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SYNTHESIS PERFORMANCE ANALYSIS ON DATA DRIVEN CONTROLLER DESIGN FOR UNKNOWN CLOSED LOOP SYSTEM

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Abstract. *During our new data era, data driven control strategy will be more widely applied than classical model based control from theory and practice for our modern times. This new paper proposes some synthesis performance analysis on data driven control. Specifically, consider an unknown closed loop system with unknown plant and unknown controller, data driven control is proposed to design that controller from the non-parametric form and parametric form, respectively, while no any modeling process for that unknown plant. All synthesis analyses hold on the condition of persistent excitation condition, guaranteeing the existence of one inverse matrix operation. From our own results, direct data driven control is more better to achieve the tracking property and good closed loop performance through choosing the approximated filter and instrumental variable, meaning the equivalence between direct data driven control and performance analysis. To show this equivalence, one filter is constructed to filter the input-output measured data. Furthermore, statistical performance analysis of data driven control is given to guarantee matching error unrelated with external input. Finally, some example simulations are exemplified to prove our theoretical results.*
Keywords: Direct data driven control, Persistent excitation, Performance analysis, Parameterized controller

1. **Introduction.** Finding rigorous and efficient ways to integrate data into control theory has been a problem of great interest for many decades. Since most of the classical contributions in control theory rely on model knowledge, the problem of finding such a model from measured data has become a mature research field, through system identification or adaptive estimation. More recently, learning controllers directly from measured data has received increasing interest, i.e., direct data driven control, meaning the unknown controller is designed from measured data directly without any information about the unknown plant or system. More specifically, controller design is divided into two main strategies, i.e., model based control and direct data driven control. Model based control firstly uses system identification theory or first principle to identify one approximated mathematical equation for unknown plant in linear or nonlinear form, then secondly this identified mathematical equation is benefit for the next controller design, so the controller obtained by model based control depends on that mathematical model greatly, meaning good controller from good mathematical model. Above is one reason why researchers spend lots of time and energy in yielding one good mathematical model. To reduce the dependence on the precise mathematical model, an innovative direct data driven control strategy was proposed during this decade, which avoids the whole model identification process and only generates a priori controller form from measured data. Based on above

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explanation, quality of measured data is important for direct data driven control, so some necessary priori steps are needed to deal with measured data, for example, filtering, and persistently exciting. From our own opinions, above two different control strategies are all applied in the corresponding different situations, determined by researcher. Specifically, model based control is good for those small size systems, such as proportion-integral-derivative (PID) controller, linear Gaussian regulator for glass of car industry, and direct data driven control is for those complex network system, for example, aircraft or rocker missile.

From 1960s to 2010s, all researches are concentrated on model based control strategy, so the number of references about model based control is vast very much. Here we do not list them, and the interested readers can refer to some classical text books, for example, system identification, feedback control system, adaptive control, robust control and model predictive control. After 2010s, direct data direct control appears with data science increases, meaning feasible for our new data era. In [1], direct data driven control method is programmed in a Python package for convenient application. Similarly, optimization theory is also embedded in direct data driven control, such as convex optimization [2], scenario optimization [3], and stochastic optimization with chance constraints [4]. Furthermore, system identification is not only for model identification, but also for control theory, i.e., constructing one new idea of identification for control. Consider the application of our considered direct data driven control, [5] combines the idea with classical PID design, i.e., generating these three PID unknown parameters through a data fitting process. [6] designs one controller for minimum and non-minimum phase system by direct data control strategy. However, before to do it, the practical engineering system must be changed into the considered minimum phase system. During these recent years, more deep researches about direct data driven control are implemented in Germany from different points of view, for example, data driven model predictive control with stability and robustness guarantee [7], dissipativity property from input-output measured data [8], one shot verification of dissipativity property from input-output measured data [9], and non-linear direct data driven control [10,11]. Their contributions and missions are to develop model free system analysis and control methods, which are only based on measured data. One approach towards these goals is to extract control theoretic system properties such as dissipativity or nonlinear measures from data, which can then be used to design controllers via direct data driven control methods. Moreover, other new fields are combined with our considered direct data driven control strategy. [12] considers modulating robustness, and robust event triggered output feedback controller is studied in [13], while guaranteeing the similarity with nonlinear direct data driven control. The detailed formulas for direct data driven control are described in [14], and its data informality is studied in [15], such as stabilization, optimality and robustness together. By the way, identification for control is mentioned in [16,17], where set membership strategy is applied to estimating the output predictor, and a new kernel approach is proposed for hybrid system identification or direct data driven switching controller design [18]. Generally, during these recent years, more and more researches about direct data driven control strategy are ongoing all over the world. Except above listed references about direct data driven control strategy, we also obtain some contributions. For example, [19] proposes adaptive iterative correlation tuning for closed loop system with two parameterized controllers, which is a special case of direct data driven control. Synthesis identification analysis is done in [20], where other interesting topics are analyzed, too, such as computational complexity, recursive feasibility and identificability. The adaptation idea is introduced into direct data driven control strategy so that the obtained controller is generated as a recursive form or adaptive modified form in [21,22]. Based on our existing results, this paper continues to solve some important aspects, being worth in research for direct data driven control, i.e., persistently exciting.

From above references and our existing contributions, we find that during our own mathematical derivations about that direct data driven controller, whatever its explicit or implicit form, one condition must hold to guarantee that direct data driven control exists. This condition corresponds to the existence of an inverse matrix, consisting of data sequence. Specifically, within the mathematical derivation for that direct data driven controller, an inverse matrix operation exists, corresponding to the called persistently excitation condition. It tells us our derived direct data driven controller exists on the condition of that persistently excitation property, being the reason of persistently exciting direct data driven control method. For the completeness, the detailed derivations about that direct data driven control and that persistently excitation condition are all given. Within the framework of persistency exciting direct data driven control, the obtained statistical performance is analyzed to check whether the matching error is related with the external input signal. By using the idea of data driven, the case of parameterized controller is turned to one question of parameter estimation, as now parameterized controller is more widely applied in practical engineering, such as PID controller. To achieve the goal of data driven controller tuning, our proposed control algorithm is given in detail. To guarantee the matching error be unrelated with that external input signal, one filter is approximately chosen to establish the equivalence between this statistical property and the main essence of direct data driven control strategy. It means our considered direct data driven control strategy has dual goals, i.e., designing the controller from measured data and satisfying the achieved statistical performance, being complete analysis for one control system. However, all of above dual goals hold on the condition of that persistently excitation, so it is important to guarantee our listed proposition is satisfied.

The main contributions of this continuous paper are formulated as follows.

1) To let the detailed derivation for direct data driven control hold, one inverse matrix operation must exist, corresponding to persistently excitation condition.

2) On basis of this direct data driven controller, designed from measured data directly, the statistical performance analysis is given to guarantee the matching error is unrelated with external input. We find statistical performance analysis is automatically satisfied due to the equivalence within them. To show this equivalence, one filter, used to filter the input-output measured data, is constructed only through our own mathematical derivations.

3) For one special case of the parameterized controller, the detailed data driven controller is given through our own description.

4) One practical engineering example is applied to designing controller through direct data driven control strategy and testifying that about whether the persistently excitation condition holds, will influence the performance.

This new paper is organized as follows. In Section 2, the considered closed loop system with unknown controller and unknown plant is given from theory and plot simultaneously. Some existing relations are easily shown. Then Section 3 proposes the detailed process about our considered direct data driven control strategy and points out one inverse matrix operation exists. This existence of inverse matrix operation corresponds to the persistently excitation condition from system identification theory. Section 4 describes the proposed data driven controller tuning in case of the parameterized controller, while not considering that unknown plant. Based on this direct data driven controller, closed loop performance analysis is done in Section 5, where we establish the equivalence between closed loop performance analysis and the main essence of direct data driven control strategy. The equivalent property depends on the approximate filter. Section 6 gives one practical example to show the efficiency of direct data driven control, and also we testify the influence about whether persistent excitation condition holds on the closed loop performance. Finally, conclusion and future work are presented in the last Section 7.

2. System Description. Although open loop system and closed loop system exist simultaneously in nature, due to bad accuracy and instability for open loop system, now most systems are working under closed loop circumstance, being plotted in the following Figure 1 with unknown plant and unknown controller.

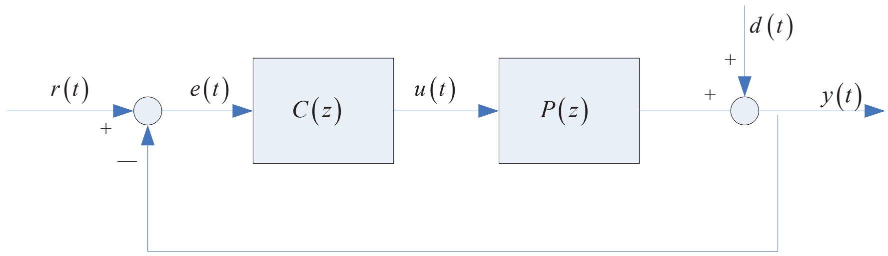


FIGURE 1. Closed loop system structure

In Figure 1, with unknown plant $P(z)$ and unknown controller $C(z)$, $r(t)$ is one external excitation signal, being used to excite the whole closed loop system. $d(t)$ is also one external disturbance or noise, not being neglected in academy and practice. $\{u(t), y(t)\}$ correspond to the input-output signal for that unknown plant $P(z)$. Error signal $e(t)$ means the deviation between external input $r(t)$ and feedback output $y(t)$, i.e., $e(t) = r(t) - y(t)$. Variable z is the shift operator, meaning $zr(t) = r(t+1)$.

From Figure 1, after simple computations, some matured relations exist, i.e.,

$$\begin{cases} y(t) = P(z)u(t) + d(t) \\ u(t) = C(z)e(t) = C(z)[r(t) - y(t)] \end{cases} \tag{1}$$

i.e.,

$$y(t) = P(z)C(z)[r(t) - y(t)] + d(t);\ y(t) = \frac{P(z)C(z)}{1 + P(z)C(z)}r(t) + \frac{1}{1 + P(z)C(z)}d(t) \tag{2}$$

From Equation (2), the closed loop output $y(t)$ is related with two signals $\{r(t), d(t)\}$ and two unknown forms $\{P(z), C(z)\}$. Due to that unknown plant $P(z)$, classical model based control is firstly to identify plant $P(z)$, then secondly design controller $C(z)$, being divided into two steps, i.e., identification and control. However, here this paper studies direct data driven control withou identifying unknown plant $P(z)$, being formulated in latter Problem 2.1.

Problem 2.1. *Consider the problem of designing controller $C(z)$ without any identification process for plant $P(z)$ in Figure 1, we apply direct data driven control strategy to achieving this goal and testifying one necessary condition.*

3. Direct Data Driven Control. Each control strategy has its own property, for example, tracking, stability, robustness and disturbance suppression. As the last Section 6 considers a practical example of aircraft tracking, during our latter mathematical derivations, tracking property is always to guarantee closed loop transfer function from external input $P(z)$ to closed loop output $y(t)$ is equal to one expected or desired reference model $M(z)$, meaning

$$\frac{P(z)C(z)}{1+P(z)C(z)} = M(z) \tag{3}$$

Above tracking property is shown explicitly in Figure 2.

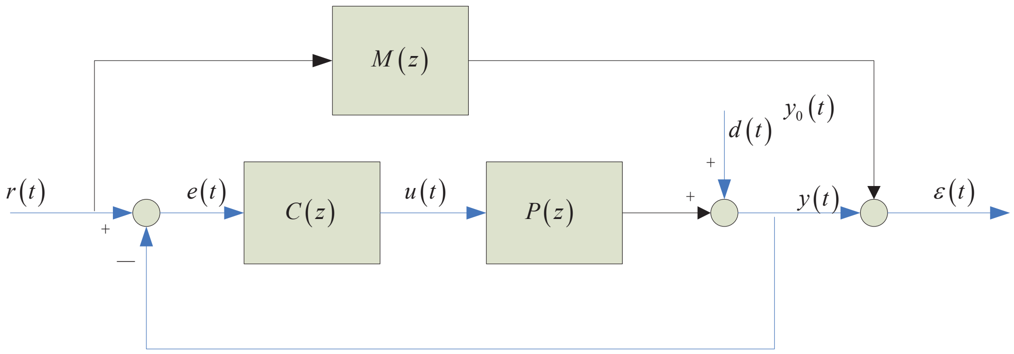


FIGURE 2. Tracking property

In Figure 2, $M(z)$ is the desired reference or tracking model. $\varepsilon(t)$ is the called matching error, i.e., $\varepsilon(t) = y(t) - y_0(t)$. Specifically, matching error is that

$$\begin{aligned}\varepsilon(t) &= y(t) - y_0(t) \\ &= \frac{P(z)C(z)}{1+P(z)C(z)} r(t) + \frac{1}{1+P(z)C(z)} d(t) - M(z)r(t) \\ &= \left[\frac{P(z)C(z)}{1+P(z)C(z)} - M(z)\right] r(t) + \frac{1}{1+P(z)C(z)} d(t)\end{aligned} \tag{4}$$

Remind that the main essence of direct data driven control strategy is used, i.e., input-output measured data are applied to designing that unknown controller $C(z)$, while avoiding the identification process for that unknown plant $P(z)$.

Observing Figure 2 again, input-output measured data are chosen around the unknown controller $C(z)$, not the unknown plant $P(z)$, i.e., $\{e(t), u(t)\}_{t=1}^{N}$, where N is the total number of measured data. Furthermore, it holds that

$$\{e(t), u(t)\}_{t=1}^{N} = \{r(t) - y(t), u(t)\}_{t=1}^{N} = \left\{\left(M^{-1}(z) - 1\right) y(t), u(t)\right\}_{t=1}^{N} \tag{5}$$

where the expected reference model $M(z)$ is used in Equation (5), i.e., $y(t) = M(z)r(t)$, and then we have $r(t) = M^{-1}(z)y(t)$ in case that $M(z)$ is inverse.

Consider the left and right side of unknown controller $C(z)$, its input-output measured data are $\{e(t), u(t)\}_{t=1}^{N}$, then the ideal controller $C(z)$ satisfies that

$$u(t) = C(z)e(t) = C(z)\left(M^{-1}(z) - 1\right) y(t);\ t = 1, 2, \ldots, N \tag{6}$$

so controller $C(z)$ is designed to satisfy above N equities, and the considered tracking property is included in that desired reference model $M(z)$. Based on above explanation, controller $C(z)$ is designed through the following optimization problem.

$$C(z) = \underset{C(z)}{\operatorname{argmin}} \frac{1}{N} \sum_{t=1}^{N} [u(t) - C(z)y_1(t)]^2;\ y_1(t) = \left(M^{-1}(z) - 1\right) y(t) \tag{7}$$

where N is the total number of observed data.

Comment. In above optimization problem (7), input-output data $\{u(t), y(t)\}_{t=1}^{N}$ are measured in priori or during the whole implement process, and $M(z)$ is priori given, so input-output data $\{u(t), y(t)\}_{t=1}^{N}$ and reference model $M(z)$ are all known, but only controller $C(z)$ is unknown. As that unknown plant $P(z)$ does not exist in Equation (7), controller $C(z)$ is yielded through solving above unconstrain optimization problem by virtue of Newton algorithm, gradient algorithm or trust region algorithm.

Whatever every optimization algorithm is used to solve Equation (7), one condition must hold to guarantee the existence of controller $C(z)$. For clarity of presentation, by differentiating with respect to $C(z)$ and by setting the derivative equal to zero in Equation (20), we find that $\{u(t), y(t)\}_{t=1}^{N}$ are collected by some physical sensors, $M(z)$ is the given reference model, only $C(z)$ is unknown, and unknown plant $P(z)$ does not exist. When to obtain one explicit form for optimal controller, by differentiating with respect to $C(z)$ and be setting the derivative equal to zero, we have

$$\frac{2}{N}\sum_{t=1}^{N}[u(t) - C(z)y_1(t)]y_1(t) = 0;\ y_1(t) = \left(M^{-1}(z) - 1\right)y(t) \tag{8}$$

i.e.,

$$\sum_{t=1}^{N}u(t)y_1(t) = C(z)\sum_{t=1}^{N}y_1(t)y_1(t);\ C(z) = \left[\sum_{t=1}^{N}y_1(t)y_1(t)\right]^{-1}\left[\sum_{t=1}^{N}u(t)y_1(t)\right] \tag{9}$$

Moreover, it holds that

$$\begin{aligned}&\sum_{t=1}^{N}y_1(t)y_1(t) = \left(M^{-1}(z) - 1\right)\sum_{t=1}^{N}y(t)y(t);\\ &y_1(t) = \left(M^{-1}(z) - 1\right)y(t) = r(t) - y(t) = e(t)\end{aligned} \tag{10}$$

From that explicit form of controller $C(z)$, we need compute one inverse matrix operation, i.e., $\left[\sum_{t=1}^{N}y_1(t)y_1(t)\right]^{-1}$. However, firstly this inverse matrix operation must exist, corresponding to the following persistent excitation condition.

Proposition 3.1 (Persistent excitation)**.** *The measured output $y(t)$ is persistent excitation with a level of excitation $\alpha_0 > 0$ if there exist constants $\alpha_1 > 0$ and $N > 0$ such that*

$$\alpha_0 \le \sum_{t=1}^{N}y(t)y(t) \le \alpha_1 \tag{11}$$

The existence of Inequality (11) is to make that inverse matrix operation $\left[\sum_{t=1}^{N}y_1(t)y_1(t)\right]^{-1}$ exist, and then controller $C(z)$ is yielded from Equation (9). Combining Equations (7) and (11), the idea of direct data driven control strategy is formulated as the following constrain optimization problem, i.e.,

$$\begin{aligned}&C(z) = \underset{C(z)}{\operatorname{argmin}}\ \frac{1}{N}\sum_{t=1}^{N}[u(t) - C(z)y_1(t)]^2;\ y_1(t) = \left(M^{-1}(z) - 1\right)y(t);\\ &\textit{subject to}\ \ \alpha_0 \le \sum_{t=1}^{N}y(t)y(t) \le \alpha_1\end{aligned} \tag{12}$$

Specifically, consider cost function in Equation (12), we rewrite

$$\sum_{t=1}^{N}[u(t) - C(z)y_1(t)]^2$$

$$= \left[\begin{array}{ccc} u(1)-C(z)y_1(1) & \cdots & u(N)-C(z)y_1(N) \end{array}\right]\left[\begin{array}{c} u(1)-C(z)y_1(1) \\ \vdots \\ u(N)-C(z)y_1(N) \end{array}\right]$$
$$= ([u(1)\ u(2)\ \cdots\ u(N)] - C(z)[y_1(1)\ y_1(2)\ \cdots\ y_1(N)])^2$$
$$= (u - C(z)y_1)^2;$$
$$u = [u(1)\ u(2)\ \cdots\ u(N)]; \quad y_1 = [y_1(1)\ y_1(2)\ \cdots\ y_1(N)] \tag{13}$$

It holds that

$$\sum_{t=1}^{N}[u(t) - C(z)y_1(t)]^2 = (u - C(z)y_1)^2 = \|u\|^2 - 2C(z)uy_1 + C^2(z)y_1y_1 \tag{14}$$

where $\|.\|$ *is the commonly used Euclidean norm.*

Taking the partial derivative with respect to $C(z)$, we have

$$2C(z)y_1y_1 - 2uy_1 = 0;\ C(z) = (y_1y_1)^{-1}uy_1 = \left[\sum_{t=1}^{N} y_1(t)y_1(t)\right]^{-1}\left[\sum_{t=1}^{N} u(t)y_1(t)\right] \tag{15}$$

Equation (15) is the same as Equation (9), where that condition of persistent excitation must hold to guarantee the existence of that inverse matrix operation.

4. **Parametrized Controller Design.** Considering that unknown controller $C(z)$ is parameterized by one parameter vector θ, then candidate controller is a function of the parameter vector θ. The general structure of the parameterized controller with θ is the following form.

$$C(z, \theta) = \alpha^T(z)\theta \tag{16}$$

where $\alpha(z)$ is a set of linear rational basis function, for example,

$$\alpha(z) = \begin{pmatrix} \alpha_1(z) & \alpha_2(z) & \cdots & \alpha_n(z) \end{pmatrix};\ \theta = \begin{pmatrix} \theta_1 & \theta_2 & \cdots & \theta_n \end{pmatrix} \tag{17}$$

where above n is the order of parameterized controller.

One reference model $M(z)$ defines the desired closed loop performance and provides the ideal linear transfer function from the reference signal $r(t)$ to closed loop output $y(t)$, i.e.,

$$y(t) = M(z)r(t) \tag{18}$$

The criterion of design controller $C(z)$ is based on the discrepancy function between the reference model and the closed loop system. Observing Equations (2) and (18), the designed goal is to let the real transfer function from reference signal $r(t)$ to closed loop output $y(t)$ track that given reference model, i.e.,

$$\frac{P(z)C(z)}{1 + P(z)C(z)} \to M(z) \tag{19}$$

To achieve above goal, the following discrepancy function is minimized by

$$J_1(C(z)) = \left\|\frac{P(z)C(z)}{1 + P(z)C(z)} - M(z)\right\|_2^2 \tag{20}$$

where $\|.\|$ is Euclidean norm.

Taking into account that parameterized controller $C(z,\theta)$ in Equation (16), above discrepancy function $J_1(C(z))$ is changed as that

$$J_1(C(z,\theta)) = \left\| \frac{P(z)C(z,\theta)}{1+P(z)C(z,\theta)} - M(z) \right\|_2^2 \tag{21}$$

Consequently, the controller design problem is changed to one parameter estimation problem, i.e.,

$$\theta = \{\arg\min\}_\theta J_1(C(z,\theta)) = \{\arg\min\}_\theta \left\| \frac{P(z)C(z,\theta)}{1+P(z)C(z,\theta)} - M(z) \right\|_2^2 \tag{22}$$

Comment. Observing that parameter estimation problem in Equation (9), this discrepancy function $J_1(C(z,\theta))$ includes some variables, such as $M(z)$, $P(z)$ and θ. Since $M(z)$ is given in priori and plant transfer function $P(z)$ is not known, we cannot minimize in practice the discrepancy function $J_1(C(z,\theta))$.

Based on above description for the parameterized controller $C(z,\theta)$, then the latter problem is concentrated on identifying that unknown parameter vector θ, corresponding to parameter tuning or controller tuning. Also due to the appearance of the unknown plant $P(z)$ in that discrepancy function $J_1(C(z,\theta))$, we think how it is possible to avoid that unknown plant $P(z)$ in the cost function, which does not require any knowledge about that unknown plant.

To show our proposed data driven controller tuning strategy, we collect the available input-output pairs $\{u(t),y(t)\}_{t=1}^N$, and based on Equation (5), we have

$$r(t) = M^{-1}(z)y(t) \tag{23}$$

which means after given reference model $M(z)$ and collected closed loop output $y(t)$, the reference signal $r(t)$ is obtained inversely, so we have some known variables, for example, $\{u(t),y(t)\}_{t=1}^N$, $M(z)$ and $r(t)$. Then we think can that parameterized controller $C(z,\theta)$ be designed or extracted from these above known variables.

In order to better understand the idea of data driven, the input-output pairs of that unknown controller are $\{e(t),u(t)\}_{t=1}^N$, and error signal $e(t)$ is solved through that

$$e(t) = r(t) - y(t) = M^{-1}(z)y(t) - y(t) = \left[M^{-1}(z) - 1\right] y(t) \tag{24}$$

It means the corresponding input-output pairs $\{e(t),u(t)\}_{t=1}^N$ about the unknown controller $C(z,\theta)$ are obtained from Equation (24), plotted in Figure 3.

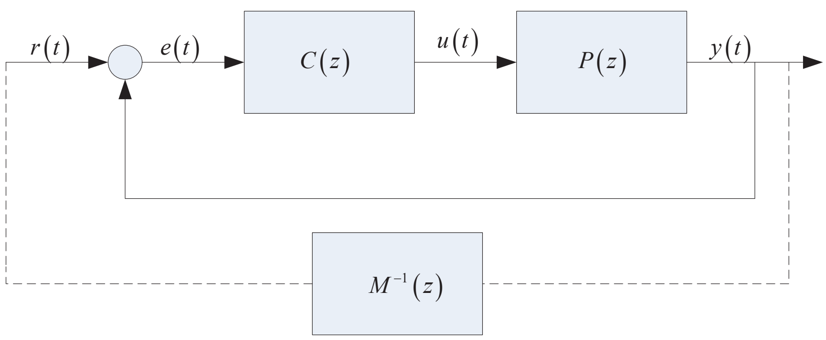


Figure 3. Idea of direct data driven

Based on input-output pairs $\{e(t),u(t)\}_{t=1}^N$ of that unknown controller $C(z,\theta)$, the objective is to design a controller that suits the closed loop behavior with the desired reference model, while avoiding any information about that unknown plant $P(z)$. Due to

$$u(t) = C(z,\theta)e(t) \tag{25}$$

substituting Equation (24) into above Equation (25), we have

$$u(t) = C(z,\theta)e(t) = C(z,\theta)\left[M^{-1}(z) - 1\right]y(t) \tag{26}$$

where in Equation (26), only controller $C(z,\theta)$ is unknown, and other variables are known in priori, for example, input-output pairs $\{u(t), y(t)\}_{t=1}^{N}$ are collected and reference model $M(z)$ is given in priori.

Choosing $t = 1, 2, \ldots, N$ in Equation (13), the other discrepancy function or cost function is selected to tune that unknown controller parameter vector θ, i.e.,

$$\begin{aligned}
\theta &= \{\arg\min\}_{\theta}\frac{1}{N}J_2(C(z,\theta)) \\
&= \{\arg\min\}_{\theta}\frac{1}{N}\left[u(t) - C(z,\theta)\left[M^{-1}(z) - 1\right]y(t)\right]^2 \\
&= \{\arg\min\}_{\theta}\frac{1}{N}[u(t) - \alpha(z)\theta e(t)]^2 \\
e(t) &= \left[M^{-1}(z) - 1\right]y(t)
\end{aligned} \tag{27}$$

Note that this cost function $J_2(C(z,\theta))$ is a purely data dependent cost. The optimal controller parameter vector $\hat{\theta}$ can be directly calculated from its least squares form, i.e.,

$$\hat{\theta} = \left[\sum_{t=1}^{N}\varphi(t)\varphi(t)\right]^{-1}\sum_{t=1}^{N}\varphi(t)u(t);\ \varphi(t) = \alpha(z)e(t) \tag{28}$$

Observing that detailed cost function $J_2(C(z,\theta))$ in Equation (27), it is convenient to collect data $\{u(t), y(t)\}_{t=1}^{N}$ and choose reference model $M(z)$, and then data driven controller tuning is to extract one optimal controller parameter vector $\hat{\theta}$ to satisfy $u(t) = C\left(z,\hat{\theta}\right)\left[M^{-1}(z) - 1\right]y(t)$, while not constructing one mathematical model for that unknown plant $P(z)$.

Finally, the complete data driven controller is formulated as the following algorithm.

Algorithm 1
Step 1: Collect input-output data $\{u(t), y(t)\}_{t=1}^{N}$;
Step 2: Choose one reference model $M(z)$;
Step 3: Compute that reference signal $r(t) = M^{-1}(z)y(t)$;
Step 4: Compute the optimal controller parameter vector $\hat{\theta}$ can be directly calculated from its least squares form.

5. **Performance Analysis.** Recall that tracking property is the basis for our considered direct data driven control in Section 3, which means one ideal case.

$$y(t) = y_0(t) = M(z)r(t) \tag{29}$$

i.e., the matching error $\varepsilon(t)$ is equal to zero.

$$\varepsilon(t) = y(t) - y_0(t) = 0 \tag{30}$$

However, this ideal case does not exist, and it is only convenient for complete theoretical analysis. In practice, we expect that matching error be independent of one priori chosen signal $\xi(t)$, whose special form will be given in latter Proposition 5.1.

To show this independent property, we construct one cross-correlation function $f(C(z))$ as that

$$f(C(z)) = \frac{1}{N}\sum_{t=1}^{N}\xi(t)\varepsilon(t) \tag{31}$$

where $\xi(t)$ is one instrumental variable, chosen according to the following two equities.

$$E[\xi(t)r(t)] \neq 0 \rightarrow \lim_{N\rightarrow\infty}\frac{1}{N}\sum_{t=1}^{N}\xi(t)r(t) \neq 0;$$

$$E[\xi(t)d(t)] = 0 \rightarrow \lim_{N\rightarrow\infty}\frac{1}{N}\sum_{t=1}^{N}\xi(t)d(t) = 0 \tag{32}$$

where operator $E[.]$ means expectation operator.

To expand Equation (18), we see that

$$\frac{P(z)C(z)}{1+P(z)C(z)} = M(z); \ \frac{1}{1+P(z)C(z)} = 1 - M(z) \tag{33}$$

Then we have

$$\varepsilon(t) = [1 - M(z)]d(t) + [1 - M(z)]C(z)y(t) - M(z)r(t) \tag{34}$$

Substituting Equation (21) into (18), it holds that

$$f(C(z)) = E\frac{1}{N}\sum_{t=1}^{N}\xi(t)\varepsilon(t) = E\frac{1}{N}\sum_{t=1}^{N}\xi(t)[[1 - M(z)]C(z)y(t) - M(z)r(t)] \tag{35}$$

Also, we have

$$y(t) - d(t) = P(z)u(t);$$

i.e.,

$$\left[\frac{P(z)C(z)}{1+P(z)C(z)} - M(z)\right]r(t) = \left[\frac{P(z)C(z)}{1+P(z)C(z)} - M(z)\right]M^{-1}(z)y(t) \tag{36}$$

i.e.,

$$\varepsilon(t) = y(t) - y_0(t) = P(z)u(t) + d(t) - y_0(t) \tag{37}$$

so

$$\begin{aligned}\frac{1}{N}\sum_{t=1}^{N}\xi(t)\varepsilon(t) &= \frac{1}{N}\sum_{t=1}^{N}\xi(t)[P(z)u(t) + d(t) - y(t)] \\ &= \frac{1}{N}\sum_{t=1}^{N}\xi(t)\left[\frac{M(z)}{C(z)(1-M(z))}u(t) - y(t)\right]; \\ P(z) &= \frac{M(z)}{C(z)(1-M(z))}\end{aligned} \tag{38}$$

Observing Equations (7) and (38), we find these two equations are equivalent with each other while choosing approximated filter $L(z)$ and instrumental variable $\xi(t)$.

Proposition 5.1. *In Equation (38), we choose*

$$L(z) = \frac{C(z)(1-M(z))}{M(z)}; \ \xi(t) = e(t) = r(t) - y(t) = \left(M^{-1}(z) - 1\right)y(t) \tag{39}$$

and then Equation (38) is equal to Equation (30).

Proof: Substituting the chosen filter $L(z)$ and instrumental variable $\xi(t)$ in Equation (38), then we have

$$\begin{aligned} f(C(z)) &= \frac{1}{N}\sum_{t=1}^{N}\left(M^{-1}(z)-1\right)y(t)\left[\frac{M(z)}{C(z)(1-M(z))}L(z)u(t)-L(z)y(t)\right] \\ &= \frac{1}{N}\sum_{t=1}^{N}\left(M^{-1}(z)-1\right)y(t)\left[u(t)-\frac{C(z)(1-M(z))}{M(z)}y(t)\right] \\ &= \frac{1}{N}\sum_{t=1}^{N}\left[u(t)-C(z)\left(M^{-1}(z)-1\right)y(t)\right]^2 \\ &= \frac{1}{N}\sum_{t=1}^{N}[u(t)-C(z)y_1(t)]^2 \end{aligned} \tag{40}$$

This completes the proof.

Proposition 5.1 tells us that the tracking property is equal to the cross-correlation function sufficiently small, i.e., zero tracking error. On basis of Proposition 5.1, to achieve the dual goals, i.e., zero tracking error and perfect performance, our considered direct data driven control strategy can be applied.

By the way, reference model $M(z)$ exists in all of our mathematical derivations. It can also be represented by a discrete time transfer function with a delay at least of 1.

$$M(z) = \frac{b_1 z^{n-1} + \cdots + b_{n-1}z + b_n}{z^n + a_1 z^{n-1} + \cdots + a_{n-1}z + a_n};\ y(t) = M(z)r(t) \tag{41}$$

where n is the order of polynomial.

Equation (28) corresponds to

$$y(k+1) = -a_1 y(k-1) - \cdots - a_n y(k-n) + b_1 r(k-1) + \cdots + b_n r(k-n) \tag{42}$$

Assume that $M(z)$ is invertible, in the sense that we can compute the sequence of reference input $r(t)$. Given the sequence of output $y(t)$, in short we can write $r(t) = M^{-1}(z)y(t)$.

6. **Simulation Example.** This section uses a practical example about unmanned aircraft vehicle (UAV) to prove the efficiency about optimal data driven controller tuning, i.e., designing the controller parameters directly from the measured input-output pairs. We all know one fact that during this whole UAV flight process, if the flight speed or velocity is too fast or slow, and the flight attitude exceeds a certain limit, then the flight state will be affected greatly, or even the possibility of crash. It means we should guarantee the safety and completeness, so it is important to design one safe controller for UAV operational completeness. Here our goal is to apply optimal data driven controller tuning strategy to designing one safe controller for UAV.

A UAV is controlled by the input variable, and an airborne measurement system can record the output response, i.e., collecting the input-output pairs. The closed loop system with feedback controller can deal with the input distortion for UAV, while restraining disturbance and keeping stability. Closed loop control structure of UAV is shown in Figure 4, where UAV sends control instructions and controls the actuator to perform the corresponding operation and feedback control.

Comparing Figure 2 and Figure 4, we see these two figures are the same as each other. For example, $C(z)$ is the controller, plant $P(z)$ is UAV, and $r(t)$ is input. The pitch channel in the flight control system of HB380 fixed-wing UAV is selected, and the simulation experiment is carried out in the MATLAB R2018B simulation platform. We collect

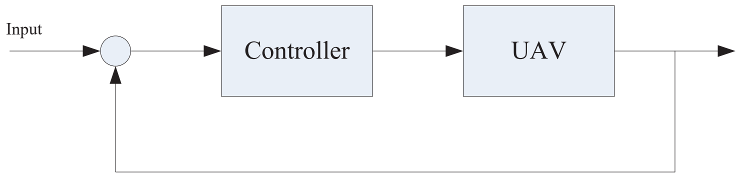
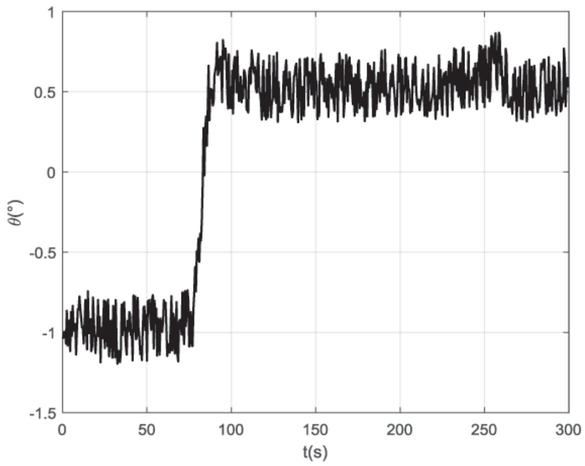
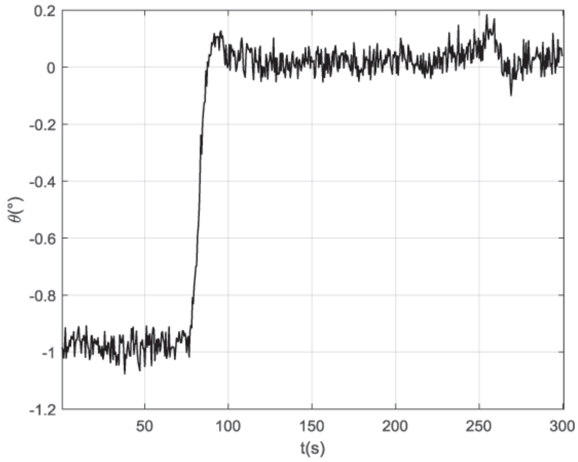


FIGURE 4. UAV closed loop control system



(a) Measured input signal



(b) Measured output signal

FIGURE 5. Input-output pairs

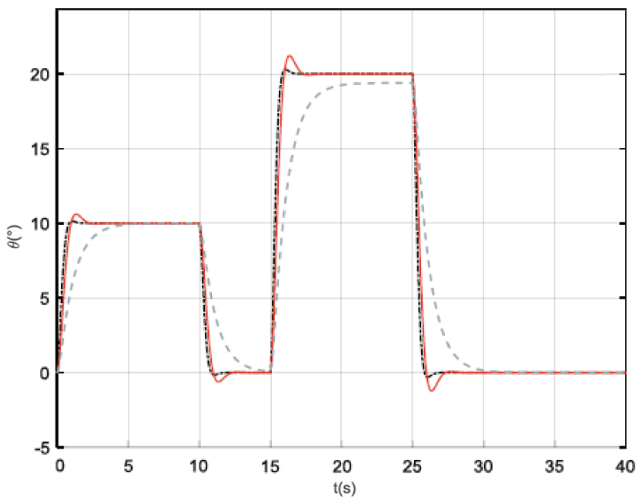


FIGURE 6. Pitch angle following curve

input-output data on both sides of the controller, shown in Figure 5. During the design of fixed-wing UAV, the aerodynamics data of UAV are usually obtained through wind tunnel experiments, so the aerodynamics parameters of UAV inevitably have errors. The smaller error brings the better performance. In order to test the performance of the identified UAV controller parameters, the response contrast curves of PID controller and the safety controller are given in Figure 6, where the pitch angle signal is 10 and 20 degrees.

Specifically, in Figure 6, the black dotted line is the given signal, the gray is PID control response curve, and the red solid line is the identification control response curve. Comparing the curves in Figure 6, the attitude angle under the control of PID controller is slow when the pitch angle is small, and the adjustment time is about 4 s, while the attitude angle response is faster. When the identification control is used, the adjustment time is about 2 s. In the case of large angle pitch signal input, the adjusting time of PID control is about 10 seconds, and there is about 0.7 degree error, but the adjusting time of identification control is about 2.8 seconds. So from this comparison, we see there is no error basically, and further visible identification of the model has a higher efficiency, meaning more accurate accuracy.

7. **Conclusion.** This paper studies direct data driven control strategy deeply from theory and practice simultaneously. Firstly, from the point of theory, all existing results hold on the condition of persistent excitation, while guaranteeing the inverse matrix operation exist and making the designed controller useful. Secondly, we establish the interesting equivalence between direct data control and closed loop performance analysis through the approximated filter and instrumental variable. Both two separated aspects are concentrated around the same goal-good tracking performance. Thirdly, after showing the main idea of data driven control for unknown closed loop system, two explicit forms of data driven controller are derived through our own mathematical derivations, i.e., nonparametric form and parameterized form, respectively. As all mathematical derivations, proposed in this new paper, only consider the linear case, all relations are linearities. However, in nature, all phenomena are nonlinearities during our normal life; in future, we will concentrate on a more general case, i.e., nonlinear data driven control based on topology and differential geometry.

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