5, 6$ (Example 4).

Table 5 presents the results of Example 3, and Table 6 shows the results of Example 4.

Table 5 shows the results of rectangular isolated footings supported on four piles with a column located in the center of the footing, and these results can be consulted in the paper presented by Farías-Montemayor et al. [35]. The constant or known parameters are c_x, c_y, D, P, M_x and M_y. The decision variables are $A_{\min}$, L_x, L_y, y_s, x_d, L_{x11}, L_{x22}, L_{y11}, L_{y22}, L_{xx11}, L_{xp1}, L_{xx12}, L_{xx22}, L_{yy11}, L_{yp1}, L_{yy12}, L_{yy22}, N_{P1}, N_{P2}, N_{P3}, N_{P4}. The results that appear for the four cases are the following: When M_y increases; All measurements and areas increase; The loads exerted by the piles in all cases are $N_{P1} = 600$ kN and $N_{P4} = 0$ kN; When $M_x = M_y$ it occurs that $N_{P2} = N_{P3} = 300$ kN, $L_x = L_y$.

Table 6 presents the results of rectangular isolated footings supported on six piles with a column located in the center of the footing, and these results can be consulted in the work presented by Farías-Montemayor et al. [35]. The constant or known parameters are c_x, c_y, D, P, M_x and M_y. The decision variables are $A_{\min}$, L_x, L_y, y_s, x_d, L_{x11}, L_{x22}, L_{y11}, L_{y22}, L_{xx11}, L_{xp1}, L_{xx12}, L_{xx22}, L_{yy11}, L_{yp1}, L_{yp2}, L_{yy12}, L_{yy13}, L_{yy22}, N_{P1}, N_{P2}, N_{P3}, N_{P4}, N_{P5}, N_{P6}. The results that appear for the six cases are the following: When M_y increases; All measurements and areas increase; The loads exerted by the piles in all cases are N_{P1}

TABLE 5. Example 3 of the four cases to obtain the minimum area

Concept	**Case I: $P = 1200$ kN; $M_x = 400$ kN-m**			**Case II: $P = 1200$ kN; $M_x = 600$ kN-m**			**Case III: $P = 1200$ kN; $M_x = 800$ kN-m**			**Case IV: $P = 1200$ kN; $M_x = 1000$ kN-m**		
M_y	400	800	1200	400	800	1200	400	800	1200	400	800	1200
$\Sigma x_{Pi}{}^2$	1.78	2.04	2.17	3.68	4.20	4.45	6.24	7.11	7.52	9.47	10.76	11.37
$\Sigma y_{Pi}{}^2$	1.78	6.24	13.35	1.94	6.77	14.46	2.04	7.11	15.16	2.12	7.34	15.65
L_x	1.93	3.10	4.25	1.99	3.20	4.40	2.03	3.27	4.49	2.05	3.31	4.56
L_y	1.93	2.03	2.07	2.52	2.65	2.71	3.10	3.27	3.34	3.68	3.88	3.97
y_s	0.97	1.01	1.04	1.26	1.33	1.35	1.55	1.63	1.67	1.84	1.94	1.99
x_d	0.97	1.55	2.13	1.00	1.60	2.20	1.01	1.63	2.25	1.03	1.66	2.28
L_{x11}	0.97	1.55	2.13	1.00	1.60	2.20	1.01	1.63	2.25	1.03	1.66	2.28
L_{x22}	0.97	1.55	2.13	1.00	1.60	2.20	1.01	1.63	2.25	1.03	1.66	2.28
L_{xx11}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
L_{xp1}	1.33	2.50	3.65	1.39	2.60	3.80	1.43	2.67	3.89	1.45	2.71	3.96
L_{xx12}	1.63	2.80	3.95	1.69	2.90	4.10	1.73	2.97	4.19	1.75	3.01	4.26
L_{xx22}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
L_{y11}	0.97	1.01	1.04	1.26	1.33	1.35	1.55	1.63	1.67	1.84	1.94	1.99
L_{y22}	0.97	1.01	1.04	1.26	1.33	1.35	1.55	1.63	1.67	1.84	1.94	1.99
L_{yy11}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
L_{yp1}	1.33	1.43	1.47	1.92	2.05	2.11	2.50	2.67	2.74	3.08	3.28	3.37
L_{yy12}	1.63	1.73	1.77	2.22	2.35	2.41	2.80	2.97	3.04	3.38	3.58	3.67
L_{yy22}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
N_{P1}	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00	600.00
N_{P2}	300.00	279.76	271.62	312.88	292.62	284.44	320.24	300.00	291.81	325.02	304.81	296.62
N_{P3}	300.00	320.24	328.38	287.12	307.38	315.56	279.76	300.00	308.19	274.98	295.19	303.38
N_{P4}	0	0	0	0	0	0	0	0	0	0	0	0
$A_{\min}$	3.74	6.29	8.82	5.02	8.49	11.93	6.29	10.67	15.02	7.55	12.85	18.09

$= 400$ kN and $N_{P6} = 0$ kN; The average of N_{P1} and N_{P5} is the value of N_{P3}; The average of N_{P2} and N_{P6} is the value of N_{P4}.

5. **Conclusions.** This paper shows a model to obtain the minimum area of rectangular foundation slabs subjected to biaxial bending in each column supported on a group of piles, assuming that the foundation slab is rigid. Also this document shows the distribution and load capacity of the piles.

The optimal model is formulated from an analytical approach based on the minimum area criterion. The independent variables or known parameters are all loads and moments acting on the foundation slab, the available allowable load capacity of the piles and the limitations of the land where the construction will be located. The dependent or unknown variables are area and sides of the foundation slab, the distribution and load capacity of the piles.

The main findings are the follows.

1) Some authors present the complete design of foundation slabs supported on a group of piles based on the knowledge of the sides and area of the foundation slab, the distribution and load capacity of the piles.

2) This study shows the simplified and accurate minimum area equations for foundation slabs supported on a group of piles to estimate the sides and area of the foundation slab, the distribution and load capacity of the piles (main contribution of this manuscript).

TABLE 6. Example 4 of the four cases to obtain the minimum area

Concept	**Case I: P = 1200 kN; M_x = 400 kN-m**			**Case II: P = 1200 kN; M_x = 600 kN-m**			**Case III: P = 1200 kN; M_x = 800 kN-m**			**Case IV: P = 1200 kN; M_x = 1000 kN-m**		
M_y	400	800	1200	400	800	1200	400	800	1200	400	800	1200
$\Sigma x_{Pi}{}^2$	3.68	4.20	4.45	7.77	8.84	9.34	13.35	15.16	16.00	20.43	23.15	24.42
$\Sigma y_{Pi}{}^2$	2.91	10.16	21.69	3.12	10.86	23.13	3.25	11.27	24.00	2.34	11.55	24.58
L_x	1.99	3.20	4.40	2.04	3.29	4.53	2.07	3.34	4.60	2.09	3.38	4.65
L_y	2.52	2.65	2.71	3.39	3.57	3.66	4.25	4.49	4.60	5.12	5.41	5.54
y_s	1.26	1.33	1.35	1.69	1.79	1.83	2.13	2.25	2.30	2.56	2.71	2.77
x_d	1.00	1.60	2.20	1.02	1.65	2.26	1.04	1.67	2.30	1.05	1.69	2.32
L_{x11}	1.00	1.60	2.20	1.02	1.65	2.26	1.04	1.67	2.30	1.05	1.69	2.32
L_{x22}	1.00	1.60	2.20	1.02	1.65	2.26	1.04	1.67	2.30	1.05	1.69	2.32
L_{xx11}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
L_{xp1}	1.39	2.60	3.80	1.44	2.69	3.93	1.47	2.74	4.00	1.49	2.78	4.05
L_{xx12}	1.69	2.90	4.10	1.74	2.99	4.23	1.77	3.04	4.30	1.79	3.08	4.35
L_{xx22}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
L_{y11}	1.26	1.33	1.35	1.69	1.79	1.83	2.13	2.25	2.30	2.56	1.69	2.77
L_{y22}	1.26	1.33	1.35	1.69	1.79	1.83	2.13	2.25	2.30	2.56	1.69	2.77
L_{yy11}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
L_{yp1}	0.96	1.03	1.05	1.39	1.49	1.53	1.83	1.95	2.00	2.26	2.41	2.47
L_{yp2}	0.96	1.03	1.05	1.39	1.49	1.53	1.83	1.95	2.00	2.26	2.41	2.47
L_{yy12}	1.26	1.33	1.35	2.69	1.79	1.83	2.13	2.25	2.30	2.56	2.71	2.77
L_{yy13}	2.22	2.35	2.41	3.09	3.27	3.36	3.95	4.19	4.30	4.82	5.11	5.24
L_{yy22}	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
N_{P1}	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00	400.00
N_{P2}	208.59	195.08	189.63	215.24	201.75	196.30	218.92	205.46	200.00	221.26	207.82	202.36
N_{P3}	295.71	302.46	305.19	292.38	299.12	301.85	290.54	297.27	300.00	289.37	296.09	298.82
N_{P4}	104.29	97.54	94.81	107.62	100.88	98.15	109.46	102.73	100.00	110.63	103.91	101.18
N_{P5}	191.41	204.92	210.37	184.76	198.25	203.70	181.08	194.54	200.00	178.74	192.18	197.64
N_{P6}	0	0	0	0	0	0	0	0	0	0	0	0
$A_{\min}$	5.02	8.49	11.93	6.92	11.76	16.55	8.82	15.02	21.16	10.71	18.27	25.76

3) The available allowable load capacity of the piles can be considered equal for all piles (see Tables 2 and 3).

4) The proposed model can be applied to rectangular isolated footings supported on a group of piles with a column located in the center of the footing (see Tables 5 and 6), and these results can be consulted in the work presented by Farías-Montemayor et al. [35].

5) The proposed model can be applied to rectangular isolated footings supported on a group of piles with a column located anywhere of the footing, just considering $M_{xT} = M_x + Pe_y$ and $M_{yT} = M_y + Pe_x$, where e_x and e_y are the coordinates of the column location.

6) The proposed model can be applied to rectangular combined footings supported on a group of piles that support two and three columns on the footing.

The main justification of this manuscript over other papers is that they do not consider adjacent constructions and in this manuscript it shows the position and the load capacity of the piles.

Suggestions for future research are

1) Optimal area for L or T-shaped foundation slabs in plan supported on a group of piles to obtain the minimum area in plan and the sides of the foundation slab, the distribution and load capacity of the piles;

2) Complete design for rectangular foundation slabs in plan supported on a group of piles to estimate the thickness and areas of reinforcing steel of the slab, since this manuscript presents the minimum area in plan and the sides of the foundation slab, the distribution and the load capacity of the piles.

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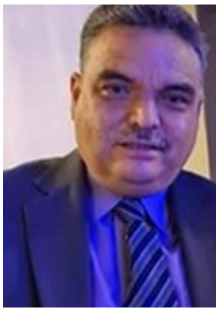

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