

ADAPTIVE DUAL-LAYER DISTURBANCE OBSERVER-BASED FULLY DISTRIBUTED FIXED-TIME CONSENSUS CONTROL FOR UNCERTAIN NONLINEAR MULTI-AGENT SYSTEMS WITH DECEPTION ATTACKS

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ABSTRACT. *A novel fully distributed fixed-time backstepping control algorithm based on an adaptive dual-layer disturbance observer (ADLDO) is proposed for nonlinear multi-agent systems with uncertainty and deception attacks. Firstly, an ADLDO is designed to compensate for the lumped disturbance composed of external system disturbance and deception attack signals. Then, based on the designed ADLDO, a new fully distributed fixed-time backstepping consensus control strategy is designed, and the filtering compensation error is introduced to improve the control accuracy of the system. Finally, the effectiveness of the proposed fixed-time backstepping control strategy is verified through simulation.*

Keywords: Fully distributed consensus control, Fixed-time control, Multi-agent systems, Backstepping control

1. Introduction. With the development of information technology and communication networks, it is difficult for a single individual to independently complete complex tasks. To address this challenge, the multi-agent systems with communication, perception, and decision-making capabilities are introduced to solve this problem [1,2]. Many scholars have investigated the problem of consensus control for multi-agent systems. Based on the knowledge of graph theory, three kinds of consensus controllers are proposed for hybrid multi-agent systems in [3]. In [4], the system transformation method is used to extend the results of [3] to second-order hybrid multi-agent systems, and two kinds of controllers are designed to realize the second-order consensus. Furthermore, [5] considers a more practical scenario by incorporating nonlinearity, and a consensus controller based on a novel Lyapunov function is proposed to ensure asymptotic consensus. In [6], an adaptive event-triggered consensus controller is designed to solve the problem of multiplicative faults for general linear multi-agent systems, and the leaderless consensus and leader-following consensus are achieved. In order to further reduce the number of triggers, a dual terminal triggered output feedback consensus controller is proposed for one-sided Lipschitz

multi-agent systems in [7]. Based on reinforcement learning and fuzzy rules emulated networks, a distributed optimal consensus control strategy is implemented for a class of heterogeneous discrete-time nonlinear multi-agent systems with unknown dynamics and uncertain control directions in [8].

In practical engineering, some tasks need to be completed within a limited time. However, the aforementioned methods do not take convergence time into account. Many scholars have conducted research on finite-time consensus control. The added power integrator method is used to achieve finite-time consensus control for multi-agent systems [9,10]. By employing the homogeneous method and neighbor-based rules, finite-time consensus control strategies are proposed for nonlinear multi-agent systems [11-13]. A consensus tracking control method based on integral sliding mode control and nonlinear disturbance observer is investigated to achieve coordination control of multiple linear motor systems with disturbance in finite time [14]. However, the convergence time of the aforementioned methods depends on initial value of the system. When the initial values are large, the convergence time will become excessively long. To solve this problem, many scholars have studied fixed-time consensus control. In [15], based on bi-limit homogeneity technique, a fixed-time consensus controller is proposed for multi-agent systems with mismatched disturbances and actuator faults. In [16], a fixed-time consensus controller is designed by using adding power integrator method, and the problem of communication closed-loop in [15] is avoided.

Due to the openness of networks, multi-agent systems are susceptible to malicious network attacks. The above methods are designed based on an ideal network environment. However, in practice, malicious network attacks can lead to performance degradation or even instability. Deception attacks are a common type of network attacks [17,18]. Based on linear iterative strategies, the distributed function calculation is designed for malicious agents in [19]. By using the improved Lyapunov-Krasovskii functional method, an adaptive neural-network controller is proposed for first-order nonlinear multi-agent systems with time delays and actuator attacks in [20], and the assumptions in [19] are relaxed. [21] extends this problem to second-order multi-agent systems. Novel adaptive control protocols are proposed to effectively mitigate the impact of these attacks by updating adaptive parameters. Based on algebraic graph theory and the Lyapunov approach, various sufficient conditions are established, which show that fixed-time consensus and finite-time flocking can be achieved for multi-agent systems. It should be noted that the control strategies in the aforementioned literature require global information about the system. As far as we know, the fully distributed fixed-time consensus control strategy for multi-agent systems under deception attack is a challenging problem, which inspires the research motivation of this article.

Based on the above discussion, the fully distributed fixed-time consensus controller for nonlinear multi-agent systems with uncertainty and deception attacks is investigated in this article. The main challenges lie in addressing the negative effects of deception attacks and achieving fixed-time consensus without relying on any global information. The main contributions are as follows. 1) Compared to [21], an ADLDO is designed to compensate for the lumped disturbance, comprising external system disturbances and deception attack signals, without requiring knowledge of the first derivative of the attack signal. 2) A new fully distributed fixed-time backstepping consensus control strategy is designed, and a filtering compensation error mechanism is introduced to improve the control accuracy of the system. The rest of this paper is organized as follows. In Section 2, some preliminaries for a class of nonlinear multi-agent systems under deception attacks are presented. In Section 3, the adaptive dual-layer disturbance observer is given. Section 4 presents the

whole fully distributed fixed-time consensus control strategy. The simulation results are shown in Section 5 and the work is summarized in Section 6.

2. Problem Statement and Preliminaries.

2.1. Communication topology. Let $G = (\Upsilon, E, A)$ denote a topology graph where $\Upsilon = [\varsigma_1, \dots, \varsigma_N]$ is the set of nodes, $E \subseteq \Upsilon \times \Upsilon$ is the set of edges, and $A = [a_{ij}] \in R^{N \times N}$ is the weighted adjacency matrix of graph G with nonnegative elements. The weight $a_{ij} > 0$ if and only if $(\varsigma_i, \varsigma_j) \in E$, and $a_{ij} = 0$ otherwise. The L is the Laplace matrix of graph G , which is a symmetric matrix. The adjacency matrix of a leader is defined as $B = \text{diag}\{b_1, b_2, \dots, b_N\}$. If there exists a path between the agent i and the leader, then $b_i > 0$; otherwise, $b_i = 0$. Define \bar{G} as the augmented topology graph of multi-agent systems containing a leader, and the Laplace matrix of \bar{G} is $H = L + B$. For any two nodes ς_i and ς_j , if there exists a path between them, then G is called a connected graph.

Assumption 2.1. *The communication topology graph \bar{G} is undirected and contains at least one directed spanning tree.*

2.2. Problem formulation. Consider a class of nonlinear second-order multi-agent systems consisting of one leader and N followers. The dynamics of the i -th follower can be described as

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = f(x_i, v_i) + u_i^a + d_i^a \end{cases} \quad (1)$$

where $x_i \in R^n$, $v_i \in R^n$ and $u_i^a \in R^n$ are the position, velocity and control input of follower i , separately. $f(x_i, v_i)$ is the inherent nonlinear function, and d_i^a is the external disturbance. The dynamic of the leader is

$$\begin{cases} \dot{x}_0 = v_0 \\ \dot{v}_0 = f(x_0, v_0) \end{cases} \quad (2)$$

where $x_0 \in R^n$ and $v_0 \in R^n$ are the position and velocity of the leader, separately. $f(x_0, v_0)$ is the nonlinear function.

Due to the openness of the network, control signals are easily affected by network attacks during transmission. In this article, the leader is regarded as a reference signal and thus the leader can be assumed to be immune from attacks. Therefore, the actual control input of follower i is considered as $u_i^a = u_i + \Delta_i \Phi_i$, and $\Delta_i = 1$ when an attack occurs, $\Delta_i = 0$ when no attack occurs, and Φ_i is the signal of deception attack.

Assumption 2.2. *The external disturbance d_i^a and signal of deception attack Φ_i are bounded, and there exist constants δ_1 and δ_2 satisfying $|\dot{d}_{il}| \leq \delta_1$, $|\ddot{d}_{il}| \leq \delta_2$, where $d_i = d_i^a + \Delta_i \Phi_i$, $d_i = [d_{i1}, d_{i2}, \dots, d_{in}]^T$, $l = 1, 2, \dots, n$.*

Lemma 2.1. [21] *For the system $\dot{x} = f(x)$, $f(0) = 0$, if there exists a continuously differentiable positive definite function $V(x)$ and constants $\kappa_1 > 0$, $\kappa_2 > 0$, $0 < p_1 < 1$, $p_2 > 1$, such that $\dot{V}(x) \leq -\kappa_1 V^{p_1}(x) - \kappa_2 V^{p_2}(x)$, then system $\dot{x} = f(x)$ is fixed-time stable, with a convergence time of $T \leq \frac{1}{\kappa_1(1-p_1)} + \frac{1}{\kappa_2(p_2-1)}$.*

Lemma 2.2. [22] *For system $\dot{x} = f(x)$, $f(0) = 0$, if there exists a continuously differentiable positive definite function $V(x)$ and constants $\kappa_3 > 0$, $\kappa_4 > 0$, $0 < \kappa_5 < \infty$, $0 < p_3 < 1$, $p_4 > 1$, such that $\dot{V}(x) \leq -\kappa_3 V^{p_3}(x) - \kappa_4 V^{p_4}(x) + \kappa_5$, then system $\dot{x} = f(x)$ is practical fixed-time stable, with a convergence time of $T \leq \frac{1}{\kappa_3(1-p_3)} + \frac{1}{\kappa_4(1-p_4)}$, where $0 < p_5 < 1$, and for $t > T$, $V \leq \min \left\{ \left(\frac{\kappa_5}{\kappa_3 p_5} \right)^{\frac{1}{p_3}}, \left(\frac{\kappa_5}{\kappa_4 p_5} \right)^{\frac{1}{p_4}} \right\}$.*

Consider deception attacks and dynamic of (1), the dynamic of follower i can be

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = f(x_i, v_i) + u_i + d_i \end{cases} \tag{3}$$

3. Design of Adaptive Dual-Layer Disturbance Observer. In this section, an ADLDO is designed to estimate the lumped disturbance d_i , whose dynamic is

$$\begin{cases} \dot{\hat{v}}_i = f(x_i, v_i) + u_i + \hat{d}_i \\ \dot{\hat{d}}_i = \Lambda_i + k_{i,1}\phi_{i,1} \end{cases} \tag{4}$$

where $\Lambda_i = k_{i,2}\phi_{i,2} + k_{i,3}sign(\eta_i)$, $\phi_{i,1} = sig^{\theta_1}(\sigma_i) + k_{i,4}sig^{\theta_2}(\sigma_i)$, $\phi_{i,2} = sig^{\theta_3}(\eta_i) + k_{i,5}sig^{\theta_4}(\eta_i)$, $\sigma_i = v_i - \hat{v}_i$, $\eta_i = \dot{\sigma}_i + k_{i,1}\phi_{i,1}$, $0 < \theta_1 < 1$, $\theta_2 > 1$, $0 < \theta_3 < 1$, $\theta_4 > 1$, $k_{i,1}, k_{i,2}, k_{i,4}, k_{i,5} > 0$, $k_{i,3} = diag\{k_{i,31}, k_{i,32}, \dots, k_{i,3n}\}$, $k_{i,3l} > \delta_1$. Since δ_1 is unknown, $k_{i,3l}$ is designed as the following adaptive law form

$$\begin{cases} \dot{k}_{i,3l} = r_0 + r_{i,l} \\ \dot{r}_{i,l} = -\ell_1 sig^{\gamma_1}(\delta_{i,l}) - \ell_2 sig^{\gamma_2}(\delta_{i,l}) \\ \dot{\delta}_{i,l} = k_{i,3l} - \frac{1}{\ell_0} |\bar{u}_{eqil}| - \vartheta_a \end{cases} \tag{5}$$

where $0 < \ell_0 < 1$, $\ell_1, \ell_2 > 0$, $0 < \gamma_1 < 1$, $\gamma_2 > 1$, $\vartheta_a > 0$, $r_0 > 0$, the term u_{eqi} is used to counteract the first-order derivative of the disturbance \dot{d}_i , where u_{eqi} is the average value of the term $k_{i,3}sign(\eta_i)$, i.e., $|u_{eqil}| = \left| \dot{d}_{il} \right|$, $u_{eqi} = [u_{eqi1}, u_{eqi2}, \dots, u_{eqin}]^T$, $l = 1, 2, \dots, n$. In addition, a low-pass filter is introduced to approximate u_{eqil} in the form of

$$\dot{u}_{eqil} = \frac{1}{\iota_0} (\iota_1 sig^{\gamma_1}(e_{ail}) + \iota_2 sig^{\gamma_2}(e_{ail})) \tag{6}$$

where $e_{ail} = k_{i,3l} - \bar{u}_{eqil}$, $\iota_0, \iota_1, \iota_2 > 0$.

According to the definition of $\sigma_i = v_i - \hat{v}_i$, its derivative can be obtained that

$$\dot{\sigma}_i = \dot{v}_i - \dot{\hat{v}}_i = d_i - \hat{d}_i \tag{7}$$

Theorem 3.1. Consider system (2) and (3) and the designed ADLDO (4) and (5), when Assumption 2.2 is satisfied, the lumped disturbances can be estimated accurately and the estimation error can converge within a fixed time.

Proof: According to the definition of η_i , the derivative of η_i is

$$\dot{\eta}_i = \dot{d}_i - \dot{\hat{d}}_i + k_{i,1}\dot{\phi}_{i,1} = \dot{d}_i - k_{i,2}\phi_{i,2} - k_{i,3}sign(\eta_i) \tag{8}$$

Selecting Lyapunov function as $V_{\eta_i} = \frac{1}{2}\eta_i^T \eta_i$, its derivative can be

$$\dot{V}_{\eta_i} = \eta_i^T \left(\dot{d}_i - k_{i,2}\phi_{i,2} - k_{i,3}sign(\eta_i) \right) \leq -k_{i,2}\eta_i^T \phi_{i,2} - k_{i,3}\eta_i^T sign(\eta_i) + \left\| \dot{d}_i \right\| \left\| \eta_i \right\| \tag{9}$$

According to $\left\| \dot{d}_i \right\| \leq \delta_1$, if $k_{i,3}$ is selected as $k_{i,3} > \delta_1$, then we can have

$$\begin{aligned} \dot{V}_{\eta_i} &\leq -k_{i,2}\eta_i^T sig^{\theta_3}(\eta_i) - k_{i,2}k_{i,5}\eta_i^T sig^{\theta_4}(\eta_i) \\ &\leq -2^{\frac{\theta_3+1}{2}} k_{i,2} V_{\eta_i}^{\frac{\theta_3+1}{2}} - 2^{\frac{\theta_4+1}{2}} k_{i,2} k_{i,5} n^{1-\frac{\theta_4+1}{2}} V_{\eta_i}^{\frac{\theta_4+1}{2}} \end{aligned} \tag{10}$$

According to Lemma 2.1, the state of η_i converges within a fixed time $T_1 \leq \frac{2^{-\frac{\theta_3+1}{2}}}{k_{i,2}(1-\frac{\theta_3+1}{2})}$

+ $\frac{2^{-\frac{\theta_4+1}{2}}}{k_{i,2}k_{i,5}n^{1-\frac{\theta_4+1}{2}}(\frac{\theta_4+1}{2}-1)}$. When $t > T_1$, $\eta_i = 0$, then

$$\dot{\sigma}_i = -k_{i,1}\phi_{i,1} = -k_{i,1}sig^{\theta_1}(\sigma_i) - k_{i,1}k_{i,4}sig^{\theta_2}(\sigma_i) \tag{11}$$

Selecting $V_{\sigma_i} = \frac{1}{2}\sigma_i^T \sigma_i$, its derivative can be

$$\dot{V}_{\sigma_i} \leq -2^{\frac{\theta_1+1}{2}} k_{i,1} V_{\sigma_i}^{\frac{\theta_1+1}{2}} - 2^{\frac{\theta_2+1}{2}} k_{i,1} k_{i,4} n^{1-\frac{\theta_2+1}{2}} V_{\sigma_i}^{\frac{\theta_2+1}{2}} \tag{12}$$

According to Lemma 2.1, the state of σ_i converges within a fixed time $T_2 \leq \frac{2^{-\frac{\theta_1+1}{2}}}{k_{i,1}(1-\frac{\theta_1+1}{2})} + \frac{2^{-\frac{\theta_2+1}{2}}}{k_{i,1} k_{i,4} n^{1-\frac{\theta_2+1}{2}} (\frac{\theta_2+1}{2}-1)}$. When $t > T_2$, $\sigma_i = \dot{\sigma}_i = 0$, then, $\sigma_i - \hat{\sigma}_i = 0$, $d_i - \hat{d}_i = 0$. Thus, it can be known that $\hat{\sigma}_i$ and \hat{d}_i can estimate σ_i and d_i within a fixed time.

Next, we will prove that $k_{i,3l} > |\dot{d}_{il}|$.

By taking the derivative of $\delta_{i,l}$, we can obtain

$$\dot{\delta}_{i,l} = \dot{k}_{i,3l} - \frac{1}{\ell_0} |\dot{u}_{eqil}| = \ell_1 sig^{\gamma_1}(\delta_{i,l}) - \ell_2 sig^{\gamma_2}(\delta_{i,l}) - \frac{1}{\ell_0} \phi_{ail} \tag{13}$$

where $\phi_{ail} = |\dot{u}_{eqil}|$ and there exists a constant $\bar{\ell}_a > 1$ such that $\|\phi_{ai}\| \leq \bar{\ell}_a \delta_3$, $\phi_{ai} = [\phi_{ai1}, \phi_{ai2}, \dots, \phi_{ain}]^T$.

According to $\delta_i = [\delta_{i,1}, \delta_{i,2}, \dots, \delta_{i,n}]^T$, it can obtain

$$\dot{\delta}_i = -\ell_1 sig^{\gamma_1}(\delta_i) - \ell_2 sig^{\gamma_2}(\delta_i) - \frac{1}{\ell_0} \phi_{ai} \tag{14}$$

Select $V_{\delta_i} = \frac{1}{2}\delta_i^T \delta_i$ and taking its derivative yields

$$\dot{V}_{\delta_i} \leq -2^{\frac{\gamma_1+1}{2}} \ell_1 V_{\delta_i}^{\frac{\gamma_1+1}{2}} - 2^{\frac{\gamma_2+1}{2}} \ell_2 n^{1-\frac{\gamma_2+1}{2}} V_{\delta_i}^{\frac{\gamma_2+1}{2}} + \ell_3 \tag{15}$$

where $\ell_3 = \frac{1}{\ell_0} \bar{\ell}_a \delta_3 \|\delta_i\|$.

According to Lemma 2.2, the state of δ_i converges within a fixed time $T_3 \leq \frac{2^{-\frac{\gamma_1+1}{2}}}{\ell_1(1-\bar{\ell}) (1-\frac{\gamma_1+1}{2})} + \frac{2^{-\frac{\gamma_2+1}{2}}}{\ell_2 n^{1-\frac{\gamma_2+1}{2}} (1-\bar{\ell}) (\frac{\gamma_2+1}{2}-1)}$, where $0 < \bar{\ell} < 1$. And there exists $t_0 > 0$ satisfying $|\delta_{i,l}| < \frac{\vartheta_a}{2}$ when $t > t_0$. According to $\delta_i = k_{i,3l} - \frac{1}{\ell_0} |\bar{u}_{eqil}| - \vartheta_a$ and $|\delta_{i,l}| < \frac{\vartheta_a}{2}$, it can be obtained that $|k_{i,3l} - \frac{1}{\ell_0} |\bar{u}_{eqil}| - \vartheta_a| < \frac{\vartheta_a}{2}$. Thus, $k_{i,3l} - \frac{1}{\ell_0} |\bar{u}_{eqil}| - \vartheta_a > -\frac{\vartheta_a}{2}$. According to $|\dot{u}_{eqil}| = |\dot{d}_{il}|$ and $\ell_0 > 1$, it can be known that $k_{i,3l} > \frac{1}{\ell_0} |\bar{u}_{eqil}| + \frac{\vartheta_a}{2} > |\dot{d}_{il}| + \frac{\vartheta_a}{2} > |\dot{d}_{il}|$; thus, $k_{i,3l} > |\dot{d}_{il}|$.

Meanwhile, it can be seen that $|k_{i,3l}| < |\delta_{i,l}| + \frac{1}{\ell_0} |\bar{u}_{eqil}| + \vartheta_a < |\delta_{i,l}| + \frac{1}{\ell_0} \delta_2 + \vartheta_a$. Thus, $k_{i,3l}$ is bounded. Therefore, the designed ADLDO (4) and (5) can estimate the lumped disturbance d_i in a fixed time.

Remark 3.1. *By introducing the adaptive law (5), the ADLDO does not need to assume that the first-order derivative of the lumped disturbances is known, thereby relaxing assumptions of the system. Compared with traditional fixed-time observers, the ADLDO designed in (4) and (5) does not require prior knowledge of lumped disturbances and is more suitable for engineering practice.*

4. Distributed Fixed-Time Consensus Control. To achieve distributed fixed-time consensus in multi-agent systems under deception attacks, combining integral sliding mode control with backstepping control, a fully distributed fixed-time consensus controller is designed in this section.

Define tracking error as

$$\begin{cases} z_{i,1} = x_i - x_0 \\ z_{i,2} = v_i - v_0 - \varsigma_{i,1} \end{cases} \tag{16}$$

where $\varsigma_{i,1}$ is the output of the fixed-time command filter, the fixed-time command filter [23] is designed as

$$\begin{cases} \dot{\varsigma}_{i,1} = \zeta_i = -k_{\varsigma,1} \text{sig}^{\frac{1}{2}}(\varsigma_{i,1} - \alpha_{i,1}) - k_{\varsigma,2} \text{sig}^{\lambda_1}(\varsigma_{i,1} - \alpha_{i,1}) + \varsigma_{i,2} \\ \dot{\varsigma}_{i,2} = -k_{\varsigma,3}(\varsigma_{i,1} - \alpha_{i,1}) - k_{\varsigma,4} \text{sig}^{2\lambda_1-1}(\varsigma_{i,1} - \alpha_{i,1}) - \text{sign}(\varsigma_{i,1} - \alpha_{i,1}) \end{cases} \quad (17)$$

where $\alpha_{i,1}$ is the input of the fixed time command filter, which will be designed later, $k_{\varsigma,1}, k_{\varsigma,2}, k_{\varsigma,3}, k_{\varsigma,4} > 0, \lambda_1 > 1$. According to [23], the filtering error $\varsigma_{i,1} - \alpha_{i,1}$ is bounded. Therefore, there exists a constant $c_e > 0$ that satisfies $\varsigma_{i,1} - \alpha_{i,1} \leq c_e$.

To compensate for filtering errors, an error compensation variable is introduced as

$$\begin{cases} \dot{\varphi}_{i,1} = -k_{\varphi,1} \text{sig}^{\lambda_2}(\varphi_{i,1}) - k_{\varphi,2} \text{sign}(\varphi_{i,1}) - \varrho_1 \varphi_{i,1} + \varphi_{i,2} + \varsigma_{i,1} - \alpha_{i,1} \\ \dot{\varphi}_{i,2} = -\varrho_2 \varphi_{i,2} - k_{\varphi,3} \text{sig}^{\lambda_2}(\varphi_{i,2}) - k_{\varphi,4} \text{sign}(\varphi_{i,2}) \end{cases} \quad (18)$$

where $k_{\varphi,1}, k_{\varphi,2}, k_{\varphi,3}, k_{\varphi,4} > 0, \lambda_2 > 1, \varrho_1 > 1, \varrho_2 > 0$.

Design integral sliding mode as

$$s_i = v_i - v_0 - \int_{t_0}^t u_{i0} d\tau \quad (19)$$

where u_{i0} is the controller to be designed later.

Based on the designed integral sliding mode, a fixed-time consensus controller is designed as

$$\begin{aligned} u_i &= u_{i0} + u_{inom} \\ u_{inom} &= -f(x_i, v_i) + f(x_0, v_0) - \hat{d}_i - k_{\mu,1} \text{sig}^{\beta_1} \left(\sum_{j=1}^N a_{ij}(s_i - s_j) + b_i s_i \right) \\ &\quad - k_{\mu,2} \text{sig}^{\beta_2} \left(\sum_{j=1}^N a_{ij}(s_i - s_j) + b_i s_i \right) \\ u_{i0} &= -\varrho_2 z_{i,2} - k_{\mu,3} \text{sig}^{\beta_3}(\xi_{i,2}) - k_{\mu,4} \text{sig}^{\beta_4}(\xi_{i,2}) + \zeta_i - k_{\varphi,3} \text{sig}^{\lambda_2}(\varphi_{i,2}) - \xi_{i,1} \\ &\quad - k_{\varphi,4} \text{sign}(\varphi_{i,2}) \end{aligned} \quad (20)$$

where $\xi_{i,1} = z_{i,1} - \varphi_{i,1}, \xi_{i,2} = z_{i,2} - \varphi_{i,2}, k_{\mu,1}, k_{\mu,2}, k_{\mu,3}, k_{\mu,4} > 0, 0 < \beta_1 < 1, \beta_2 > 1, 0 < \beta_3 < 1, \beta_4 > 1$.

Theorem 4.1. Consider system (1) and (2) with the external disturbances and deception attacks, under the designed ADLDO (4) and (5) and fully distributed fixed-time consensus controller (20), the tacking errors of the system can converge within a fixed time.

Proof: By taking the derivative of s_i , we can obtain

$$\begin{aligned} \dot{s}_i &= d_i - \hat{d}_i - k_{\mu,1} \text{sig}^{\beta_1} \left(\sum_{j=1}^N a_{ij}(s_i - s_j) + b_i s_i \right) \\ &\quad - k_{\mu,2} \text{sig}^{\beta_2} \left(\sum_{j=1}^N a_{ij}(s_i - s_j) + b_i s_i \right) \end{aligned} \quad (21)$$

According to the designed ADLDO, when $t > T_1 + T_2, d_i - \hat{d}_i = 0$. Defining $s = [s_1^T, s_2^T, \dots, s_N^T]^T$, and selecting $V_s = \frac{1}{2} s^T (H \otimes I_n) s$, it can be known that

$$\begin{aligned} \dot{V}_s &= -s^T (H \otimes I_n) (k_{\mu,1} \text{sig}^{\beta_1}((H \otimes I_n) s) + k_{\mu,2} \text{sig}^{\beta_2}((H \otimes I_n) s)) \\ &\leq -k_{\mu,1} (2\lambda_{\max}(H))^{\frac{\beta_1+1}{2}} V_s^{\frac{\beta_1+1}{2}} - k_{\mu,2} (n + N)^{\frac{1-\beta_2}{2}} (2\lambda_{\max}(H))^{\frac{\beta_2+1}{2}} V_s^{\frac{\beta_2+1}{2}} \end{aligned} \quad (22)$$

According to Lemma 2.1, the state s can converge within a fixed time T_4 . When $t > T_4$, $\dot{s}_i = 0$. Thus, according to the definition of s_i , we can have $u_{i0} = \dot{v}_i - \dot{v}_0$.

Define $\xi_1 = [\xi_{1,1}^T, \xi_{2,1}^T, \dots, \xi_{N,1}^T]^T$, and select $V_1 = \frac{1}{2}\xi_1^T(H \otimes I_n)\xi_1$, then

$$\dot{V}_1 = \sum_{i=1}^N \chi_{i,1} (\xi_{i,2} + \varphi_{i,2} + \varsigma_{i,1} - \alpha_{i,1} + \alpha_{i,1} - \dot{\varphi}_{i,1}) \quad (23)$$

where $\chi_{i,1} = \sum_{j=1}^N a_{ij}(\xi_{i,1} - \xi_{j,1}) + b_i \xi_{i,1}$.

The virtual control law is designed as

$$\alpha_{i,1} = -k_{\xi,1} \text{sig}^{\beta_3}(\xi_{i,1}) - k_{\xi,2} \text{sig}^{\beta_4}(\xi_{i,1}) - \varrho_1 z_{i,1} - k_{\varphi,1} \text{sig}^{\lambda_2}(\varphi_{i,1}) - k_{\varphi,2} \text{sign}(\varphi_{i,1}) \quad (24)$$

where $k_{\xi,1}, k_{\xi,2} > 0$.

Substituting $\alpha_{i,1}$ into (23) yields

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^N \chi_{i,1}^T (\xi_{i,2} - k_{\xi,1} \text{sig}^{\beta_3}(\xi_{i,1}) - k_{\xi,2} \text{sig}^{\beta_4}(\xi_{i,1}) - \varrho_1 \xi_{i,1}) \\ &\leq \sum_{i=1}^N \chi_{i,1}^T \xi_{i,2} - \bar{c}_1 V_1^{\frac{\beta_3+1}{2}} - \bar{c}_2 V_1^{\frac{\beta_4+1}{2}} - 2\varrho_1 V_1 \end{aligned} \quad (25)$$

where $\bar{c}_1 = k_{\xi,1} 2^{\frac{\beta_3+1}{2}} \lambda_{\min}(H) \lambda_{\max}^{-\frac{\beta_3+1}{2}}(H)$, $\bar{c}_2 = k_{\xi,2} 2^{\frac{\beta_4+1}{2}} (n+N)^{\frac{1-\beta_4}{2}} \lambda_{\min}(H) \lambda_{\max}^{-\frac{\beta_4+1}{2}}(H)$.

Select $V_2 = \frac{1}{2}\xi_2^T(H \otimes I_n)\xi_2$, $\xi_2 = [\xi_{1,2}^T, \xi_{2,2}^T, \dots, \xi_{N,2}^T]^T$, considering $u_{i0} = \dot{v}_i - \dot{v}_0$, then

$$\begin{aligned} \dot{V}_2 &= \sum_{i=1}^N \chi_{i,2}^T (-\varrho_2 \xi_{i,2} - k_{\mu,3} \text{sig}^{\beta_3}(\xi_{i,2}) - k_{\mu,4} \text{sig}^{\beta_4}(\xi_{i,2}) - \xi_{i,1}) \\ &\leq -2\varrho_2 V_2 - \bar{c}_3 V_2^{\frac{\beta_3+1}{2}} - \bar{c}_4 V_2^{\frac{\beta_4+1}{2}} - \sum_{i=1}^n \chi_{i,2}^T \xi_{i,1} \end{aligned} \quad (26)$$

where $\bar{c}_3 = k_{\mu,3} (2\lambda_{\max}(H))^{\frac{\beta_3+1}{2}} \lambda_{\max}^{\frac{\beta_3+1}{2}}(H)$, $\bar{c}_4 = k_{\mu,4} (n+N)^{\frac{1-\beta_4}{2}} (2\lambda_{\max}(H))^{\frac{\beta_4+1}{2}} \lambda_{\max}^{\frac{\beta_4+1}{2}}(H)$.

Then, selecting $V = V_1 + V_2$, it can be seen that

$$\dot{V} \leq -\bar{c}_5 V^{\frac{\beta_3+1}{2}} - \bar{c}_6 V^{\frac{\beta_4+1}{2}} \quad (27)$$

where $\bar{c}_5 = \min\{\bar{c}_1, \bar{c}_3\}$, $\bar{c}_6 = \min\{\bar{c}_2, \bar{c}_4\}$.

According to Lemma 2.1, $\xi_{i,1}$ and $\xi_{i,2}$ can converge within a fixed time.

Select $V_{\varphi_1} = \frac{1}{2}\varphi_1^T \varphi_1$, where $\varphi_1 = [\varphi_{1,1}^T, \varphi_{2,1}^T, \dots, \varphi_{N,1}^T]^T$. According to $\varrho_1 > 1$ and $\varsigma_{i,1} - \alpha_{i,1} \leq c_e$, then

$$\begin{aligned} \dot{V}_{\varphi_1} &= \sum_{i=1}^N \varphi_{i,1}^T (-k_{\varphi,1} \text{sig}^{\lambda_2}(\varphi_{i,1}) - k_{\varphi,2} \text{sign}(\varphi_{i,1}) - \varrho_1 \varphi_{i,1} + \varphi_{i,2} + \varsigma_{i,1} - \alpha_{i,1}) \\ &\leq -2^{\frac{\lambda_2+1}{2}} (n+N)^{\frac{1-\lambda_2}{2}} k_{\varphi,1} V_{\varphi_1}^{\frac{\lambda_2+1}{2}} - \sqrt{2} k_{\varphi,2} V_{\varphi_1}^{\frac{1}{2}} + \frac{1}{2} c_e^2 \end{aligned} \quad (28)$$

Select $V_{\varphi} = V_{\varphi_1} + V_{\varphi_2}$, where $V_{\varphi_2} = \frac{1}{2}\varphi_2^T \varphi_2$, $\varphi_2 = [\varphi_{1,2}^T, \varphi_{2,2}^T, \dots, \varphi_{N,2}^T]^T$, then

$$\begin{aligned} \dot{V}_{\varphi} &= \dot{V}_{\varphi_1} + \sum_{i=1}^N \varphi_{i,2}^T (-\varrho_2 \varphi_{i,2} - k_{\varphi,3} \text{sig}^{\lambda_2}(\varphi_{i,2}) - k_{\varphi,4} \text{sign}(\varphi_{i,2})) \\ &\leq \dot{V}_{\varphi_1} - 2^{\frac{\lambda_2+1}{2}} (n+N)^{\frac{1-\lambda_2}{2}} k_{\varphi,3} V_{\varphi_2}^{\frac{\lambda_2+1}{2}} - \sqrt{2} k_{\varphi,4} V_{\varphi_2}^{\frac{1}{2}} \\ &\leq -\bar{c}_7 V_{\varphi}^{\frac{\lambda_2+1}{2}} - \bar{c}_8 V_{\varphi}^{\frac{1}{2}} + \frac{1}{2} c_e^2 \end{aligned} \quad (29)$$

where $\bar{c}_7 = (n + N)^{\frac{1-\lambda_2}{2}} \min\{k_{\varphi,1}, k_{\varphi,3}\}$, $\bar{c}_8 = \sqrt{2} \min\{k_{\varphi,2}, k_{\varphi,4}\}$.

According to Lemma 2.2, error compensation variables $\varphi_{i,1}$ and $\varphi_{i,2}$ can converge to the neighborhood near the origin within a fixed time. Thus, the tracking error $z_{i,1}$ and $z_{i,2}$ can converge to the neighborhood near the origin within a fixed time.

Remark 4.1. *By combining integral sliding mode with fixed-time backstepping control, the controller parameters designed in this paper do not depend on the global information of the system, thereby enhancing both flexibility and robustness.*

5. Numerical Example. An example of a multi-spacecraft attitude control systems is worked out to verify the proposed control strategy. Consider a target spacecraft situated in an Earth orbit, and three follower spacecrafts orbit around this target spacecraft. The communication topology between the target spacecraft and three follower spacecrafts is

$$L = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The i -th follower spacecraft with respect to the target spacecraft can be modeled by

$$M_i(q_i)\ddot{q}_i = C_i(q_i, \dot{q}_i) \dot{q}_i = u_i + d_i \tag{30}$$

where $q_i = [q_{i,1}, q_{i,2}, q_{i,3}]^T$, $\dot{q}_i = Z_i(q_i)\omega_i$, $Z_i = \frac{1}{2} \left[\frac{1-q_i^T q_i}{2} I_3 + q_i q_i^T \right]$, $M_i(q_i) = Z^{-T}(q_i) J_i Z_i^{-1}(q_i)$, $C_i(q_i, \dot{q}_i) = -Z_i^{-T}(q_i) (J_i Z_i^{-1}(q_i) \dot{q}_i)^\times Z_i^{-1}(q_i) - Z_i^{-T}(q_i) J_i Z^{-1}(q_i) \dot{Z}_i(q_i) Z_i^{-1}(q_i)$, u_i is the control input, d_i are external disturbances, J_i is the moment of inertia, q_i^\times is the antisymmetric matrix of q_i . The dynamic of target spacecraft is

$$\begin{cases} \dot{q}_0 = v_0 \\ \dot{v}_0 = [-0.2^2 \sin(0.2t), 0.2^2 \cos(0.2t), 0.2^2 \cos(0.2t)] \end{cases} \tag{31}$$

Select $J_1 = \text{diag}\{17, 12, 9\}$, $J_2 = \text{diag}\{14, 13, 10\}$, $J_3 = \text{diag}\{20, 10, 12\}$. The external disturbances are set as $d_i = 0.01[\cos(0.2t), -\sin(0.2t), \cos(0.3t)]^T$. Spacecraft 1 has been subjected to a deception attack in the form of $u_1^a = u_1 + 0.03[\cos(0.3t), -\cos(0.2t), \cos(0.5t)]^T$.

The parameters of designed observer are $k_{i,1} = 7.5$, $k_{i,2} = 2$, $k_{i,4} = 3.8$, $k_{i,5} = 2.5$, $\theta_1 = 0.3$, $\theta_2 = 1.8$, $\theta_3 = 0.45$, $\theta_4 = 1.25$, $r_0 = 0.1$, $\ell_0 = 0.85$, $\ell_1 = 2.5$, $\ell_2 = 2$, $\vartheta_a = 0.1$, $\gamma_1 = 0.55$, $\gamma_2 = 1.35$.

The parameters of filter are $k_{\varsigma,1} = 5$, $k_{\varsigma,2} = 2$, $k_{\varsigma,3} = 2$, $k_{\varsigma,4} = 0.2$, $\lambda_1 = 4.5$. The parameters of controller are $k_{\varphi,1} = 2$, $k_{\varphi,2} = 0.2$, $k_{\varphi,3} = 2$, $k_{\varphi,4} = 2$, $\lambda_2 = 5.5$, $\varrho_1 = 1.5$, $\varrho_2 = 5.5$, $k_{\mu_1} = 0.01$, $k_{\mu_2} = 0.2$, $k_{\mu_3} = 2$, $k_{\mu_4} = 0.1$, $\beta_1 = 0.3$, $\beta_2 = 4.5$, $\beta_3 = 0.05$, $\beta_4 = 1.5$, $k_{\xi_1} = 0.03$, $k_{\xi_2} = 0.2$.

Figure 1 shows the attitude and angular velocity tracking curve under the designed controller. As can be seen from the figure, the attitude and angular velocity of the system can track the state of the leader accurately within 3 seconds.

The time evolution of the integral sliding mode manifold s_i and control input u_i are illustrated in Figure 2. It is observed that s_i is fixed-time stable. The curve of u_i is smooth and without obvious oscillation, which verifies the effectiveness and stability of the control strategy. Furthermore, a simulation comparison with the adaptive fixed-time control scheme (28) in [24] is provided. With the same initial states and other control parameters, the trajectories under the two control schemes are shown in Figure 3, where $\Omega_1 = \sqrt{\sum_{i=1}^N \|q_i - q_0\|^2}$ and $\Omega_2 = \sqrt{\sum_{j=1}^N \sum_{i=1}^N \|q_i - q_j\|^2}$ are the attitude consensus errors. From the figure, it can be observed that the control scheme designed in this article

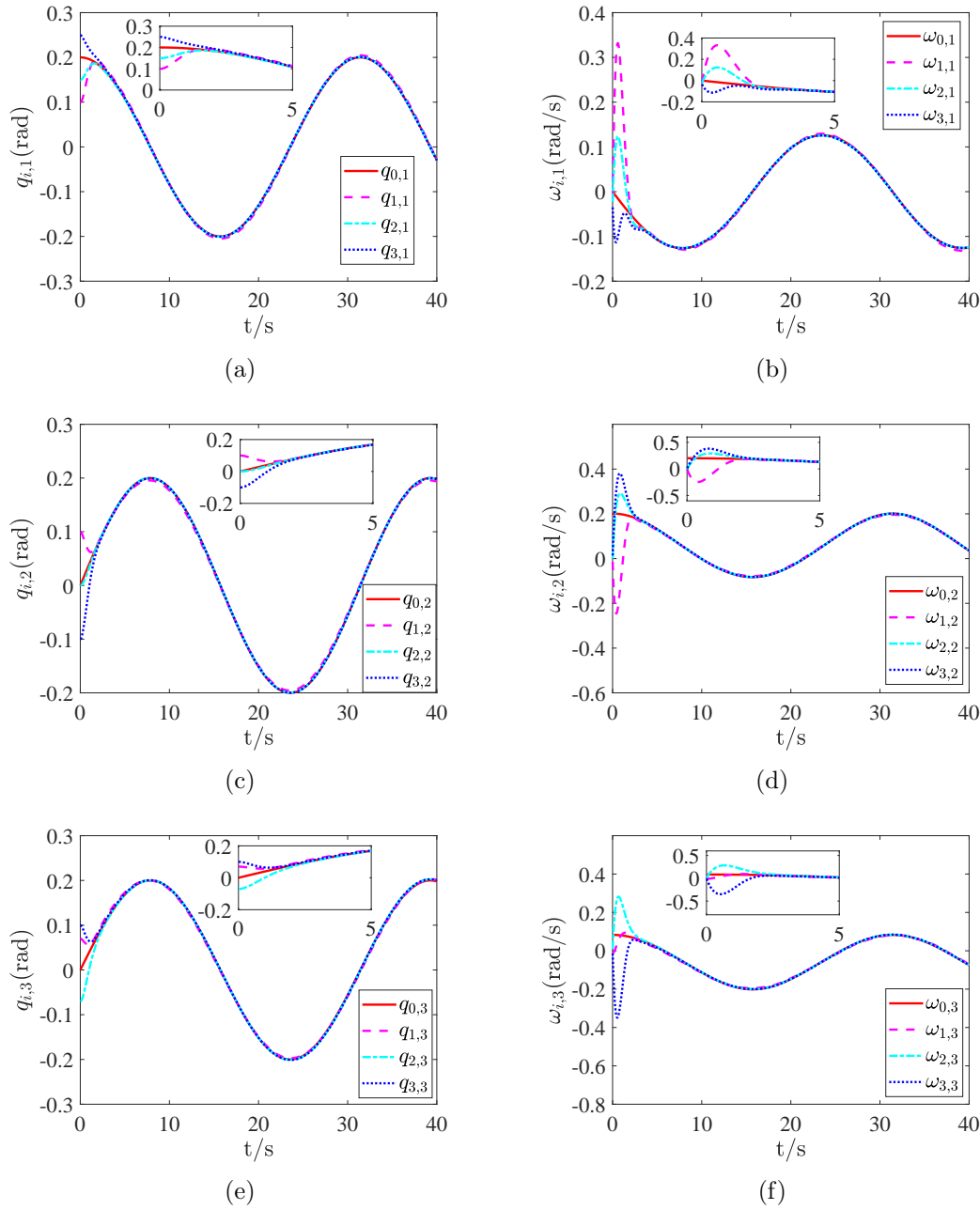


FIGURE 1. Attitude and angular velocity tracking curve

possesses a faster convergence rate than the control scheme (28) in [24]. Consequently, compared to the control scheme (28) in [24], the proposed scheme in this article is superior to some extent.

6. Conclusions. This paper investigates the distributed leader following consensus control problem for second-order nonlinear multi-agent systems under the influence of deception attacks. Firstly, a fixed-time ADLDO is constructed to estimate deception attack signals, thereby relaxing the assumptions on the attack signals. Then, based on the observation values of the attack signal, a fully distributed fixed-time backstepping integral sliding mode controller is designed to reduce the computational complexity and communication consumption of the system. Finally, the fixed-time stability theory is employed to prove that the states of the multi-agent systems can achieve consensus quickly under

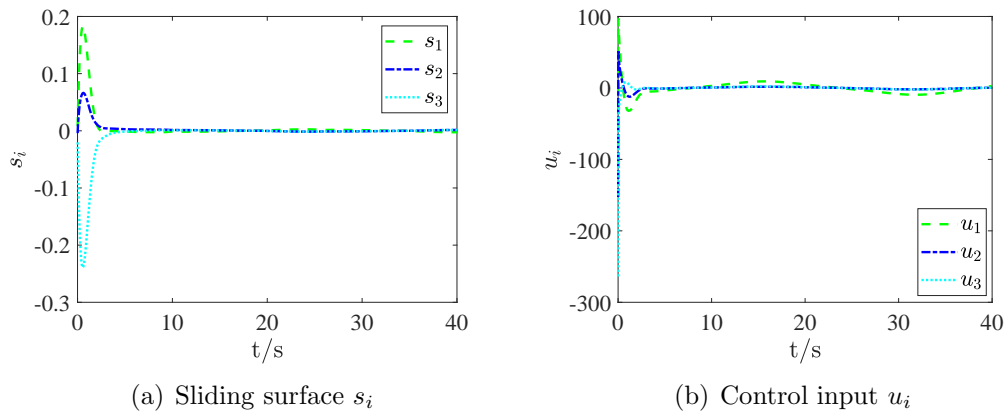


FIGURE 2. Sliding surface and control input curve

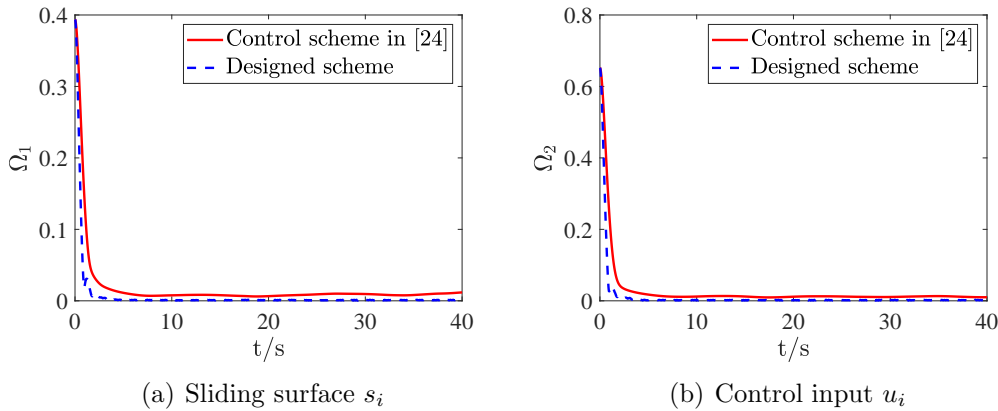


FIGURE 3. Attitude consensus errors by two control schemes

deception attacks. In future work, the Denial-of-Service attacks and switching topologies in multi-agent systems will be taken into account, and the proposed scheme will be applied to a more general system.

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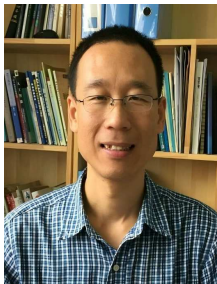
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