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DISTURBANCE OBSERVER-BASED PREDEFINED-TIME TERMINAL SLIDING MODE TRACKING CONTROL FOR UNMANNED SURFACE VEHICLES

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Abstract. *This paper proposes a predefined-time terminal sliding mode control method for unmanned surface vehicles (USVs) with external disturbances. A predefined-time controller is first designed based on a sliding mode surface, ensuring that the tracking errors of USVs reach zero within the desired settling time. To address the external disturbances, a finite-time disturbance observer is further designed. The disturbance observer achieves stability at the initial time by effectively setting the initial value, thereby ensuring the predefined-time stability of the closed-loop system. Subsequently, the application of the proposed approach in USV tracking control is investigated by using predefined-time stability theory and Lyapunov stability theory. Theoretical analysis proves that the closed-loop system achieves predefined time stability. Finally, simulation experiments on USVs validate the effectiveness of the proposed method.*
Keywords: Disturbance observer, Sliding mode surface, Unmanned surface vehicle, Tracking control, Predefined-time stability

1. **Introduction.** Over the past few years, with the continuous advancement of automation technology, the demand for unmanned surface vehicles (USVs) has significantly increased. USVs are renowned for their autonomy, cost-effectiveness, flexibility, safety, and strong concealment capabilities. Their versatility enables widespread deployment in applications spanning aquaculture, water quality surveillance, oceanic exploration, naval patrols, emergency response operations, and defense missions [1]. However, in the maritime environment, external environmental disturbances typically include wind, waves, currents, and other hydrodynamic forces. These factors affect the accuracy of the model and, consequently, the precision of control. In particular, when the disturbance intensity is relatively high, they may also impact the stability of the system. To address these issues, the development of advanced control strategies is critical to enhance USVs' resilience to disturbances and improve efficiency. Several control methodologies, including fuzzy control [2], sliding mode control [3], backstepping control [4], and model predictive control [5], have been explored to achieve precise autonomous control. Due to its strong robustness against disturbances and parameter uncertainties, along with a straightforward design and fast response, sliding mode control has been extensively utilized in this field.

In the 1960s, sliding mode control (SMC) was first introduced by Soviet scholar Emelyanov and his colleagues [6]. During the 1990s, efforts to improve the convergence speed of sliding mode control (SMC) led to the integration of neural network-based terminal

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attractors, which effectively mitigated the issue of infinite convergence time present in classical SMC. This progress facilitated the emergence of terminal sliding mode (TSM) control strategies aimed at ensuring convergence within a finite time. In recent decades, notable advancements have emerged in terminal sliding mode methodologies. For instance, to address the singularity issues in traditional designs, nonsingular terminal sliding mode approaches were introduced [7, 8], and fast terminal sliding mode (FTSM) strategies were formulated to accelerate convergence [9]. Additionally, these techniques have been extended from second-order systems to multi-input multi-output and high-order configurations, broadening their applicability [10].

Although TSM methods achieved finite-time convergence with theoretically predictable timing, the actual duration remained influenced by variables like the system's initial conditions and controller parameters. Notably, the controller lacked a mechanism to explicitly predefine convergence time. To resolve this limitation, Polyakov proposed fixed-time control method [11], which ensures convergence within a predetermined upper bound $T_{\max}$, independent of the system's starting condition. In contrast to finite-time approaches, fixed-time control establishes $T_{\max}$ as a universal upper limit for all initial conditions, significantly decoupling convergence time from initial states [12]. However, the actual convergence time T within fixed-time control still depended on the initial state, though this dependency was bounded by $T_{\max}$. Designing such controllers poses inherent difficulties, primarily due to the absence of a direct correlation between convergence time and system parameters. Furthermore, tuning parameters often fails to reduce $T_{\max}$ below a certain threshold, thereby impeding the practical versatility of fixed-time strategies [13].

In scenarios requiring more precise time efficiency, in which the system must converge to equilibrium at a predefined moment, predefined-time control was introduced [14]. Distinct from fixed-time control, this methodology guarantees that convergence time is entirely decoupled from initial conditions while establishing an explicit link between control parameters and the system's convergence duration [15]. Such attributes render it particularly advantageous for controller design. Studies in [16, 17] devised time-varying functions to enforce convergence to equilibrium or its vicinity at a specified moment, enabling predefined-time stabilization. Further refining this concept, [18] formulated a Lyapunov-based framework to characterize predefined-time convergence and elucidated its relationship with finite-time and fixed-time methodologies. Motivated by the previous discussion, [19] investigated the predefined-time synchronization of two chaotic systems by utilizing the FTSM. Grounded in predefined-time theory, they developed a new Lyapunov function and a new FTSM, both of which offer adequate conditions to attain predefined-time synchronization. Through setting predefined parameters, the expected convergence time can be efficiently achieved [20].

In the research field of USVs, key focus areas include trajectory tracking [21], path following [22], course control [23], and dynamic positioning [24]. Among these, the trajectory tracking task imposes especially high demands on controller performance. This is because it is necessary to achieve precise position tracking within restricted time periods. To address trajectory tracking challenges, researchers have increasingly leveraged sliding mode techniques [25]. For instance, [26] integrated prescribed performance control (PPC) with super-twisting sliding mode algorithms and disturbance rejection strategies, proposing a composite controller that incorporates a disturbance observer and prescribed performance specifications [27, 28]. The research found that the utilization of predefined-time terminal sliding mode (PTSM) control in USVs is limited. While PTSM ensures time-bound convergence, most research focus on traditional sliding mode control, lacking this feature. This paper explores PTSM control for USV trajectory tracking, aiming to improve time efficiency and robustness.

Inspired by these advancements, this study introduces a disturbance observer-integrated predefined-time terminal sliding mode (DO-PTSM) controller tailored for USVs. The main contributions of this study are summarized below.

1) A novel control method is proposed based on PTSM control theory for USVs, ensuring that trajectory tracking errors converge to zero within a predefined time. This design improves control accuracy and introduces time-constrained performance capabilities for trajectory tracking.

2) To mitigate the impact of external disturbances, a disturbance observer is integrated into the system. The observer achieves instantaneous stability at the initial moment, ensuring the system's predefined-time stability. Meanwhile, it significantly enhances the system's robustness in complex and uncertain environments. The results demonstrate that the proposed DO-PTSM controller not only achieves high-precision trajectory tracking but also significantly improves the system's robustness in complex environments while effectively reducing control chattering.

The structure of this paper is organized as follows. Section 2 introduces the mathematical model of unmanned surface vehicle and summarizes the preliminary results that will be referenced throughout the paper. Section 3 presents the design method of the DO-PTSM controller, along with proofs of system stability and predefined-time stability. Section 4 provides a simulation analysis of the DO-PTSM controller and a comparative simulation with the conventional PTSM controller [29] and the FTSM controller [19]. Section 5 provides a synthesis of the research findings and proposes directions for future work.

2. **Problem Formulation.**

2.1. **System models.** Considering the external disturbances, the following three-degree-of-freedom fully-driven USV motion model, including surge, sway, and yaw, is presented [30].

$$\begin{cases} \dot{\eta} = R(\psi)\boldsymbol{v} \\ M\dot{\boldsymbol{v}} + C(\boldsymbol{v})\boldsymbol{v} + D(\boldsymbol{v})\boldsymbol{v} = \tau + d \end{cases} \tag{1}$$

where, $\tau = [\tau_1, \tau_2, \tau_3]^{\mathrm{T}}$ is the control vector, $\eta(t) = [x(t), y(t), \psi(t)]^{\mathrm{T}}$ is the position vector of the unmanned surface vehicle in the Earth-fixed coordinate system. Within the ship's body-fixed reference frame, the vector $\boldsymbol{v}(t) = [u(t), v(t), r(t)]^{\mathrm{T}}$ encapsulates three key kinematic quantities: the longitudinal velocity along the ship's heading, the lateral velocity perpendicular to its longitudinal axis, and the yaw angular velocity describing rotational motion around the vertical axis, and $d = [d_1, d_2, d_3]^{\mathrm{T}}$ is the vector of external disturbances. The rotation matrix $R(\psi)$ is used for coordinate transformation between the ship's body-fixed reference frame and the geographic coordinate system. The rotation matrix $R(\psi)$, inertia matrix M, centripetal matrix $C(\boldsymbol{v})$, damping matrix $D(\boldsymbol{v})$, and skew-symmetric matrix $S(r)$ are defined as follows [30], and their definitions follow the standard marine craft model in Fossen, which is widely adopted in USV dynamic modeling.

$$R(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix}$$

$$C(\boldsymbol{v}) = \begin{bmatrix} 0 & 0 & -m_{22}v \\ 0 & 0 & m_{11}u \\ m_{22}v & -m_{11}u & 0 \end{bmatrix}, \quad D(\boldsymbol{v}) = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix}, \quad S(r) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The matrix $R(\psi)$ satisfies $\dot{R}(\psi) = R(\psi)S(r)$, $\|R(\psi)\| = 1$, $R(\psi)R^\top(\psi) = I$, and $R^\top(\psi)S(r)R(\psi) = R(\psi)S(r)R^\top(\psi) = S(r)$.

2.2. **Preliminaries.** Let us examine the equation of a nonlinear system as follows:

$$\dot{\mathbf{x}} = f(t, \mathbf{x}; \boldsymbol{\lambda}) \tag{2}$$

where, $\mathbf{x} \in \mathbb{R}^n$ serves as the state-vector of the system. The vector $\boldsymbol{\lambda} \in \mathbb{R}^b$ represents the system's parameters, and since $\dot{\boldsymbol{\lambda}} = 0$, these parameters remain constant over time. The function $f : \mathbb{R}^n \to \mathbb{R}^n$ describes the dynamics of the nonlinear system. The variable t is a time variable within the interval $[t_0, \infty)$, where $t_0 \in [0, \infty)$ denotes the starting time, and $\mathbf{x}_0 = \mathbf{x}(t_0)$ represents the system's initial state.

Definition 2.1. [31] *Predefined-time synchronization. Suppose the system described by Equation (2) has fixed-time stability. Then, the function $f(t, \mathbf{x}_0; \boldsymbol{\lambda})$ is capable of converging to an equilibrium point within a prespecified time. Specifically,*

$$\lim_{t \to T(\boldsymbol{\lambda})} \|\mathbf{x}(t)\| = 0 \tag{3}$$

In this situation, the settling time $T(\boldsymbol{\lambda})$ depends solely on $\boldsymbol{\lambda}$ and is independent of the initial state $\mathbf{x}_0$. Moreover, $T(\boldsymbol{\lambda})$ is globally bounded. That is, for any starting condition $\mathbf{x} \in \mathbb{R}^n$, there exists a $T(\boldsymbol{\lambda}) \in [0, \infty)$ such that $T(\boldsymbol{\lambda}) \le T_{\max}$. As a result, the system defined by Equation (2) shows predefined-time stability, guaranteeing synchronization within the preset time limit.

Lemma 2.1. [19] *Consider a nonlinear system with non-Lipschitz continuity, expressed as $\dot{x} = f(x, t)$, where $f(0) = 0$. Let $V(x)$ be a Lyapunov function, positive definite for all $x \neq 0$, together with positive constants $A, B, \Gamma > 0$ and $0 < \delta < 1$, satisfying $4B\Gamma = A^2$. Then, the following condition holds:*

$$\dot{V} \le -\frac{4}{T_p A(1-\delta)} \left(AV + BV^{\frac{1+\delta}{2}} + \Gamma V^{\frac{3-\delta}{2}} \right) \tag{4}$$

Based on the above condition, the system described by $\dot{x} = f(x)$ achieves predefined-time stability. The convergence time T_c is entirely governed by the predefined-time parameter T_p, so as to ensure that the actual settling time meets $T_c(x_0) \le T_p$.

Lemma 2.2. [19] *The PTSM for synchronization is defined by the equation below:*

$$s = \dot{x} + \alpha x + \beta x^{\frac{q}{p}} + \gamma x^{2-\frac{q}{p}} = 0 \tag{5}$$

where, $x(t)$ represents scalar real variable, and the constants α, β, γ are positive. Consider two odd integers q and p, where holds.

If the parameters in Equation (5) satisfy the following inequalities, then the system $\dot{x} = f(x)$ will exhibit predefined-time stability.

$$\alpha \ge \frac{4}{T_p\left(1-\frac{q}{p}\right)}, \quad \beta \ge \frac{2\mu}{T_p\left(1-\frac{q}{p}\right)}, \quad \gamma \ge \frac{2}{T_p\mu\left(1-\frac{q}{p}\right)} \tag{6}$$

where T_p is the predefined-time parameter, with $\mu > 0$ being a positive constant. In Equation (5), q and p are required to be odd integers, and the solution under consideration is limited to the real domain. This ensures that the terms $\gamma x^{2-q/p}$, $\beta x^{q/p}$, and x always produce real values [14].

Lemma 2.3. [32] *For a particular second-order nonlinear differential operator, it is given by*

$$\dot{\theta}_1 = -m_1 \text{sig}^{\frac{1}{2}}(\omega(t)) + \theta_2, \quad \dot{\theta}_2 = -m_2 \text{sign}(\omega(t)) \tag{7}$$

where $\omega(t) = f(t) - \theta_1$ and the condition $|f(t)| < \rho$ holds. When m_1 is set to $1.5\sqrt{\rho}$ and m_2 is equal to 1.1ρ, the solution of $\omega(t)$ and its derivative $\dot{\omega}(t)$ exhibit globally fixed-time stability with a pre-established finite convergence time.

$$t_f \leq \frac{7.6\omega(0)}{m_2 - \rho} \tag{8}$$

Assumption 2.1. *The desired trajectory and its first and second derivatives are bounded.*

2.3. **Control objective.** This study aims to develop a controller for USVs that ensures trajectory tracking errors vanish within a predefined time, independent of the initial conditions. This controller is aimed at achieving precise trajectory tracking for USVs while ensuring robustness against external disturbances.

3. **Controller Design and Stability Analysis.** This section outlines the core contributions of the study. Subsection 3.1 derives the trajectory tracking error equation for the USV. In Subsection 3.2, we propose a state observer that achieves stability at the initial time, thereby ensuring that it does not affect the predefined-time stability of the controller. Subsection 3.3 details the design of the DO-PTSM controller. Finally, the stability and predefined-time stability of the closed-loop system are proven. The architecture of the control system put forward in this study is illustrated in Figure 1.

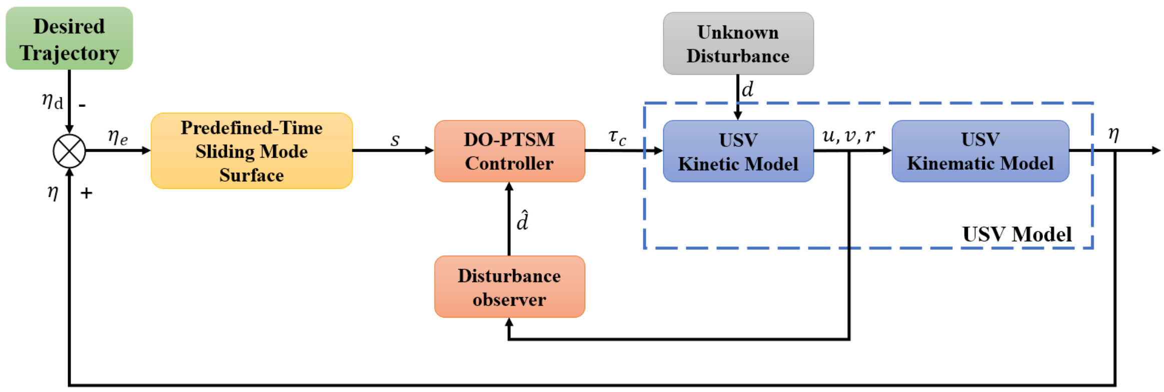

FIGURE 1. Architecture of the USV trajectory tracking control system

3.1. **Tracking error equation of the unmanned surface vessel.** Define the desired position vector of the unmanned surface vehicle as $\eta_d = [x_d(t), y_d(t), \psi_d(t)]^{\mathrm{T}}$. The trajectory tracking error for the USV is defined as follows:

$$e_1 = \eta - \eta_d, \quad e_2 = \dot{\eta} - \dot{\eta}_d \tag{9}$$

where, $e_1 = [e_{11}, e_{12}, e_{13}]^{\mathrm{T}}$ is the position and heading angle tracking error, and $e_2 = [e_{21}, e_{22}, e_{23}]^{\mathrm{T}}$. Taking the derivative of Equation (9) with respect to Equation (1), the following USV tracking error equation is obtained:

$$\dot{e}_1 = e_2, \quad \dot{e}_2 = R(\psi)M^{-1}\tau + f + d_0 - \ddot{\eta}_d \tag{10}$$

where, $f = R(\psi)M^{-1}(-C(\boldsymbol{v})\boldsymbol{v} - D(\boldsymbol{v})\boldsymbol{v}) + R(\psi)S(r)\boldsymbol{v}$, and $d_0 = R(\psi)M^{-1}d$ is the external disturbance, $f = [f_1, f_2, f_3]^{\mathrm{T}}$.

3.2. **The design of the disturbance observer.** To address external disturbances while ensuring the predefined time stability of the controller, this paper designs a disturbance observer that achieves stability from the initial time. The observer is employed to assess external disturbances. Firstly, let

$$\omega_i(t) = \bar{\boldsymbol{v}}_i(t) - (\boldsymbol{v}_i(t) - \boldsymbol{v}_i(0)) \tag{11}$$

$$\dot{\bar{\boldsymbol{v}}}_i(t) = -m_1 \mathrm{sig}^{\frac{1}{2}}(\omega_i(t)) + \theta_i(t) + F_i \tag{12}$$

$$\dot{\theta}_i(t) = -m_2 \mathrm{sign}(\omega_i(t)) \tag{13}$$

$$M^{-1}\hat{d} = -m_1 \mathrm{sig}^{\frac{1}{2}}(\omega_i(t)) + \theta_i(t) \tag{14}$$

where, $\boldsymbol{v}(t) = [u(t), \nu(t), r(t)]^{\mathrm{T}}$ is the velocity state vector of the USV in the body-fixed frame, $\bar{\nu}_i(t)$ and $\theta_i(t)$ represent the system state variables, and $\omega_i(t)$ is an observation error variable, which characterizes the deviation between the observer state and the actual velocity variation. $F_i = -M^{-1}C(\boldsymbol{v})\nu - M^{-1}D(\nu)\nu + M^{-1}\tau$ with $i = 1, 2, 3$. In Equation (12), there are $\bar{\nu}_i(0) = 0$, $\theta_i(0) = 0$, and m_1, m_2 are design parameters. It can be known from Equation (14) that the observed value of the system disturbance is $\hat{d} = M\left(-m_1 \mathrm{sig}^{\frac{1}{2}}(\omega_i(t)) + \theta_i(t)\right)$.

Subsequently, we will conduct an analysis on the dynamics of $\omega_i(t)$. Firstly, it can be deduced from Equation (11) that $\omega_i(0) = 0$. Based on Lemma 2.3, it follows that $\dot{\omega}_i(t) = 0$ and $\omega_i(t) = 0$ for $t \geq \frac{7.6\omega_i(0)}{m_2 - \rho}$, where $\rho_i = \sup_{t\geq 0} |\dot{v}_i(t)|$. In this paper, the state observer is designed in such a way that it will reach a stable state at the moment of $t = 0$ and thus will not affect the predefined time stability of the entire system.

Remark 3.1. *To improve the robustness of the USV control system against external disturbances, a disturbance observer is designed to estimate such disturbances. Setting the initial values of $\bar{\nu}_i(t)$, $\nu_i(t)$ and $\nu_i(0)$ in Equation (11) to be 0. Combined with Lemma 2.3, it can be deduced that the observer can be stabilized in $t \geq 0$. This design successfully combines the ability to handle uncertain disturbances with the predefined-time stability criterion, which is one of the contributions in this paper. At the same time, the predefined-time stability performance of the controller is maintained.*

3.3. **Predefined-time terminal sliding mode controller design.** The PTSM controller designed in this study includes a novel PTSM reaching law and a new PTSM sliding surface for the controller design.

Step 1. A new sliding mode surface with PTSM is adopted, and is shown below:

$$s(t) = e_2 + \alpha_0 e_1 + \beta_0 e_1^{\frac{q_0}{p_0}} + \gamma_0 e_1^{2-\frac{q_0}{p_0}} \tag{15}$$

where $\alpha_0, \beta_0, \gamma_0 > 0$, q_0, p_0 are positive odd integers, and $q_0 < p_0$.

Since Equation (15) can be transformed into the form of Equation (5), it inherits the predefined-time stability property established in Lemma 2.2. Therefore, when the parameters α_0, β_0, γ_0 in Equation (15) satisfy the inequalities in Equation (19), the new sliding mode surface $s(t)$ will ensure predefined-time stability. The convergence time $T_{c0}(s(t_0))$ from any initial condition $s(t_0) \neq 0$ to the equilibrium $s(t_s) = 0$ is bounded by the predefined time T_{p0}, such that $T_{c0}(s(t_1)) \leq T_{p0}$.

Equation (15) can be transformed into the form of Equation (5). The demonstration of its predefined time stability employs an approach analogous to that used in Lemma 2.2, and is thus omitted here for brevity.

Step 2. A new reaching law incorporating PTSM is introduced as below:

$$\frac{d}{dt}s(t) = -\alpha s(t) - \beta s(t)^{\frac{q}{p}} - \gamma s(t)^{2-\frac{q}{p}} = 0 \tag{16}$$

where, $\alpha, \beta, \gamma > 0$, and p, q are odd positive integers, satisfying $p > q$.

When the parameters in Equation (16) satisfy the inequalities in Equation (6), the reaching law secures predefined-time stability. The time $T_c(s_0)$ needed for $s(t)$ to evolve from any initial condition $s(t_0) \neq 0$ to the equilibrium point $s(t_c) = 0$ is constrained by the predefined parameter T_p, satisfying $T_c(s_0) \leq T_p$.

Equation (16) can be rewritten in the form of Equation (5), and its stability proof follows the methodology of Lemma 2.2, thus avoiding repetition.

Step 3. Considering the USV motion model with external disturbances, based on the tracking error equation shown in Equation (10), a PTSM robust controller is designed. This design incorporates a novel PTSM-based reaching law, a PTSM-based sliding surface, and a disturbance observer, guaranteeing the convergence of the trajectory-tracking error to zero within a predefined time.

$$\tau = -MR^{-1}U + \hat{d} \tag{17}$$

where $U = [u_1, u_2, u_3]^{\mathrm{T}}$ is defined as

$$U_i = f_i - \ddot{\eta}_{di} + \alpha_0 e_{2i} + \frac{\beta_0 q_0 e_{2i} e_{1i}^{\frac{q_0}{p_0}-1}}{p_0} + \gamma_0 \left(2 - \frac{q_0}{p_0}\right) e_{2i} e_{1i}^{1-\frac{q_0}{p_0}} + \alpha_1 s_i + \beta_1 s_i^{\frac{q_1}{p_1}} + \gamma_1 s_i^{2-\frac{q_1}{p_1}} \tag{18}$$

where $i = 1, 2, 3$, $\alpha_j, \beta_j, \gamma_j > 0$, q_j and p_j are strictly positive and odd and $q_j < p_j$, $j = 0, 1$.

Predefined-time stability of System (10) is ensured when the conditions in Equation (19) are satisfied by the parameters in Equation (18). Specifically, the settling time for the reaching motion stage, denoted as T_{c1}, is bounded by the predefined parameter T_{p1}, such that $T_{c1} \leq T_{p1}$. The sliding mode phase settling time T_{c0} is limited by the predefined-time parameter T_{p0}, ensuring $T_{c0} \leq T_{p0}$. The overall settling time T_c, given by $T_c = T_{c0} + T_{c1}$, is constrained by the total predefined-time parameter $T_p = T_{p0} + T_{p1}$, satisfying $T_c(x_0) \leq T_p$. This guarantees that the system maintains robust and accurate tracking performance despite external disturbances.

$$\alpha_j \geq \frac{4}{T_{pj}\left(1 - \frac{q_j}{p_j}\right)}, \ \beta_j \geq \frac{2\mu_j}{T_{pj}\left(1 - \frac{q_j}{p_j}\right)}, \ \gamma_j \geq \frac{2}{T_{pj}\mu_j\left(1 - \frac{q_j}{p_j}\right)} \tag{19}$$

where $\mu_j = \sqrt{\beta_j/\gamma_j}$, $\mu_j > 0$, and T_{pj} is the predefined-time parameter with $j = 0, 1$.

Theorem 3.1. *Consider the motion model of a USV with external disturbances. This model is described by Equation (1) and satisfies Assumption 2.1. Under the PTSM robust controller defined by Equation (18), the system achieves predefined-time stability if the control parameters α_j, β_j, and γ_j satisfy the inequality conditions in Equation (19). Within the specified time T_p, the trajectory tracking error reaches zero.*

Proof: Stability and convergence analysis: The Lyapunov function is constructed as follows:

$$V(t) = \frac{1}{2}s(t)^2 \tag{20}$$

The derivative of Equation (15) can be obtained as follows:

$$\frac{d}{dt}s(t) = \dot{e}_2(t) + \alpha_0 e_2 + \frac{\beta_0 q_0 e_2 e_1^{\frac{q_0}{p_0}-1}}{p_0} + \gamma_0\left(2 - \frac{q_0}{p_0}\right) e_2 e_1^{1-\frac{q_0}{p_0}} \tag{21}$$

The time derivative of $V(t)$ can be derived by integrating Equation (10), Equation (17), Equation (18), and Equation (21) as follows:

$$\frac{d}{dt}V(t) = s(t)\left(\dot{e}_2(t) + \alpha_0 e_2 + \frac{\beta_0 q_0 e_2 e_1^{\frac{q_0}{p_0}-1}}{p_0} + \gamma_0\left(2 - \frac{q_0}{p_0}\right) e_2 e_1^{1-\frac{q_0}{p_0}}\right)$$

$$= s(t)\left(R(\psi)M^{-1}\tau + f + d_0 - \ddot{\eta}_d + \alpha_0 e_2 + \frac{\beta_0 q_0 e_2 e_1^{\frac{q_0}{p_0}-1}}{p_0} + \gamma_0\left(2 - \frac{q_0}{p_0}\right) e_2 e_1^{1-\frac{q_0}{p_0}}\right)$$
$$= s(t)\left(-\alpha_1 s - \beta_1 s^{\frac{q_1}{p_1}} - \gamma_1 s^{2-\frac{q_1}{p_1}}\right)$$
$$= -\alpha_1 s(t)^2 - \beta_1 s(t)^{\frac{p_1+q_1}{p_1}} - \gamma_1 s(t)^{\frac{3p_1-q_1}{p_1}} \tag{22}$$

Since both $(p_1 + q_1)$ and $(3p_1 - q_1)$ are positive even integers, it follows that the inequalities $s(t)^{\frac{p_1+q_1}{p_1}} > 0$ and $s(t)^{\frac{3p_1-q_1}{p_1}} > 0$ in Equation (22) hold true. Thus, it follows that $\dot{V}(t) \leq 0$. With the successful completion of stability and convergence analysis, the uncertain system in Equation (1), under the influence of the robust PTSM controller defined in Equation (17), demonstrates Lyapunov stability.

Predefined-time stability analysis: From Equation (22), the time derivative of $V(t)$ is expressed as

$$\dot{V}(t) = -\left(\alpha_1 V + \beta_1 V^{\frac{1+\delta}{2}} + \gamma_1 V^{\frac{3-\delta}{2}}\right) \tag{23}$$

where $\delta = q/p$ and $0 < \delta < 1$.

By comparing with the parameter condition in Equation (6) and referring to Lemma 2.1 and Lemma 2.2, the system parameter conditions for the Lyapunov function in Equation (23) are obtained as

$$\alpha_1 \geq \frac{4}{T_p(1-\delta)}, \quad \beta_1 \geq \frac{2\sqrt{B}}{T_p\sqrt{\Gamma}(1-\delta)}, \quad \gamma_1 \geq \frac{2\sqrt{\Gamma}}{T_p\sqrt{B}(1-\delta)} \tag{24}$$

It follows that Equation (24) is equivalent to Equation (6), where $\mu = \sqrt{B}/\sqrt{\Gamma}$ and $\mu > 0$. The detailed proof is omitted for brevity.

As a result, the predefined-time stability is established, leading to the derivation of the condition expressed in Equation (19).

The proof is finished.

Remark 3.2. *It is important to highlight that, when the parameters of the proposed PTSM-based sliding mode surface and the innovative reaching law satisfy Equation (6), both exhibit predefined-time stability. This property not only simplifies the parameter design of the sliding mode surface to adjust its sliding motion, but also enables straightforward tuning of the reaching law parameters to control the rate of convergence of the variable index-based reaching law. Moreover, the reaching law successfully mitigates chattering effects and preserves robust performance by eliminating sign functions and adding power terms to the sliding mode function.*

4. **Simulation Results.** This chapter primarily focuses on the selection of parameters for unmanned surface vehicle models and controllers. Simulation studies and comparative evaluations were then performed to verify the effectiveness and advantages of the proposed approach.

4.1. **Simulation parameters.** To demonstrate the performance and advantages of the proposed controller, MATLAB simulations are performed using the USV dynamic model outlined in Equation (1). The key parameters of the USV dynamic system are as follows [33]: $d_{11} = 12$, $d_{22} = 17$, $d_{33} = 0.5$, $m_{11} = 25.8$, $m_{22} = 33.8$, $m_{33} = 2.76$.

The parameter settings for the predefined-time terminal sliding mode controller are as follows: $\mu_0 = 0.5$, $q_0/p_0 = 93/99$, $T_{p0} = 16.5$, $\mu_1 = 0.5$, $q_1/p_1 = 7/9$, $T_{p1} = 1.8$, $\alpha_0 = 4$, $\beta_0 = 1$, $\gamma_0 = 4$, $\alpha_1 = 10$, $\beta_1 = 2.5$, $\gamma_1 = 10$.

4.2. **Simulation verification and comparison.** Two simulation scenarios were constructed to evaluate the performance and resilience of the proposed control method.

In Scenario 1, simulations were conducted for the proposed controller and compared against both the PTSM controller without a disturbance observer [29] and the FTSM controller [19], aiming to evaluate the proposed approach in terms of robustness and control performance. The closed-loop system was initialized with $\eta(0) = [0.1, 1.01, 0.1]^{\mathrm{T}}$ and $\boldsymbol{v}(0) = [0, 0, 0]^{\mathrm{T}}$, while the reference trajectory was defined as $\eta_d = [\sin(0.2t), 2 - \cos(0.2t), 0.2t]^{\mathrm{T}}$. In this scenario, white Gaussian noise is introduced as the external disturbance. Such a disturbance emulates random and rapidly varying environmental forces acting on USVs in practice, including small-scale turbulent wave action, high-frequency wind gusts, and sensor measurement noise. These stochastic factors can cause rapid fluctuations in vessel motion, making them representative of unpredictable maritime operating conditions. The external disturbance was modeled as $\dot{d}_i = l_i d_i + G_i$, $i = u, v, r$, where l_u, l_v, l_r are constants with a value of -0.01, and G_u, G_v, G_r represent normally distributed Gaussian random signals.

Figures 2 and 3 compare the tracking performance of the proposed predefined-time terminal sliding mode robust controller with that of the conventional PTSM controller and the FTSM controller. The results clearly indicate that the proposed controller provides more accurate control performance than the PTSM and FTSM controllers. The simulation findings demonstrate that the proposed controller delivers markedly improved tracking performance compared to both the PTSM and FTSM controllers. Since Scenario 1 is conducted under external disturbances, this demonstrates that the proposed controller exhibits stronger robustness. Figure 4 provides a zoomed-in view of Figure 3, showing that

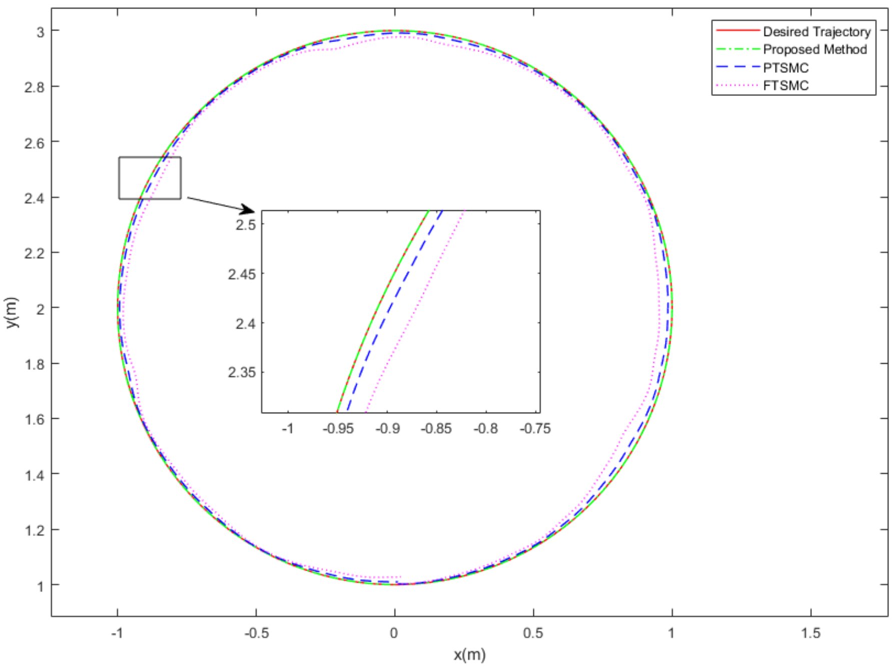

FIGURE 2. USV position (x, y) and desired trajectory (x_d, y_d)

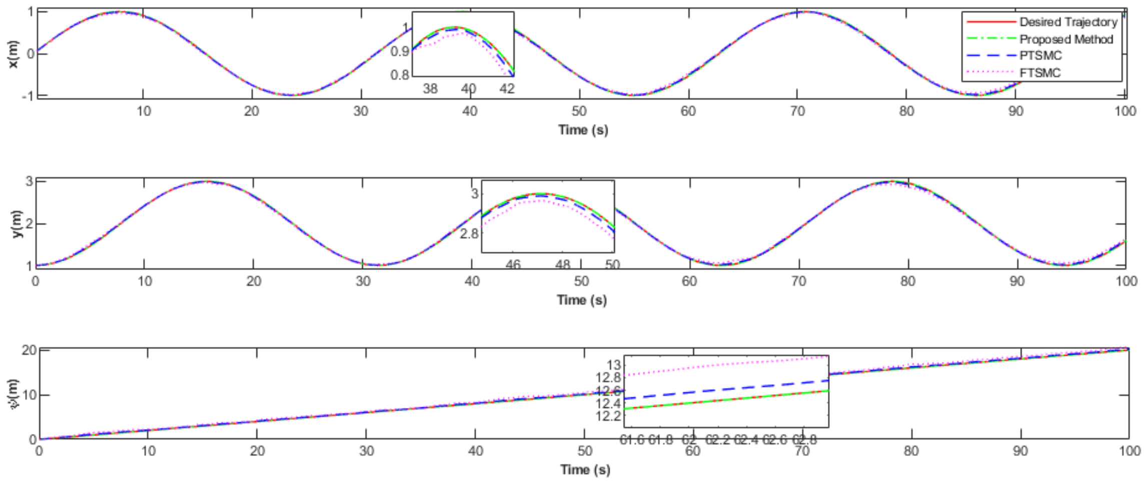

FIGURE 3. Tracking performance of x, y, ψ track the desired trajectory x_d, y_d, ψ_d

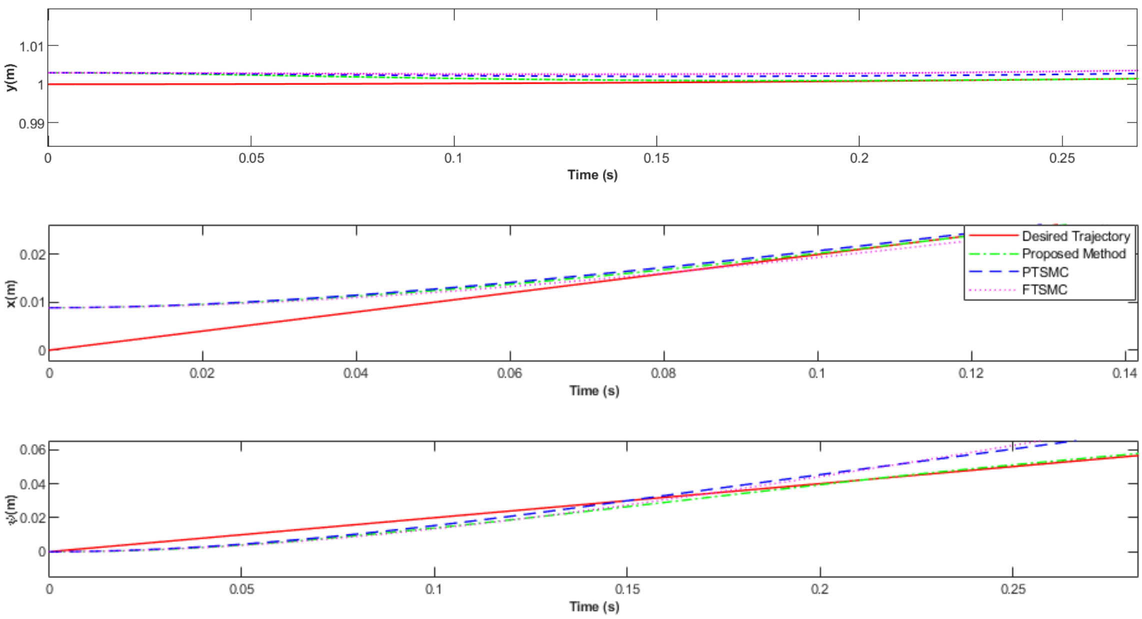

FIGURE 4. Tracking performance of x, y, ψ track the desired trajectory x_d, y_d, ψ_d

x_d, y_d, and ψ_d all stabilize and track the desired trajectory within 1 second, indicating that the system reaches a stable state within 1 second, thereby achieving predefined-time trajectory tracking. The total convergence time satisfies $T_c < T_{p0} + T_{p1}$, confirming that the system's settling time $T_c = T_{c0} + T_{c1}$ is governed by the predefined-time bound $T_p = T_{p0} + T_{p1}$, thereby ensuring predefined-time stability. Figure 5 shows a comparison of the tracking errors among the three controllers under Scenario 1. The results show that the proposed method maintains errors close to zero, which are significantly smaller than those of the other two methods. Figure 6 illustrates the disturbance tracking results of the proposed observer under the reference white noise disturbance. As shown in Figure 6, the disturbance observer employed in this study is capable of quickly and accurately estimating external disturbances. Figure 7 compares the control inputs of the controllers

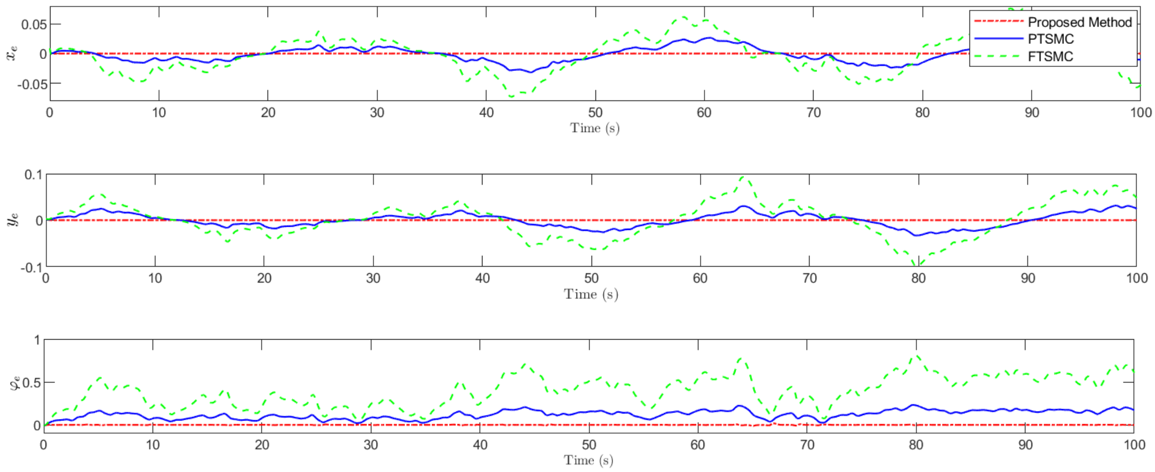

FIGURE 5. Control errors x_e, y_e, and ψ_e

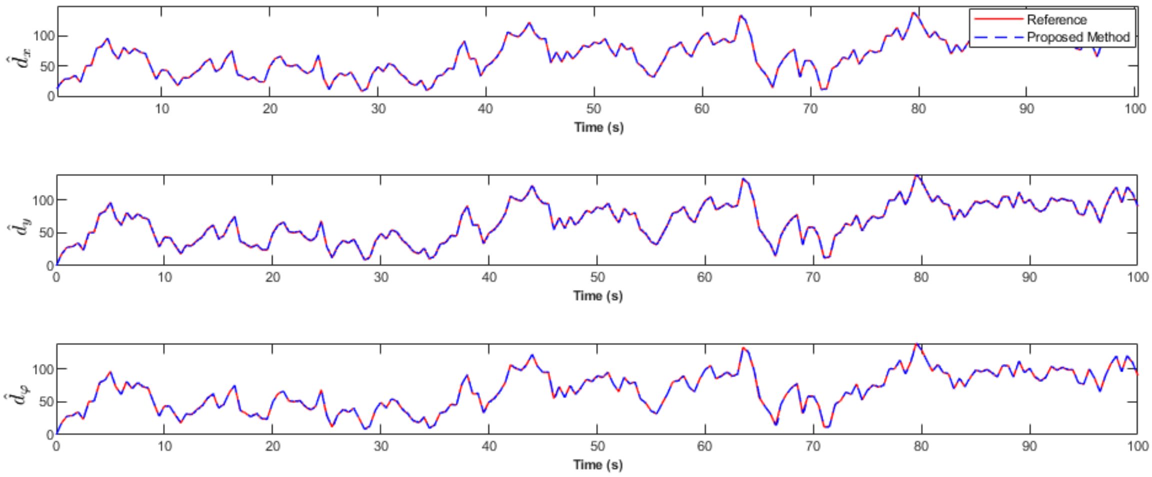

FIGURE 6. Disturbance estimation of the predefined-time terminal sliding mode robust controller

in Scenario 1. The results indicate that the control inputs of the proposed controller are smoother and have smaller amplitudes. Evidently, the proposed method outperforms both the PTSM and FTSM control methods, demonstrating superior robustness, higher tracking accuracy of the reference trajectory, and more stable control inputs in the presence of external disturbances.

In Scenario 2, to evaluate the performance of the proposed controller under different environments, the disturbances were modified to $d_u = 5\sin(2t)$, $d_v = \sin(2t)$, and $d_r = \sin(t)$, which are referred to as the second type of disturbances (STD). These sinusoidal disturbances represent periodic environmental forces such as regular ocean waves, tidal currents, and repetitive wind patterns. Such periodic factors are common in coastal waters and open seas, and their amplitudes and frequencies directly affect the vessel's heading stability and trajectory tracking accuracy. Figure 8 presents the tracking results of the disturbances $\hat{d}_u$, $\hat{d}_v$, and $\hat{d}_r$, demonstrating that the proposed robust controller can rapidly and accurately track the second type of disturbances. Figure 9 depicts the tracking errors $\tilde{d}_u$, $\tilde{d}_v$, and $\tilde{d}_r$, showing that the disturbance tracking errors stabilize at zero from the beginning. This indicates that the proposed observer reaches a stable state at the initial moment, thereby not affecting the predefined-time stability of the entire system.

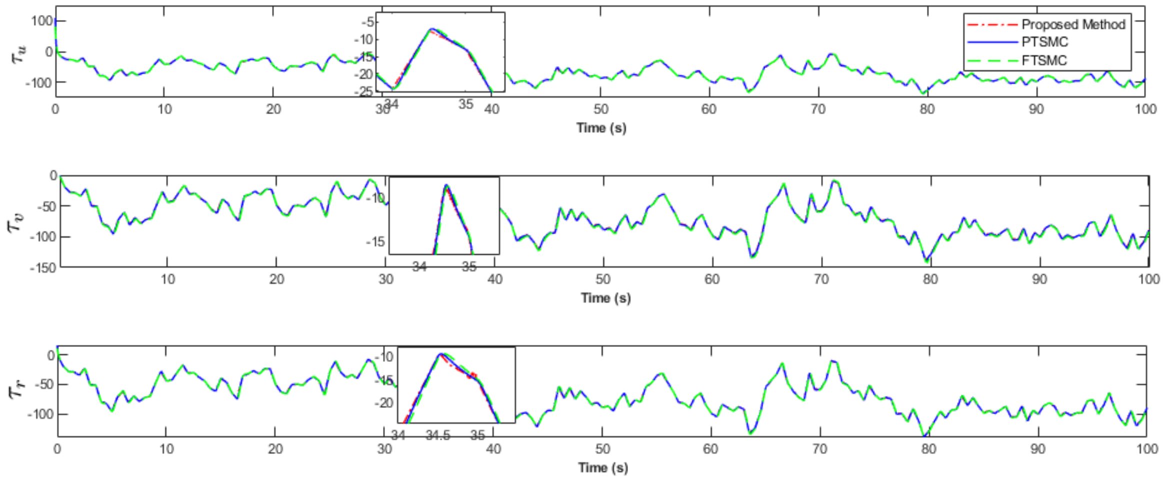

FIGURE 7. The control inputs τ_u, τ_v, τ_r

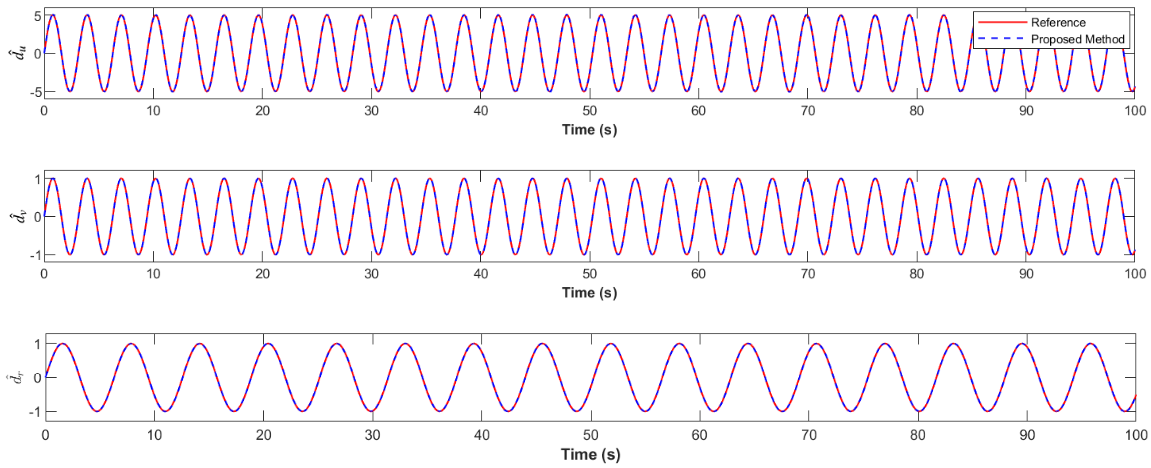

FIGURE 8. Disturbance estimation of the predefined-time terminal sliding mode robust controller

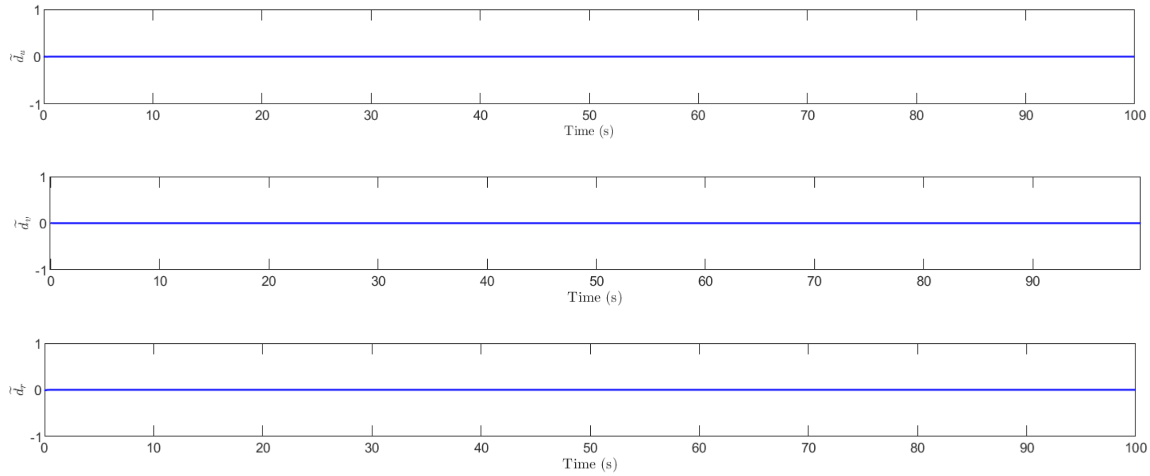

FIGURE 9. Tracking error of the disturbance observer

It can be concluded that the proposed robust controller performs excellently in various environments, capable of quickly and accurately estimating different types of external disturbances.

5. **Conclusion.** This paper presented a DO-PTSM controller for the trajectory tracking control of complex USV. The proposed controller demonstrates robust performance across various scenarios, effectively addressing trajectory tracking issues under external disturbances. The study's principal contributions are outlined below: The proposed controller ensures that trajectory tracking errors converge to zero within a predefined time, as validated in Scenario 1. Simulation results confirm that the total convergence time satisfies $T_c < T_p$, demonstrating the effectiveness of the predefined-time stability design. By incorporating a disturbance observer, the controller achieves rapid and precise estimation of external disturbances, even under challenging dynamic environments. This significantly enhances system robustness, as observed in Scenarios 1 and 2. Simulation results further indicate that, compared to traditional PTSM and FTSM controllers, the proposed method achieves higher trajectory tracking accuracy and effectively reduces control chattering under the influence of white noise and other dynamic disturbances. The controller's ability to handle different types of external disturbances, including sinusoidal variations (Scenario 2), demonstrates its adaptability and robustness in complex maritime conditions. In conclusion, the proposed controller provides a novel solution for trajectory tracking of USV, offering predefined-time stability, robustness against disturbances, and superior tracking accuracy. Future work will focus on further reducing the potential chattering effects caused by sliding mode control and improving the smoothness of the control inputs. Moreover, experimental validation on a real USV platform will be conducted to further evaluate the effectiveness and engineering feasibility of the proposed method.

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