International Journal of Innovative Computing, Information and Control
Volume **22**, Number **3**, June **2026**

ICIC International ©2026 ISSN 1349-4198
pp. 673–685

THE PARAMETERIZATION OF ALL UNKNOWN INPUT DISTURBANCE OBSERVERS FOR PLANTS WITH GENERAL EXOGENOUS OUTPUT DISTURBANCES

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Received August 2025; revised December 2025

Abstract. *Disturbance observer-based control has been recognized as one of the most promising approaches for disturbance attenuation, and many papers on design methods of disturbance observers have been published. Recently, the parameterization of all disturbance observers for plants with any disturbances has been clarified. However, these methods generally require the availability of the control input. Since there are systems which the control input of system is not unavailable, it is essential to design unknown input disturbance observers. In this paper, we clarify the parameterization of all unknown input disturbance observers for plants with general exogenous output disturbances. Furthermore, we present a design method applicable to such observers.*
Keywords: Disturbance observer, Parameterization, General exogenous disturbance, Unknown input

1. Introduction.

Disturbance observers have been widely applied in the motion-control field to reject disturbances or to make a closed-loop system robustly stable [1, 2, 3, 4, 5]. Generally, the disturbance observer includes the disturbance signal generator and an observer. The disturbance, which is usually assumed to be step disturbance, is estimated by the observers. Because the disturbance observer has a simple structure and is easy to understand, it has been used in many applications [6, 7, 8, 9].

Disturbance observer-based control has been seen as the most promising approaches to attenuate disturbances [10]. A variety of disturbance observers have been proposed. For example, Li et al. proposed frequency domain disturbance observers and time domain disturbance observers in different domains for linear systems [10]. In addition, Chen et al. proposed nonlinear disturbance observers for constant disturbances and nonlinear disturbance observers for general exogenous disturbances based on different disturbances in

DOI: 10.24507/ijicic.22.03.673

[11]. However, no one examines the parameterization of all disturbance observers. Therefore, Yamada et al. proposed the parameterization of linear disturbance observers by using control input and output of the system [12]. Juntawongso et al. proposed the parameterization for all linear disturbance observers with constant disturbances [13]. In addition, Juntawongso et al. proposed the parameterization for all linear disturbance observers with general exogenous disturbances [14]. Looking through those disturbance observers, the control input is necessary for designing disturbance observers. There exist plants of which the control input is not available [15]. The problem to design an unknown input disturbance observer is important to solve.

Recently, approaches based on parameterization have been applied to more structured disturbance and control problems. For example, Phukapak et al. investigated the parameterization of all disturbance observers which handle periodic output disturbances [18]. In addition, Kimura et al. studied the parameterization of extended semi-strongly stabilizing controllers [19]. These works have significantly improved our theoretical understanding of parameterization frameworks used for disturbance rejection and controller design.

However, these existing results mainly focus on disturbance observers based on input-output information or controller structures. They do not clearly discuss how to parameterize unknown input disturbance observers that use both available state and output data. Moreover, the role of tunable stable filters in shaping disturbance attenuation performance has not been fully clarified. This motivates the development of a new parameterization framework for unknown input disturbance observers with general exogenous output disturbances.

In this paper, we clarify the parameterization of all unknown input disturbance observers for plants with general exogenous output disturbances. By using the available state variable and the output of the system, the parameterization of all unknown input disturbance observer is obtained. In addition, when the state variable is not available, we present a design method for unknown input disturbance observers. This paper is organized as follows. In Section 2, the problem considered in this paper is explained. In Section 3, when the state variable and the output are available, the parameterization of all unknown input disturbance observers is clarified. In Section 4, the parameterization of all linear functional unknown input observers is presented. In Section 5, a design procedure of unknown input observer is shown. In Section 6, numerical examples are illustrated to show the effectiveness of the proposed parameterization and applicability to constant output disturbances. Section 7 gives concluding remarks.

2. **Problem Formulation.** Considering the plant described by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + d(t), \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state variable, $u(t) \in R^p$ is the control input, $y(t) \in R^m$ is the output, $d(t) \in R^m$ is the disturbance, $A \in R^{n\times n}$, $B \in R^{n\times p}$, $C \in R^{m\times n}$. It is assumed that (A, B) is stabilizable, (C, A) is detectable, $x(t)$ and $y(t)$ are available, but $u(t)$ and $d(t)$ are unavailable, and $d(t)$ is general exogenous output disturbances expressed by

$$\begin{cases} \dot{\xi}(t) = \bar{A}\xi(t) \\ d(t) = \bar{C}\xi(t), \end{cases} \tag{2}$$

where $\bar{A} \in R^{q\times q}$ and $\bar{C} \in R^{m\times q}$ are unknown.

When the disturbance $d(t)$ is not available, a disturbance estimator called the disturbance observer is frequently used. The disturbance observer estimates the disturbance

$d(t)$ in (1) using available measurements. Based on the assumption that $y(t)$ and $x(t)$ are available measurements, the general form of the unknown input disturbance observer $\tilde{d}(s)$ for (1) and (2) is written as

$$\tilde{d}(s) = F_1(s)y(s) + F_2(s)x(s), \tag{3}$$

where $F_1(s) \in R^{m\times m}(s)$, $F_2(s) \in R^{m\times n}(s)$, $\tilde{d}(s) = \mathcal{L}\left(\tilde{d}(t)\right)$, $\tilde{d}(t) \in R^m(t)$, $x(s) = \mathcal{L}(x(t))$ and $y(s) = \mathcal{L}(y(t))$. We call the system $\tilde{d}(s)$ in (3) an unknown input disturbance observer for general exogenous output disturbances $d(t)$, if

$$\lim_{t\to\infty}\left(d(t) - \tilde{d}(t)\right) \simeq 0 \tag{4}$$

is satisfied for any $y(0)$, $u(t)$ and $\xi(0)$.

The problem considered in this paper is to clarify the parameterization of all unknown input disturbance observers $\tilde{d}(s)$ in (3) for general exogenous output disturbances in (2).

3. **Parameterization of All Unknown Input Disturbance Observers.** In this section, we clarify the parameterization of all unknown input disturbance observers.

The parameterization of all unknown input disturbance observers $\tilde{d}(s)$ in (3) for general exogenous output disturbances in (2) is summarized in the following theorem.

Theorem 3.1. *The system $\tilde{d}(s)$ in (3) is an unknown input disturbance observer for general exogenous output disturbances if and only if $F_1(s)$ and $F_2(s)$ are written as*

$$F_1(s) = I - Q(s), \tag{5}$$

and

$$F_2(s) = -\left(I - Q(s)\right)C, \tag{6}$$

respectively, where $Q(s) \in RH_\infty$ is any function satisfying

$$\bar{\sigma}\left\{I - Q(j\omega_{\max})\right\} \simeq 0, \tag{7}$$

where $\bar{\sigma}\{\cdot\}$ denotes the maximum singular value, and $\omega_{\max}$ is the maximum frequency component contained in $d(s)$.

Proof: First, the necessity is shown. That is, we show that if the system $\tilde{d}(s)$ in (3) satisfies (4), then $F_1(s)$ and $F_2(s)$ in (3) are written by (5) and (6), respectively.

From the assumption in (4), $d(s) - \tilde{d}(s)$ can be written as

$$d(s) - \tilde{d}(s) = Q(s)d(s), \tag{8}$$

where $Q(s) \in RH_\infty$. From (8), $\tilde{d}(s)$ is rewritten as

$$\tilde{d}(s) = (I - Q(s))d(s). \tag{9}$$

From (1), $d(s)$ is written as

$$d(s) = y(s) - Cx(s). \tag{10}$$

Substituting (10) into (9), we obtain

$$\tilde{d}(s) = (I - Q(s))y(s) - (I - Q(s))Cx(s). \tag{11}$$

This equation corresponds to (5) and (6). Since $Q(s) \in RH_\infty$, it follows that $(I - Q(s)) \in RH_\infty$ and $-(I - Q(s))C \in RH_\infty$. From (4) and (11), (7) is satisfied. In this way, the necessity has been proved.

Next, the sufficiency is shown. That is, we show that if $F_1(s)$ and $F_2(s)$ are described by (5) and (6), and $Q(s) \in RH_\infty$ is any function, then $\tilde{d}(s)$ in (3) satisfies (4).

Substituting (5) and (6) into (3), $\tilde{d}(s)$ is written as

$$\begin{aligned}\tilde{d}(s) &= (I - Q(s))y(s) - (I - Q(s))Cx(s)\\ &= (I - Q(s))(y(s) - Cx(s))\\ &= (I - Q(s))d(s). \end{aligned} \tag{12}$$

From (12), we obtain

$$d(s) - \tilde{d}(s) = Q(s)d(s). \tag{13}$$

Because $Q(s) \in RH_\infty$, the error mapping is stable. Furthermore, under the condition in (7), we obtain

$$\lim_{t\to\infty} \left(d(t) - \tilde{d}(t)\right) \simeq 0, \tag{14}$$

which means that (4) is satisfied. In this way, the sufficiency has been proved.

Thus, Theorem 3.1 has been proved. □

Section 3 clarifies the parameterization of all unknown input disturbance observers under the assumption that the disturbance can be asymptotically estimated. However, in practical applications, exact disturbance estimation is often unnecessary or even infeasible due to modeling uncertainty or measurement noise.

To address this issue, we extend the framework in Section 3 to linear functional unknown input disturbance observers in Section 4. This extension allows controlled approximation of the disturbance through a stable filtering structure, which significantly enhances practical applicability while preserving the parameterization structure.

4. **Parameterization of All Linear Functional Unknown Input Disturbance Observers.** In this section, we define a linear functional unknown input disturbance observer for any plant $G(s) \in R(s)$ in (1) with the output disturbance $d(s)$ in (2) and clarify the existence condition for a linear functional unknown input disturbance observer for plant $G(s) \in R(s)$ with any output disturbance and the parameterization of all linear functional disturbance observers for plant $G(s)$ in (1) with any output disturbances $d(s)$ in (2).

For any $d(s)$, $x(0)$ and $u(s)$, we call $\tilde{d}(s)$ the linear functional unknown input disturbance observer for $G(s)$ in (1) if

$$d(s) - \tilde{d}(s) = F(s)d(s) \tag{15}$$

is satisfied, where $F(s) \in RH_\infty$.

Next, we clarify the parameterization of all linear functional disturbance observers with any output disturbances, which is summarized in the following theorem.

Theorem 4.1. *The system $\tilde{d}(s)$ in (3) is a linear functional disturbance observer for $G(s)$ in (1) with general exogenous output disturbances $d(s)$ in (2) if and only if $F_1(s)$, $F_2(s)$ and $F(s)$ are written by*

$$F_1(s) = I + Q(s), \tag{16}$$

$$F_2(s) = -C - Q(s)C, \tag{17}$$

and

$$F(s) = I - F_1(s) = -Q(s), \tag{18}$$

where $Q(s) \in RH_\infty^{m\times m}$ is any function. In particular, $F_1(s), F_2(s) \in RH_\infty$ and the estimation error satisfies $d(s) - \tilde{d}(s) = F(s)\,d(s)$.

To establish the sufficiency of Theorem 4.1, it is necessary to eliminate the state-dependent term in the estimation error dynamics. For this purpose, we introduce the following lemma, which provides a key algebraic condition for characterizing all solutions of the associated linear equation.

Lemma 4.1. *Assume that* $A(s) \in RH_\infty^{m\times n}$, $B(s) \in RH_\infty^{q\times p}$, $C(s) \in RH_\infty^{m\times p}$ *and*

$$\text{rank}\begin{bmatrix} A^T(s) & B^T(s) \end{bmatrix} = \gamma \tag{19}$$

are satisfied. Then there exist $X(s) \in RH_\infty^{m\times m}$ *and* $Y(s) \in RH_\infty^{m\times q}$ *satisfying*

$$X(s)A(s) + Y(s)B(s) = C(s), \tag{20}$$

if and only if there exists $U(s) \in \mathcal{U}$ *satisfying*

$$\begin{bmatrix} A(s) \\ B(s) \\ C(s) \end{bmatrix} = U(s)\begin{bmatrix} A(s) \\ B(s) \\ 0 \end{bmatrix}. \tag{21}$$

When $X_0(s) \in RH_\infty^{m\times m}$ *and* $Y_0(s) \in RH_{m\times q}$ *are solutions of (20), then all solutions of (20) are given by*

$$[X(s) \;\; Y(s)] = [X_0(s) \;\; Y_0(s)] + Q(s)[W_1(s) \;\; W_2(s)], \tag{22}$$

where $W_1(s) \in RH_\infty$ *and* $W_2(s) \in RH_\infty$ *satisfy*

$$W_1(s)A(s) + W_2(s)B(s) = 0 \tag{23}$$

and

$$\text{rank}\ [W_1(s) \;\; W_2(s)] = n + q - \gamma, \tag{24}$$

and $Q(s) \in RH_\infty^{p\times(n+q-\gamma)}$ *is any function* [21].

Using Lemma 4.1, we will show the proof of Theorem 4.1.

Proof: First, necessity is shown. That is, we show that if the system $\tilde{d}(s)$ in (3) satisfies (15), then $F_1(s), F_2(s)$ and $F(s)$ are written by (16), (17) and (18).

From (3) and $y(s) = Cx(s) + d(s)$ in (1), we have

$$\tilde{d}(s) = (F_1(s)C + F_2(s))x(s) + F_1(s)d(s). \tag{25}$$

By the definition of linear functional unknown input disturbance observer (15), the estimation error must depend only on $d(s)$ and be stable.

Therefore, the state-dependent term in (25) must vanish, which implies

$$F_1(s)C + F_2(s) = 0. \tag{26}$$

Then we have $\tilde{d}(s) = F_1(s)d(s)$ and $F(s) = I - F_1(s)$.

In order to obtain the parameterization of all $(F_1(s), F_2(s))$ satisfying (26), we apply Lemma 4.1. From Lemma 4.1, all solutions of (26) are written as

$$[F_1(s) \;\; F_2(s)] = [I \;\; -C] + Q(s)[I \;\; -C], \tag{27}$$

with arbitrary $Q(s) \in RH_\infty^{m\times m}$. This corresponds to (16), (17) and (18). This proves the necessity.

Next, the sufficiency is shown. That is, we show that if $F_1(s)$, $F_2(s)$ and $F(s)$ are given by (16), (17) and (18), then $\tilde{d}(s)$ in (3) satisfies (15). Substituting (16) and (17) into (3) gives

$$\tilde{d}(s) = (I + Q(s))y(s) + (-C - Q(s)C)x(s). \tag{28}$$

Since $y(s) = Cx(s) + d(s)$, we obtain $\tilde{d}(s) = (I + Q(s))d(s)$. Hence, the estimation error becomes

$$d(s) - \tilde{d}(s) = (I - F_1(s))d(s) = F(s)d(s). \tag{29}$$

Thus, $\tilde{d}(s)$ satisfies (15).

Thus, Theorem 4.1 has been proved. □

5. **Design Procedure.** This section shows a design procedure for constructing a disturbance observer based on Theorem 3.1. From Theorem 3.1, the observer can be parameterized by a stable, proper transfer matrix $Q(s) \in RH_\infty$.

Before introducing the two design cases, we clarify the relationship between the choice of $Q(s)$ and the core condition in Theorem 3.1. According to Theorem 3.1, the disturbance estimation error satisfies $d(s) - \tilde{d}(s) = Q(s)d(s)$, and the asymptotic estimation performance depends on the magnitude of $I - Q(j\omega)$ around the dominant frequency range of the disturbance. The condition in Equation (7),

$$\bar{\sigma}\{I - Q(j\omega_{\max})\} \simeq 0,$$

ensures that the dominant disturbance components are sufficiently preserved in the estimation process while maintaining stability. The following two design cases illustrate how different selections of $Q(s)$ satisfy this condition from both idealized and practical viewpoints.

5.1. **Case I: Unfiltered design with** $Q(s) = 0$**.** In this first case, we set $Q(s)$, which yields the simplified estimator:

$$\tilde{d}(t) = y(t) - Cx(t). \tag{30}$$

This structure directly subtracts the known output component $Cx(t)$ from the measured output $y(t)$ assuming full knowledge of the state $x(t)$ and no filtering is applied to the innovation signal. According to Theorem 3.1, this configuration ensures that

$$\lim_{t\to\infty} \left\{ d(t) - \tilde{d}(t) \right\} \simeq 0. \tag{31}$$

5.2. **Case II: Filtered design with** $Q(s) = I - \bar{Q}(s)$**.** In the second case, we introduce a normalized low-pass filter into the design. Specifically, we define

$$Q(s) = I - \bar{Q}(s), \tag{32}$$

where

$$\bar{Q}(s) = \operatorname{diag}\left\{ \frac{1}{(1+\tau_1 s)^{n_1}} \quad \cdots \quad \frac{1}{(1+\tau_m s)^{n_m}} \right\}, \tag{33}$$

where $\tau_i > 0$ $(i = 1, \ldots, m)$ is a small number and n_i $(i = 1, \ldots, m)$ is arbitrary positive integer.

In this design, $I - Q(s) = \bar{Q}(s)$ holds. By selecting $\bar{Q}(s)$ as a low-pass filter, the condition $\bar{\sigma}\{I - Q(j\omega_{\max})\} \simeq 0$ in Equation (7) is approximately satisfied for disturbances whose dominant frequency components lie within the passband of $\bar{Q}(s)$. Consequently, high-frequency noise components are attenuated while the low-frequency disturbance information is preserved.

This leads to the filtered estimator:

$$\tilde{d}(s) = \left(I - \bar{Q}(s)\right)(y(s) - Cx(s)). \tag{34}$$

This form introduces a normalized low-pass filter on the innovation signal, where the time constant τ_i $(i = 1, \ldots, m)$ controls the trade-off between tracking speed and noise attenuation. A smaller τ_i yields faster response, while a larger τ_i $(i = 1, \ldots, m)$ leads to smoother but slower estimation.

6. **Simulation Results.** This section illustrates simulation results to show the effectiveness of the proposed method and to compare two design cases for the disturbance observer derived in Section 5. The objective is to validate the performance of the observer under different choices of the filter matrix $Q(s)$ using a known, externally generated disturbance.

6.1. **System and disturbance setup.** Consider the problem of obtaining the parameterization of all unknown input disturbance observers under general exogenous disturbances for the plant expressed by

$$\begin{cases} \dot{\xi}(t) = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \xi(t) \\ d(t) = [1 \quad 0]\xi(t) \end{cases}, \tag{35}$$

where

$$\xi(0) = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}. \tag{36}$$

The control input $u(t)$ is given by

$$u(t) = \sin(t). \tag{37}$$

6.2. **Result of case I.** In this subsection, we show the response of $d(t)$ and $\tilde{d}(t)$ when $Q(s)$ is designed using case I.

The response of $d(t)$ and $\tilde{d}(t)$ is illustrated in Figure 1. Here, the solid line shows the response of $d(t)$ and the dotted line shows that of $\tilde{d}(t)$. Figure 1 shows that $\tilde{d}(t)$ accurately reconstructs $d(t)$ with negligible error and no observable delay. Figure 2 illustrates the estimation error, which converges rapidly and remains zero throughout the simulation.

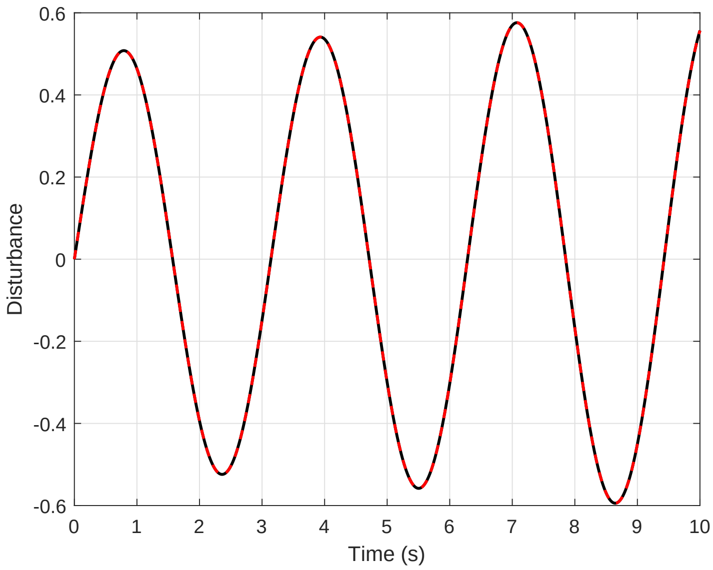

FIGURE 1. Response of disturbance $d(t)$ and estimated disturbance $\tilde{d}(t)$ using $Q(s) = 0$

6.3. **Result of case II.** In this subsection, we show the response of $d(t)$ and $\tilde{d}(t)$ when $Q(s)$ is designed using case II.

We now apply a normalized low-pass filter with a very small time constant with $\tau = 0.0001$ as

$$\bar{Q}(s) = \frac{1}{1 + 0.0001s}. \tag{38}$$

This $\bar{Q}(s)$ has an extremely high cutoff frequency, effectively passing nearly all frequency components of the innovation signal. As a result, the observer behaves almost identically to the unfiltered case $Q(s) = 0$.

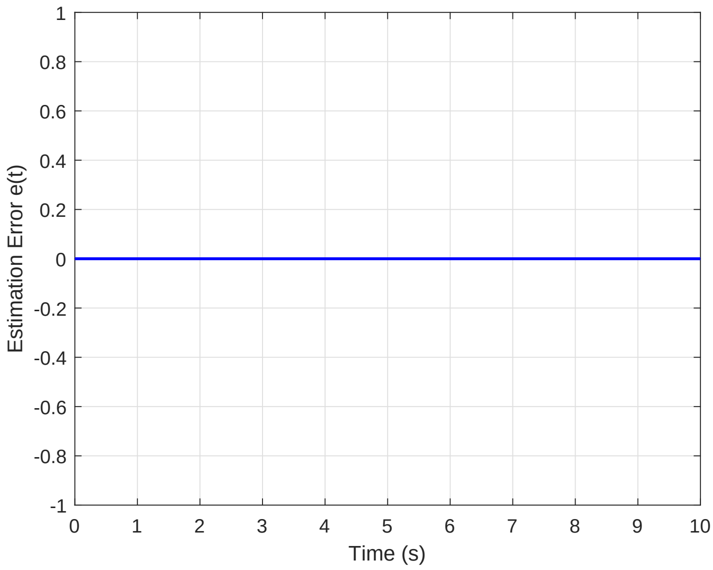

FIGURE 2. Response of estimation error $e(t)$ for the case $Q(s) = 0$

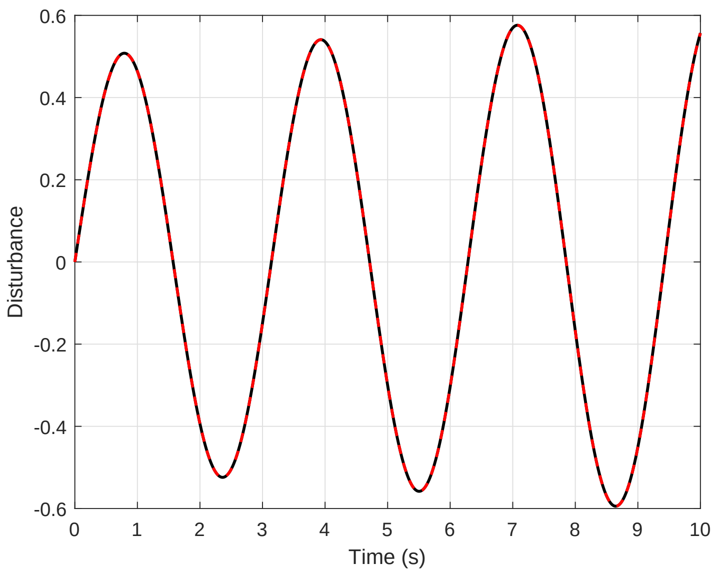

FIGURE 3. Response of disturbance $d(t)$ and estimated disturbance $\tilde{d}(t)$ using $Q(s) = I - \bar{Q}(s)$

Simulation result is shown in Figure 3. Here, the solid line shows the response of $d(t)$ and the dotted line shows that of $\tilde{d}(t)$. From Figure 3, we confirm that the estimated disturbance $\tilde{d}(t)$ closely matches the disturbance $d(t)$ with negligible delay and error. The estimation error as shown in Figure 4 also remains extremely small and comparable to case I.

6.4. **Linearized pendulum system.** To further demonstrate the applicability of the proposed method to systems beyond simple linear examples, we consider a nonlinear pendulum system and apply the proposed observer to its linearized model. This example illustrates that the proposed framework can be effectively used for nonlinear systems through standard linearization techniques.

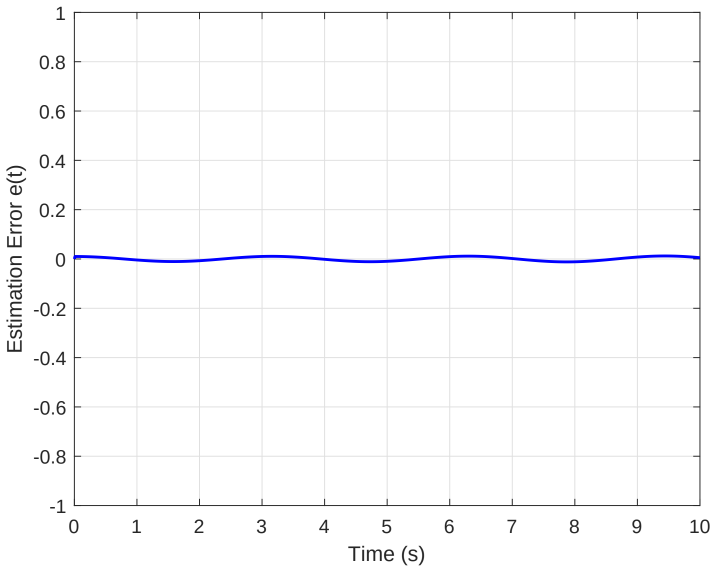

FIGURE 4. Response of estimation error $e(t)$ for the case $Q(s) = I - \bar{Q}(s)$

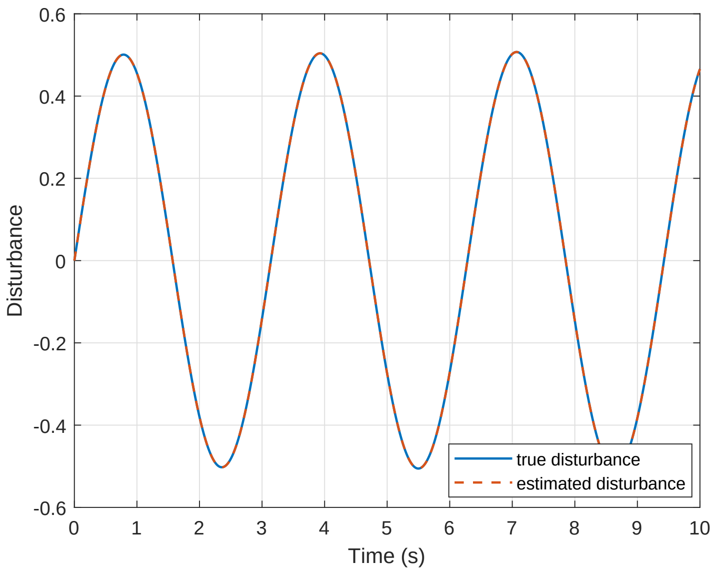

FIGURE 5. Response of disturbance $d(t)$ and estimated disturbance $\tilde{d}(t)$ for the linearized pendulum system

The nonlinear pendulum dynamics are given by

$$\ddot{\theta}(t) + \frac{g}{l}\sin\theta(t) = u(t) + d(t), \tag{39}$$

where $\theta(t)$ is the angular displacement, $u(t)$ is the control input, $d(t)$ is an unknown output disturbance, g is the gravitational acceleration, and l is the pendulum length.

Assuming small angular motion around the equilibrium point $\theta(t) \approx 0$, the system can be linearized as

$$\ddot{\theta}(t) + \frac{g}{l}\theta(t) = u(t) + d(t). \tag{40}$$

Defining the state variables as $x_1(t) = \theta(t)$ and $x_2(t) = \dot{\theta}(t)$, the linearized state-space representation becomes

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -\dfrac{g}{l} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = [1 \quad 0] x(t) + d(t) \end{cases} . \tag{41}$$

In this simulation, the filtered observer design described in case II is applied to the linearized model. The free parameter $Q(s)$ is chosen in the same manner and no additional tuning is required. This ensures consistency with the previously presented design procedure.

Simulation results demonstrate that the estimated disturbance $\tilde{d}(t)$ accurately tracks the true disturbance $d(t)$ for the linearized pendulum system. These results confirm that the proposed parameterization-based observer can be effectively applied to nonlinear systems through linearization around an equilibrium point, thereby demonstrating the generality of the proposed method.

7. **Conclusions.** This paper has clarified the parameterization of all unknown input disturbance observers for plants with general exogenous output disturbances. In addition, the parameterization of all linear functional unknown input disturbance observers is presented. The proposed method enables the estimation of unknown output disturbances using only the measurable system output $y(t)$ and known system state $x(t)$, without requiring any model of the disturbance generator. A key feature of this approach is the freedom to choose a stable filter $Q(s) \in RH_\infty$, which directly determines the structure of the disturbance estimator. Two design cases are investigated:

1) Unfiltered estimator ($Q(s) = 0$), which provides exact and fast estimation in the ideal case where full-state feedback is available and no measurement noise is present.
2) Filtered estimator $\left(Q(s) = I - \bar{Q}(s)\right)$, with $\bar{Q}(s)$ as a normalized low-pass filter. This design is expected to offer robustness to high-frequency noise at the expense of amplitude attenuation and phase lag.

Simulation results using a general exogenous disturbance confirm the theoretical convergence property: $\lim_{t\to\infty}\left(d(t) - \tilde{d}(t)\right) \simeq 0$. In the unfiltered case, the disturbance estimate perfectly matches the true disturbance. In the filtered case, the estimate remains accurate when the filter time constant is small (e.g., $\tau = 0.0001$), and performance degrades gradually with larger τ.

The findings demonstrate that the parameterized observer structure is flexible and effective. This design enables the engineer to achieve a balance between estimation accuracy and robustness, making it suitable for real-world applications where sensor noise or model mismatch may occur. In practical implementations, the tuning of the free parameter $Q(s)$ can be performed by selecting the filter bandwidth according to the dominant frequency range of the disturbance and the noise level. Specifically, a smaller filter time constant improves noise attenuation but simultaneously leads to estimation delay, while a larger bandwidth enhances tracking performance even though it increases noise sensitivity.

Future work includes extending this framework to the case where $x(t)$ is not directly measurable and must be estimated. In the current formulation, the availability of the state variable is essential for constructing the unknown input disturbance observer, and the proposed parameterization cannot be directly applied when only output measurements are available. A possible extension is to integrate a state observer or output-based estimation scheme into the proposed parameterization framework, which would allow disturbance estimation under partial state information.

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