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A DESIGN METHOD FOR CONTROL SYSTEM USING DISTURBANCE OBSERVER FOR PERIODIC DISTURBANCE

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Abstract. *In this paper, we examine a design method for the control system using a disturbance observer for periodic disturbance. To attenuate the influence of periodic disturbances on the input-output property, the disturbance observer for periodic disturbance is considered by several researchers. Yamada et al. have clarified the parameterization of all disturbance observers and all functional disturbance observers for all disturbance observers for a periodic disturbance. Using the result of Yamada et al., we can construct control systems attenuating the periodic disturbances. However, the result of Yamada et al. does not guarantee that the control system is stable. To design the control system using the disturbance observer for periodic disturbances, it is required to obtain the parameterization of all feedback controllers to maintain stability. In this paper, we clarify the parameterization of all feedback controllers for the control system using a disturbance observer. In addition, we proposed a design method for the control system using disturbance observer.*

Keywords: Parameterization, Disturbance observer, Non-periodic reference input, Periodic disturbance

1. Introduction.

In this paper, we examine a design method for control system using a disturbance observer for periodic disturbance proposed by Yamada et al. [28].

In order to attenuate the periodic disturbance, repetitive control is a well-known method of suppressing disturbances [1], and is effective when the reference input is a periodic reference input and the disturbance is a periodic disturbance and the output follows the

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reference input without steady state error. However, when the reference input is non-periodic input, repetitive control is not effective [2]. In order to overcome this problem, a robust higher-order repetitive control [3] is proposed. However, this remains the difficulty to reduce the order of controller. There is a possibility to overcome the problem to design control systems to follow non-periodic reference input and to attenuate periodic disturbance without using repetitive control, we have possibility to use the disturbance observer, which is effective to attenuate periodic disturbance effectively.

A disturbance observer is used to estimate the disturbances in the factory plant [9], and various papers are proposed [5, 6, 7, 8]. In addition, many papers are proposing on disturbance observer control systems that utilize these papers [10, 11, 12, 13, 14], and the applications of disturbance observers have been proposed in many control systems such as a motion-control field [15, 16, 17]. A disturbance observer is used in motion control field to cancel the disturbance or to make the closed-loop system robustly stable [18, 19, 20]. Typically a disturbance observer includes a disturbance signal generator and an observer, and the disturbance that is normally considered as a step disturbance is estimated by the observer. Since the disturbance observer is simple to understand the structure, it is used in many cases [18, 19, 21].

Mita et al. pointed out that disturbance observers are not only alternative design of complete controllers [20]. Extended H_∞ control in [20] has therefore been proposed as an effective motion control method that cancels disturbances. From another point of view, Kobayashi et al. considered an observer design method for obtaining phase compensation based on disturbance observers [21].

Another important control problem is the parameterization problem which is the problem of finding all stable controllers for the plant [22]. Since if the parameterization of all disturbance observers for any disturbances could be obtained, we could express results from previous studies of disturbance observers in a uniform manner, and in addition, disturbance observers for any disturbances could be designed systematically, Yamada et al. examined the parameterization of all disturbance observers [23]. There exists another study that motion control realization is based on the disturbance observer and the Kalman filter [15]. This study realizes high robustness against disturbance, parameter variations, effective noise suppression, and wide band force sensing by using a disturbance observer and Kalman filter.

However, the disturbance observer is not limited to estimating only non-periodic disturbances. The methods proposed in [18, 23] can estimate disturbances with a finite number of frequency components but are ineffective for disturbances with an infinite number of frequency components. In practical control systems, many disturbances manifest as periodic disturbances. For instance, in false data injection attacks targeting load frequency control systems, denial-of-service attacks are often modeled as periodic disturbances [30]. Phukapak et al. further clarified the parameterization of all disturbance observers for periodic disturbances [4].

Using this parameterization in [4], we have a possibility to design a control system to follow non-periodic reference input and to attenuate periodic disturbance without using repetitive control. A design method for non-periodic reference input is proposed by Yamada et al. [28]. Yamada et al. proposed a design method using a disturbance observer for non-periodic input without using repetitive control [28]. In addition, Yamada et al. proposed a design method using a disturbance observer for non-periodic input and non-minimum phase plant without using repetitive control [29]. However, a method in [28, 29] remains a difficulty. Their control system has a disturbance observer for periodic disturbance, a controller for disturbance observer and a feedback controller. In their method, at first disturbance observer for periodic disturbance observer is designed. However, the feedback controller does not always stabilize the disturbance observer controller, making the system potentially unstable.

In this paper, we propose a design method for the control system using the disturbance observer for the periodic disturbance. First we clarify the parameterization of all stabilizing controllers using the disturbance observer for the periodic disturbance. Using obtained parameterization, we present a design procedure for the control system using a disturbance observer for periodic disturbance. This paper is organized as follows. In Section 2, the preliminary result and the problem formulation in this paper are explained. In Section 3, we examine the parameterization of all feedback controllers to make functions in the controller for the disturbance observer stable. In Section 4, we clarify that obtained feedback controllers in Section 3 make the plant stable. In Section 5, we summarize the control characteristics of the control system. In Section 6, a design procedure of the control system is presented. In Section 7, we provide a numerical example to illustrate the effectiveness of the proposed method. Section 8 gives concluding remarks.

2. Preliminary Result and Problem Formulation.

Consider the plant written by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + d(t) \end{cases}, \tag{1}$$

where $x(t) \in R^n$ is the state variable, $u(t) \in R$ is the control input, $y(t) \in R$ is the output, $r(t) \in R$ is the non-periodic reference input for $y(t)$, $d(t) \in R$ is the periodic disturbance with period $T > 0$ satisfying

$$d(t+T) = d(t) \quad (\forall t \geq 0), \tag{2}$$

$A \in R^{n\times n}$, $B \in R^n$ and $C \in R^{1\times n}$. It is assumed that (A, B) is stabilizable, (C, A) is detectable, A has no eigenvalue on the imaginary axis, and

$$\det \begin{bmatrix} A - sI & B \\ C & 0 \end{bmatrix} = 0 \tag{3}$$

has no roots in the closed right half plane. In addition, we assume that $u(t)$ and $y(t)$ are available, but $d(t)$ is unavailable. The transfer function in Equation (1) is denoted by

$$y(s) = G(s)u(s) + d(s), \tag{4}$$

where

$$G(s) = C(sI - A)^{-1}B \in R(s), \tag{5}$$

$y(s) = \mathcal{L}\{y(t)\}$, $u(s) = \mathcal{L}\{u(t)\}$ and $d(s) = \mathcal{L}\{d(t)\}$. Note that the assumption in Equation (3) implies that $G(s)$ has no zeroes in the closed right half plane, that is, $G(s)$ in Equation (5) is of minimum phase.

Under these assumptions, Yamada et al. [28] proposed a design method for control system in Figure 1 for the output $y(t)$ to follow the non-periodic reference input $r(t)$ without steady state error, for the periodic disturbance $d(t)$ to be attenuated, and for the transfer function from $d(s)$ to $y(s)$ to have a finite number of poles. Here, $\tilde{d}(s)$ is disturbance observer for periodic disturbance [4] written by

$$\tilde{d}(s) = F_1(s)e^{-sT}y(s) + F_2(s)e^{-sT}u(s). \tag{6}$$

According to [4], the parameterization of all $F_1(s)$ and $F_2(s)$ satisfying

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} \left(d(t) - \tilde{d}(t)\right) = 0 \tag{7}$$

for any initial state $x(0)$, control input $u(t)$ and periodic disturbance $d(t)$ is given by

$$F_1(s) = D(s) + Q(s)D(s) \in RH_\infty \tag{8}$$

and

$$F_2(s) = -N(s) - Q(s)N(s) \in RH_\infty, \tag{9}$$

where $D(s) \in RH_\infty$ and $N(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)}, \tag{10}$$

respectively, and $Q(s) \in RH_\infty$ is function satisfying

$$D(s_i) + Q(s_i)D(s_i) = 1 \quad \forall s_i \ (i = 0, 1, \ldots), \tag{11}$$

$$s_i = j\omega_i, \tag{12}$$

$$\omega_i = \frac{2\pi i}{T} \quad (i = 0, 1, \ldots) \tag{13}$$

and j is the imaginary unit. $C_1(s) \in R(s)$ is the feedback controller, $C_2(s) \in R(s)$ is the controller for disturbance observer [24] and given by

$$C_2(s) = \frac{C_n(s)}{1 + C_d(s)e^{-sT}}, \tag{14}$$

where $C_n(s) \in RH_\infty$ and $C_d(s) \in RH_\infty$. $C_2(s)$ in Equation (14) is used to make the transfer function from $d(s)$ to $y(s)$ have a finite number of poles. It is assumed that $G(s)$, $F_1(s) \in RH_\infty$ and $F_2(s) \in RH_\infty$ are of minimum phase. $F_1(s)$ and $F_2(s)$ are designed using the design method in [4].

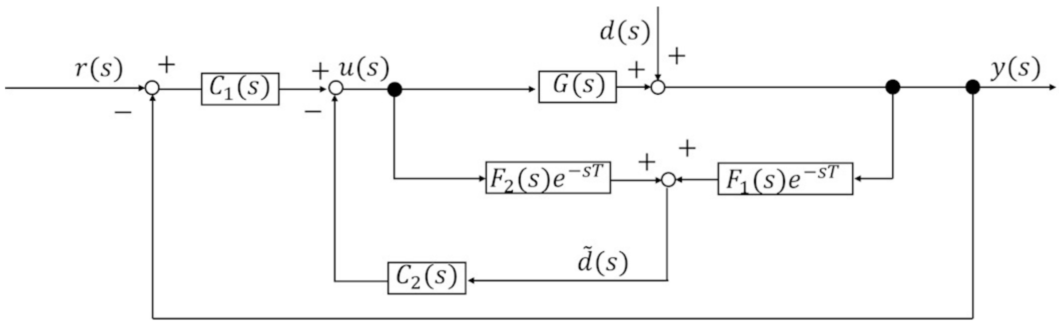

Figure 1. Structure of a disturbance observer system

Here, transfer function from $d(s)$ to $y(s)$ in Figure 1 is given by

$$\frac{y(s)}{d(s)} = \frac{1 + (C_d(s) + F_2(s)C_n(s))e^{-sT}}{1 + C_1(s)G(s) + \{(1 + C_1(s)G(s))C_d(s) + (F_1(s)G(s) + F_2(s))C_n(s)\}e^{-sT}}. \tag{15}$$

According to [28], $C_d(s)$ and $C_n(s)$ in Equation (14) to attenuate disturbances and to make the transfer function from $d(s)$ to $y(s)$ have finite number of poles are written by

$$C_d(s) = \frac{(F_1(s)N(s) + F_2(s)D(s))q(s)D_c(s)}{(F_2(s)N_c(s) - F_1(s)D_c(s))N(s)} \tag{16}$$

and

$$C_n(s) = -\frac{q(s) + C_d(s)}{F_2(s)}, \tag{17}$$

respectively, where $C_d(s)$ and $C_n(s)$ satisfy the following equations

$$1 + (C_d(s) + F_2(s)C_n(s))|_{s_i = j\frac{2\pi i}{T}} = 0 \quad (i = 0, 1, 2, \ldots) \tag{18}$$

and

$$(1 + C_1(s)G(s))C_d(s) + (F_1(s)G(s) + F_2(s))C_n(s) = 0, \tag{19}$$

respectively, to attenuate disturbances and to make the transfer function from $d(s)$ to $y(s)$ have finite number of poles. Here, $N_c(s) \in RH_\infty$ and $D_c(s) \in RH_\infty$ are coprime factors of $C_1(s)$ satisfying

$$C_1(s) = \frac{N_c(s)}{D_c(s)}, \tag{20}$$

$q(s) \in RH_\infty$ is a strictly proper low-pass filter to make $C_d(s)$ in Equation (16) proper and written by

$$q(s) = \frac{1}{(1 + \tau s)^{n_q}}, \tag{21}$$

$n_q > 0$ is positive integer and $\tau > 0$ is small real positive number and n_q is positive integer to make $q(s)/G(s)$ and $q(s)/F_2(s)$ proper. Note that in the frequency range satisfying

$$q(s_i) \simeq 1 \quad (i = 0, 1, \ldots, k_{\max}), \tag{22}$$

frequency component of the periodic disturbance $d(s)$ is attenuated, where $k_{\max}$ is maximum integer. In order to satisfy Equation (18), $C_d(s)$ and $C_n(s)$ are designed by

$$C_d(s) + F_2(s)C_n(s) = -q(s). \tag{23}$$

In addition, Yamada et al. clarified the stability conditions of the control system in Figure 1 summarized in the following lemma [28].

Lemma 2.1. *The control system in Figure 1 is stable if and only if the following conditions are satisfied.*

1) $C_1(s)$ stabilizes $G(s)$. That is, all transfer functions $C_1(s)G(s)/(1+C_1(s)G(s))$, $G(s)/(1 + C_1(s)G(s))$, $C_1(s)/(1 + C_1(s)G(s))$ and $1/(1 + C_1(s)G(s))$ are stable.
2) $C_d(s) \in RH_\infty$ in Equation (14).
3) $C_n(s) \in RH_\infty$ in Equation (14).

When using the method proposed in [28], even if the controller $C_1(s)$ stabilizes the plant $G(s)$, $C_1(s)$ does not necessarily make $C_d(s)$ in Equation (16) stable. In this case, from Equation (19), transfer functions from the periodic disturbance $d(s)$ to the output $y(s)$ written by

$$y(s) = \frac{1 + (C_d(s) + F_2(s)C_n(s))e^{-sT}}{1 + C_1(s)G(s)} d(s), \tag{24}$$

and the periodic disturbance $d(s)$ to the control input $u(s)$ written by

$$u(s) = \frac{-C_1(s) + (-C_1(s)C_d(s) - F_1(s)C_n(s))e^{-sT}}{1 + C_1(s)G(s)} d(s), \tag{25}$$

are unstable, respectively.

The problem considered in this paper is to overcome this problem and clarify the parameterization of all $C_1(s)$ to make $C_d(s)$ stable.

3. Parameterization of all $C_1(s)$ to Make $C_d(s)$ in Equation (16) Stable.

In this section, we clarify the parameterization of all $C_1(s)$ that makes $C_d(s)$ in Equation (16) stable.

The parameterization is summarized in the following theorem.

Theorem 3.1. *$C_1(s)$ written in Equation (20) makes $C_d(s)$ in Equation (16) stable if and only if $N_c(s)$ and $D_c(s)$ in Equation (20) are written by*

$$N_c(s) = \bar{X}(s) - F_1(s)\hat{Q}(s) \tag{26}$$

and

$$D_c(s) = \bar{Y}(s) - F_2(s)\hat{Q}(s), \tag{27}$$

where $\bar{X}(s) \in RH_\infty$ and $\bar{Y}(s) \in RH_\infty$ are satisfying

$$\bar{X}(s)F_2(s) - \bar{Y}(s)F_1(s) = I \tag{28}$$

and $\hat{Q}(s) \in RH_\infty$ is any function.

Proof: First, the necessity is shown, that is, we show that if $C_d(s)$ in Equation (16) is stable, $N_c(s)$ and $D_c(s)$ in Equation (20) are written by Equations (26) and (27), respectively. Since $q(s)$ is written by Equation (21), the assumptions $D_c(s) \in RH_\infty$, $F_1(s) \in RH_\infty$, $N(s) \in RH_\infty$, $F_2(s) \in RH_\infty$, $D(s) \in RH_\infty$, and $N(s)$ are of minimum phase, if $C_d(s) \in RH_\infty$, then

$$F_2(s)N_c(s) - F_1(s)D_c(s) \in \mathcal{U}. \tag{29}$$

According to [25], this equation is equivalent to

$$F_2(s)N_c(s) - F_1(s)D_c(s) = I. \tag{30}$$

Using the result in [25, 31] and simple manipulations, all solutions of $N_c(s)$ and $D_c(s)$ to Equation (30) are written by Equations (26) and (27), respectively, where $\bar{X}(s) \in RH_\infty$ and $\bar{Y}(s) \in RH_\infty$ are satisfying Equation (28) and $\hat{Q}(s) \in RH_\infty$ is any function. The necessity was shown.

Next, the sufficiency is shown. That is, we show that if $N_c(s)$ and $D_c(s)$ are written by Equations (26) and (27), respectively, then $C_d(s)$ in Equation (16) is stable. Substituting Equations (26) and (27) to Equation (16), we have

$$C_d(s) = \frac{(F_1(s)N(s) + F_2(s)D(s))q(s)\left(\bar{Y}(s) - F_2(s)\hat{Q}(s)\right)}{\left(\bar{X}(s)F_2(s) - \bar{Y}(s)F_1(s)\right)N(s)}. \tag{31}$$

Since $\bar{X}(s)F_2(s) - \bar{Y}(s)F_1(s) = I$ and $q(s)/N(s) \in RH_\infty$, $C_d(s) \in RH_\infty$ in Equation (31). The sufficiency was shown.

In this way, we have thus proved Theorem 3.1. □

Using the result in Theorem 3.1, we can design $C_1(s)$ to make $C_d(s)$ stable. However, $C_1(s)$ also needs to make $G(s)$ stable. In the next section, we will clarify that $C_1(s)$ designed in Theorem 3.1 makes $G(s)$ stable.

4. **Stability of Control System.** In this section, we will clarify that $C_1(s)$ satisfying conditions in Theorem 3.1 make $G(s)$ stable.

From the definition of internal stability, if all transfer functions $C_1(s)G(s)/(1+C_1(s)G(s))$, $G(s)/(1+C_1(s)G(s))$, $C_1(s)/(1+C_1(s)G(s))$ and $1/(1+C_1(s)G(s))$ are stable, then $C_1(s)$ makes $G(s)$ stable. From Equations (20), (26), (27) and (28), all transfer functions $C_1(s)G(s)/(1+C_1(s)G(s))$, $G(s)/(1+C_1(s)G(s))$, $C_1(s)/(1+C_1(s)G(s))$ and $1/(1+C_1(s)G(s))$ are written by

$$\frac{C_1(s)G(s)}{(1+C_1(s)G(s))} = -\left(\bar{X}(s) - F_1(s)\hat{Q}(s)\right)N(s), \tag{32}$$

$$\frac{G(s)}{(1+C_1(s)G(s))} = -\left(\bar{Y}(s) - F_2(s)\hat{Q}(s)\right)N(s), \tag{33}$$

$$\frac{C_1(s)}{(1+C_1(s)G(s))} = -\left(\bar{X}(s) - F_1(s)\hat{Q}(s)\right) D(s) \tag{34}$$

and

$$\frac{1}{(1+C_1(s)G(s))} = -\left(\bar{Y}(s) - F_2(s)\hat{Q}(s)\right) D(s). \tag{35}$$

Since all transfer functions $C_1(s)G(s)/(1+C_1(s)G(s))$, $G(s)/(1+C_1(s)G(s))$, $C_1(s)/(1+C_1(s)G(s))$ and $1/(1+C_1(s)G(s))$ are stable, $C_1(s)$ stabilizes $G(s)$.

Thus, we have found that $C_1(s)$ satisfying Theorem 3.1 makes $G(s)$ stable.

5. **Control Characteristics.** In this section, we explain control characteristics of the control system in Figure 1.

First, the input output characteristic of control system in Figure 1 is shown. The transfer function from the reference input $r(s)$ to the output $y(s)$ and that from the reference input $r(s)$ to the error $e_r(s) = r(s) - y(s)$ are written by

$$y(s) = \frac{C_1(s)G(s)}{1+C_1(s)G(s)} r \tag{36}$$

and

$$e_r(s) = r(s) - y(s) = \frac{1}{1+C_1(s)G(s)} r(s), \tag{37}$$

respectively. In order for the output $y(s)$ to follow the non-periodic reference input $r(s)$ without steady state error, from the internal model principle [26], $C_1(s)$ is written by the form

$$C_1(s) = C_r(s)\bar{C}_1(s), \tag{38}$$

where $C_r(s)$ is internal model of the reference input $r(s)$ and assumed to be of minimum phase and bi-proper and $\bar{C}_1(s) \in R(s)$, and $C_r(s)\bar{C}_1(s)$ is assumed to have no unstable pole-zero cancellation. In order for $C_1(s)$ to satisfy Equation (38), from Equation (27), $\hat{Q}(s)$ is designed by

$$\hat{Q}(s) = \frac{1}{F_2(s)} \left(\bar{Y}(s) - \frac{\tilde{Q}(s)}{C_r(s)} \right), \tag{39}$$

where $\tilde{Q}(s) \in RH_\infty$ is any function to make Equation (39) proper. Thus, substituting Equation (39) to Equation (27) gives the form in (38).

Next, the disturbance attenuation characteristic is shown. Substituting Equations (26), (27) and (38) to Equation (24), transfer function from the periodic disturbance $d(s)$ to the output $y(s)$ is written by

$$y(s) = \frac{\left(1 + \left(C_d(s) + F_2(s)C_n(s)\right) e^{-sT}\right) C_r^{-1}(s)D(s)}{\bar{X}(s)N(s) + \bar{Y}(s)D(s)} d(s). \tag{40}$$

From Equation (18), the disturbance characteristic is specified using controllers $C_d(s)$ and $C_n(s)$ of $C_2(s)$ in Equation (14). From Equation (23), the periodic disturbance with period T is attenuated effectively.

In this way, the purpose of $C_1(s)$ is to specify the input/output characteristics, and that of $C_2(s)$ is to specify the disturbance attenuation characteristics. This implies that the control system in Figure 1 is a kind of two-degree-of-freedom control system, which make the system flexible in optimizing both transient and steady state performance. By independently tuning $C_1(s)$ and $C_2(s)$, the system can achieve a well-balanced trade-off between fast response and disturbance attenuation, leading to improved stability and performance under varying operating conditions.

6. **Design Procedure.** In this section, we present the design procedure for the control system shown in Figure 1. The process consists of six main steps, with each step breaking down into smaller tasks to guide the implementation.

A design procedure of the control system in Figure 1 is summarized as follows.

Step 1) Obtain the coprime factors $N(s)$ and $D(s)$ of $G(s)$ satisfying Equation (10) using the method in [32].

Step 2) Design $F_1(s)$ and $F_2(s)$:
- (a) Settle function $Q(s)$ to satisfy Equation (11).
- (b) Design $F_1(s) \in RH_\infty$ and $F_2(s) \in RH_\infty$ by Equations (8) and (9) using the method in [11].

Step 3) Define $q(s)$ by Equation (21), where τ is small positive number and n_q is chosen to make $q(s)/G(s)$ and $q(s)/F_2(s)$ proper.

Step 4) Obtain $\hat{Q}(s)$ to make $C_1(s)$ have the form in Equation (38):
- (a) Design $\tilde{Q}(s)$ to make $\bar{Y}(s) - \tilde{Q}(s)/C_r(s)$ strictly proper.
- (b) Obtain $\hat{Q}(s)$ by Equation (39).

Step 5) Find $N_c(s)$ and $D_c(s)$:
- (a) Find $\bar{X}(s)$ and $\bar{Y}(s)$ satisfying Equation (28).
- (b) Obtain $N_c(s)$ and $D_c(s)$ by Equations (26) and (27) with $\hat{Q}(s)$ which is obtained by previous step.

Step 6) Design $C_d(s)$ and $C_n(s)$ by Equations (16) and (17).

7. **Numerical Example.** In this section, a numerical example is shown to illustrate the effectiveness of the proposed method.

Consider the problem to design the control system in Figure 1 to attenuate periodic disturbances $d(t)$ with period $T = \pi$ [sec] and to follow the reference input $r(t) = t$ for the minimum phase plant $G(s)$ written by

$$G(s) = \frac{s+1}{s^2+7s+10}. \tag{41}$$

Coprime factors $N(s)$ and $D(s)$ of $G(s)$ in Equation (41) satisfying Equation (10) are written by

$$N(s) = \frac{s+1}{s^2+7s+10} \tag{42}$$

and

$$D(s) = 1. \tag{43}$$

Using the method in [8], $F_1(s)$ and $F_2(s)$ are designed by Equations (8) and (9). We have

$$F_1(s) = \frac{-5s^2-500s+50000}{s^2+1100s+100000} \tag{44}$$

and

$$F_2(s) = \frac{-s-1}{s^2+7s+10}, \tag{45}$$

where $Q(s)$ is settled by

$$Q(s) = 0. \tag{46}$$

From Equation (21), $q(s)$ is given by

$$q(s) = \frac{1}{0.001s + 1}. \tag{47}$$

From Equation (39), $\tilde{Q}(s)$ is given by

$$\tilde{Q}(s) = \frac{s^2 + 2s + 1}{s^2 + 3s + 2.25}, \tag{48}$$

and $\hat{Q}(s)$ is obtained by

$$\hat{Q}(s) = \frac{3s^3 + 23.25s^2 + 45.75s + 22.5}{s^3 + 4s^2 + 5.25s + 2.25}, \tag{49}$$

and $C_r(s)$ is given by

$$C_r(s) = \frac{-s^2 - 2s - 1}{s^2}. \tag{50}$$

A pair of solution $\bar{X}(s)$ and $\bar{Y}(s)$ to Equation (28) is

$$\bar{X}(s) = 0 \tag{51}$$

and

$$\bar{Y}(s) = -1. \tag{52}$$

From Equations (20), (26) and (27), we have

$$C_1(s) = \frac{3s^8 + 56.25s^7 + 431.2s^6 + 1768s^5 + 4268s^4 + 6242s^3 + 5423s^2 + 2565s + 506.2}{s^8 + 12s^7 + 54.25s^6 + 122.2s^5 + 147.2s^4 + 90.75s^3 + 22.5s^2}, \tag{53}$$

where

$$N_c(s) = \frac{-3s^3 - 23.25s^2 - 45.75s - 22.5}{s^3 + 4s^2 + 5.25s + 2.25}, \tag{54}$$

and

$$D_c(s) = \frac{-s^5 - 8s^4 - 17s^3 - 10s^2}{s^5 + 11s^4 + 43.25s^3 + 79s^2 + 68.25s + 22.5}. \tag{55}$$

$C_1(s)$ in Equation (53) has the form in Equation (38), making the output $y(s)$ follow the non-periodic reference input $r(s)$ without steady state error.

Using above parameters, $C_d(s)$ and $C_n(s)$ are given by Equations (16) and (17), respectively.

Using designed system in Figure 1, the response of the error $e_r(t) = r(t) - y(t)$ for ramp reference input $r(t) = t$ is shown in Figure 2.

In order to show the effectiveness of the proposed method, the error $e(t) = r(t) - \mathcal{L}^{-1}[y(s)]$ of the repetitive control system for the ramp input $r(t) = t$ is shown in Figure 3. Here, the repetitive controller in [33] is designed as

$$C(s) = \frac{F(s)e^{-sT}}{1 - F(s)e^{-Ts}}\hat{C}(s), \tag{56}$$

where $F(s)$ is the low pass filter given by

$$F(s) = \frac{1}{0.001s + 1} \tag{57}$$

and $\hat{C}(s)$ is given by

$$\hat{C}(s) = \frac{1}{0.0001s + 1}\frac{1}{G(s)}. \tag{58}$$

Figure 2 shows that the output $y(t)$ follows the reference input $r(t)$ without steady state error. By contrast, Figure 3 shows that the repetitive control system in [33] has the steady state error. This is an advantage of the proposed control system in Figure 1.

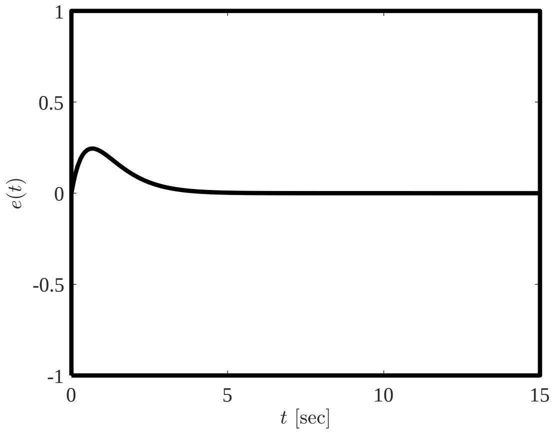

FIGURE 2. The response of the error $e_r(t) = r(t) - y(t)$ for $r(t) = t$ in the control system in Figure 1

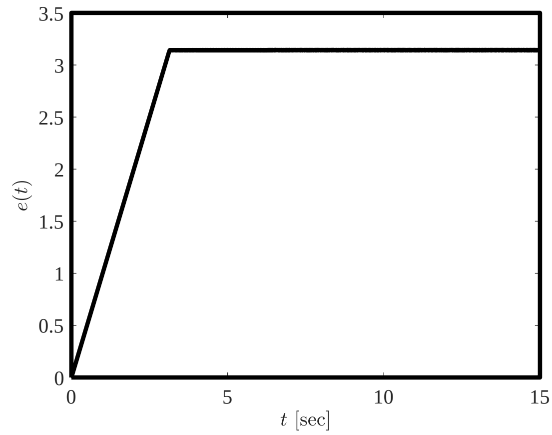

FIGURE 3. The response of the error $e_r(t) = r(t) - y(t)$ for $r(t) = t$ in the repetitive control system

Next, the disturbance attenuation characteristic is shown. When the periodic disturbance $d(t)$ is given by

$$d(t) = \begin{cases} \frac{2}{\pi}t & 0 \leq t - kt < \frac{\pi}{2} \\ \frac{2}{\pi}(\pi - t) & \frac{\pi}{2} \leq t - kt < \pi \end{cases}, \tag{59}$$

where k is the maximum integer which will not exceed t/T, the disturbance $d(t)$ in Equation (59) is a triangular wave with the period T. The response of the output $y(t)$ for the disturbance $d(t)$ in Equation (59) is shown in Figure 4, which shows that the periodic disturbance $d(t)$ in Equation (59) is attenuated effectively.

Moreover, when the periodic disturbance $d(t)$ with the period T is

$$d(t) = \sum_{i=1}^{2} \sin\left(\frac{2i\pi}{T}t + \frac{\pi}{4}\right) + \frac{\pi}{4}. \tag{60}$$

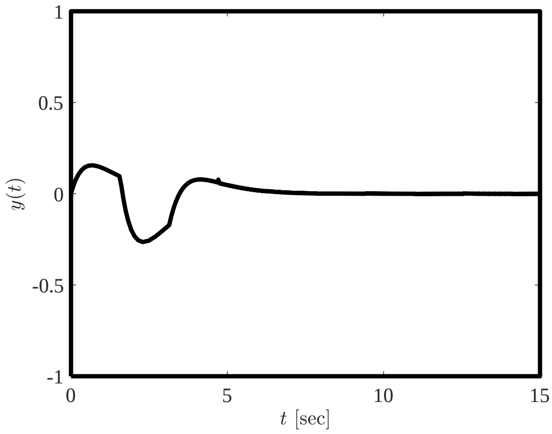

FIGURE 4. The response of the output $y(t)$ for the triangular wave disturbance $d(t)$ in Equation (59)

The response of the output $y(t)$ for the disturbance $d(t)$ in Equation (60) is shown in Figure 5. It is clearly seen that the periodic disturbance $d(t)$ in Equation (60) is also attenuated effectively.

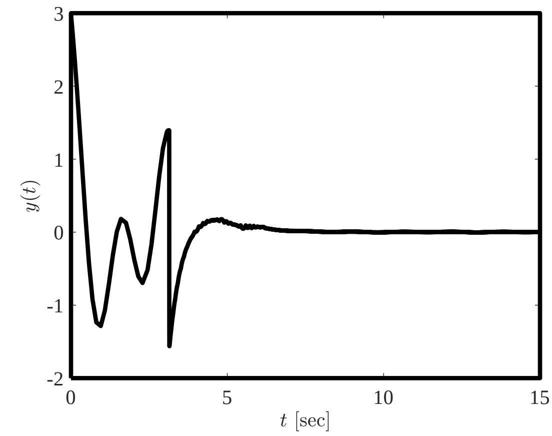

FIGURE 5. The response of the output $y(t)$ for the disturbance $d(t)$ in Equation (60)

In this way, using the proposed method, we can design the stable control system to attenuate the periodic disturbance and to follow the reference input without steady state error.

8. **Conclusion.** In this paper, we have proposed a design method for the control system using the disturbance observer for the periodic disturbance by using the parameterization of all feedback controllers to make the plant stable and to make functions in the controller for the disturbance observer stable. We examined the parameterization and proved the stability of the designed control system. Next, the control characteristics of the proposed control system and a design procedure of the proposed control system are shown. Finally, we show features of the proposed design method through a numerical example. This example demonstrates that our designed system can effectively handle ramp reference inputs, which is a known limitation of traditional repetitive control systems. Moreover, our approach is anticipated to have practical applications in industrial operations, such

as temperature control, where ramp inputs are commonly used to gradually increase the temperature to a desired target point. Additionally, in this paper, the plant is assumed to be of minimum phase. A design method for control system in Figure 1 for the non-mininum-phase plant will be discussed in another article.

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