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IMPULSIVE SYNCHRONIZATION CONTROL OF POSITIVE COMPLEX DYNAMIC NETWORKS WITH NONLINEAR COUPLINGS

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Abstract. *This study investigates the impulsive synchronization control of positive complex dynamical networks with nonlinear couplings. A generalized network model is formulated by incorporating nonlinear interactions, preserving the positivity constraint inherent in such systems. An isolated node is designated as the reference trajectory for synchronization convergence. Building upon this framework, two distinct control strategies are developed: an impulsive synchronization controller requiring full-state measurement, and an observer-based synchronization controller for scenarios with partial state observations. Through systematic integration of these control architectures, both network positivity and global asymptotic stability are theoretically ensured. The main contributions of the paper are as follows: (i) A novel impulsive synchronization controller and an observer-based synchronization controller are constructed, (ii) A tractable approach is given to obtain the gains of controller and observer, and (iii) Linear programming and copositive Lyapunov function are employed to analyze the synchronization and stability of positive complex dynamical networks. Finally, several illustrative examples are given to verify the effectiveness of the proposed theoretical results.*
Keywords: Positive complex dynamical networks, Impulsive synchronization control, Nonlinearities, Linear programming

1. **Introduction.** Positive systems, characterized by non-negative state variables and outputs, have attracted sustained research attention over the past three decades [1-3]. The stabilization problem for such systems was systematically addressed through linear programming (LP) in [4], establishing foundational methodologies. Subsequent advancements include a distributed fault detection filter design for positive semi-Markovian jump systems using copositive Lyapunov functions (CLF) and LP techniques [5], as well as a unified stability and optimal control framework for switched positive systems based on CLF theory [6]. Notably, stochastic stabilization of positive semi-Markovian jump systems was achieved through synergistic applications of LP and CLF [7]. In [8,9], the switched linear CLF and the time-varying linear CLF were proposed to address the stability problem of switched systems. These collective efforts [4-9] demonstrate that CLF and LP constitute powerful tools for tackling fundamental challenges in positive system analysis. In parallel, complex dynamical networks (CDNs) have emerged as a dominant modeling

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paradigm for large-scale interconnected systems, with applications spanning social network analysis, biological system modeling, and epidemic transmission prediction [10-12]. The intrinsic positivity constraints in these applications necessitate specialized analysis of CDNs, particularly in domains where non-negativity is physically mandated [13]. While existing studies provided comprehensive solutions for centralized positive systems, their extension to network-based configurations introduces unique challenges. Although CLF and LP have proven effective in analyzing positive complex dynamical networks (PCDNs), their implementation in distributed network architectures reveals critical research gaps regarding scalability and computational complexity.

The synchronization control of CDNs constitutes a critical research domain, as these systems exhibit inherent limitations in achieving spontaneous synchronization [14-16]. In [17-20], significant progress has been made through various control paradigms: [17] constructed the design of nonlinear controllers for a wind turbine, while a novel intermittent control strategy was developed for tracking synchronization of multiple robot manipulators with a directed graph in [18]. Further advancements include two aperiodic intermittent control schemes introduced in [19] – one utilizing pinning control and the other operating without nodal constraints – which optimized control expenditure through reducing actuation frequency and minimal controller deployment. However, these methodologies were found inadequate in addressing dynamic network topology variations. To mitigate this limitation, an impulsive synchronization framework was pioneered in [20], representing a class of discontinuous control strategies that simultaneously reduced both operational costs and computational overhead. In [21-24], theoretical framework was further advanced through several key contributions: 1) the formulation of time-delay impulsive controllers [21], 2) the establishment of dual stability criteria for impulsive systems [22], and 3) the development of delay-dependent impulsive stabilization techniques [23]. Despite these advances, research on impulsive control remains notably scarce for positive systems. The recent contribution in [24] presented an impulsive positive system model with a CLF-based positivity criterion, though it focused exclusively on stability analysis rather than stabilization synthesis. This identified research gap underscored the pressing need to design impulsive controllers for PCDNs through systematic integration of CLF theory and LP techniques. Such integration is anticipated to bridge the current theoretical-practical divide while preserving system positivity constraints.

Contemporary engineering systems are characterized by networked configurations of nonlinearly interacting agents, as evidenced in large-scale infrastructure projects and multi-agent robotic systems [25]. In CDNs, node-edge interactions rarely conform to linear superposition principles, but rather exhibit nonlinear interdependencies manifested through synergistic cooperation, competitive resource allocation, and adaptive feedback mechanisms. These nonlinear dynamics constitute fundamental properties of real-world systems, which linear approximations fail to adequately represent [26-28]. Consequently, nonlinear CDN models provide superior descriptive capability for practical system complexity.

Despite these advancements, critical open challenges persist regarding impulsive synchronization of nonlinear PCDNs. The primary research questions are formulated as follows. First, how to design the impulsive synchronization controllers of PCDNs under complex nonlinear inputs? The system may have non-monotonic properties under complex nonlinear inputs. This will increase the complexity of the impulsive synchronization controller. Second, how to construct a framework using CLF and LP to solve the considered design? This is a new topic in the field of positive systems.

Motivated by these foundational findings, this paper focuses on the impulsive synchronization control of PCDNs with complex nonlinear inputs. The main contributions of this paper include that (i) A novel impulsive controller is designed to achieve synchronization of PCDNs, (ii) An observer-based impulsive controller is constructed to enhance the robustness of PCDNs, and (iii) A unified framework on the positivity and stability of PCDNs is established using LP and CLF. The rest of the paper is structured as follows. Section 2 introduces the foundational concepts and presents the preliminaries. Section 3 elaborates on the main results. Section 4 provides two illustrative examples to demonstrate the findings. Finally, Section 5 concludes with a summary of the paper.

The notations used in this paper are standardized as shown in Table 1.

Table 1. Notations

Notations	Expression
$\mathcal{R}^n$	n-dimensional Euclidean space
$\mathcal{R}^{n\times m}$	$n\times m$ real matrices
a_{ij}	The element in the ith row and jth column of matrix A
$A\succeq 0$ ($\succ 0$)	All elements in A are nonnegative (or positive)
$A^\top$	The transposition of matrix A
I_N	The $N\times N$ identity matrix
$\mathbf{1}_n$	An n-dimensional matrix with all elements being 1
$\mathbf{1}_\wp^{(r)}$	$\mathbf{1}_\wp^{(r)} = (\underbrace{0,\dots,0}_{r-1},1,\underbrace{0,\dots,0}_{\wp-r})^\top$
$\bigotimes$	The Kronecker product
$\Vert e_i(t)\Vert_1$	The 1-norm of $e_i(t)$

2. **Problem Statement and Preliminaries.** Consider a complex dynamical network with N coupled nonlinear nodes described as

$$\dot{x}_i(t) = Ax_i(t) + \sum_{z=1}^{M} D^{(z)} f_i^{(z)}(x_i(t)) + \sum_{j\in\mathcal{N}} \omega_{ij}\Gamma x_j(t) + Bu_i(t), \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^\top \in \mathcal{R}^n$ and $u_i(t) = (u_{i1}(t), u_{i2}(t), \dots, u_{im}(t))^\top \in \mathcal{R}^m$, $i\in\{1,2,\dots,N\}$ denote the state and control input, respectively; $f_i^{(z)}(x_i(t))\in\mathcal{R}^n$, $z\in\{1,2,\dots,M\}$ represents a nonlinear function; $A\in\mathcal{R}^{n\times n}$, $D^{(z)}\in\mathcal{R}^{n\times n}$, and $B\in\mathcal{R}^{n\times m}$ are the system matrices; $W=(\omega_{ij})\in\mathcal{R}^{N\times N}$ is the coupled configuration matrix of the network whose elements satisfy $\omega_{ij}>0$ ($i\neq j$) but not all zeros, and W is assumed to be symmetric with $\omega_{ii} = -\sum_{j=1,j\neq i}^{N}\omega_{ij}$; $\Gamma = \mathrm{diag}\{\gamma_1,\gamma_2,\dots,\gamma_n\}\in\mathcal{R}^{n\times n}$ is an inner-coupling matrix; the system matrix A is a Metzler matrix. Throughout this paper, B and $D^{(z)}$ have appropriate dimension and it is assumed that $B\succeq 0$ and $D^{(z)}\succeq 0$.

An isolated node is given as

$$\dot{s}_0(t) = As_0(t) + \sum_{z=1}^{M} D^{(z)} f_0^{(z)}(s_0(t)), \tag{2}$$

where $s_0(t) = (s_{01}(t), s_{02}(t), \dots, s_{0n}(t))^\top\in\mathcal{R}^n$ is regarded as the target state that will be achieved by the system (1) and $f_0^{(z)}(s_0(t)) = \left(f_{01}^{(z)}(s_{01}(t)), f_{02}^{(z)}(s_{02}(t)), \dots, f_{0n}^{(z)}(s_{0n}(t))\right)^\top \in \mathcal{R}^n$ is the nonlinear function.

Assumption 2.1. *The nonlinear function* $f_i^{(z)}(x_i(t))$ *in this paper satisfies*

$$a_1^{(z)} \leq \frac{f_{ij}^{(z)}(x_{ij}(t)) - f_{0j}^{(z)}(s_{0j}(t))}{x_{ij}(t) - s_{0j}(t)} \leq a_2^{(z)},$$

where $x_{ij}(t)$ *represents the* j *element of* $x_i(t)$ *for* $j \in \{1, 2, \ldots, n\}$; $a_1^{(z)}$ *and* $a_2^{(z)}$ *are known scalars with* $0 < a_1^{(z)} < a_2^{(z)}$.

Remark 2.1. *The function defined in Assumption 2.1 is called a sector nonlinear function, which is capable of restricting the nonlinear term to a specific range. It provides an effective characterization of this nonlinear phenomenon compared to Lipschitz function [17]. It is well known that few studies have been devoted to positivity of nonlinear systems. According to Assumption 2.1, the positivity of nonlinear systems with sector nonlinearity can be guaranteed.*

Definition 2.1. *A system is positive if all its states and outputs are nonnegative for any nonnegative initial conditions and inputs.*

Definition 2.2. *System (1) is said to be globally exponentially stable if for any* $o > 0$, *and scalars* $\xi > 0$, $\eta > 0$, $\|x(t_0)\|_1 < o$ *such that* $\|x(t)\|_1 \leq \xi e^{-\eta(t-t_0)}$, $t \geq t_0$.

Lemma 2.1. *A matrix* A *is Metzler if and only if there exists a constant* α *such that* $A + \alpha I \succeq 0$.

Lemma 2.2. *The continuous-time system*

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned}$$

is positive if and only if A *is Metzler,* $B \succeq 0$, *and* $C \succeq 0$.

3. **Main Results.** In this section, an impulsive synchronization controller and an observer-based synchronization controller are designed for PCDNs such that Equation (1) is positive and synchronizes with Equation (2).

3.1. **Impulsive synchronization control.** An impulsive controller is established as

$$u_i(t) = \begin{cases} K_1(x_i(t) - s_0(t)), & t \neq t_k, \\ \sum_{k=1}^{\infty} K_2(x_i(t) - s_0(t))\delta(t - t_k), & t = t_k, \end{cases} \tag{3}$$

where $K_1 \in \mathcal{R}^{m\times n}$ and $K_2 \in \mathcal{R}^{m\times n}$ are gain matrices; $\delta(\cdot)$ is the Dirac delta function; $\{t_k\}$ denotes impulse time sequences with control input, which satisfies $0 \leq t_0 < t_1 < \cdots < t_k \to +\infty$. Let $e_i(t) = x_i(t) - s_0(t)$. Based on the controller (3) and system (1) and (2), the error dynamics can be expressed as

$$\begin{aligned} &\dot{e}_i(t) = Ae_i(t) + \sum_{z=1}^{M} D^{(z)}\widehat{f}_i^{(z)}(e_i(t)) + \sum_{j\in\mathcal{N}} \omega_{ij}\Gamma(t)e_j(t) + BK_1e_i(t),\ t \neq t_k, \\ &\Delta e_i(t) = BK_2e_i(t),\ t = t_k, \end{aligned} \tag{4}$$

where $\widehat{f}_i^{(z)}(e_i(t)) = f_i^{(z)}(x_i(t)) - f_0^{(z)}(s_0(t))$. Using the Kronecker product, system (4) can be rewritten as

$$\begin{aligned} &\dot{e}(t) = (I_N \otimes A + W \otimes \Gamma + I_N \otimes BK_1)e(t) + I_N \otimes \sum_{z=1}^{M} D^{(z)}\widehat{f}(e(t)),\ t \neq t_k, \\ &\Delta e(t) = (I_N \otimes BK_2)e(t),\ t = t_k. \end{aligned} \tag{5}$$

Theorem 3.1. *If there exist constants* $\alpha > 0$, $\theta < 0$, $\varsigma > 1$ *and* $\mathcal{R}^n$ *vectors* $v \succ 0$, $\psi_r \prec 0$, $\phi_r \succ 0$ *such that*

$$\sum_{r=1}^{\wp} \mathbf{1}_{\wp}^{(r)\top} B^\top v A + \sum_{r=1}^{\wp} \mathbf{1}_{\wp}^{(r)\top} B^\top v \omega_{ii} \Gamma + \sum_{r=1}^{\wp} \mathbf{1}_{\wp}^{(r)\top} B^\top v \times \sum_{z=1}^{M} a_1^z D^{(z)} + B \sum_{r=1}^{\wp} \mathbf{1}_{\wp}^{(r)} \psi_r^\top$$
$$+ \alpha \sum_{r=1}^{\wp} \mathbf{1}_{\wp}^{(r)\top} B^\top v I_n \succeq 0, \tag{6a}$$
$$\mathbf{1}_N \otimes A^\top v + W^\top \mathbf{1}_N \otimes \Gamma^\top v + \mathbf{1}_N \otimes \sum_{r=1}^{\wp} \psi_r + \mathbf{1}_N \otimes \sum_{z=1}^{M} \left(a_2^z D^{(z)}\right)^\top v - \theta \mathbf{1}_N \otimes v \preceq 0, \tag{6b}$$
$$(1-\varsigma) v + \sum_{r=1}^{\wp} \phi_r \preceq 0, \tag{6c}$$

hold, then the system (1) is positive and stable under the impulsive synchronization controller (3) with

$$K_1 = \sum_{r=1}^{\wp} \frac{\mathbf{1}_{\wp}^{(r)} \psi_r^\top}{\mathbf{1}_{\wp}^{(r)\top} B^\top v}, \; K_2 = \sum_{r=1}^{\wp} \frac{\mathbf{1}_{\wp}^{(r)} \phi_r^\top}{\mathbf{1}_{\wp}^{(r)\top} B^\top v}. \tag{7}$$

Proof: The proof is divided into two steps: Positivity and Stability.

Step 1 (Positivity): First, consider the positivity of the system (5). Give the initial state $e(t_0) \succeq 0$ and we obtain $e_i(t_0) \succeq 0$ for each i. According to Assumption 2.1, we have $a_1^{(z)} e(t) \leq \widehat{f}(e(t))$ and

$$\dot{e}_i(t_0^+) \succeq \mathcal{A} e(t_0),$$

where $\mathcal{A} = \begin{pmatrix} \Theta_1 & \omega_{12}\Gamma & \cdots & \omega_{1N}\Gamma \\ \omega_{21}\Gamma & \Theta_2 & \cdots & \omega_{2N}\Gamma \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{N1}\Gamma & \omega_{N2}\Gamma & \cdots & \Theta_N \end{pmatrix}$ and $\Theta_i = A + BK_1 + \sum_{z=1}^{M} a_1^{(z)} D^{(z)} + \omega_{ii}\Gamma$.

From Equations (6a) and (7), it derives that $A + BK_1 + \sum_{z=1}^{M} a_1^{(z)} D^{(z)} + \omega_{ii}\Gamma + \alpha I_n \succeq 0$. By Lemma 2.1, $\mathcal{A}$ is Metzler. Since $BK_2 \succeq 0$, it follows that $BK_2 + I \succ 0$. Thus, we have $\dot{e}(t_0^+) \succeq 0$ for $e(t_0) \succeq 0$. Then, it follows $e(t) \succeq 0$ for any initial state $e(t_0) \succeq 0$ by recursive derivation. Therefore, the system (5) is positive by Lemma 2.2.

Step 2 (Stability): Let $\mu = (\overbrace{v, \ldots, v}^{N})$ and construct the CLF $V(e(t)) = e^\top(t)\mu = e^\top(t)(\mathbf{1}_N \otimes v)$. Using Assumption 2.1 yields that $\widehat{f}(e(t)) \leq a_2^{(z)} e(t)$. Substituting Equation (5) into the CLF gives that

$$\begin{aligned} \dot{V}(e(t)) &= \dot{e}^\top(t)\mu \\ &= e^\top(t)(I_N \otimes A + W \otimes \Gamma + I_N \otimes BK_1)^\top \mu + \widehat{f}(e(t))^\top I_N \otimes \sum_{z=1}^{M} {D^{(z)}}^\top \mu \\ &\leq e^\top(t) \left(I_N \otimes A + W \otimes \Gamma + I_N \otimes BK_1 + I_N \otimes \sum_{z=1}^{M} a_2^{(z)} D^{(z)} \right)^\top \mu. \end{aligned} \tag{8}$$

Combining Equations (7) and (8) gives that

$$\begin{aligned} &\dot{V}(e(t)) - \theta V(e(t)) \\ &\leq e^\top(t) \left(I_N \otimes A + W \otimes \Gamma + I_N \otimes BK_1 + I_N \otimes \sum_{z=1}^{M} a_2^{(z)} D^{(z)} - \theta I_N \otimes I_n \right)^\top \mu \end{aligned}$$

$$\leq e^{\top}(t)\left(\mathbf{1}_N \otimes A^{\top}v + W^{\top}\mathbf{1}_N \otimes \Gamma^{\top}v + \mathbf{1}_N \otimes \sum_{r=1}^{\wp}\psi_r + \mathbf{1}_N \otimes \sum_{z=1}^{M}\left(a_2^{(z)}D^{(z)}\right)^{\top}v \right.$$
$$\left. - \theta\mathbf{1}_N \otimes v\right). \tag{9}$$

By Equations (6b) and (9), it gives that $\dot{V}(e(t)) \leq \theta V(e(t))$. It can be rewritten as $V(e(t)) \leq V(e(t_{k-1}))e^{\theta(t-t_{k-1})}$. When $t = t_k$, $k \in N$, it follows that

$$\begin{aligned} V(e(t_k)) &= e^{\top}(t_k)\mu = \left(e^{\top}\left(t_k^-\right) + \Delta e^{\top}(t)\right)\mu \\ &= \left(e^{\top}\left(t_k^-\right) + \left((I_N \otimes BK_2)e\left(t_k^-\right)\right)^{\top}\right)\mu \\ &= \left(e^{\top}\left(t_k^-\right) + e^{\top}\left(t_k^-\right)\left(I_N \otimes K_2^{\top}B^{\top}\right)\right)\mu \\ &= e^{\top}\left(t_k^-\right)\left(I_N \otimes I + I_N \otimes K_2^{\top}B^{\top}\right)\mu. \end{aligned} \tag{10}$$

Based on Equations (7) and (10), we have

$$\begin{aligned} V(e(t_k)) - \varsigma V\big(e\left(t_k^-\right)\big) &= e^{\top}\left(t_k^-\right)\left(I_N \otimes I + I_N \otimes K_2^{\top}B^{\top}\right)\mu - \varsigma e^{\top}\left(t_k^-\right)\mu \\ &= e^{\top}\left(t_k^-\right)\left(I_N \otimes \mu + I_N \otimes \sum_{r=1}^{\wp}\frac{\phi_r\mathbf{1}_{\wp}^{(r)\top}}{\mathbf{1}_{\wp}^{(r)\top}B^{\top}v}B^{\top}\mu - \varsigma I_N \otimes \mu\right) \\ &= e^{\top}\left(t_k^-\right)\left((1-\varsigma)\mathbf{1}_N \otimes v + \mathbf{1}_N \otimes \sum_{r=1}^{\wp}\phi_r\right). \end{aligned} \tag{11}$$

Using Equations (6c) and (11) gives $V(e(t_k)) \leq \varsigma V\big(e\left(t_k^-\right)\big)$. Together with $V(e(t)) \leq V(e(t_{k-1}))e^{\theta(t-t_{k-1})}$, it derives that $V(e(t)) \leq V(e(t_0))e^{\theta(t-t_0)}$ for $t \in [t_0, t_1)$. Then, $V(e(t_1)) \leq V(e(t_0))e^{\theta(t_1-t_0)}$. When $t = t_1$, we have $V(e(t_1)) \leq \varsigma V(e(t_1^-)) \leq \varsigma e^{\theta(t_1-t_0)}V(e(t_0))$. It holds that $V(e(t)) \leq V(e(t_1))e^{\theta(t-t_1)} \leq \varsigma e^{\theta(t-t_0)}V(e(t_0))$ for $t \in [t_1, t_2)$. Thus, it is clear that $V(e(t)) \leq \varsigma^{k-1}e^{\theta(t-t_0)}V(e(t_0))$ for $t \in [t_{k-1}, t_k)$ $(k \in N)$. Moreover, the following inequality holds:

$$e^{\top}(t)v \leq \varsigma^{k-1}e^{\theta(t-t_0)}e^{\top}(t_0)v. \tag{12}$$

By Definition 2.2, it gives $\|e_i(t)\|_1 \leq c\varsigma^{k-1}e^{\theta(t-t_0)}$, where $a = \min\{v\}$, $b = \max\{v\}$, and $c = \|e_i(t_0)\|_1\frac{b}{a} > 0$. Define $\alpha < 0$ and a constant γ $(0 \leq \gamma \leq -\alpha)$. It follows that $\ln\varsigma - \gamma(t_k - t_{k-1}) \leq 0$, $k \in N$. Then,

$$\begin{aligned} \|e_i(t)\|_1 &\leq c\varsigma^{k-1}e^{-\gamma(t-t_0)}e^{(\theta+\gamma)(t-t_0)} \\ &\leq c\varsigma^{k-1}e^{-\gamma(t_{k-1}-t_0)}e^{(\theta+\gamma)(t-t_0)} \\ &= c\varsigma e^{-\gamma(t_1-t_0)}\varsigma e^{-\gamma(t_2-t_1)}\cdots\varsigma e^{-\gamma(t_{k-1}-t_{k-2})}e^{(\theta+\gamma)(t-t_0)} \\ &\leq ce^{(\theta+\gamma)(t-t_0)},\ t \in [t_{k-1}, t_k). \end{aligned} \tag{13}$$

Define a constant $\tau = t_k - t_{k-1}$ for $k \in N$. We can obtain the lower bound of impulsive interval $\tau_{lower} \geq \frac{\ln\varsigma}{\gamma}$. By Definition 2.1, the system (5) is globally exponentially stable. □

Remark 3.1. *The definition of positivity is essential for the investigation of positive systems. Up to now, a unified framework for the positivity across various systems has not been established completely. Although numerous results have been reported on CDNs, limited research has been dedicated to PCDNs. Existing findings on positive systems are not directly applicable to PCDNs. In Theorem 3.1, a design approach to the positivity of PCDNs is presented by introducing an impulsive controller (3). Its objective is to adjust and optimize the behavior and performance of the network by controlling the nodes or edges. Compared* *with continuous control, discrete control can greatly reduce the interference to the network system. It helps to maintain the long-term stability of the system.*

Remark 3.2. *The study of isolated nodes can help to reveal topological properties of the network such as node degree distribution, clustering coefficients, and community structure. Isolated nodes are common in many real networks. In social networks, certain users may not be connected to other users. In protein interaction networks, certain proteins may not have been found to interact with other proteins. Thus, isolated nodes can model these systems realistically. In previous works, such as [15,16], isolated nodes were introduced into the synchronization control of CNs. These isolated nodes can influence both the coupling strength and the synchronization speed of the network. Specifically, the state information of these isolated nodes is fed back to the entire network. This feedback mechanism adjusts the dynamics of the network, thereby affecting the overall synchronization performance. As a result, the synchronization behavior of the entire system can be controlled and improved.*

3.2. **Observer-based synchronization control.** Consider CDNs with N coupled nodes described as

$$\begin{aligned}&\dot{x}_i(t) = Ax_i(t) + \sum_{z=1}^{M} D^{(z)} f_i^{(z)}(x_i(t)) + \sum_{j\in\mathcal{N}} \omega_{ij}\Gamma x_j(t) + Bu_i(t),\\ &y_i(t) = Cx_i(t),\end{aligned} \tag{14}$$

where $y_i(t) \in \mathcal{R}^s$ denotes the output; $C \in \mathcal{R}^{s\times n}$ is the system matrix, and it is assumed that $C \succeq 0$.

First, a Luenberger observer is constructed:

$$\begin{aligned}&\dot{\widehat{x}}_i(t) = A\widehat{x}_i(t) + \sum_{z=1}^{M} D^{(z)} f_i^{(z)}(\widehat{x}_i(t)) + \sum_{j\in\mathcal{N}} \omega_{ij}\Gamma \widehat{x}_j(t) + L(\widehat{y}_i(t) - y_i(t)) + Bu_i(t),\\ &\widehat{y}_i(t) = C\widehat{x}_i(t),\end{aligned} \tag{15}$$

where $\widehat{x}_i(t) \in \mathcal{R}^n$ is the estimation of the state, $\widehat{y}_i(t) \in \mathcal{R}^s$ is the observer output, and $L \in \mathcal{R}^{n\times s}$ denotes the observer gain to be designed. For the ith node, the observation-based synchronization controller is designed as

$$u_i(t) = \begin{cases} K_1(\widehat{x}_i(t) - s_0(t)), & t \neq t_k,\\ \sum_{k=1}^{\infty} K_2(\widehat{x}_i(t) - s_0(t))\delta(t - t_k), & t = t_k. \end{cases} \tag{16}$$

Let $\widehat{e}_i(t) = \widehat{x}_i(t) - x_i(t)$ and $\widehat{f}_i^{(z)}(\widehat{e}_i(t)) = f_i^{(z)}(\widehat{x}_i(t)) - f_i^{(z)}(x_i(t))$. Then,

$$\begin{aligned}\dot{e}_i(t) &= \dot{x}_i(t) - \dot{s}_0(t) = Ae_i(t) + \sum_{z=1}^{M} D^{(z)} \widehat{f}_i^{(z)}(e_i(t)) + \sum_{j\in\mathcal{N}} \omega_{ij}\Gamma e_j(t) + Bu_i(t),\\ \dot{\widehat{e}}_i(t) &= \dot{\widehat{x}}_i(t) - \dot{x}_i(t)\\ &= A\widehat{e}_i(t) + \sum_{z=1}^{M} D^{(z)} \widehat{f}_i^{(z)}(\widehat{e}_i(t)) + \sum_{j\in\mathcal{N}} \omega_{ij}\Gamma \widehat{e}_j(t) + LC\widehat{e}_i(t)\\ &= (A + LC)\widehat{e}_i(t) + \sum_{j\in\mathcal{N}} \omega_{ij}\Gamma \widehat{e}_j(t) + \sum_{z=1}^{M} D^{(z)} \widehat{f}_i^{(z)}(\widehat{e}_i(t)).\end{aligned} \tag{17}$$

Substituting Equation (16) into (17), we have

$$\begin{aligned}
&\dot{e}_i(t) = Ae_i(t) + \sum_{z=1}^{M} D^{(z)} \widehat{f}_i^{(z)}(e_i(t)) + \sum_{j\in\mathcal{N}} \omega_{ij}\Gamma(t)e_j(t) + BK_1e_i(t) + BK_1\widehat{e}_i(t),\ t \neq t_k,\\
&\Delta e_i(t) = BK_2e_i(t) + BK_2\widehat{e}_i(t),\ t = t_k,\\
&\dot{\widehat{e}}_i(t) = (A + LC)\widehat{e}_i(t) + \sum_{j\in\mathcal{N}} \omega_{ij}\Gamma\widehat{e}_j(t) + \sum_{z=1}^{M} D^{(z)} \widehat{f}_i^{(z)}(\widehat{e}_i(t)).
\end{aligned} \tag{18}$$

Using the Kronecker product, the error system (18) can be expressed:

$$\begin{aligned}
&\dot{e}(t) = (I_N \otimes (A + BK_1) + W \otimes \Gamma)e(t) + I_N \otimes \sum_{z=1}^{M} D^{(z)} \widehat{f}(e(t)) + (I_N \otimes BK_1)\widehat{e}(t),\ t \neq t_k,\\
&\Delta e(t) = (I_N \otimes BK_2)e(t) + (I_N \otimes BK_2)\widehat{e}(t),\ t = t_k,\\
&\dot{\widehat{e}}(t) = (I_N \otimes (A + LC) + W \otimes \Gamma)\widehat{e}(t) + I_N \otimes \sum_{z=1}^{M} D^{(z)} \widehat{f}(\widehat{e}(t)).
\end{aligned} \tag{19}$$

Let $X(t) = \left(e(t)^\top \widehat{e}(t)^\top\right)^\top$, and we obtain

$$\begin{aligned}
&\dot{X}(t) = \mathcal{A}X(t) + \mathcal{B}F(X(t)),\ t \neq t_k,\\
&\Delta X(t) = \mathcal{C}X(t),\ t = t_k,
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
&\mathcal{A} = \begin{pmatrix} I_N \otimes (A + BK_1) + W \otimes \Gamma & I_N \otimes BK_1 \\ 0 & I_N \otimes (A + LC) + W \otimes \Gamma \end{pmatrix},\\
&\mathcal{B} = \begin{pmatrix} I_N \otimes \sum_{z=1}^{M} D^{(z)} & 0 \\ 0 & I_N \otimes \sum_{z=1}^{M} D^{(z)} \end{pmatrix},\ \mathcal{C} = \begin{pmatrix} I_N \otimes BK_2 & I_N \otimes BK_2 \\ 0 & 0 \end{pmatrix},\\
&F(X(t)) = \left(\widehat{f}(e(t))^\top, \widehat{f}(\widehat{e}(t))^\top\right)^\top.
\end{aligned}$$

Theorem 3.2. *If there exist constants $\alpha > 0$, $\theta < 0$, $\varsigma > 1$, $\mathcal{R}^n$ vectors $v \succ 0$, $s \succ 0$, $\psi_r \succ 0$, $\phi_r \succ 0$, and $\mathcal{R}^s$ $\varrho_r \prec 0$ such that*

$$\begin{aligned}
&\sum_{r=1}^{\wp} \mathbf{1}_{\wp}^{(r)\top} B^\top vA + \sum_{r=1}^{\wp} \mathbf{1}_{\wp}^{(r)\top} B^\top v\omega_{ii}\Gamma + \sum_{r=1}^{\wp} \mathbf{1}_{\wp}^{(r)\top} B^\top v \sum_{z=1}^{M} a_1^{(z)} D^{(z)} + B \sum_{r=1}^{\wp} \mathbf{1}_{\wp}^{(r)} \psi_r^\top\\
&+ \alpha \sum_{r=1}^{\wp} \mathbf{1}_{\wp}^{(r)\top} B^\top vI_n \succeq 0,
\end{aligned} \tag{21a}$$

$$\begin{aligned}
&\sum_{r=1}^{\kappa} \mathbf{1}_{\kappa}^{(r)\top} sA + \sum_{r=1}^{\kappa} \mathbf{1}_{\kappa}^{(r)\top} s\omega_{ii}\Gamma + \sum_{r=1}^{\kappa} \mathbf{1}_{\kappa}^{(r)\top} s \sum_{z=1}^{M} a_1^{(z)} D^{(z)} + \sum_{r=1}^{\kappa} \mathbf{1}_{\kappa}^{(r)} \varrho_r^\top C\\
&+ \alpha \sum_{r=1}^{\kappa} \mathbf{1}_{\kappa}^{(r)\top} sI_n \succeq 0,
\end{aligned} \tag{21b}$$

$$\begin{aligned}
&\mathbf{1}_N \otimes A^\top v + \mathbf{1}_N \otimes \sum_{r=1}^{\wp} \psi_r + W^\top \mathbf{1}_N \otimes \Gamma^\top v + \mathbf{1}_N \otimes \sum_{z=1}^{M} \left(a_2^{(z)} D^{(z)}\right)^\top v\\
&- \theta \mathbf{1}_N \otimes v \preceq 0,
\end{aligned} \tag{21c}$$

$$\mathbf{1}_N \otimes \sum_{r=1}^{\wp} \psi_r + \mathbf{1}_N \otimes A^\top s + \mathbf{1}_N \otimes C^\top \sum_{r=1}^{\kappa} \varrho_r + W^\top \mathbf{1}_N \otimes \Gamma^\top s + \mathbf{1}_N \otimes \sum_{z=1}^{M} \left(a_2^{(z)} D^{(z)} \right)^\top s$$
$$- \theta \mathbf{1}_N \otimes s \preceq 0, \tag{21d}$$

$$(1-\varsigma)v + \sum_{r=1}^{\wp} \phi_r \preceq 0, \tag{21e}$$

$$(1-\varsigma)s + \sum_{r=1}^{\wp} \phi_r \preceq 0, \tag{21f}$$

hold, then the system (14) is positive and synchronous with the isolated node under the controller (16) with

$$K_1 = \sum_{r=1}^{\wp} \frac{\mathbf{1}_\wp^{(r)} \psi_r^\top}{\mathbf{1}_\wp^{(r)\top} B^\top v}, \ K_2 = \sum_{r=1}^{\wp} \frac{\mathbf{1}_\wp^{(r)} \phi_r^\top}{\mathbf{1}_\wp^{(r)\top} B^\top v}, \ L = \sum_{r=1}^{\kappa} \frac{\mathbf{1}_\kappa^{(r)} \varrho_r^\top}{\mathbf{1}_\kappa^{(r)\top} s}. \tag{22}$$

Proof: The proof is divided into two steps: Positivity and Stability.

Step 1 (Positivity): First, consider the positivity of the system (20). Give the initial state $X(t_0) \succeq 0$ and we obtain $X_i(t_0) \succeq 0$ for each ith node. Under Assumption 2.1, it is easy to have

$$\dot{X}_i\left(t_0^+\right) \succeq \mathcal{A} X_i(t_0) + \mathcal{B} F(X_i(t_0)) \succeq \mathcal{Q} X(t_0), \ t \neq t_k, \tag{23}$$

where

$$\mathcal{Q} = \begin{pmatrix} \begin{pmatrix} I_N \otimes (A + BK_1) + W \otimes \Gamma \\ + I_N \otimes \sum_{z=1}^{M} a_1^{(z)} D^{(z)} \end{pmatrix} & I_N \otimes BK_1 \\ 0 & \begin{pmatrix} I_N \otimes (A + LC) + W \otimes \Gamma \\ + I_N \otimes \sum_{z=1}^{M} a_1^{(z)} D^{(z)} \end{pmatrix} \end{pmatrix},$$

$$I_N \otimes (A + BK_1) + W \otimes \Gamma + I_N \otimes \sum_{z=1}^{M} a_1^{(z)} D^{(z)} = \begin{pmatrix} \Lambda_1 & \omega_{12}\Gamma & \cdots & \omega_{1N}\Gamma \\ \omega_{21}\Gamma & \Lambda_2 & \cdots & \omega_{2N}\Gamma \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{N1}\Gamma & \omega_{N2}\Gamma & \cdots & \Lambda_N \end{pmatrix},$$

$$I_N \otimes (A + LC) + W \otimes \Gamma + I_N \otimes \sum_{z=1}^{M} a_1^{(z)} D^{(z)} = \begin{pmatrix} \Xi_1 & \omega_{12}\Gamma & \cdots & \omega_{1N}\Gamma \\ \omega_{21}\Gamma & \Xi_2 & \cdots & \omega_{2N}\Gamma \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{N1}\Gamma & \omega_{N2}\Gamma & \cdots & \Xi_N \end{pmatrix},$$

$\Lambda_i = A + BK_1 + \sum_{z=1}^{M} a_1^{(z)} D^{(z)} + \omega_{ii}\Gamma$, and $\Xi_i = A + LC + \sum_{z=1}^{M} a_1^{(z)} D^{(z)} + \omega_{ii}\Gamma$. By (22), we can have

$$\begin{aligned} \Lambda_i + \alpha I_n = & \frac{1}{\sum_{r=1}^{\wp} \mathbf{1}_\wp^{(r)\top} B^\top v} \left(\sum_{r=1}^{\wp} \mathbf{1}_\wp^{(r)\top} B^\top v A + \sum_{r=1}^{\wp} \mathbf{1}_\wp^{(r)\top} B^\top v \omega_{ii} \Gamma + \sum_{r=1}^{\wp} \mathbf{1}_\wp^{(r)\top} \right. \\ & \left. \times B^\top v \sum_{z=1}^{M} a_1^{(z)} D^{(z)} + B \sum_{r=1}^{\wp} \mathbf{1}_\wp^{(r)} \psi_r^\top + \alpha \sum_{r=1}^{\wp} \mathbf{1}_\wp^{(r)\top} B^\top v I_n \right) \\ = & A + B \sum_{r=1}^{\wp} \frac{\mathbf{1}_\wp^{(r)} \psi_r \top}{\mathbf{1}_\wp^{(r)\top} B^\top v} + \sum_{z=1}^{M} a_1^{(z)} D^{(z)} + \omega_{ii}\Gamma + \alpha I_n, \end{aligned}$$

$$\Xi_i + \alpha I_n = \frac{1}{\sum_{r=1}^{\kappa} \mathbf{1}_\kappa^{(r)\top} s} \left(\sum_{r=1}^{\kappa} \mathbf{1}_\kappa^{(r)\top} s A + \sum_{r=1}^{\kappa} \mathbf{1}_\kappa^{(r)\top} s \omega_{ii} \Gamma + \sum_{r=1}^{\kappa} \mathbf{1}_\kappa^{(r)\top} \times s \sum_{z=1}^{M} a_1^{(z)} D^{(z)} \right.$$

$$+\sum_{r=1}^{\kappa}\mathbf{1}_{\kappa}^{(r)}\varrho_r^{\top}C+\alpha\sum_{r=1}^{\kappa}\mathbf{1}_{\kappa}^{(r)\top}sI_n\Bigg)$$
$$=A+\sum_{r=1}^{\kappa}\frac{\mathbf{1}_{\kappa}^{(r)}\varrho_r^{\top}}{\mathbf{1}_{\kappa}^{(r)\top}s}C+\sum_{z=1}^{M}a_1^{(z)}D^{(z)}+\omega_{ii}\Gamma+\alpha I_n. \tag{24}$$

From Equations (21a) and (21b) gives that $\Lambda_i+\alpha I_n\succeq 0$ and $\Xi_i+\alpha I_n\succeq 0$. Then, Λ_i and Ξ_i are Metzler. Since $BK_1\succeq 0$ and $I_N\otimes\sum_{z=1}^{M}D^{(z)}\succeq 0$, we can obtain $\mathcal{Q}$ is Metzler and $\mathcal{B}\succeq 0$. Thus, it derives that $\dot{X}(t_0^+)\succeq 0$ for $X(t_0)\succeq 0$. Then, it follows $X(t)\succeq 0$ for any initial state $X(t_0)\succeq 0$ by recursive derivation. Since $I+BK_2\succeq 0$, the system (20) is positive by Lemma 2.2.

Step 2 (Stability): Construct the CLF $V(X(t))=X^{\top}(t)\left(\mu^{\top},\lambda^{\top}\right)^{\top}$, where $\mu=(\overbrace{v,\ldots,v}^{N})$ and $\lambda=(\overbrace{s,\ldots,s}^{N})$. Under Assumption 2.1, we obtain

$$\begin{aligned}
&\dot{V}(X(t))\\
&=\dot{X}^{\top}(t)\left(\mu^{\top},\lambda^{\top}\right)^{\top}\\
&=X^{\top}(t)\mathcal{A}^{\top}\left(\mu^{\top},\lambda^{\top}\right)^{\top}+\widehat{f}(e(t))^{\top}\mathcal{B}^{\top}\left(\mu^{\top},\lambda^{\top}\right)^{\top}\\
&\le X^{\top}(t)\begin{pmatrix}\begin{pmatrix}I_N\otimes(A+BK_1)+W\otimes\Gamma\\+I_N\otimes\sum_{z=1}^{M}a_2^{(z)}D^{(z)}\end{pmatrix} & I_N\otimes BK_1\\ 0 & \begin{pmatrix}I_N\otimes(A+LC)+W\otimes\Gamma\\+I_N\otimes\sum_{z=1}^{M}a_2^{(z)}D^{(z)}\end{pmatrix}\end{pmatrix}^{\top}\begin{pmatrix}\mu\\ \lambda\end{pmatrix}.
\end{aligned}\tag{25}$$

Combining Equations (22) and (25) yields that

$$\begin{aligned}
&\dot{V}(X(t))-\theta V(X(t))\\
&\le X^{\top}(t)\begin{pmatrix}\begin{pmatrix}\mathbf{1}_N\otimes A^{\top}v+\mathbf{1}_N\otimes\sum_{r=1}^{\wp}\psi_r\\+W^{\top}\mathbf{1}_N\otimes\Gamma^{\top}v+\mathbf{1}_N\otimes\sum_{z=1}^{M}\left(a_2^{(z)}D^{(z)}\right)^{\top}v-\theta\mathbf{1}_N\otimes v\end{pmatrix}\\ \begin{pmatrix}\mathbf{1}_N\otimes\sum_{r=1}^{\wp}\psi_r+\mathbf{1}_N\otimes A^{\top}s+\mathbf{1}_N\otimes C^{\top}\sum_{r=1}^{\kappa}\varrho_r\\+W^{\top}\mathbf{1}_N\otimes\Gamma^{\top}s+\mathbf{1}_N\otimes\sum_{z=1}^{M}\left(a_2^{(z)}D^{(z)}\right)^{\top}s-\theta\mathbf{1}_N\otimes s\end{pmatrix}\end{pmatrix}.
\end{aligned}\tag{26}$$

Using Equations (21c) and (21d) gives $\dot{V}(X(t))-\theta V(X(t))\le 0$. Then, we can obtain $V(X(t))\le V(X(t_{k-1}))e^{\theta(t-t_{k-1})}$. When $t=t_k$, $k\in N$, it follows that

$$V(X(t_k))=X^{\top}(t_k)\begin{pmatrix}\mu\\ \lambda\end{pmatrix}=X^{\top}\left(t_k^{-}\right)\begin{pmatrix}I_N\otimes\mu+I_N\otimes K_2^{\top}B^{\top}\mu\\ I_N\otimes\lambda+I_N\otimes K_2^{\top}B^{\top}\mu\end{pmatrix}. \tag{27}$$

Furthermore, based on Equation (22), we have

$$\begin{aligned}
V(X(t_k))-\varsigma V\left(X\left(t_k^{-}\right)\right)&=X^{\top}\left(t_k^{-}\right)\begin{pmatrix}I_N\otimes\mu+I_N\otimes K_2^{\top}B^{\top}\mu\\ I_N\otimes\lambda+I_N\otimes K_2^{\top}B^{\top}\mu\end{pmatrix}-\varsigma X^{\top}\left(t_k^{-}\right)\begin{pmatrix}\mu\\ \lambda\end{pmatrix}\\
&=X^{\top}\left(t_k^{-}\right)\begin{pmatrix}(1-\varsigma)\mathbf{1}_N\otimes v+\mathbf{1}_N\otimes\sum_{r=1}^{\wp}\phi_r\\ (1-\varsigma)\mathbf{1}_N\otimes s+\mathbf{1}_N\otimes\sum_{r=1}^{\wp}\phi_r\end{pmatrix}.
\end{aligned}\tag{28}$$

Combining Equations (21e) and (21f) gives $V(X(t_k))-\varsigma V\big(X\left(t_k^{-}\right)\big)\le 0$. It follows that $V(X(t))\le V(X(t_0))e^{\theta(t-t_0)}$ for $t\in[t_0,t_1)$. Then, we have $V(X(t_1))\le V(X(t_0))e^{\theta(t_1-t_0)}$. When $t=t_1$, we have $V(X(t_1))\le\varsigma V(X(t_1^{-}))\le\varsigma e^{\theta(t_1-t_0)}V(X(t_0))$. It holds that $V(X(t))$

$\leq V(X(t_1))e^{\theta(t-t_1)} \leq \varsigma e^{\theta(t-t_0)}V(e(t_0))$ for $t \in [t_1, t_2)$. Thus, it is clear that $V(X(t)) \leq \varsigma^{k-1}e^{\theta(t-t_0)}V(X(t_0))$ for $t \in [t_{k-1}, t_k)$ $(k \in N)$. Then,

$$X^\top(t)\begin{pmatrix}\mu\\\lambda\end{pmatrix} \leq \varsigma^{k-1}e^{\theta(t-t_0)}X^\top(t_0)\begin{pmatrix}\mu\\\lambda\end{pmatrix}. \tag{29}$$

By Definition 2.2 and Equation (29), it gives $\|X_i(t)\|_1 \leq r\varsigma^{k-1}e^{\theta(t-t_0)}$, where $d = \min\{v\}$, $h = \max\{v\}$, $k = \min\{s\}$, $j = \max\{s\}$, and $r = \|X_i(t_0)\|_1 \begin{pmatrix}\frac{h}{d}\\\frac{j}{k}\end{pmatrix}$. Define $\alpha < 0$ and a constant γ $(0 \leq \gamma \leq -\alpha)$. It follows that $\ln\varsigma - \gamma(t_k - t_{k-1}) \leq 0$, $k \in N$. Then,

$$\begin{aligned}\|X_i(t)\|_1 &\leq r\varsigma^{k-1}e^{-\gamma(t-t_0)}e^{(\theta+\gamma)(t-t_0)}\\&\leq r\varsigma^{k-1}e^{-\gamma(t_{k-1}-t_0)}e^{(\theta+\gamma)(t-t_0)}\\&= r\varsigma e^{-\gamma(t_1-t_0)}\varsigma e^{-\gamma(t_2-t_1)}\cdots\varsigma e^{-\gamma(t_{k-1}-t_{k-2})}e^{(\theta+\gamma)(t-t_0)}\\&\leq re^{(\theta+\gamma)(t-t_0)}.\end{aligned} \tag{30}$$

Define a constant $t_k - t_{k-1} = \tau$ for $k \in N$. Then, it can be obtained the lower bound of impulsive interval $\tau_{lower} \geq \frac{\ln\varsigma}{\gamma}$. By Definition 2.1, the system (20) is globally stable. □

Remark 3.3. *In many practical applications, it is difficult to obtain the information of states directly. This problem may stem from a variety of reasons such as the high cost of sensors, the complexity of the measurement environment, and the presence of noise interference in the measurement process. Therefore, it is essential to introduce an observer to realize estimation of the system state. The observer estimates the state of the system and provides the necessary state feedback. In addition, the design method of state feedback observer has wide applicability. It can generalize to nonlinear systems.*

4. Illustrative Examples.

In rolling systems, the continuity of materials, high-speed operation, and the complex coupling interactions between rolling mills make it important to analyze the dynamic behavior of the entire systems. Traditional centralized control strategies often fail to capture the dynamic characteristics of subsystems and their coupling relationships. The introduction of CDNs provides a novel perspective for addressing this challenge. It enables to analyze the rolling process by simulating the rolling systems with CDNs.

The rolling systems consist of three subsystems: roughing mills, intermediate mills, and finishing mills. These subsystems can be represented by three nodes. Each node exhibits its own dynamic characteristics such as rotational speed, rolling force, and tension. CDNs provide a descriptive framework for the structure. There exist physical connections or control relationships between the subsystems. This representation of node-edge allows CDNs to describe the dynamic interactions.

The state variables of the rolling systems are non-negative due to their physical properties. Therefore, it is reasonable to model the rolling system using PCDNs. Furthermore, the performance of the system may be affected by external factors such as power fluctuations and mechanical vibrations. An impulsive controller can act on the discrete structure of the network and update control signals at specific triggering moments. When the system deviates from its equilibrium state, the impulsive controller effectively restores stability through discrete adjustments. $x(t)$ denotes the actual state vector of each node, $s_0(t)$ represents the state of isolated node, and $\hat{x}(t)$ denotes the observer state. Additionally, let $e(t)$ and $\hat{e}(t)$ represent the synchronization error and the observation error, respectively. The following two examples validate the effectiveness of the proposed design.

Example 4.1. *Consider the CDNs (1) consisting of three nodes with the following parameters:*

$$A = \begin{pmatrix} -1.1 & 0.1 & 0.1 \\ 0.1 & -1 & 0.1 \\ 0.1 & 0.2 & -1.2 \end{pmatrix},\ D_1 = \begin{pmatrix} 0.5 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.3 \end{pmatrix},\ D_2 = \begin{pmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.3 \end{pmatrix},$$
$$D_3 = \begin{pmatrix} 0.7 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.3 \end{pmatrix},\ B = \begin{pmatrix} 0.1 & 0.1 \\ 0.3 & 0.3 \\ 0.2 & 0.2 \end{pmatrix}.$$

The nonlinear functions are defined as $f^{(z)}(x(t)) = \sin^2 x(t)$, $z = 1, 2, 3$, *and the parameters of the coupled network configuration matrices are set as*

$$W = \begin{pmatrix} -0.1 & 0.05 & 0.05 \\ 0.05 & -0.1 & 0.05 \\ 0.05 & 0.05 & -0.1 \end{pmatrix},\ \Gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Choose the constants $\alpha = 2$, $\theta = -0.1$, $\varsigma = 5$, $a_1^{(z)} = 0.3$, *and* $a_2^{(z)} = 0.4$. *By Theorem 3.1, we obtain the gain matrices:*

$$K_1 = \begin{pmatrix} -0.6169 & -0.8780 & -0.4530 \\ -0.6169 & -0.8780 & -0.4530 \end{pmatrix},\ K_2 = \begin{pmatrix} 1.9044 & 2.3328 & 2.2152 \\ 1.9044 & 2.3328 & 2.2152 \end{pmatrix}.$$

The synchronization error trajectories are shown in Figures 1-3 with $x_1(0) = (40, 35, 30)^\top$, $x_2(0) = (25, 20, 15)^\top$, $x_3(0) = (10, 7, 4)^\top$ *and* $s_0(0) = (7.5, 3.5, 0.5)^\top$. *The synchronization error trajectories in Figures 1-3 are non-negative and converge quickly and smoothly to zero under the proposed impulsive control strategy. This indicates that the system state is stable. Notably, despite varied and nontrivial initial states across nodes, all synchronization errors reduce significantly within a few impulsive instants. The consistent convergence patterns across different state components highlight the robustness of the control design. Furthermore, the sparsity of the impulses emphasizes the practical feasibility of the method in systems where continuous actuation is costly or infeasible.*

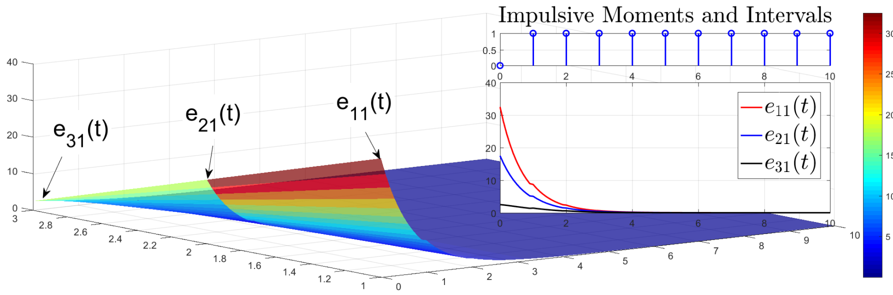

FIGURE 1. Synchronization error for e_{i1}, $i = 1, 2, 3$ with time on x-axis and index i on y-axis

Example 4.2. *Consider the CDNs (14) consisting of three nodes with*

$$A = \begin{pmatrix} -1.1 & 0.1 & 0.1 \\ 0.1 & -1 & 0.1 \\ 0.1 & 0.2 & -1.2 \end{pmatrix},\ D_1 = \begin{pmatrix} 0.3 & 0.1 & 0.1 \\ 0.2 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.3 \end{pmatrix},\ D_2 = \begin{pmatrix} 0.3 & 0.1 & 0.1 \\ 0.2 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.3 \end{pmatrix},$$
$$D_3 = \begin{pmatrix} 0.3 & 0.1 & 0.1 \\ 0.2 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.3 \end{pmatrix},\ B = \begin{pmatrix} 0.01 & 0.01 \\ 0.03 & 0.03 \\ 0.02 & 0.02 \end{pmatrix},\ C = \begin{pmatrix} 0.01 & 0.03 & 0.02 \\ 0.02 & 0.03 & 0.01 \end{pmatrix}.$$

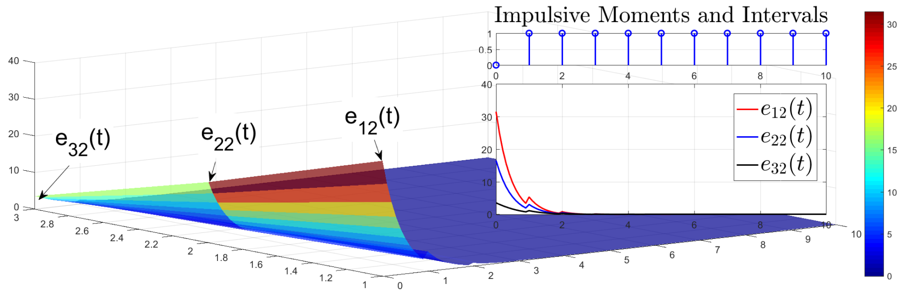

FIGURE 2. Synchronization error for e_{i2}, $i = 1, 2, 3$ with time on x-axis and index i on y-axis

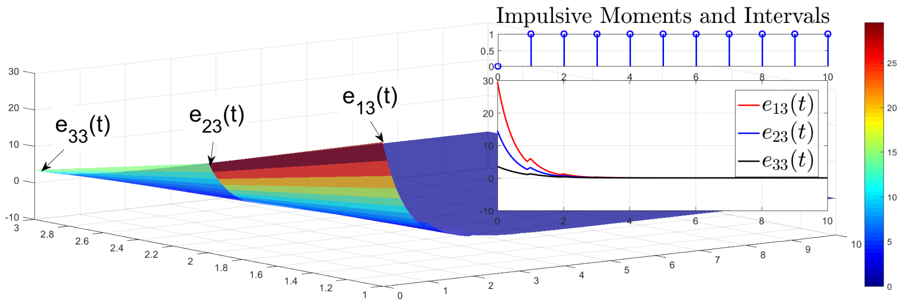

FIGURE 3. Synchronization error for e_{i3}, $i = 1, 2, 3$ with time on x-axis and index i on y-axis

The nonlinear functions are defined as $f^{(z)}(x(t)) = \sin^2 x(t)$, $z = 1, 2, 3$, *and the parameters of the coupled network configuration matrices are set as*

$$W = \begin{pmatrix} -0.1 & 0.05 & 0.05 \\ 0.05 & -0.1 & 0.05 \\ 0.05 & 0.05 & -0.1 \end{pmatrix}, \ \Gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Choose the constants $\alpha = 2$, $\theta = -0.1$, $\varsigma = 8$, $a_1^{(z)} = 0.3$, *and* $a_2^{(z)} = 0.4$. *By Theorem 3.2, we can obtain that the controller gain matrices and observer gain matrices:*

$$K_1 = \begin{pmatrix} 0.1567 & 0.1120 & 0.1256 \\ 0.1567 & 0.1120 & 0.1256 \end{pmatrix}, \ K_2 = \begin{pmatrix} 35.7967 & 32.6147 & 28.3277 \\ 35.7967 & 32.6147 & 28.3277 \end{pmatrix},$$

$$L = \begin{pmatrix} -1.2382 & -1.2866 \\ -1.6009 & -1.5823 \\ -1.7778 & -1.6676 \end{pmatrix}.$$

Simulations are conducted to evaluate both the system state and the observer state. Let $N = 3$, *with initial state conditions* $x_1(0) = (35, 34, 33)^\top$, $x_2(0) = (32, 31, 30)^\top$, $x_3(0) = (29, 28, 27)^\top$, $s_0(t) = (12.5, 11.5, 10.5)^\top$, $\hat{x}_1(0) = (50, 49, 48)^\top$, $\hat{x}_2(0) = (47, 46, 45)^\top$, *and* $\hat{x}_3(0) = (44, 43, 42)^\top$. *Figures 4-6 illustrate the trajectories of the actual system states and their corresponding observer states. It is evident that the observer accurately approximates the true state of the system with high precision. Specifically, all components exhibit rapid decay and reach indistinguishable alignment within approximately* $t = 6$. *This fast convergence demonstrates the high estimation accuracy and responsiveness of the observer. The synchronization error trajectories are shown in Figures 7-9. The error trajectories remain non-negative and converge to zero by* $t =$ *6-7 under the designed impulsive controller. The*

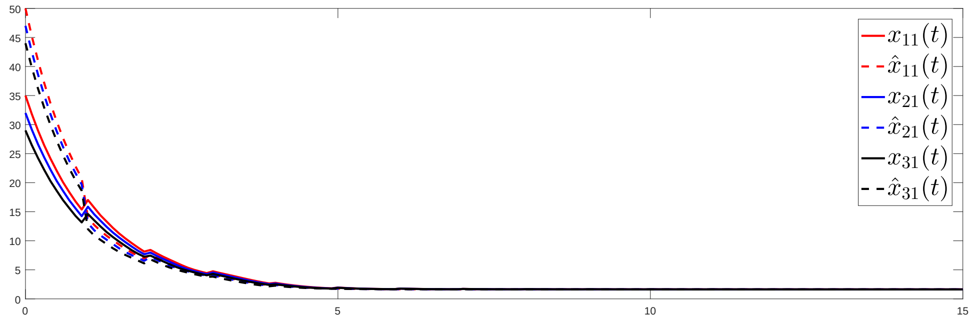

FIGURE 4. Trajectories of system states x_{i1} and observer estimates $\hat{x}_{i1}$ for $i = 1, 2, 3$

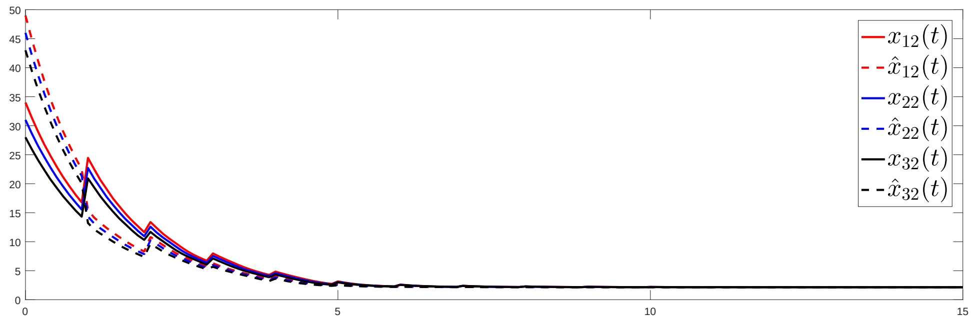

FIGURE 5. Trajectories of system states x_{i2} and observer estimates $\hat{x}_{i2}$ for $i = 1, 2, 3$

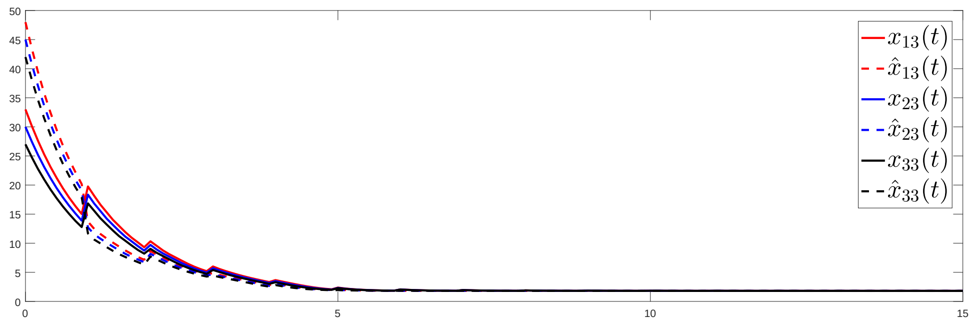

FIGURE 6. Trajectories of system states x_{i3} and observer estimates $\hat{x}_{i3}$ for $i = 1, 2, 3$

results show that synchronization is not only theoretically guaranteed but also practically achievable with sparse and well-scheduled impulsive interventions.

5. **Conclusions.** This paper focuses on the design of impulsive controller of PCDNs with nonlinearities based on a state feedback observer. Isolated nodes are employed to regulate the synchronization speed of the network. A matrix decomposition approach is utilized to derive the gain matrices for both the observer and synchronization controller. In addition, an LP method is proposed to solve all conditions. To validate the theoretical findings, two numerical examples are provided. Future research will focus on addressing the stability of impulsive positive systems with time-varying topology.

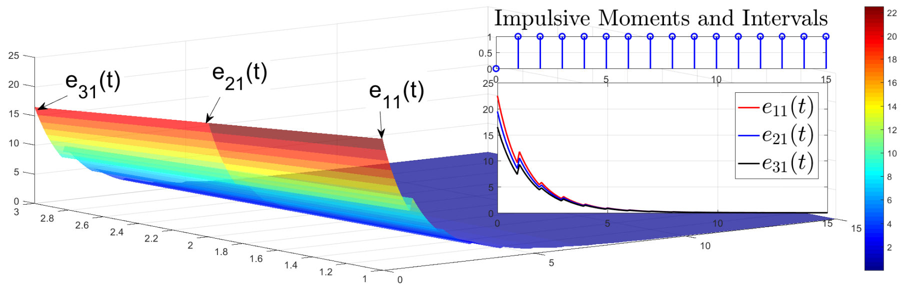

FIGURE 7. Synchronization error for e_{i1}, $i = 1, 2, 3$ with time on x-axis and index i on y-axis

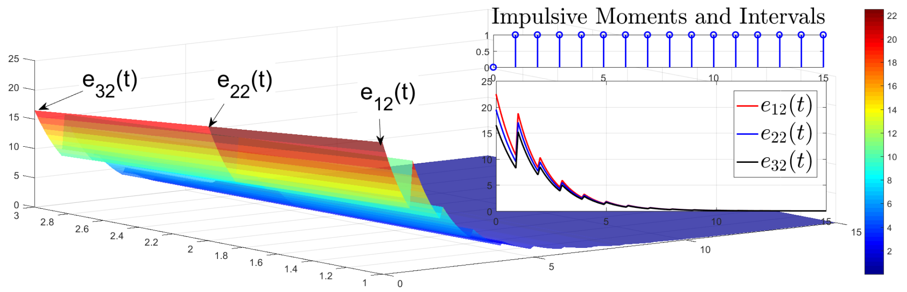

FIGURE 8. Synchronization error for e_{i2}, $i = 1, 2, 3$ with time on x-axis and index i on y-axis

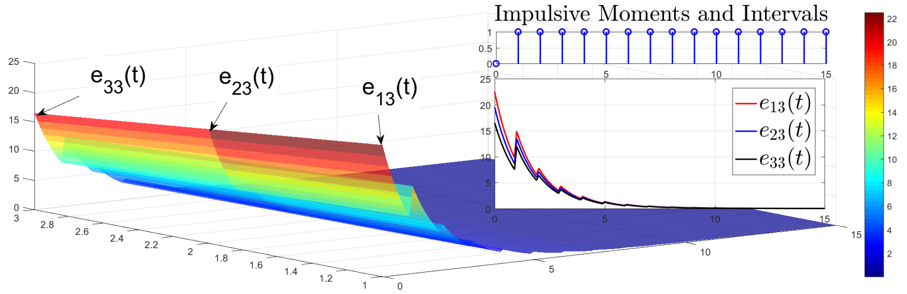

FIGURE 9. Synchronization error for e_{i3}, $i = 1, 2, 3$ with time on x-axis and index i on y-axis

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