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DUAL PERFECT TRACKING AND VALIDATION FOR DATA DRIVEN TUNING CONTROL

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Abstract. *Due to the popularly studied subject about data driven strategy during this big data era, this paper extends our previous contributions about data driven tuning control into an in-depth research from both academia and practical industry. Specifically, first after reviewing one interesting closed loop system with unknown plant and two controllers, i.e., feed forward controller and feedback controller, our mission is to design these two controllers in order to achieve dual control goals, i.e., perfect tracking and disturbance rejection. Second, one algorithm is proposed to implement through our own derivation, depending on the unknown plant. Third, to avoid the modeling process of the unknown plant, data driven tuning control is applied to designing the two parameterized controllers, embodying in tuning the unknown controller parameters from three sequent aspects, i.e., data driving tuning algorithm, perfect tracking analysis and controller validation, respectively.*

Keywords: Data driven tuning control, Perfect tracking, Disturbance rejection, Controller validation

1. **Introduction.** During this more advanced society, everyday we need to contact with others, such as human being or phenomenon, existing in the natural world. When we or human being confront with some natural phenomenon, it is urgent to conquest them, for example, driving the car, ship or plane, making machine work well according to our expected function, wishing everything better. All above thinkings are formulated as the control idea or mission, i.e., letting some plants work within our expected range. The idea of control was originated from [1], and then experienced more than one century from the classical control theory to the present modern control theory. More specifically, classical control theory, depending on transfer function form, analyzes the considered plant output response, and designs the commonly used proportion integration differentiation (PID) controller to guarantee plant work well. Here plant may be factory, robot, plane or other motion machines. To improve the control accuracy, tracking efficiency or disturbance rejection, etc., the modern control theory is proposed to be dependent of state space form, which leads to lots of advanced control strategies, such as robust control, adaptive control, sliding mode control, model predictive control, neural network control, and fuzzy control. In our opinions, each control strategy has its own advantage and shortcoming, so our goal is to display its advantage and alleviate its shortcoming. Whatever for the classical control theory and modern control theory, the first premise is about the knowledge of the considered plant, i.e., trying to obtain some knowledge of the unknown plant and describing them into the editable forms, such as picture, graph and mathematical form. The whole

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process of obtaining the editable form corresponds to mathematical modeling, i.e., constructing one mathematical equation to represent the motion principle for the unknown plant through physical principle modeling or system identification modeling. Based on this priori information as the mathematical equation, then it is applied to latter controller design, benefiting for classical or modern control theory. Above detailed description of controller design is named as model based control, depending on the constructed model greatly, so before to design controller, firstly we must spend lots of time and energy in modeling process, thus leading to one interesting subject on system identification, which applies the measured data to constructing one mathematical model for the unknown plant. As system identification experienced more than sixty years, it is very mature from both the theory and practical application, corresponding to yielding one mathematical model from the measured data directly without any physical principle. Within this new data era, researchers are thinking about how to apply the measured data to designing controllers, i.e., extracting one rough controller from the measured data directly, and similar to the main essence of system identification. This idea brings an innovative subject of data driven control strategy. Generally, data driven control is to design one controller from the measured data without any complex modeling process for the unknown plant.

After data driven control is proposed to replace the original model based control, lots of detailed data driven control strategies appear, for example, virtual reference feedback control, subspace control, and iterative feedback control, embodying the essence of data driven control strategy, i.e., designing the controller from the measured data directly. After 2010s, data direct control appears with data science increases, meaning feasible for our new data era. In [2], data driven control method is programmed in a Python package for convenient application. Similarly, optimization theory is also embedded in data driven control, such as convex optimization [3], scenario optimization [4], and stochastic optimization with chance constraints [5]. Furthermore, system identification is not only for model identification, but also for control theory, i.e., constructing one new idea of identification for control. Consider the application of our considered data driven control, [6] combines the idea with classical PID design, i.e., generating these three PID unknown parameters through a data fitting process. [7] designs one controller for minimum and non-minimum phase system by data driven control strategy. However, before to do it, the practical engineering system must be changed into the considered minimum phase system. During these recent years, more deep researches about data driven control are implemented in Germany from different points of view, for example, data driven model predictive control with stability and robustness guarantee [8], dissipativity property from input-output measured data [9], one shot verification of dissipativity property from input-output measured data [10], and nonlinear data driven control [11]. Their contributions and missions are to develop model free system analysis and control methods, which are only based on the measured data. One approach towards these goals is to extract control theoretic system properties such as dissipativity or nonlinear measures from data, which can then be used to design controllers via data driven control methods. Moreover, other new fields are combined with our considered data driven control strategy. [12] considers modulating robustness, and robust event triggered output feedback controller is studied in [13], while guaranteeing the similarity with nonlinear direct data driven control. The detailed formulas for data driven control are described and its data informality is studied in [14], such as stabilization, optimality and robustness together. By the way, identification for control is mentioned in [15], where set membership for estimating the output predictor, and a new kernel based approach for hybrid system identification and data driven design for switching controllers. Generally, during these recent years, more and more researches about data driven control strategy are ongoing all over the world. Moreover in

[16], to alleviate the problem of blackout or damaging the grid equipment, a robust data driven frequency domain grid connected converter controller design with the possibility of defining robust passivity condition on one performance channel.

As a consequence, due to the widely studied references and our previous contributions about data driven control strategy, this new paper continues to do some better improvements, so that guaranteed performance and other extension are achieved through our own mathematical derivations and practical application in engineering. More specifically, for the sake of completeness, after the considered closed loop system with one unknown plant and two unknown controllers is required, the detailed control goal is mentioned to be the dual forms, i.e., perfecting tracking or model matching and disturbance rejection. To achieve these dual goals, firstly we propose a control algorithm to design both the feed forward controller and feedback controller iteratively. From our mathematical derivations, we see that all derivations hold on the condition of the priori information about the unknown plant, meaning physical modeling or system identification modeling is needed to identify that unknown plant, so it corresponds to the classical model based control strategy. To avoid this additional modeling process, and design those two unknown controllers directly from the measured data, data driven control strategy is benefited. Within this framework of data driven idea, those two unknown controllers are parameterized by tow different parameter vectors, so the problem of controller design is transformed into the parameter estimation, bringing the data driven control to our named data driven tuning. Here, the meaning of tuning means controller parameters change with environment of time varying. Considering the problem of tuning controller parameter, data driven tuning control is proposed to achieve the dual goals, and further its detailed algorithm, corresponding analysis are also given to complete the research of data driven tuning control. To testify whether the obtained parameterized controller works well, i.e., guaranteeing the whole closed loop system works normally, one controller validation process is added. All above described topics correspond to our named better improvements from the theory and convenient application.

2. System Structure Review.

Here consider the following closed loop system in Figure 1.

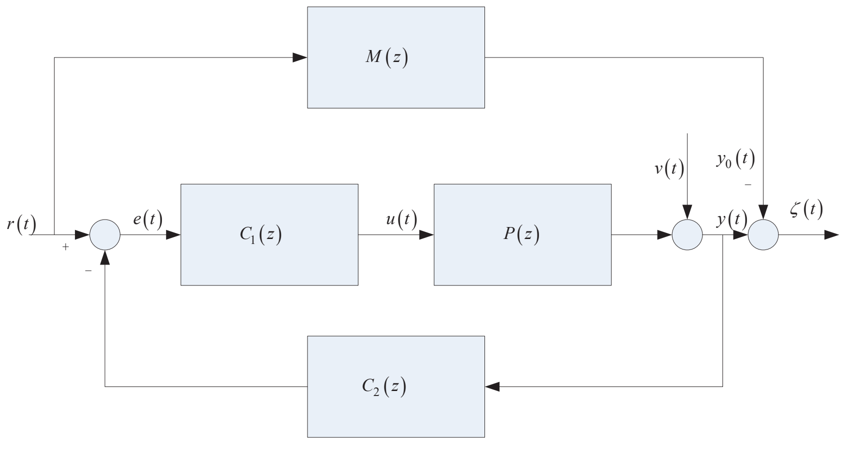

FIGURE 1. Closed loop system structure

In above Figure 1, three different modulars exist, corresponding to plant $P(z)$, feed forward controller $C_1(z)$ and feedback controller $C_2(z)$, and each modular of $\{P(z), C_1(z), C_2(z)\}$ is unknown. $r(t)$ and $y(t)$ are closed loop input-output signal, $u(t)$ is the external disturbance or noise. $e(t)$ is the feedback error signal, i.e., $e(t) = r(t) - C_2(z)y(t)$, where

z is the backward shift operator. $v(t)$ is the external noise or disturbance, described in deterministic or statistical case. Due to its existence, it will affect the entire control performance greatly, so one of our goals in this paper is to reject this external noise.

Moreover, in above Figure 1, a reference model $M(z)$ is given in priori by the designer, so the corresponding reference output or named expected closed loop output $y_0(t)$ is that $y_0(t) = M(z)r(t)$. Then the final closed loop output error $\zeta(t) = y(t) - y_0(t) = y(t) - M(z)r(t)$, so the whole control objective is to design two approximate controllers $\{C_1(z), C_2(z)\}$ such that the final closed loop output error $\zeta(t)$ is sufficiently small of zero. From Figure 1, some relations exist, for example,

$$\begin{aligned} &y_0(t) = M(z)r(t); \quad y(t) = P(z)C_1(z)r(t) - P(z)C_1(z)C_2(z)y(t) + v(t); \\ &e(t) = r(t) - C_2(z)y(t) \end{aligned} \tag{1}$$

i.e.,

$$\begin{aligned} y(t) &= \frac{P(z)C_1(z)}{1 + P(z)C_1(z)C_2(z)} r(t) + \frac{1}{1 + P(z)C_1(z)C_2(z)} v(t); \\ \zeta(t) &= \left[\frac{P(z)C_1(z)}{1 + P(z)C_1(z)C_2(z)} - M(z) \right] r(t) + \frac{1}{1 + P(z)C_1(z)C_2(z)} v(t) \end{aligned} \tag{2}$$

Then the control objective, i.e., $\zeta(t) = y(t) - y_0(t) = y(t) - M(z)r(t) \to 0$ is changed to the following two conditions.

$$\frac{P(z)C_1(z)}{1 + P(z)C_1(z)C_2(z)} \to M(z); \quad \frac{1}{1 + P(z)C_1(z)C_2(z)} \to 0 \tag{3}$$

so our mission in this paper is to design these two controllers $\{C_1(z), C_2(z)\}$ to satisfy above two conditions in Equation (3).

3. Model Tracking Problem.

From above description about our control objective, those two conditions in Equation (3) are equivalent to that zero error, i.e., $\zeta(t) = y(t) - y_0(t) = 0$, corresponding to model matching problem or model tracking problem.

3.1. Dual control goals.

Here consider the parameterized controllers $\{C_1(z, \theta), C_2(z, \eta)\}$ by two different controller parameter vectors $\{\theta, \eta\}$, respectively, as the parameterized controller is widely applied in practical engineering, for example, PID control or adaptive control. Based on the parameterized case, Figure 1 is transformed into the following Figure 2.

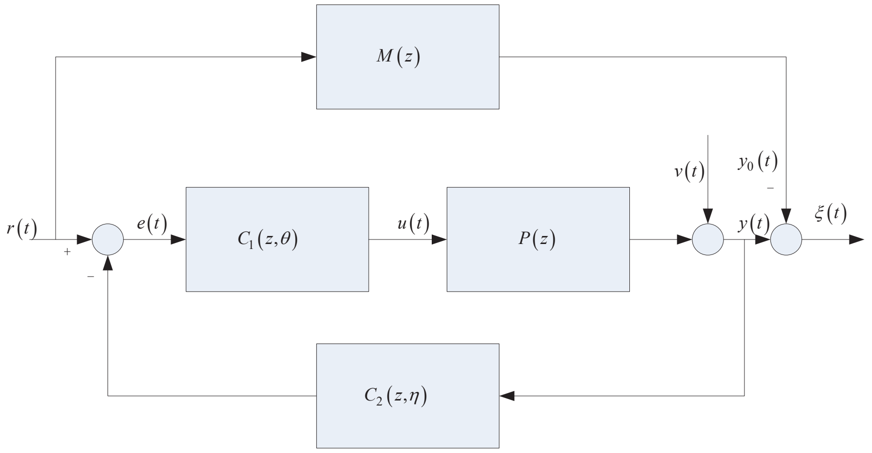

Figure 2. The parameterized closed loop structure

Similarly, Equation (2) is rewritten as

$$\begin{aligned} y(t) &= \frac{P(z)C_1(\theta)}{1+P(z)C_1(\theta)C_2(\eta)}r(t) + \frac{1}{1+P(z)C_1(\theta)C_2(\eta)}v(t); \\ \zeta(t) &= \left[\frac{P(z)C_1(\theta)}{1+P(z)C_1(\theta)C_2(\eta)} - M(z)\right]r(t) + \frac{1}{1+P(z)C_1(\theta)C_2(\eta)}v(t) \end{aligned} \tag{4}$$

Similarly Equation (3) is changed

$$\frac{P(z)C_1(\theta)}{1+P(z)C_1(\theta)C_2(\eta)} \to M(z); \quad \frac{1}{1+P(z)C_1(\theta)C_2(\eta)} \to 0 \tag{5}$$

To simplify notation, variable z is abbreviated through latter mathematical derivations. Observing Equation (5), the first equation means perfect tracking or model matching, and the second one is disturbance rejection. The combinations of perfect tracking and disturbance rejection are our control goals, i.e., our called dual control goals.

3.2. **Algorithm 1.** As that disturbance rejection condition

$$\frac{1}{1+P(z)C_1(\theta)C_2(\eta)} \to 0$$

is an ideal case, in practice analysis, we can limit it within a tolerable range, such as $[-0.5, 0.5]$. Without loss of generality, we set one constant a from interval $[-0.5, 0.5]$ to satisfy

$$\frac{1}{1+P(z)C_1(\theta)C_2(\eta)} = a \tag{6}$$

Then we have

$$\begin{aligned} &\frac{P(z)C_1(\theta)}{1+P(z)C_1(\theta)C_2(\eta)} = P(z)C_1(\theta)a = M(z); \quad \frac{1}{1+P(z)C_1(\theta)C_2(\eta)} = a; \\ &P(z)C_1(\theta)C_2(\eta) = \frac{1}{a} - 1; \quad C_1(\theta) = \frac{M(z)}{aP(z)} = P^{-1}(z)\frac{M(z)}{a} \end{aligned} \tag{7}$$

The advantage of the parameterized control is that original controller design problem is also changed to the controller parameter identification problem, i.e., identifying those two controller parameter vectors $\{\theta, \eta\}$ while satisfying those dual control goals in Equation (5). From Equation (7), one parameter vector θ can be identified from the following optimization problem.

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \left\| C_1(\theta) - \frac{M(z)}{aP(z)} \right\|^2 \tag{8}$$

where $\hat{\theta}$ denotes the estimation value, and notation $\|.\|$ is the commonly used Euclidean norm.

From Equation (7), it holds that

$$C_2(\eta) = \frac{\frac{1}{a} - 1}{P(z)C_1(\theta)} = P^{-1}(z)\frac{1-a}{a}C_1^{-1}(\theta) \tag{9}$$

So the other parameter vector η is solved from Equation (9), i.e.,

$$\hat{\eta} = \underset{\eta}{\operatorname{argmin}} \left\| C_2(\eta) - P^{-1}(z)\frac{1-a}{a}C_1^{-1}(\theta) \right\|^2 \tag{10}$$

where similarly $\hat{\eta}$ is the parameter estimation for that controller parameter vector η. However, parameter vector θ also exists in that right cost function of Equation (10), and an easy way to solve it is to replace it with its estimation value $\hat{\theta}$, i.e.,

$$\hat{\eta} = \underset{\eta}{\operatorname{argmin}} \left\| C_2(\eta) - P^{-1}(z)\frac{1-a}{a}C_1^{-1}\left(\hat{\theta}\right) \right\|^2 \tag{11}$$

Combining Equations (8) and (11) together, the detailed algorithm, used to achieve the dual control goals, is formulated as the following Algorithm 1.

```
Algorithm 1
Step 1: Given the reference model $M(z)$;
Step 2: Modeling plant $P(z)$;
Step 3: Consider the disturbance rejection, we set $a = 0.2$;
Step 4: Compute the feed forward controller parameter vector $\theta$ through Equation (8) to get its parameter estimation $\hat{\theta}$;
Step 5: Based on above feed forward controller parameter estimation $\hat{\theta}$, substitute $\hat{\theta}$ into feed forward controller $C_1\left(\hat{\theta}\right)$ and then identify the other feedback controller parameter vector $\hat{\eta}$ through Equation (11).
Step 6: Testify whether those dual control goals are satisfied, if yes, then terminate above algorithm, or turn to step 4 until Equation (5) is satisfied.
Step 7: Choose those two parameter vectors $\left\{\hat{\theta}, \hat{\eta}\right\}$ as the final two controllers, whose final forms are the linear multiplies forms with respect to two vectors $\left\{\hat{\theta}, \hat{\eta}\right\}$.
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For the sake of completeness, the detailed parameterized controller forms $C_1(\theta)$ and $C_2(\eta)$ are chosen as

$$C_1(\theta) = \alpha(z)\theta; \quad C_2(\eta) = \beta(z)\eta \tag{12}$$

where $\alpha(z)$ and $\beta(z)$ are two prior known basis function vectors with approximate dimension, for example,

$$\alpha(z) = \beta(z) = [1 \; z \; z^2 \; \cdots \; z^n]^T \tag{13}$$

Then the two parameter vectors $\{\theta, \eta\}$ are described as follows.

$$\theta = [\theta_1 \; \theta_2 \; \theta_3 \; \cdots \; \theta_n]^T; \quad \eta = [\eta_1 \; \eta_2 \; \eta_3 \; \cdots \; \eta_n]^T \tag{14}$$

where n is the dimension of the considered two controllers.

Specifically, step 4 in Algorithm 1 concerns on the model matching performance, and step 3 limits the requirement of disturbance rejection. The combinations of them correspond to our called dual control goals, not mentioned the other disturbance rejection in our previous papers.

4. **Data Driven Tuning Control.**

Observing that optimization problem (8), used to identify the feed forward controller parameter vector θ in Algorithm 1, plant $P(z)$ exists, so Algorithm 1 depends on the priori information about that unknown plant $P(z)$, meaning the model based control. It tells us before to implement Algorithm 1, firstly we must spend time and energy in modeling that unknown plant $P(z)$. To avoid this modeling process, and design those two unknown controllers $\{C_1(z, \theta), C_2(z, \eta)\}$ directly from the measured data, this section proposes the idea of data driven tuning control, corresponding to tuning the controller parameters from the collected data.

4.1. **Algorithm 2.**

For better understanding the idea of data driven tuning control, we replot Figure 2 into Figure 3.

Figure 3 shows that ideal case, i.e., $\zeta(t) = y(t) - y_0(t) = y(t) - M(z)r(t) = 0$ for given reference model $M(z)$, requires the external input $r(t)$ satisfy that $r(t) = M^{-1}(z)y(t)$, where $M^{-1}(z)$ is an inverse model. Remember the main essence of data driven strategy

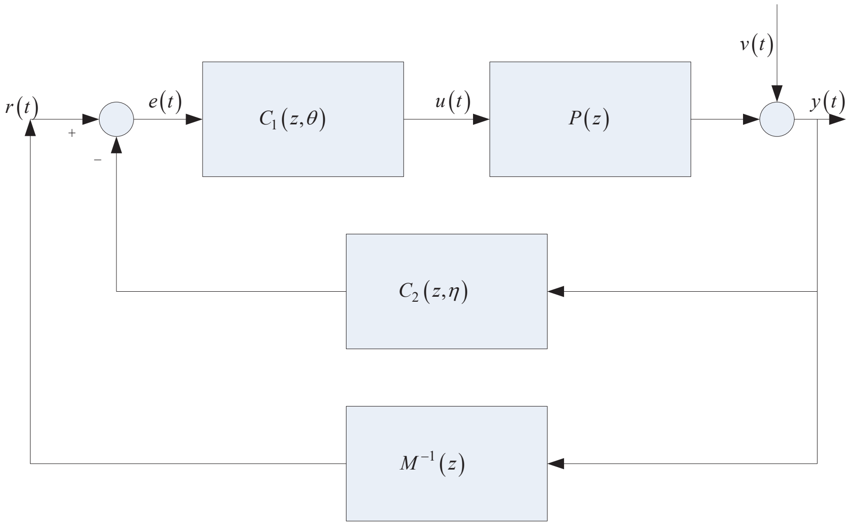


Figure 3. Modified closed loop structure

is to extract some useful information for two controllers $\{C_1(z,\theta), C_2(z,\eta)\}$ from the measured data; consequently we collect both sides of that feed forward controller $C_1(z,\theta)$ to form its input-output data set, i.e., $\{e(t), u(t)\}_{t=1}^{N}$, where N is the total number of data. Due to the relation about $e(t)$, we have

$$\begin{aligned} e(t) &= r(t) - C_2(\eta)y(t) = M^{-1}(z)y(t) - C_2(\eta)y(t) = \left[M^{-1}(z) - C_2(\eta)\right] y(t); \\ u(t) &= C_1(\theta)e(t) = C_1(\theta)\left[M^{-1}(z) - C_2(\eta)\right] y(t) \end{aligned} \tag{15}$$

so the input-output of feed forward controller $C_1(z,\theta)$ is similar to the whole closed loop input-output data, i.e., $\{e(t), u(t)\}_{t=1}^{N} \to \{y(t), u(t)\}_{t=1}^{N}$, meaning after we collect the whole closed loop input-output data $\{y(t), u(t)\}_{t=1}^{N}$ and one given reference model $M(z)$, those two unknown controllers $\{C_1(z,\theta), C_2(z,\eta)\}$ must satisfy

$$u(t) = C_1(\theta)\left[M^{-1}(z) - C_2(\eta)\right] y(t), \quad t = 1, 2, \ldots, N$$

Thanks for the parameterized controllers in Equation (12), data driven tuning control is yielded to tune those two unknown parameter vectors $\{\theta, \eta\}$, while satisfying above ideal equity, and thus giving the following optimization problem.

$$\left(\hat{\theta}, \hat{\eta}\right) = \underset{\theta,\eta}{\operatorname{argmin}} \frac{1}{N}\sum_{t=1}^{N}\left[u(t) - C_1(\theta)\left[M^{-1}(z) - C_2(\eta)\right] y(t)\right]^2 \tag{16}$$

Substituting Equation (12) into Equation (16), it holds that

$$\begin{aligned} &C_1(\theta) = \alpha(z)\theta, \quad C_2(\eta) = \beta(z)\eta; \quad \varphi_1(t,\eta) = \alpha(z)\left[M^{-1}(z) - \beta(z)\eta\right] \\ &u(t) - C_1(\theta)\left[M^{-1}(z) - C_2(\eta)\right] y(t) = u(t) - \varphi_1(t,\eta)\theta \end{aligned} \tag{17}$$

Similarly, we have

$$\begin{aligned} u(t) - C_1(\theta)\left[M^{-1}(z) - C_2(\eta)\right] y(t) &= u(t) - \alpha(z)\theta M^{-1}(z)y(t) + \alpha(z)\theta y(t)\beta(z)\eta \\ &= u(t) - \varphi_2(t,\theta)\eta; \\ \varphi_2(t,\theta) = \alpha(z)\theta M^{-1}(z)y(t) - \alpha(z)\theta y(t)\beta(z) & \end{aligned} \tag{18}$$

From Equation (17), it holds that

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \frac{1}{N}\sum_{t=1}^{N}\left[u(t) - \varphi_1(t,\eta)\theta\right]^2 \tag{19}$$

i.e.,

$$\hat{\theta} = \left[\sum_{t=1}^{N} \varphi_1(t,\eta)\varphi_1(t,\eta)\right]^{-1} \left[\sum_{t=1}^{N} \varphi_1(t,\eta)u(t)\right] \tag{20}$$

Similarly from Equation (18), we have

$$\hat{\eta} = \operatorname*{argmin}_{\eta} \frac{1}{N}\sum_{t=1}^{N}[u(t) - \varphi_2(t,\theta)\eta]^2 \tag{21}$$

i.e.,

$$\hat{\eta} = \left[\sum_{t=1}^{N} \varphi_2(t,\theta)\varphi_2(t,\theta)\right]^{-1} \left[\sum_{t=1}^{N} \varphi_2(t,\theta)u(t)\right] \tag{22}$$

Generally, from above mathematical derivation for Equation (16), only input-output data $\{y(t), u(t)\}_{t=1}^{N}$ are collected, and other $M(z)$ is given in priori, no any information about plant $P(z)$ is needed in above analysis. Moreover, although parameter vectors $\{\theta, \eta\}$ exist in cost function as a nonlinearity, they can be identified with each other as the called iterative estimation, which leads to the following Algorithm 2.

Algorithm 2

Step 1: Given reference model $M(z)$;

Step 2: Choose two basis function vectors $\alpha(z)$ and $\beta(z)$;

Step 3: Collect input-output data set $\{y(t), u(t)\}_{t=1}^{N}$;

Step 4: Choose two initial parameter vectors θ_0 and η_0;

Step 5: Compute the next controller parameter vectors $\hat{\theta}_1$ and $\hat{\eta}_1$ within the first iteration process, i.e.,

$$\begin{aligned}\hat{\theta}_1 &= \left[\sum_{t=1}^{N} \varphi_1(\eta_0)\varphi_1(\eta_0)\right]^{-1} \left[\sum_{t=1}^{N} \varphi_1(\eta_0)u(t)\right]; \\ \hat{\eta}_1 &= \left[\sum_{t=1}^{N} \varphi_2(\theta_0)\varphi_2(\theta_0)\right]^{-1} \left[\sum_{t=1}^{N} \varphi_2(\theta_0)u(t)\right]\end{aligned} \tag{23}$$

Step 6: Continue to identify the next controller parameter vectors $\hat{\theta}_2$ and $\hat{\eta}_2$ within the first iteration process, i.e.,

$$\begin{aligned}\hat{\theta}_2 &= \left[\sum_{t=1}^{N} \varphi_1\left(\hat{\eta}_1\right)\varphi_1\left(\hat{\eta}_1\right)\right]^{-1} \left[\sum_{t=1}^{N} \varphi_1\left(\hat{\eta}_1\right)u(t)\right]; \\ \hat{\eta}_2 &= \left[\sum_{t=1}^{N} \varphi_2\left(\hat{\theta}_1\right)\varphi_2\left(\hat{\theta}_1\right)\right]^{-1} \left[\sum_{t=1}^{N} \varphi_2\left(\hat{\theta}_1\right)u(t)\right]\end{aligned} \tag{24}$$

$\vdots$

Step N: Repeat above iterative identification process to generate two controller parameter vectors $\hat{\theta}_N$ and $\hat{\eta}_N$ using the similar Equations (23) and (24), i.e.,

$$\left\{\hat{\theta}_0, \hat{\theta}_1, \ldots, \hat{\theta}_{N-1}, \hat{\theta}_N\right\}; \quad \{\hat{\eta}_0, \hat{\eta}_1, \ldots, \hat{\eta}_{N-1}, \hat{\eta}_N\} \tag{25}$$

Step N+1: Testify whether the following terminated condition is satisfied, i.e.,

$$\left\|\hat{\theta}_N - \hat{\theta}_{N-1}\right\| + \|\hat{\eta}_N - \hat{\eta}_{N-1}\| \leq 0.5 \tag{26}$$

if yes then terminate above iterative algorithm, but or not then return to step 4.

The main difference between Algorithm 1 and Algorithm 2 is whether that unknown plant $P(z)$ appears in cost function. As no any plant exists in Algorithm 2, Algorithm 2 is suited to the idea of data driven tuning control, i.e., tuning the two controller parameter vectors $\hat{\theta}_N$ and $\hat{\eta}_N$ to satisfy the closed relation between both sides of feed forward controller $C_1(\theta)$.

4.2. **Analysis.** For clarity of presentation, assume a pair of parameter vectors $\{\theta_0, \eta_0\}$ exist and satisfy that ideal case, corresponding to the perfect tracking or model matching property, i.e.,

$$\frac{P(z)C_1(\theta_0)}{1 + P(z)C_1(\theta_0)C_2(\eta_0)} = M(z) \tag{27}$$

Take substraction operation to be

$$\begin{aligned} &\frac{P(z)C_1(\theta)}{1 + P(z)C_1(\theta)C_2(\eta)} - \frac{P(z)C_1(\theta_0)}{1 + P(z)C_1(\theta_0)C_2(\eta_0)} \\ &= \frac{P(z)[C_1(\theta) + C_1(\theta)P(z)C_1(\theta_0)C_2(\eta_0)] - C_1(\theta_0) - P(z)C_1(\theta)C_2(\eta)C_1(\theta_0)}{[1 + P(z)C_1(\theta)C_2(\eta)][1 + P(z)C_1(\theta_0)C_2(\eta_0)]} \end{aligned} \tag{28}$$

Compute the numerator term to be

$$Numerator = [C_1(\theta) - C_1(\theta_0)] + P(z)C_1(\theta)C_1(\theta_0)[C_2(\eta_0) - C_2(\eta)] \tag{29}$$

Applying Taylor series expansion on both $C_1(\theta)$ and $C_2(\eta)$, we have

$$\begin{aligned} C_1(\theta) &= C_1(\theta_0) + \frac{\partial C_1(\theta)}{\partial \theta}(\theta - \theta_0) + high\ order\ term; \\ C_2(\eta) &= C_2(\eta_0) + \frac{\partial C_2(\eta)}{\partial \eta}(\eta - \eta_0) + high\ order\ term \end{aligned} \tag{30}$$

Substituting Equation (3) into Equation (29), it holds that

$$Numerator = \frac{\partial C_1(\theta)}{\partial \theta}(\theta - \theta_0) + P(z)C_1(\theta)C_1(\theta_0)\frac{\partial C_2(\eta)}{\partial \eta}(\eta - \eta_0) + high\ order\ term \tag{31}$$

Based on the parameterized controllers in Equation (12), above Equation (31) is reduced to

$$Numerator = \alpha(z)(\theta - \theta_0) + P(z)\left[C_1^2(\theta_0) + C_1(\theta_0)\alpha(z)(\theta - \theta_0)\right]\beta(z)(\eta - \eta_0) \tag{32}$$

During Algorithm 1 and Algorithm 2, the terminated condition is to guarantee the considered parameter estimation error be sufficiently small, i.e., there exist two small values ξ_1 and ξ_2 such that

$$\|\theta - \theta_0\| \leq \xi_1; \quad \|\eta - \eta_0\| \leq \xi_2 \tag{33}$$

Combining Equations (28), (32) and (33), we have easily that

$$\begin{aligned} &\frac{P(z)C_1(\theta)}{1 + P(z)C_1(\theta)C_2(\eta)} - \frac{P(z)C_1(\theta_0)}{1 + P(z)C_1(\theta_0)C_2(\eta_0)} \\ &= \frac{P(z)[\alpha(z)(\theta - \theta_0)] + P(z)\left[C_1^2(\theta_0) + C_1(\theta_0)\alpha(z)(\theta - \theta_0)\right]\beta(z)(\eta - \eta_0)}{[1 + P(z)C_1(\theta)C_2(\eta)][1 + P(z)C_1(\theta_0)C_2(\eta_0)]} \end{aligned} \tag{34}$$

Taking absolute operation on both sides of Equation (34), it yields

$$\begin{aligned} &\frac{P(z)C_1(\theta)}{1 + P(z)C_1(\theta)C_2(\eta)} - \frac{P(z)C_1(\theta_0)}{1 + P(z)C_1(\theta_0)C_2(\eta_0)} \\ &\leq \frac{|P(z)||\alpha(z)|\xi_1}{[1 + P(z)C_1(\theta_0)C_2(\eta_0)]^2} + \frac{|P(z)||C_1^2(\theta_0) + C_1(\theta_0)\alpha(z)\xi_1||\beta(z)|\xi_2}{[1 + P(z)C_1(\theta_0)C_2(\eta_0)]^2} \end{aligned} \tag{35}$$

Take regularization on two basis function vectors $\alpha(z)$ and $\eta(z)$, i.e.,

$$|\alpha(z)| = |\eta(z)| = 1 \tag{36}$$

Then Equation (35) is reduced to

$$\begin{aligned}&\left|\frac{P(z)C_1(\theta)}{1+P(z)C_1(\theta)C_2(\eta)} - \frac{P(z)C_1(\theta_0)}{1+P(z)C_1(\theta_0)C_2(\eta_0)}\right| \\ &\leq \frac{|P(z)|\xi_1}{[1+P(z)C_1(\theta_0)C_2(\eta_0)]^2} + \frac{|P(z)||C_1^2(\theta_0)+C_1(\theta_0)\xi_1|\xi_2}{[1+P(z)C_1(\theta_0)C_2(\eta_0)]^2}\end{aligned} \tag{37}$$

Considering the mission of disturbance rejection, we have

$$\begin{aligned}&\left|\frac{P(z)C_1(\theta)}{1+P(z)C_1(\theta)C_2(\eta)} - \frac{P(z)C_1(\theta_0)}{1+P(z)C_1(\theta_0)C_2(\eta_0)}\right| \leq |P(z)|a^{-2}\left[\xi_1 + \|\theta_0\|^2\right]\xi_2; \\ &C_1^2(\theta_0) = \|\alpha(z)\theta_0\|^2 = \|\alpha(z)\|^2\|\theta_0\|^2 = \|\theta_0\|^2\end{aligned} \tag{38}$$

Observing that upper bound in Equation (38), $|P(z)|$ and $\|\theta_0\|$ are fixed values. As that disturbance rejection condition $\frac{1}{1+P(z)C_1(\theta)C_2(\eta)} \to 0$, meaning in a tolerable range, for example, $[-0.5, 0.5]$, then we guarantee $\frac{1}{1+P(z)C_1(\theta)C_2(\eta)} = a = 0.5$. So if the two parameter estimation errors are sufficiently small, then the model tracking property is achieved, i.e.,

$$\begin{aligned}&\left|\frac{P(z)C_1(\theta)}{1+P(z)C_1(\theta)C_2(\eta)} - \frac{P(z)C_1(\theta_0)}{1+P(z)C_1(\theta_0)C_2(\eta_0)}\right| \to 0; \\ &\xi_1 \textit{ and } \xi_2 \textit{ are sufficiently small}\end{aligned} \tag{39}$$

Generally, above analysis is suited to our dual control goals.

4.3. **Validation.** After the detailed algorithm and analysis about data driven tuning control strategy are given through our own mathematical derivation, another issue appears about how to determine the performance of the designed parameterized controllers $\{C_1(\theta), C_2(\eta)\}$, i.e., controller validation process. For the sake of completeness, here we use the cross correlation method to implement it, i.e., checking the cross correlation function between the external input $r(t)$ and closed loop output error $\zeta(t)$ be zero, then leading to white and uncorrelated testify. To compute that cross correlation between the external input $r(t)$ and closed loop output error $\zeta(t)$, firstly, we need to give an explicit form for closed loop output error $\zeta(t)$. From Figure 1 again, we have

$$\zeta(t) = \left[\frac{P(z)C_1(\theta)}{1+P(z)C_1(\theta)C_2(\eta)} - M(z)\right] r(t) + \frac{1}{1+P(z)C_1(\theta)C_2(\eta)} v(t) \tag{40}$$

From Equation (27), some relations hold, such as

$$\frac{P(z)C_1(\theta_0)}{1+P(z)C_1(\theta_0)C_2(\eta_0)} = M(z); \quad \frac{1}{1+P(z)C_1(\theta_0)C_2(\eta_0)} = 1 - C_2(\eta_0)M(z) \tag{41}$$

Substitute Equation (41) into Equation (40) to get

$$\begin{aligned}&\frac{P(z)C_1(\theta_0)}{1+P(z)C_1(\theta_0)C_2(\eta_0)} \times [1+P(z)C_1(\theta_0)C_2(\eta_0)] = P(z)C_1(\theta_0) = \frac{M(z)}{1-C_2(\eta_0)M(z)}; \\ &\frac{P(z)C_1(\theta)}{1+P(z)C_1(\theta)C_2(\eta)} \approx \frac{P(z)C_1(\theta)}{1+P(z)C_1(\theta_0)C_2(\eta_0)} \\ &\qquad = P(z)C_1(\theta)[1-C_2(\eta_0)M(z)] \\ &\qquad = \frac{M(z)}{[1-C_2(\eta_0)M(z)]C_1(\theta_0)} C_1(\theta)[1-C_2(\eta_0)M(z)]\end{aligned}$$

$$= M(z)\frac{C_1(\theta)}{C_1(\theta_0)};$$

$$P(z) = \frac{M(z)}{[1 - C_2(\eta_0)M(z)]C_1(\theta_0)} \tag{42}$$

i.e.,

$$\frac{P(z)C_1(\theta)}{1 + P(z)C_1(\theta)C_2(\eta)} = M(z)\frac{C_1(\theta)}{C_1(\theta_0)} \tag{43}$$

Substituting Equation (43) into Equation (40), we have

$$\zeta(t) = y(t) - y_0(t) = M(z)\left[\frac{C_1(\theta)}{C_1(\theta_0)} - 1\right]r(t) + [1 - C_2(\eta_0)M(z)]v(t) \tag{44}$$

Take the cross correlation to yield

$$Er(t)\zeta(t) = \lim_{N\to\infty}\frac{1}{N}\sum_{t=0}^{N} r(t)\zeta(t) = M(z)\left[\frac{C_1(\theta)}{C_1(\theta_0)} - 1\right]\phi_r(w) \tag{45}$$

where the condition about external input $r(t)$ being independent of external noise $v(t)$ is used, and notation $\phi_r(w) = Er^2(t)$ is the power spectrum for external input $r(t)$.

From the derived result in Equation (45), we see the uncorrelation is guaranteed on virtue of $C_1(\theta) \approx C_1(\theta_0)$, i.e., $\theta \approx \theta_0$, which means the controller parameter estimation is unbiased. From Section 4.2, this unbiased condition holds from Algorithm 2. On the other hand, from the cross correlation testify, the other nonparametric controller is also obtained. Specifically, the output error for feed forward controller $C_1(z)$ is defined as follows.

$$\zeta_1(t) = u(t) - C_1(z)e(t) = u(t) - C_1(z)\left(M^{-1}(z) - C_2(z)\right)y(t) \tag{46}$$

One ideal case is

$$E\zeta_1(t) = E[u(t) - C_1(z)e(t)] = E\left[u(t) - C_1(z)\left(M^{-1}(z) - C_2(z)\right)y(t)\right] = 0 \tag{47}$$

To satisfy above ideal case, we expand it as follows.

$$Eu(t)y(t) = E\left[C_1(z)\left(M^{-1}(z) - C_2(z)\right)y(t)\right]y(t);$$

$$\phi_{uy}(w) = C_1(z)\left(M^{-1}(z) - C_2(z)\right)\phi_y(w); \quad C_1(z)\left(M^{-1}(z) - C_2(z)\right) = \frac{\phi_{uy}(w)}{\phi_y(w)} \tag{48}$$

where notation E denotes the expectation operation, $\phi_{uy}(w)$ and $\phi_y(w)$ are cross spectrum and auto-spectrum.

From Equation (48), we see after two controllers are designed whatever in their parameterized forms or nonparametric forms, we collect a second group of data $\{y(t), u(t)\}_{t=1}^{N}$ to get the cross spectrum $\phi_{uy}(w)$ and auto spectrum $\phi_y(w)$; thus, two designed controllers $\{C_1(z), C_2(z)\}$ must satisfy that equity in Equation (48).

5. **Simulation Example.**

To show the efficiency of our proposed better improvement for data driven tuning control, i.e., tuning the controller parameters from the measured input-output data through Algorithm 2 while achieving that perfect tracking, we consider the following plant model $P(z)$ as follows.

$$P(z) = \frac{(z - 2.4)(z - 1.2)(z - 0.5)}{z(z - 0.8)(z - 0.4)(z - 0.6)} \tag{49}$$

Remember the main essence of data driven tuning control, that reference model $M(z)$ and the measured input-output data $\{y(t), u(t)\}_{t=1}^{N}$ are needed.

$$M(z) = \frac{0.6z^4 - 0.12z^3 + 0.8z^2 + 0.4z + 6}{z^5 - z^4 + 0.24z^3 + 0.2z^2 - 2z + 10} \tag{50}$$

During the latter simulation, those feed forward controller $C_1(z)$ and feedback controller $C_2(z)$ are all parameterized by those two separate parameter vectors $\{\theta, \eta\}$, i.e.,

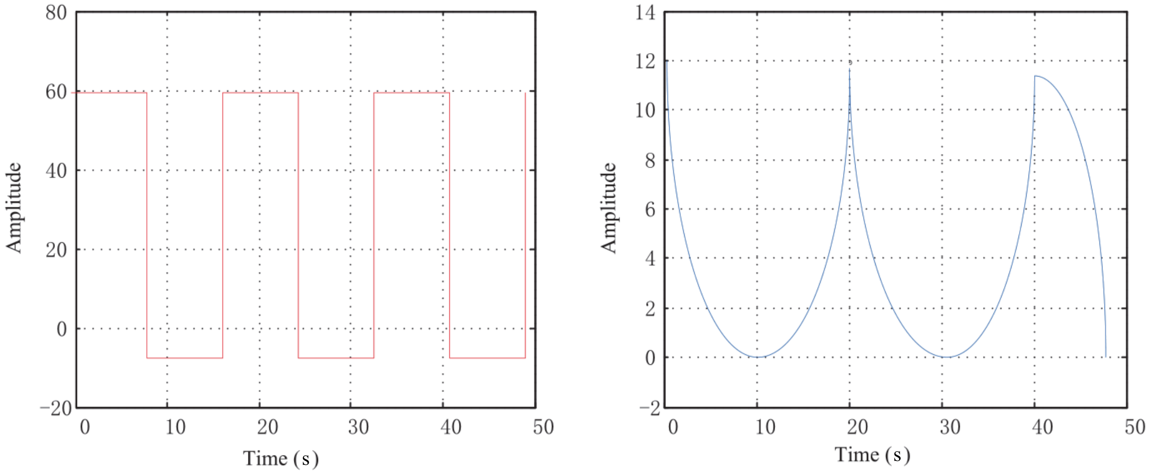
$$C_1(z) = C_1(\theta) = \begin{bmatrix} 1 & z & z^2 & z^3 & z^4 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix};$$

$$C_2(z) = C_2(\eta) = \begin{bmatrix} 1 & z & z^2 & z^3 & z^4 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{bmatrix}; \quad \alpha(z) = \beta(z) = \begin{bmatrix} 1 & z & z^2 & z^3 & z^4 \end{bmatrix} \tag{51}$$

To compute the simulation results, the true two controller parameter vectors $\{\theta, \eta\}$ are chosen as

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.8 \\ 0.2 \\ -0.4 \\ -0.3 \end{bmatrix}; \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 1.8 \\ 2.0 \\ 0.4 \\ -0.6 \end{bmatrix} \tag{52}$$

Based on plant model $P(z)$ in Equation (49), reference model $M(z)$ in Equation (50), and those two parameterized controllers $\{C_1(\theta), C_2(\eta)\}$, the considered closed loop system is determined. After imposing one kind of sine signal into this considered closed loop system, both signals between that feed forward controller $C_1(\theta)$ are collected together to constitute the named measured input-output data set $\{y(t), u(t)\}_{t=1}^{50}$, plotted in Figure 4. Applying that reference model $M(z)$ in Equation (50) and the measured input-output data set in



(a) The measured input signal (b) The measured output signal

FIGURE 4. The measured input-output data

Figure 4, Algorithm 2 is applied to identifying those two controller parameter vectors iteratively, through minimizing one cost function consisted by the measured input-output data.

As the identification of two controller parameter vectors $\{\theta, \eta\}$ is to minimize that cost function in Equation (16) through our proposed algorithm, when the iterative step is increased, that cost function will decrease to its minimum value, i.e., 0. At this moment, the corresponding final parameter vectors $\{\theta, \eta\}$ are chosen as their estimation values. Figure 5 plots the varied curve of cost function with iterative step increases. Due to the existence of two controllers, i.e., feed forward controller and feedback controller, the entire tuning processes for those two controller parameter vectors $\{\theta, \eta\}$ are shown in Figure 6, where Figure 6(a) gives the convergence curves for feed forward controller parameters, and Figure 6(b) shows the convergence curves for feedback controller parameters. From these ten convergence curves, corresponding to those total ten controller parameters, we find each controller parameter will converge to its true value respectively with iterative step increases. Remember above process is an inverse process, i.e., identifying parameter vectors $\{\theta, \eta\}$ only through the measured input-output data and the given reference model without using that plant model.

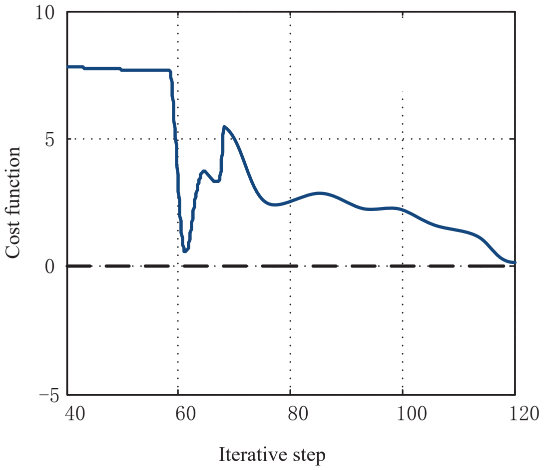
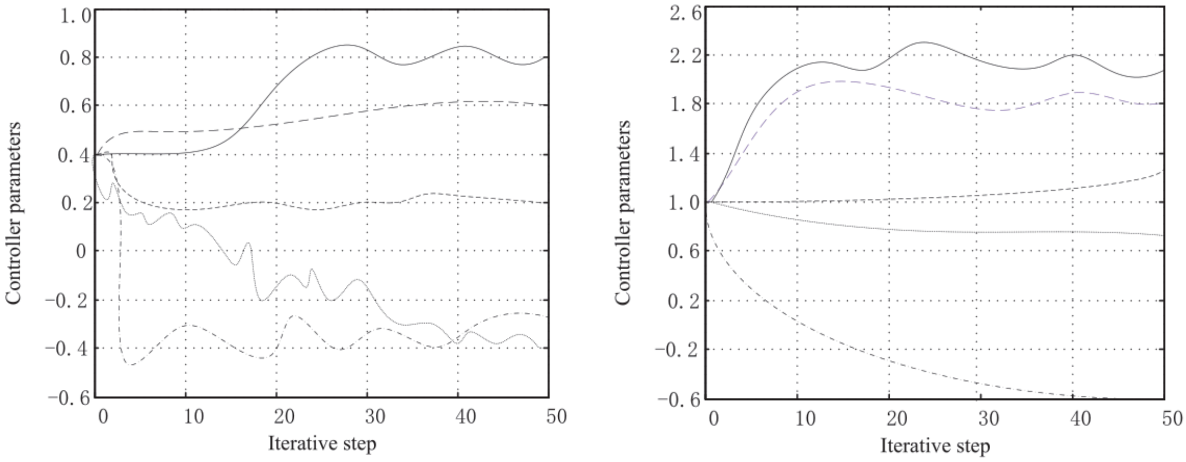


FIGURE 5. Cost function varies with iterative step increases



(a) Feed forward controller parameters (b) Feedback controller parameters

FIGURE 6. Controller parameters tuning

Further, Figure 7 shows the desired or expected perfect tracking property, i.e., guaranteeing the closed loop output $y(t)$ tracks the expected output $y_0(t)$. Specifically, in Figure 7, the black line denotes the expected output $y_0(t)$, and the red line is the real closed loop output $y(t)$, substituting our identified controller parameters $\left\{\hat{\theta}, \hat{\eta}\right\}$. During time interval [0, 70s], error $\zeta(t)$ exists. However, after 70s, this error $\zeta(t)$ will be very small, and later approach to zero, suiting to our considering perfect tracking property.

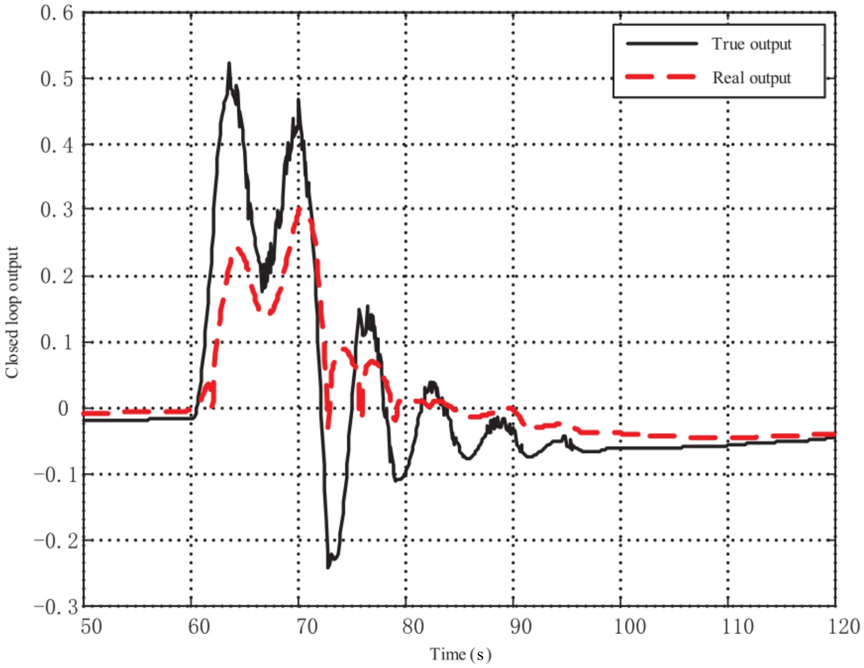


Figure 7. Comparison of true closed loop output and its real one

6. **Conclusion.** Within this new big data era, we make an in-depth research on data driven control strategy, i.e., designing controller from data directly without any modeling process. When considering the parameterized controller, it reduces to data driven tuning control, tuning the controller parameters to satisfy the dual goals, i.e., perfect tracking and disturbance. Furthermore, the deep research on data driven tuning control is completed from the tuning algorithm, analysis and controller validation only through our own derivations. Later, our future work will be centered around adaptive data driven control.

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