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OPTIMAL DATA DRIVEN CONTROL TUNING FOR UNKNOWN CLOSED LOOP SYSTEM

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Abstract. *This paper proposes an innovative closed loop controller design strategy from the measured input-output pairs, while avoiding the complex modelling process of the unknown plant. More specifically, firstly considering one commonly used closed loop system with the unknown plant and unknown controller simultaneously, to achieve the tracking property with one reference model, the parameterized controller design problem is transformed into one parameter estimation problem. Furthermore, the controller parameters are identified only through the measured data without any prior knowledge about the unknown plant, corresponding to our named optimal data driven control tuning. Secondly, the test implemented to verify the stability of closed loop system is described, and the term unfalsification is satisfied on an inequality condition. Thirdly, for a nonlinear closed loop system with nonlinear plant and nonlinear controller together, nonlinear controller is replaced by a linear approximation form with sufficient accuracy. Then combinations with least squares algorithm and power spectrum theory are applied to identifying these unknown weights or parameters, existing in linear approximation form while avoiding the modelling process of nonlinear plant. Finally, the performance of optimal data driven control tuning is simulated through the pitch channel in flight attitude angle of unmanned aerial vehicle (UAV).*

Keywords: Data driven, Control tuning, Stability analysis, Optimality, Nonlinear controller, Linear approximation

1. **Introduction.** Where planning is considered with determining a trajectory for one practical plant, for example, vehicle, ship, and quadcopter, from user-specified goals, control is concerned with determining the required actuator setpoints in order to follow the desired trajectory. However, during the road of controller design, one necessary element is urgently needed, i.e., one mathematical equation for the considered plant. This mathematical equation can describe the intrinsic principle, embedding in the considered plant, so some laws of motion are applied to analyzing the intrinsic mechanism, meaning that the latter controller design must be done on the basis of this mathematical equation. Above description corresponds to model-based control strategy. To obtain this important mathematical equation for the considered plant, two methods are beneficial, i.e., system identification and physical modeling. System identification is to measure the input-output data sequences, and then extract some useful information from these data sequences by using some existing ways, for example, statistical learning, Bayesian inference, deep learning and machine learning. The reason about why system identification is feasible is that data includes information in our data era, and our work concerns on how to find these useful information from vast data. Specifically, for system identification, used to model

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the unknown plant, the total number of input-output data is the mentioned sample size, and then we want to extract the intrinsic principle about plant from these input-output data. In case of the number of observations more exceeding this sample size, then the input is persistent excitation, while the identification model satisfies the expected accuracy. From the knowledge of system identification theory, the situation with observed disturbance or noise in the output corresponds to the robust system identification, which is also extended to robust optimal control.

The design of controllers based on measured data is of great importance in theoretical research and practical applications, because this approach allows to save from time consuming and costly modelling process. Data driven methods have been formulated to overcome limitations of model based controller synthesis, avoiding the definition of a system model, either derived from first principles or experimental data. Other names by which these methods are known include model free and data based, highlighting the fact that only experimental data is used. Based on these measured data, our task is to extract some useful information about the unknown controller, but not for the unknown system model.

The limitations and uncertainties associated with models and assumptions, on the one hand, and the emergence of progressively complex systems, on the other hand, have sparked a paradigm shift towards data-driven control methodologies. The exponentially increasing number of research papers in this field and the growing number of courses offered in universities worldwide on the subject clearly show this trend. The new data-driven control system design paradigm has re-emerged to circumvent the necessity of deriving offline or online plant models. Many plants regularly generate and store huge amounts of operating data at specific instants of time. Such data encompasses all the relevant plant information required for control, estimation, performance assessment, decision-making and fault diagnosis. This data availability has facilitated the design of data-driven control systems.

The main features of the data-driven control approaches can be categorized as follows.

1) Control system design and analysis employ only the measured plant input-output data. Such data are the controller design's starting point and end criteria for control system performance.

2) No priori information and assumptions on the plant's dynamics or structure are required.

3) The controller structure can be predetermined.

4) The closed-loop stability, convergence and safe operation issues should be addressed in a data-driven context.

5) A designer-specified cost function is minimized using the measured data to derive the controller parameters.

Data driven control methods can be classified into two groups: direct methods such as iterative feedback tuning [1], virtual reference feedback tuning [2], correlation based tuning [3], and indirect methods which may rely on recurrent neural networks [4], Hammerstein-Wiener models [5], AutoRegressive models with exogenous variables for the plant identification phase [6]. Among them, direct methods have a number of advantages, since they allow to prevent problems caused by unmodeled dynamics and overcome model mismatch problems. Other than indirect methods, the mathematical modeling process in direct methods is not required. Indeed, direct methods directly target the final aim of tuning the controller parameters. Current algorithms implementations of this concept about data driven have mostly focused around open loop experiment of single variable system which have obtained significant results in [7]. In addition, all of the proposed solutions are applied to fixed structure controllers, leading to a result which is a set of parameters or gains for the given controller. The choice of the controller structure is beyond here, and the analysis of parameterized controller structure [8].

The data driven methodology in the framework of control can be thought of as a robust controller designed according to an approach having two levels of abstractions, shown in Figure 1. The upper level prescribes that the controller is formed by two units with different aims: one unit (control unit) selects the control action to apply to the system. This unit is not determined though as it contains parameters that must be tuned; a second unit (tuning unit) is on duty to perform the parameter tuning. This fundamental split is at the basis of the concept itself of an adaptive system. The lower level consists instead in selecting specific algorithms for the two units.

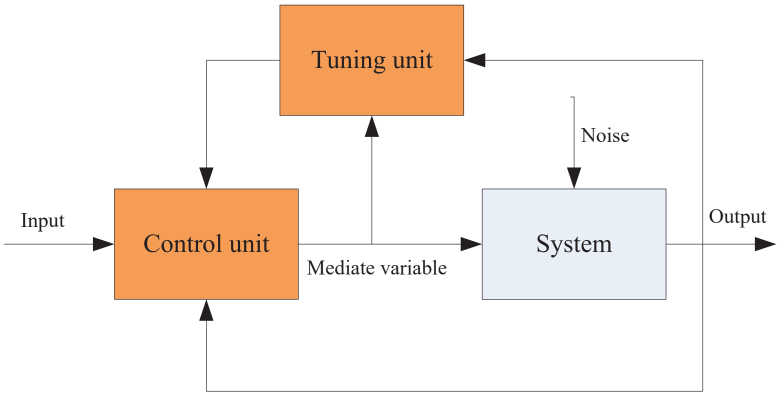


Figure 1. Data driven control

Nonetheless, data based methodology covers a very large spectrum of applicability, so it is necessary to define or classify the type of controllers that can be generated [9]. Consequently, different aspects are considered, such as the variability concerning the time in which the controller estimates are tuned [10]. In this particular case, there is the online type of adaptability [11]. The tuning unit provides updated estimates of the system at each instant of time [12], and these new estimates are incorporated and used in the controller unit for the selection of control actions [13]. In contrast, when the controller is sporadically, updated, or the controller implementation and data acquisition phases are widely separated, it is considered off-line adaptation. After data collection, the operation of the control system stops, and the controller is readjusted according to the tuning system parameters. Our previous paper [14] studies the latter direct data driven control, i.e., designing that feedback controller based on measured input-output data sequence directly.

Based on above our own descriptions about data driven control strategy, this new note continues to do some deep theoretical research about data driven controller tuning through our own mathematical derivations. More specifically, consider one commonly used closed loop system structure with unknown plant and unknown controller simultaneously, the mission of data driven idea is to design that unknown controller directly only through the measured data, while avoiding the modelling process for that unknown plant. By using the idea of data driven, the case of parameterized controller is turned to one question of parameter estimation, as now parameterized controller is more widely applied in practical engineering, such as proportion-integral-derivative (PID) controller. To achieve the goal of data driven controller tuning, our proposed control algorithm is given in detail. Here the notion of controller tuning means the controller parameter is updated or tuned through its simplified least squares form. Then the test implemented to verify the stability of the closed loop system is described, while using the term unfalsification to denote an approach that allows to discard iteratively controllers, i.e., closed loop stability analysis. Moreover, as we all know that linear system of linear controller does not exist in real life, and all phenomena are nonlinear, the mission of this new paper is to collect the input-output data firstly, and then yield a rough controller only using the collected data sequence, i.e., giving a detailed expression for that nonlinear controller on basis of input-output spectrums without any knowledge about that unknown nonlinear plant.

Generally, the main original contributions are listed as follows.

1) The detailed data driven controller tuning is given through our own description.

2) The corresponding closed loop stability analysis is proposed to test the term unfalsification.

3) Linear approximation is used to replace the original nonlinear controller, and all unknown parameters in linear approximation are identified through power spectrum theory and least squares algorithm.

This paper is organized follows. In Section 2, the control problem is defined, and some existing mathematical relations are reviewed. Section 3 describes the proposed data driven controller tuning in case of the parameterized controller, while not considering that unknown plant. In Section 4, closed loop stability analysis is developed to satisfy other requirement or fact. To design that nonlinear controller in case of no any information about nonlinear plant, the idea of data driven control tuning is also applied to using a linear approximation to replace that nonlinear controller in Section 5, and the detailed derivation of linear approximation is introduced to guarantee one optimal approximation condition. Section 6 shows the application of the proposed theoretical contributions to one simulation example, while conclusions are drawn in Section 7.

2. **System Description.** The proposed closed loop system structure is plotted in Figure 2, where $e(t)$ denotes the error signal, $r(t)$ is the reference signal, $C(z)$ is the controller, and $P(z)$ is the considered plant. Furthermore, $P(z)$ and $C(z)$ are all unknown, $y(t)$ is the system output, and z is the unit backward shift operator.

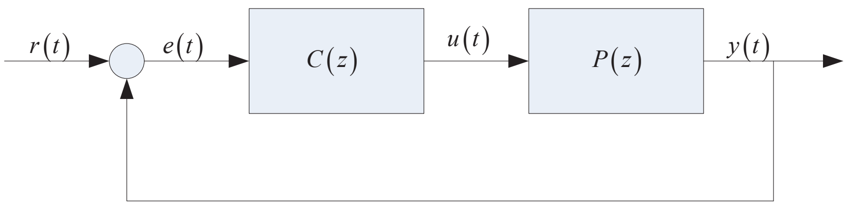


FIGURE 2. Closed loop system structure

In above Figure 2, some existing mathematical relations are listed easily.

$$y(t) = P(z)u(t) = P(z)C(z)e(t) = P(z)C(z)[r(t) - y(t)] \tag{1}$$

i.e.,

$$y(t) = \frac{P(z)C(z)}{1 + P(z)C(z)} r(t) \tag{2}$$

One data set of input-output pairs is required and data set is obtained from an open loop experiment $\{u(t), y(t)\}_{t=1}^{N}$, where N is the total number of input-output pairs. The problem consists of developing a suitable controller conferring to system performances as similar as possible to the ones of a reference model directly from data. The reference model is chosen by researcher based on the desired closed loop performance.

Considering that unknown controller $C(z)$ is parameterized by one parameter vector θ, then candidate controller is a function of the parameter vector θ. The general structure of the parameterized controller with θ is the following form.

$$C(z, \theta) = \alpha^T(z)\theta \tag{3}$$

where $\alpha(z)$ is a set of linear rational basis function, for example,

$$\alpha(z) = \begin{pmatrix} \alpha_1(z) & \alpha_2(z) & \cdots & \alpha_n(z) \end{pmatrix}; \quad \theta = \begin{pmatrix} \theta_1 & \theta_2 & \cdots & \theta_n \end{pmatrix} \tag{4}$$

where above n is the order of parameterized controller.

One reference model $M(z)$ defines the desired closed loop performance and provides the ideal linear transfer function from the reference signal $r(t)$ to closed loop output $y(t)$, i.e.,

$$y(t) = M(z)r(t) \tag{5}$$

The criterion of design controller $C(z)$ is based on the discrepancy function between the reference model and the closed loop system. Observing Equations (2) and (5), the designed goal is to let the real transfer function from reference signal $r(t)$ to closed loop output $y(t)$ track that given reference model, i.e.,

$$\frac{P(z)C(z)}{1 + P(z)C(z)} \to M(z) \tag{6}$$

To achieve above goal, the following discrepancy function is minimized by

$$J_1(C(z)) = \left\| \frac{P(z)C(z)}{1 + P(z)C(z)} - M(z) \right\|_2^2 \tag{7}$$

where $\|.\|$ is Euclidean norm.

Taking account of that parameterized controller $C(z, \theta)$ in Equation (3), above discrepancy function $J_1(C(z))$ is changed as

$$J_1(C(z, \theta)) = \left\| \frac{P(z)C(z, \theta)}{1 + P(z)C(z, \theta)} - M(z) \right\|_2^2 \tag{8}$$

Consequently, the controller design problem is changed to one parameter estimation problem, i.e.,

$$\theta = \{\arg\min\}_\theta J_1(C(z, \theta)) = \{\arg\min\}_\theta \left\| \frac{P(z)C(z, \theta)}{1 + P(z)C(z, \theta)} - M(z) \right\|_2^2 \tag{9}$$

Comment: Observing that parameter estimation problem in Equation (9), this discrepancy function $J_1(C(z, \theta))$ includes some variables, such as $M(z)$, $P(z)$ and θ. Since $M(z)$ is given in priori and plant transfer function $P(z)$ is not known, we cannot minimize in practice the discrepancy function $J_1(C(z, \theta))$.

3. **Data Driven Control Tuning.** Based on above description for the parameterized controller $C(z, \theta)$, then the latter problem is concentrated on identifying that unknown parameter vector θ, corresponding to parameter tuning or controller tuning. Also due to the appearance of the unknown plant $P(z)$ in that discrepancy function $J_1(C(z, \theta))$, we think how it possible to avoid that unknown plant $P(z)$ in the cost function, which does not require any knowledge about that unknown plant.

To show our proposed data driven controller tuning strategy, we collect the available input-output pairs $\{u(t), y(t)\}_{t=1}^N$, and based on Equation (5), we have

$$r(t) = M^{-1}(z)y(t) \tag{10}$$

which means after given reference model $M(z)$ and collected closed loop output $y(t)$, the reference signal $r(t)$ is obtained inversely, so we have some known variables, for example, $\{u(t), y(t)\}_{t=1}^{N}$, $M(z)$ and $r(t)$. Then we think can that parameterized controller $C(z,\theta)$ be designed or extracted from these above known variables.

In order to better understand the idea of data driven, the input-output pairs of that unknown controller are $\{e(t), u(t)\}_{t=1}^{N}$, and error signal $e(t)$ is solved through

$$e(t) = r(t) - y(t) = M^{-1}(z)y(t) - y(t) = \left[M^{-1}(z) - 1\right] y(t) \tag{11}$$

It means the corresponding input-output pairs $\{e(t), u(t)\}_{t=1}^{N}$ about the unknown controller $C(z,\theta)$ are obtained from Equation (11), plotted in Figure 3.

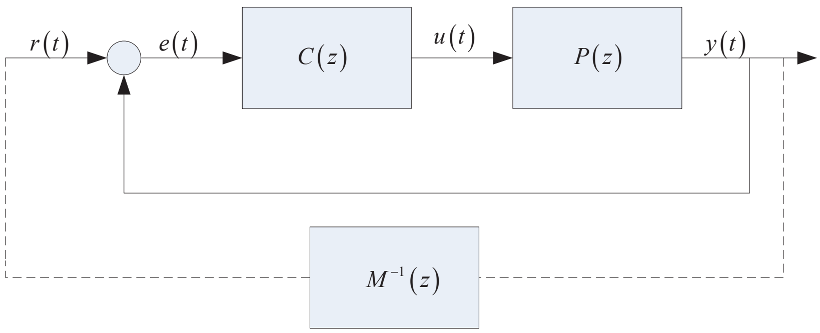


Figure 3. Idea of direct data

Based on input-output pairs $\{e(t), u(t)\}_{t=1}^{N}$ of that unknown controller $C(z,\theta)$, the objective is to design a controller that suits the closed loop behavior with the desired reference model, while avoiding any information about that unknown plant $P(z)$.

Due to

$$u(t) = C(z,\theta)e(t) \tag{12}$$

substituting Equation (11) into above Equation (12), we have

$$u(t) = C(z,\theta)e(t) = C(z,\theta)\left[M^{-1}(z) - 1\right] y(t) \tag{13}$$

where in Equation (13), only controller $C(z,\theta)$ is unknown, and other variables are known in priori, for example, input-output pairs $\{u(t), y(t)\}_{t=1}^{N}$ are collected and reference model $M(z)$ is given in priori.

Choosing $t = 1, 2, \ldots, N$ in Equation (13), the other discrepancy function or cost function is selected to tune that unknown controller parameter vector θ, i.e.,

$$\begin{aligned} \theta &= \{\arg\min\}_{\theta} \frac{1}{N} J_2(C(z,\theta)) \\ &= \{\arg\min\}_{\theta} \frac{1}{N} \left[u(t) - C(z,\theta)\left[M^{-1}(z) - 1\right] y(t)\right]^2 \\ &= \{\arg\min\}_{\theta} \frac{1}{N} [u(t) - \alpha(z)\theta e(t)]^2 \\ e(t) &= \left[M^{-1}(z) - 1\right] y(t) \end{aligned} \tag{14}$$

Furthermore, reference model $M(z)$ exists in above mathematical derivations, and it can also be represented by a discrete time transfer function with a delay at least of 1.

$$M(z) = \frac{b_1 z^{n-1} + \cdots + b_{n-1}z + b_n}{z^n + a_1 z^{n-1} + \cdots + a_{n-1}z + a_n}; \quad y(t) = M(z)r(t)$$

where n is the order of polynomial. Assume that $M(z)$ is invertible, in the sense that we compute the sequence of reference input $r(t)$. Given the sequence of output $y(t)$, in short we can write $r(t) = M^{-1}(z)y(t)$.

Note that this cost function $J_2(C(z,\theta))$ is a purely data dependent cost. The optimal controller parameter vector $\hat{\theta}$ can be directly calculated from its least squares form, i.e.,

$$\hat{\theta} = \left[\sum_{t=1}^{N} \varphi(t)\varphi(t)\right]^{-1} \sum_{t=1}^{N} \varphi(t)u(t); \quad \varphi(t) = \alpha(z)e(t) \tag{15}$$

Observing that detailed cost function $J_2(C(z,\theta))$ in Equation (14), it is convenient to collect data $\{u(t), y(t)\}_{t=1}^{N}$ and choose reference model $M(z)$, and then data driven controller tuning is to extract one optimal controller parameter vector $\hat{\theta}$ to satisfy $u(t) = C\left(z, \hat{\theta}\right)\left[M^{-1}(z) - 1\right] y(t)$, while not constructing one mathematical model for that unknown plant $P(z)$.

Finally, the complete data driven controller tuning is formulated as the following algorithm.

Step 1:	Collect input-output data $\{u(t), y(t)\}_{t=1}^{N}$;
Step 2:	Choose one reference model $M(z)$;
Step 3:	Compute that reference signal $r(t) = M^{-1}(z)y(t)$;
Step 4:	Compute the optimal controller parameter vector $\hat{\theta}$ can be directly calculated from its least squares form.

4. **Closed Loop Stability Analysis.** This section gives the data driven controller tuning based on controller unfalsification with stability analysis. To have both satisfactory performance and stabilization, our approach needs input sensitivity function, defined as follows, respectively.

$$W_1(z,\theta) = \frac{C(z,\theta)}{1 + P(z)C(z,\theta)}; \quad W_2(z,\theta) = \frac{P(z)C(z,\theta)}{1 + P(z)C(z,\theta)} \tag{16}$$

and other real forms are given as follows.

$$W_1(z) = \frac{C(z)}{1 + P(z)C(z)}; \quad W_2(z) = \frac{P(z)C(z)}{1 + P(z)C(z)} \tag{17}$$

We formulate the discrepancy between each reference model and related sensitivity function, i.e.,

$$J_3(C(z,\theta)) = \|W_1(z) - W_1(z,\theta)\|_2^2; \quad J_4(C(z,\theta)) = \|W_2(z) - W_2(z,\theta)\|_2^2 \tag{18}$$

To explain how to obtain above two discrepancy functions, we have

$$r(t) = C(z,\theta)^{-1}u(t) + y(t), \quad t = 1, 2, \ldots, N \tag{19}$$

Then input-output are yielded.

$$u(t,\theta) = W_1(z)r(t); \quad y(t,\theta) = W_2(z)r(t) \tag{20}$$

So $J_3(C(z,\theta))$ and $J_4(C(z,\theta))$ are minimized by solving the following cost functions.

$$J_3^N(C(z,\theta)) = \frac{1}{N}\sum_{t=1}^{N}[u(t) - u(t,\theta)]^2; \quad J_4^N(C(z,\theta)) = \frac{1}{N}\sum_{t=1}^{N}[y(t) - y(t,\theta)]^2 \tag{21}$$

Due to

$$y(t) - y(t,\theta) = [1 - W_2(z)]P(z)u(t) - C(z,\theta)^{-1}W_2(z)u(t)$$

$$u(t) - u(t,\theta) = u(t) - W_1(z)P(z)u(t) - C(z,\theta)^{-1}W_1(z)u(t) \tag{22}$$

As controller parameter vector θ exists in both $J_3(C(z,\theta))$ and $J_4(C(z,\theta))$, the optimal controller parameter vector $\hat{\theta}$ is defined as

$$\hat{\theta} = \{\arg\min\}_\theta [J_3(C(z,\theta)) + J_4(C(z,\theta))] \tag{23}$$

According to the term unfalsification for closed loop stability, we construct one important relation between closed loop stability and input discrepancy $u(t) - u(t,\theta)$, expressed as

$$\begin{aligned} u(t) - u(t,\theta) &= \left(C(z)^{-1} - C(z,\theta)^{-1}\right) W_1(z)u(t) = \Delta C(z,\theta) W_1(z) u(t) \\ \Delta C(z,\theta) &= \left(C(z)^{-1} - C(z,\theta)^{-1}\right) \end{aligned} \tag{24}$$

By virtue of small gain theorem, the following theorem about closed loop stability is formulated.

Theorem 4.1. *Consider data driven controller tuning for closed loop system, the optimal controller parameter vector $\hat{\theta}$ is estimated from input-output pairs $\{u(t), y(t)\}_{t=1}^N$. If*

$$\|W_1(z)\Delta C(z,\theta)\| \leq 1 \tag{25}$$

then the designed controller $C\left(z,\hat{\theta}\right)$ will internally stabilizes that unknown plant $P(z)$.

5. Nonlinear Analysis.

During above mathematical derivations, the unknown controller $C(z)$ is a linear controller, i.e.,

$$u(t) = C(z)e(t) = C(q)[r(t) - y(t)] \tag{26}$$

However, $C(z)$ is a nonlinear controller, so above equation is rewritten as

$$u(t) = C(z, e(t)) = C(z, r(t) - y(t)) \tag{27}$$

Also Equation (2) can be rewritten similarly to embody the existence of nonlinear element.

Problem: Observing Figure 2, i.e., nonlinear closed loop system, we want to design that feed forward nonlinear controller $C(z)$ while avoiding the modelling process for nonlinear plant $P(z)$. This problem of controller design is dependent of our mentioned direct data driven control strategy. Observing both sides of nonlinear controller $C(z)$ in Figure 2, the input-output for nonlinear controller $C(z)$ are $e(t)$ and $u(t)$, i.e., $r(t) - y(t)$ and $u(t)$, so we need to construct one relation between these two input-output.

5.1. Our idea.

Due to nonlinear form is a fuzzy idea, in reality we need make it as an explicit form, i.e., applying one linear form to approximate or replace the original nonlinear form. Our idea is plotted in Figure 4.

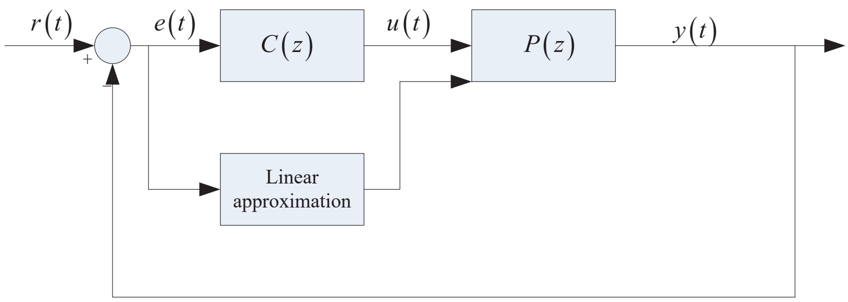


FIGURE 4. Linear approximation

Based on linear approximation, we assume the linear form as

$$C(z,e(t)) = C(z,r(t)-y(t)) = a_0 + \sum_{i=1}^{t} b_i u(i) \tag{28}$$

where linear form (28) is a direct weight form about controller output $u(t)$, and t is the order of the term. For more complex dynamical system, for example, quadrotor UAV, employing linearized models that struggle to accurately characterize nonlinear dynamics, we propose a modeling approach based on Multi-dimensional Taylor Network to approximate the nonlinear UAV model.

Moreover, to embody the input-output about that nonlinear controller $C(z)$, we rewrite that linear form as that.

$$C(z,e(t)) = C(z,r(t)-y(t)) = a_0 + \sum_{i=1}^{t} b_i u(i) + \sum_{i=1}^{t} a_i e(i) \tag{29}$$

Linear form of Equation (29) is linear with respect to the input-output $\{e(i), u(i)\}_{i=1}^{t}$.

Rewrite Equation (29) as

$$C(z,e(t)) = \begin{pmatrix} e(1) & u(1) & \cdots & e(t) & u(t) & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \\ \vdots \\ a_t \\ b_t \\ a_0 \end{pmatrix} = \varphi^T(t)\theta \tag{30}$$

where regressor variable $\varphi(t)$ and controller parameters θ are defined as follows, respectively.

$$e(t) = \begin{pmatrix} e(1) & u(1) & \cdots & \varepsilon(t) & u(t) & 1 \end{pmatrix}; \quad \theta = \begin{pmatrix} a_1 \\ b_1 \\ \vdots \\ a_t \\ b_t \\ a_0 \end{pmatrix} \tag{31}$$

The merit of linear approximation is to change the controller design into one parameter estimation problem, i.e., identifying that unknown parameter vector θ.

For convenience, controller parameter θ is solved through the following optimization problem, i.e.,

$$\theta = \{\arg\min\}_{\theta} \sum_{i=1}^{N} \left[u(t) - \varphi^T(t)\theta\right]^2 \tag{32}$$

where N is the total number of input-output.

It is easy to obtain that

$$\theta = \left[\sum_{i=1}^{N} \varphi(t)\varphi^T(t)\right]^{-1} \left[\sum_{i=1}^{N} \varphi(t)u(t)\right] \tag{33}$$

where above Equation (33) corresponds to the least squares identification.

5.2. **Our algorithm.** For notational simplicity, we use that simplified linear form in Equation (28), i.e.,

$$\begin{aligned} C(z,e(t)) &= C(z,r(t)-y(t)) = a_0 + \sum_{i=1}^{t} b_i u(i) \\ u(t) &= C(z,e(t))e(t) = \left[a_0 + \sum_{i=1}^{t} b_i u(i)\right] e(t) \end{aligned} \tag{34}$$

Post-multiplying both sides by $e(t)$ and taking the expectation operation gives that

$$\phi_{eC(e)} = a_0\phi_e(0) + \sum_{i=1}^{t} b_i \phi_{ue}(t-i) \tag{35}$$

i.e.,

$$\phi_{eu}(t) = a_0\phi_e(0) + \sum_{i=1}^{t} b_i\phi_{ue}(t-i) = \begin{pmatrix} \phi_e(0) & \phi_{ue}(t-1) & \cdots & \phi_{ue}(0) \end{pmatrix} \begin{pmatrix} a_0 \\ b_1 \\ \vdots \\ b_t \end{pmatrix} \tag{36}$$

Choosing $t = 0, 1, \ldots, N$, then we have

$$\begin{aligned} \phi_{eu}(0) &= \begin{pmatrix} \phi_e(0) & \phi_{ue}(-1) & \cdots & \phi_{ue}(-N) \end{pmatrix}; \\ \phi_{eu}(1) &= \begin{pmatrix} \phi_e(0) & \phi_{ue}(0) & \cdots & \phi_{ue}(1-N) \end{pmatrix}\theta \\ &\vdots \\ \phi_{eu}(N) &= \begin{pmatrix} \phi_e(0) & \phi_{ue}(N-1) & \cdots & \phi_{ue}(0) \end{pmatrix}\theta; \quad \theta = \begin{pmatrix} a_0 \\ b_1 \\ \vdots \\ b_N \end{pmatrix} \end{aligned} \tag{37}$$

where ϕ_{eu} and ϕ_e are cross-spectrum and auto-spectrum for input-output data $\{e(t), u(t)\}$.

Combining those N equations in (37), we have

$$\Phi_{eu} = \begin{pmatrix} \phi_{eu}(0) \\ \phi_{eu}(1) \\ \vdots \\ \phi_{eu}(N) \end{pmatrix}; \quad \Phi = \begin{pmatrix} \phi_e(0) & \phi_{ue}(-1) & \cdots & \phi_{ue}(-N) \\ \phi_e(0) & \phi_{ue}(0) & \cdots & \phi_{ue}(1-N) \\ \vdots & \vdots & \vdots & \vdots \\ \phi_e(0) & \phi_{ue}(N-1) & \cdots & \phi_{ue}(0) \end{pmatrix}$$

$$\theta = \begin{pmatrix} a_0 \\ b_1 \\ \vdots \\ b_N \end{pmatrix}; \quad \Phi_{eu} = \Phi\theta \tag{38}$$

Then the unknown parameter vector θ is solved through the following least squares algorithm.

$$\begin{aligned} \theta &= \{\arg\min\}_\theta \frac{1}{N}\sum_{i=1}^{N}[\Phi_{eu} - \Phi\theta]^2 \\ &= \{\arg\min\}_\theta \frac{1}{N}(\Phi_{eu} - \Phi\theta)^T(\Phi_{eu} - \Phi\theta) \\ &= \{\arg\min\}_\theta \frac{1}{N}\|\Phi_{eu} - \Phi\theta\|^2 \end{aligned} \tag{39}$$

i.e., the parameter estimation θ is deemed as

$$\theta = \left[\frac{1}{N}\Phi^T\Phi\right]^{-1}\left[\frac{1}{N}\Phi^T\Phi_{eu}\right] \tag{40}$$

Combining above mathematical derivations, we introduce some other interesting subjects together, for example, least squares algorithm, linea approximation theory and power spectrum theory. To the best of our knowledge, the approximation accuracy is tolerable in case of sufficiently large t in that linear approximation form.

Comparing Equations (33) and (40), we find although their forms are the same as each other, coming from the least squares algorithm, but Equation (33) means regressor variable, and Equation (40) requires regressor matrix. From a statistical point of view, more input-output data are collected to construct that regressor matrix in Equation (40), i.e., meaning that more data lead to more accuracy. Consequently, Equation (40) is better than Equation (33) in practice.

6. **Simulation Example.** This sections uses a practical example about UAV to prove the efficiency about optimal data driven control tuning, i.e., designing the controller parameters directly from the measured input-output pairs.

We all know one fact that during this whole UAV flight process, if the flight speed or velocity is too fast or slow, and the flight attitude exceeds a certain limit, then the flight state will be affected greatly, or even the possibility of crash. It means we should guarantee the safety and completeness, so it is important to design one safe controller for UAV operational completeness. Here our goal is to apply optimal data driven controller tuning strategy to designing one safe controller for UAV.

A UAV is controlled by the input variable, and an airborne measurement system can record the output response, i.e., collecting the input-output pairs. The closed loop system with feedback controller can deal with the input distortion for UAV, while restraining disturbance and keeping stability. Closed loop control structure of UAV is shown in Figure 5, where UAV sends control instructions and controls the actuator to perform the corresponding operation and feedback control.

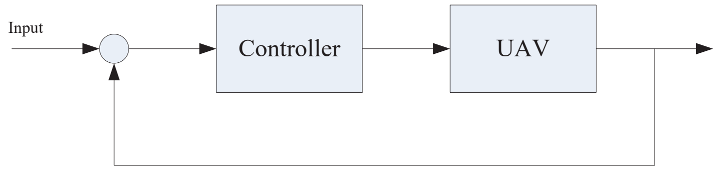


FIGURE 5. UAV closed-loop control system

In Figure 5, model $P(z)$ is considered as follows, and the detailed modeling process is neglected here.

$$P(z) = \frac{(z-2.4)(z-1.2)(z-0.5)}{z(z-0.8)(z-0.4)(z-0.6)} \tag{41}$$

Comparing Figure 2 and Figure 5, we see these two figures are the same as each other. For example, $C(z)$ is the controller, plant $P(z)$ is UAV, and $r(t)$ is input. Remember the main essence of data driven tuning control, that reference model $M(z)$ and the measured input-output data $\{y(t), u(t)\}_{t=1}^N$ are needed.

$$M(z) = \frac{0.6z^4 - 0.12z^3 + 0.8z^2 + 0.4z + 6}{z^5 - z^4 + 0.24z^3 + 0.2z^2 - 2z + 10} \tag{42}$$

During the latter simulation, the feed forward controller $C(z)$ is parameterized by one parameter vector θ, i.e.,

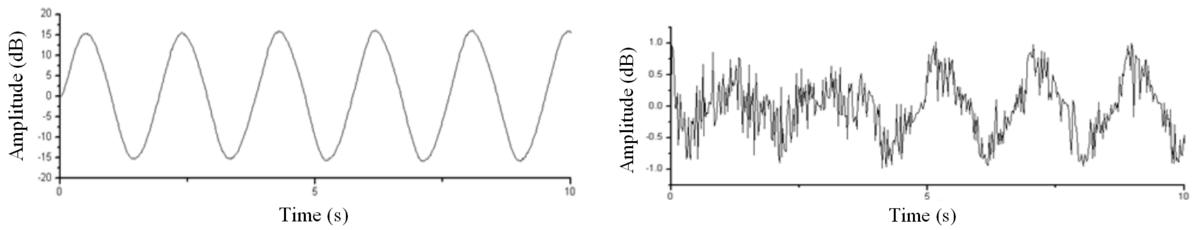
$$C(z) = C(\theta) = \begin{bmatrix} 1 & z & z^2 & z^3 & z^4 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix};$$

$$\alpha(z) = \begin{bmatrix} 1 & z & z^2 & z^3 & z^4 \end{bmatrix} \tag{43}$$

To compute the simulation results, the true controller parameter vector θ is chosen as

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.8 \\ 0.2 \\ -0.4 \\ -0.3 \end{bmatrix} \tag{44}$$

Based on plant model $P(z)$, reference model $M(z)$, and the parameterized controller $C(\theta)$, the considered closed loop system is determined. After imposed one kind of sine signal into this considered closed loop system, both signals between that feed forward controller $C(\theta)$ are collected together to constitute the named measured input-output data set $\{y(t), u(t)\}_{t=1}^{50}$, plotted in Figure 6. Applying that reference model $M(z)$ and the measured input-output data set, our proposed algorithm is applied to identifying that controller parameter vector, through minimizing one cost function consisted by the measured input-output data.



(a) Input signal (b) Output signal

FIGURE 6. Input-output pairs

The pitch channel in the flight control system of HB380 fixed-wing UAV is selected, and the simulation experiment is carried out in the MATLAB R2018B simulation platform. We collect input-output data on both sides of the controller, shown in Figure 6. During the design of fixed-wing UAV, the aerodynamics data of UAV are usually obtained through wind tunnel experiments, so the aerodynamics parameters of UAV inevitably have errors. The smaller error brings to the better performance. In order to test the performance of the identified UAV controller parameters, the response contrast curves of PID controller and the safety controller are given in Figure 7, where the pitch angle signal is 10 and 20 degrees.

Specifically, in Figure 7, the black dotted line is the given signal, the gray is the PID control response curve, and the red solid line is the identification control response curve. Compared with the curves in Figure 7, the attitude angle under the control of PID controller is slow when the pitch angle is small, and the adjustment time is about 4 s, while the attitude angle response is faster. When the identification control is used, the adjustment time is about 2 s. In the case of large angle pitch signal input, the adjusting time

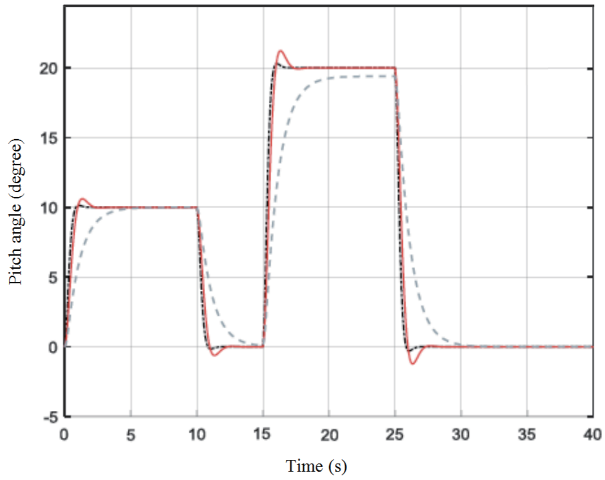


Figure 7. Pitch angle following curve

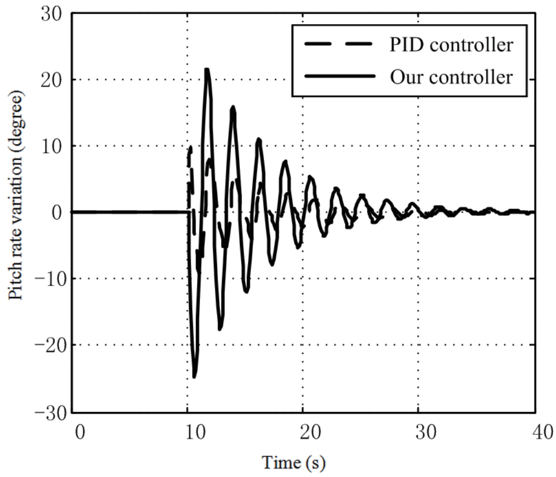


Figure 8. Pitch rate variation curve

of PID control is about 10 seconds, and there is about 0.7 degree error, but the adjusting time of identification control is about 2.8 seconds. So from this comparison, we see there is no error basically, and further visible identification of the model has a higher efficiency, meaning more accurate accuracy.

Figure 8 shows the pitch rate curve. From the curve, we see that the pitch rate of the identification controller is higher than the PID control, and the fall speed is faster than PID control. The variation of the whole pitch rate is in the safe range and meets the requirements of the safety controller.

As the identification of controller parameter vector θ is to minimize that cost function through our proposed algorithm, when the iterative step is increased, that cost function will decrease to its minimum value, i.e., 0. At this moment, the corresponding final parameter vector θ is chosen as its estimation values. For that parameterized controller, the entire identification process for controller parameter vector θ is shown in Figure 9, where we find each controller parameter will converge to its true value respectively with iterative step increases. Remember above process is an inverse process, i.e., identifying parameter vector θ only through the measured input-output data and the given reference model without applying that plant model.

After controller parameter vector θ is identified from the data set about UAV roll process, then to testify whether this controller is efficient, we extend this same controller

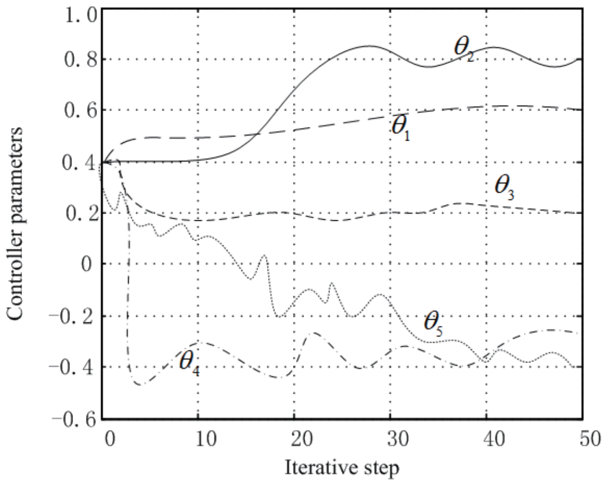
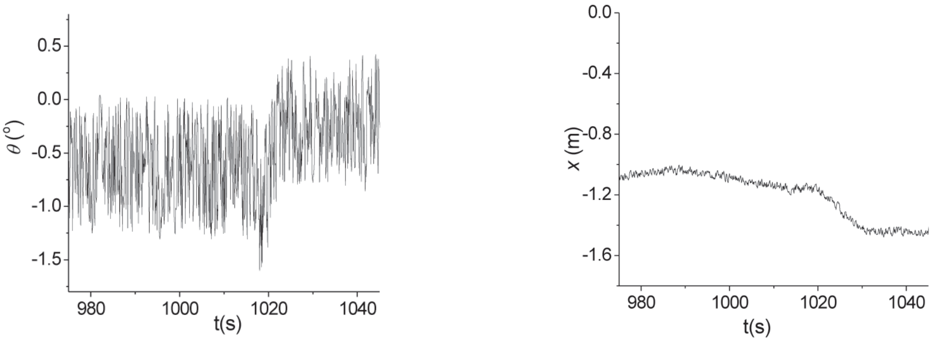


Figure 9. Controller parameters



(a) Change in roll angle (b) Longitudinal slip distance

Figure 10. Extending to roll control

to UAV roll control process. Specifically, when UAV returns over the landing site, the remote control software sends the roll command, and the roll angle and longitudinal slip curves are shown in Figure 10. From Figure 10, it can be seen that the pitch angle and longitudinal sidelobe distance do not change much during the roll process. The roll angle changes from $-0.6°$ step to $0°$ at 1018 s, which is due to the force sensor judging that one of UAV's feet has touched the ground. According to roll control strategy, the total rotor roll is rapidly reduced at this time, so the roll angle will have a step.

7. **Conclusion.** By virtue of the idea of data driven, the case of parameterized controller is turned to one question of parameter estimation. To achieve the goal of data driven controller tuning, our proposed control algorithm is given in detail. Then the test implemented to verify the stability of the closed loop system is described, while using term unfalsification to denote an approach that allows to discard iteratively controllers, i.e., closed loop stability analysis. Moreover, for a simple nonlinear closed loop system, linear approximation form is used to replace the original nonlinear controller, and then least squares algorithm and power spectrum theory are combined to derive the detailed linear approximation form, while avoiding any knowledge of nonlinear plant. So later we want to consider the nonlinear direct data driven control strategy from the theory and practice respectively, paving a new road for further research on nonlinear data driven control tuning for more complex system.

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