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A LINGUISTIC INTERVAL-VALUED SPHERICAL FUZZY POWER WEIGHTED AVERAGE OPERATOR AND ITS APPLICATION IN MULTI-ATTRIBUTE GROUP DECISION-MAKING

Liwei Sha[1,2], Huaizhe Zhao[1,2], Jianzhong Huang[3], Tie Hou[1,*] and Li Zou[1,2]

[1]School of Computer and Artificial Intelligence
[2]Computational Intelligence Center
Shandong Jianzhu University
No. 1000, Fengming Road, Lingang Development Zone, Jinan 250101, P. R. China
{ 2023115017; 2023115009}@stu.sdjzu.edu.cn; zouli20@sdjzu.edu.cn
[*]Corresponding author: houtie2017@sdjzu.edu.cn

[3]Shandong Medical Device and Pharmaceutical Packaging Inspection Institute
No. 15166, Century Avenue, High Tech District, Jinan 250101, P. R. China
617131785@qq.com

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Abstract. *Aggregation and decision-making of linguistic evaluation information in uncertain environments has garnered considerable research attention. This paper proposes a novel Linguistic Interval-Valued Spherical Fuzzy Power Weighted Average operator and develops a multi-attribute group decision-making method based on this operator to tackle qualitative linguistic assessments. First, we define a support degree function between linguistic interval-valued spherical fuzzy numbers to quantify the correlation among expert evaluations. Subsequently, we construct a Linguistic Interval-Valued Spherical Fuzzy Power Weighted Average operator that incorporates information relevance to effectively fuse multi-expert evaluations. Finally, we develop an objective attribute weight determination method based on similarity analysis of expert assessments, establish a complete multi-attribute group decision-making method framework, and validate the proposed method's rationality and effectiveness through an illustrative example.*

Keywords: Linguistic interval-valued spherical fuzzy sets, Multi-attribute group decision-making, Power weighted average operator

1. **Introduction.** In real-world decision-making scenarios, decision-makers often rely on qualitative linguistic evaluations due to the inherent uncertainty and fuzziness of objective things. Recognizing the need to effectively manage the inherent fuzziness and incompleteness commonly encountered in real-world information, Zadeh proposed fuzzy sets in 1965 [1], using real numbers between 0 and 1 to quantify the relationship between alternatives and various rules. To address the limitation of fuzzy sets considering only membership degrees, Atanassov extended fuzzy sets in 1986 by simultaneously considering membership, non-membership, and hesitation degrees, introducing intuitionistic fuzzy sets [2]. To account for cases where membership degrees and other values are interval numbers, Atanassov and Gargov proposed interval-valued intuitionistic fuzzy sets in 1989 [3]. In real-world decision-making processes, the sum of membership and non-membership degrees provided by decision-makers for alternatives may exceed 1, whereas the sum of their

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squares may be less than or equal to 1. In light of this, Atanassov further extended intuitionistic fuzzy sets in 1989, proposing Type-2 intuitionistic fuzzy sets [4], which provide decision-makers with a broader preference domain.

In 2019, Kutlu Gündoğdu and Kahraman introduced spherical fuzzy sets [5], a novel extension of fuzzy sets represented by four functions: membership, non-membership, hesitation, and abstention degrees. Spherical fuzzy sets have been widely applied in decision-making [6, 7, 8, 9], medical diagnosis [10], pattern recognition [11, 12], and other research areas. However, as research progresses and theoretical studies are applied to real-world problems, many issues in reality are too complex for precise numerical descriptions to fully capture, especially in scenarios involving human cognition and decision-making. To address this challenge, Zadeh's linguistic variables [13] enable decision-makers to solve practical problems through qualitative evaluations. Consequently, numerous scholars have proposed extensions of spherical fuzzy sets to enhance their modeling capabilities [14, 15, 16]. Jin et al. [17] combined the notion of linguistic fuzzy set and spherical fuzzy set and proposed linguistic spherical fuzzy weighted averaging and geometric operators based on spherical fuzzy numbers. While their work facilitates the direct use of language in evaluations, a drawback is its confinement to single-point linguistic terms rather than intervals. This lack of an interval-valued structure limits the model's flexibility. Liu et al. [18] proposed the concept of the linguistic interval-valued spherical fuzzy set (LIVSFS) and discussed the basic operational laws. Nevertheless, the aggregation of LIVSFS information remains an underexplored area.

The theoretical frameworks and methods of multi-attribute group decision-making (MAGDM) have found extensive applications across diverse domains including engineering, education, economics, management, and healthcare. Recently, many scholars have proposed various MAGDM methodologies based on the above research to address various challenges in complex decision environments [19, 20, 21, 22, 23]. Xu et al. [24] discussed the concept of the chi-square divergence measure for spherical fuzzy sets, and proposed an MAGDM method based on the TOPSIS method. While their approach demonstrates proficiency in managing numerical spherical fuzzy data, it lacks the necessary capability to process linguistic or interval-valued linguistic information, thus failing to accommodate the qualitative nature of human expert assessments. While Hussain et al. [25] made significant contributions to the development of interval-valued T-spherical fuzzy aggregation operators for healthcare decision-making, their approach remains constrained by its dependence on numerical interval representations, which cannot effectively capture and process the qualitative linguistic evaluations naturally provided by human experts in real-world group decision-making contexts. While Garg and Kumar [26] introduced linguistic interval-valued intuitionistic fuzzy sets for group decision-making, their model's fundamental limitation lies in its restriction to a two-dimensional membership-nonmembership framework, which fails to capture the crucial hesitancy dimension and richer uncertainty representation offered by linguistic spherical fuzzy models.

To represent decision-makers' preferences using linguistic interval values and apply LIVSFS to decision-making problems, this paper firstly proposes the Linguistic Interval-Valued Spherical Fuzzy Power Weighted Average (*LIVSFPWA*) operator and the Linguistic Interval-Valued Spherical Fuzzy Power Average (*LIVSFPA*) operator based on the fundamental operations and properties of LIVSFS, aiming to aggregate evaluation information. Secondly, a method for determining the weights of unknown attributes is proposed by utilizing the similarity among multi-expert correlation information. Finally, an MAGDM method under the linguistic interval-valued spherical fuzzy environment is developed based on the proposed operators, and the effectiveness and rationality of the proposed method are demonstrated through an illustrative example of teacher selection.

The rest of this paper is organized as follows. Section 2 reviews the fundamental operations and properties of LIVSFS, as well as aggregation operators for MAGDM methods. Section 3 introduces the *LIVSFPWA* operator and the *LIVSFPA* operator for aggregating evaluation information, proposes a method for determining unknown attribute weights and investigates the MAGDM method under linguistic interval-valued spherical fuzzy environment. Section 4 validates the rationality and effectiveness of the proposed method through an illustrative example. Finally, Section 5 concludes the paper and suggests potential directions for future research.

2. **Preliminaries.** To better handle uncertain information in decision-making processes, this paper adopts linguistic interval values to represent decision-makers' preferences, thereby enhancing their freedom of preference expression and assisting them in addressing decision problems under linguistic uncertainty. This section briefly presents the relevant theoretical foundations, including linguistic term sets, linguistic interval-valued spherical fuzzy sets, and related concepts, which provide the theoretical basis for the following sections.

Definition 2.1. [27] *Let $S = \{s_0, s_1, \ldots, s_g\}$ be a finite set of linguistic items consisting of an odd number of items, where g is a positive integer. If S satisfies the following conditions:*

1. *Orderliness: $s_i \geq s_j \Leftrightarrow i \geq j$;*
2. *Negative operator: $\neg(s_{-i}) = s_i$, especially, $\neg(s_0) = s_0$;*
3. *Maximum operator: $s_i \geq s_j \Leftrightarrow \max(s_i, s_j) = s_i$;*
4. *Minimum operator: $s_i \leq s_j \Leftrightarrow \min(s_i, s_j) = s_i$.*

then S is called a linguistic term set.

Definition 2.2. [18] *Let $X \neq \varphi$ be a finite universe and $\overline{S} = \{s_z | s_0 \leq s_z \leq s_h, z \in [0, h]\}$ be a continuous linguistic term set. A Linguistic Interval-Valued Spherical Fuzzy Set (LIVSFS) A in X is defined as*

$$A = \{\langle x, (\tilde{s}_{uA}(x), \tilde{s}_{\pi A}(x), \tilde{s}_{vA}(x))\rangle | x \in X\}, \tag{1}$$

where $\tilde{s}_{uA}(x) = \left[s^L_{uA}(x), s^U_{uA}(x)\right]$, $\tilde{s}_{\pi A}(x) = \left[s^L_{\pi A}(x), s^U_{\pi A}(x)\right]$ and $\tilde{s}_{vA}(x) = \left[s^L_{vA}(x), s^U_{vA}(x)\right]$ represent the membership degree, the hesitancy degree and the non-membership degree of x to A, respectively, such that for each $x \in X$,

$$0 \leq \left(\tilde{s}^U_{uA}(x)\right)^2 + \left(\tilde{s}^U_{\pi A}(x)\right)^2 + \left(\tilde{s}^U_{vA}(x)\right)^2 \leq h^2 \tag{2}$$

Then $\tilde{s}_{rA}(x) = \left[r^L_A(x), r^U_A(x)\right]$ is the waiver degree of x to A, where

$$r^L_A(x) = \sqrt{h^2 - \left(\left(u^L_A(x)\right)^2 + \left(\pi^L_A(x)\right)^2 + \left(v^L_A(x)\right)^2\right)},$$

$$r^U_A(x) = \sqrt{h^2 - \left(\left(u^U_A(x)\right)^2 + \left(\pi^U_A(x)\right)^2 + \left(v^U_A(x)\right)^2\right)}.$$

The set of all LIVSFS in X is denoted by LIVSF(X).

Usually, $\alpha_A = \left(\left[s^L_{uA}, s^U_{uA}\right], \left[s^L_{\pi A}, s^U_{\pi A}\right], \left[s^L_{vA}, s^U_{vA}\right]\right)$ is called a Linguistic Interval-Valued Spherical Fuzzy Number (LIVSFN) of A.

For the sake of readability, we denote the LIVSFN by $\alpha = ([s_a, s_b], [s_c, s_d], [s_e, s_f])$, where $[s_a, s_b] \subseteq [s_0, s_h]$, $[s_c, s_d] \subseteq [s_0, s_h]$, $[s_e, s_f] \subseteq [s_0, s_h]$, $0 \leq b^2 + d^2 + f^2 \leq h^2$ and also $s_a, s_b, s_c, s_d, s_e, s_f \in \overline{S}$.

Definition 2.3. [18] *Let $\alpha_1 = ([s_{a_1}, s_{b_1}], [s_{c_1}, s_{d_1}], [s_{e_1}, s_{f_1}])$, $\alpha_2 = \left([s_{a_2}, s_{b_2}], [s_{c_2}, s_{d_2}], [s_{e_2}, s_{f_2}]\right)$ be two LIVSFNs, λ is a positive integer, then*

1. $\alpha_1 \oplus \alpha_2 = \left(\left[s_{\sqrt{a_1^2+a_2^2-\frac{a_1^2a_2^2}{h^2}}}, s_{\sqrt{b_1^2+b_2^2-\frac{b_1^2b_2^2}{h^2}}}\right], \left[s_{\frac{c_1c_2}{h}}, s_{\frac{d_1d_2}{h}}\right], \left[s_{\frac{e_1e_2}{h}}, s_{\frac{f_1f_2}{h}}\right]\right)$;
2. $\alpha_1 \otimes \alpha_2$
$= \left(\left[s_{\frac{a_1a_2}{h}}, s_{\frac{b_1b_2}{h}}\right], \left[s_{\sqrt{c_1^2+c_2^2-\frac{c_1^2c_2^2}{h^2}}}, s_{\sqrt{d_1^2+d_2^2-\frac{d_1^2d_2^2}{h^2}}}\right], \left[s_{\sqrt{e_1^2+e_2^2-\frac{e_1^2e_2^2}{h^2}}}, s_{\sqrt{f_1^2+f_2^2-\frac{f_1^2f_2^2}{h^2}}}\right]\right)$;
3. $\lambda\alpha_1 = \left(\left[s_{\sqrt{h^2-h^2\left(1-\frac{a_1^2}{h^2}\right)^\lambda}}, s_{\sqrt{h^2-h^2\left(1-\frac{b_1^2}{h^2}\right)^\lambda}}\right], \left[s_{h\left(\frac{c_1}{h}\right)^\lambda}, s_{h\left(\frac{d_1}{h}\right)^\lambda}\right], \left[s_{h\left(\frac{e_1}{h}\right)^\lambda}, s_{h\left(\frac{f_1}{h}\right)^\lambda}\right]\right)$;
4. $\alpha_1^\lambda = \begin{pmatrix} \left[s_{h\left(\frac{a_1}{h}\right)^\lambda}, s_{h\left(\frac{b_1}{h}\right)^\lambda}\right], \left[s_{\sqrt{h^2+h^2\left(1-\frac{c_1^2}{h^2}\right)^\lambda}}, s_{\sqrt{h^2+h^2\left(1-\frac{d_1^2}{h^2}\right)^\lambda}}\right], \\ \left[s_{\sqrt{h^2+h^2\left(1-\frac{e_1^2}{h^2}\right)^\lambda}}, s_{\sqrt{h^2-h^2\left(1-\frac{f_1^2}{h^2}\right)^\lambda}}\right] \end{pmatrix}$.

Definition 2.4. [18] *Let $\alpha = ([s_a, s_b], [s_c, s_d], [s_e, s_f])$ be an LIVSFN. The score function and the accuracy function of α are respectively defined as*

$$S(\alpha) = s_{\sqrt{\frac{2h^2+a^2+b^2-e^2-f^2-\left(\frac{c}{2}\right)^2-\left(\frac{d}{2}\right)^2}{4}}}, \tag{3}$$

$$H(\alpha) = s_{\sqrt{\frac{a^2+b^2+e^2+f^2+c^2+d^2}{2}}}. \tag{4}$$

Then, the comparison laws based on the score function and the accuracy function between two different LIVSFNs will be defined as follows.

Theorem 2.1. [18] *Let $\alpha_1 = ([s_{a_1}, s_{b_1}], [s_{c_1}, s_{d_1}], [s_{e_1}, s_{f_1}])$ and $\alpha_2 = ([s_{a_2}, s_{b_2}], [s_{c_2}, s_{d_2}], [s_{e_2}, s_{f_2}])$ be two different LIVSFNs, and the comparison laws be defined as follows:*

1. *If $S(\alpha_1) > S(\alpha_2)$, then $\alpha_1 > \alpha_2$;*
2. *If $S(\alpha_1) < S(\alpha_2)$, then $\alpha_1 < \alpha_2$;*
3. *If $S(\alpha_1) = S(\alpha_2)$, then*
   (a) *If $H(\alpha_1) > H(\alpha_2)$, then $\alpha_1 > \alpha_2$;*
   (b) *If $H(\alpha_1) < H(\alpha_2)$, then $\alpha_1 < \alpha_2$;*
   (c) *If $H(\alpha_1) = H(\alpha_2)$, then $\alpha_1 = \alpha_2$.*

Definition 2.5. [18] *The Power Average (PA) operator is a mapping $PA : \Omega^n \to \Omega$, satisfying*

$$PA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{i=1}^{n} \frac{(1+T(\alpha_i))\alpha_i}{\sum_{j=1}^{n}(1+T(\alpha_j))}, \tag{5}$$

where $T(\alpha_i) = \sum_{j=1}^{n} Sup(\alpha_i, \alpha_j)$, and $Sup(\alpha_i, \alpha_j)$ denotes the support degree of α_i to α_j, and the following properties hold:

1. *$Sup(\alpha_i, \alpha_j) \in [0, 1]$;*
2. *$Sup(\alpha_i, \alpha_j) = Sup(\alpha_j, \alpha_i)$;*
3. *If $|\alpha_i - \alpha_j| \le |\alpha_s - \alpha_t|$, then $Sup(\alpha_i, \alpha_j) \ge Sup(\alpha_s, \alpha_t)$.*

Clearly, the more similar α_i and α_j are (i.e., the closer their distance), the greater their support degree will be.

Definition 2.6. [18] *The Ordered Weighted Averaging (OWA) operator is a mapping* $OWA : \Omega^n \to \Omega$*, satisfying*

$$OWA(\alpha_1, \alpha_2, \dots, \alpha_n) = \sum_{i=1}^{n} \beta_i \varpi_i, \tag{6}$$

where β_i $(1 \le i \le n)$ *denotes the* i*-th largest element in* $(\alpha_1, \dots, \alpha_n)$, $\varpi = \{\varpi_1, \dots, \varpi_n\}$ *represents an* n*-dimensional positional weight vector satisfying* $\varpi_i \in [0, 1]$ *and* $\sum_{i=1}^{n} \varpi_i = 1$.

The OWA operator first sorts the input data $(\alpha_1, \dots, \alpha_n)$, in descending order, yielding the reordered sequence $(\beta_1, \dots, \beta_n)$. The aggregated result is then computed by applying the weights ϖ_i $(1 \le i \le n)$ to this reordered sequence. Importantly, the weight ϖ_i is unrelated to the original data point α_i, but is instead exclusively tied to the position i in the descending-sorted sequence β_i.

3. The *LIVSFPWA* Operator and the Proposed MAGDM Framework.

This section establishes the theoretical foundation of our proposed framework. We first introduce the Linguistic Interval-Valued Spherical Fuzzy Power Weighted Average (*LIVSFPWA*) operator, which extends Yager's power average operator [28] to effectively handle linguistic interval-valued spherical fuzzy information. Furthermore, we develop a similarity-based method for determining attribute weights objectively from expert evaluation data and an MAGDM framework.

3.1. *LIVSFPWA* operator and properties.

Definition 3.1. *Let* $\alpha_i = ([s_{a_i}, s_{b_i}], [s_{c_i}, s_{d_i}], [s_{e_i}, s_{f_i}])$ $(i = 1, 2, \dots, n)$ *be a collection of* n *LIVSFNs, and* $\boldsymbol{\omega} = (\omega_1, \omega_2, \dots, \omega_n)^\top$ *be their associated weight vector satisfying* $\omega_i > 0$ *and* $\sum_{i=1}^{n} \omega_i = 1$*. The LIVSFPWA operator is a mapping* $LIVSFPWA : \Omega^n \to \Omega$ *defined as*

$$LIVSFPWA(\alpha_1, \alpha_2, \dots, \alpha_n) = \bigoplus_{i=1}^{n} \frac{\omega_i (1 + T(\alpha_i))}{\sum_{j=1}^{n} \omega_j (1 + T(\alpha_j))} \alpha_i. \tag{7}$$

The total support measure $T(\alpha_i)$ *for* α_i *is computed as*

$$T(\alpha_i) = \sum_{\substack{j=1 \\ j \ne i}}^{n} \omega_i Sup(\alpha_i, \alpha_j), \tag{8}$$

where $Sup(\alpha_i, \alpha_j) = 1 - d(\alpha_i, \alpha_j)$ *represents the support degree between* α_i *and* α_j.

The support degree $Sup(\alpha_i, \alpha_j)$ *satisfies the following properties:*

1. *Normalization:* $Sup(\alpha_i, \alpha_j) \in [0, 1]$;
2. *Symmetry:* $Sup(\alpha_i, \alpha_j) = Sup(\alpha_j, \alpha_i)$;
3. *Monotonicity: If* $d(\alpha_i, \alpha_s) < d(\alpha_i, \alpha_t)$*, then* $Sup(\alpha_i, \alpha_s) \ge Sup(\alpha_i, \alpha_t)$.

where $d(\alpha_i, \alpha_j)$ *denotes the metric distance between linguistic interval-valued spherical fuzzy numbers* α_i *and* α_j.

Clearly, $Sup(\alpha_i, \alpha_j)$ *serves as a similarity measure – The more similar two linguistic interval-valued spherical fuzzy numbers are, the greater their support degree becomes.*

When the weight vector is uniformly distributed as $\boldsymbol{\omega} = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)^\top$*, the LIVSFPWA operator reduces to the Linguistic Interval-Valued Spherical Fuzzy Power Average (LIVSFPA) operator:*

$$LIVSFPA(\alpha_1, \alpha_2, \dots, \alpha_n) = \bigoplus_{i=1}^{n} \frac{1 + T(\alpha_i)}{\sum_{j=1}^{n} (1 + T(\alpha_j))} \alpha_i, \tag{9}$$

where the total support degree $T(\alpha_i)$ is calculated as

$$T(\alpha_i) = \frac{1}{n}\sum_{\substack{j=1\\ j\neq i}}^{n} Sup(\alpha_i, \alpha_j). \tag{10}$$

Theorem 3.1. *Let $\alpha_i = ([s_{a_i}, s_{b_i}], [s_{c_i}, s_{d_i}], [s_{e_i}, s_{f_i}])$ $(i = 1, 2, \ldots, n)$ be a collection of LIVSFNs, and $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_n)^\top$ be their associated weight vector satisfying $\omega_i > 0$ and $\sum_{i=1}^{n}\omega_i = 1$. Let $\phi_i = \frac{\omega_i(1+T(\alpha_i))}{\sum_{j=1}^{n}\omega_j(1+T(\alpha_j))}$, and then the LIVSFPWA operator is formulated as*

$$\begin{aligned}
&LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n)\\
&= \bigoplus_{i=1}^{n} \frac{\omega_i(1+T(\alpha_i))\alpha_i}{\sum_{j=1}^{n}\omega_j(1+T(\alpha_j))}\\
&= \left(\begin{bmatrix} s_{\sqrt{h^2-h^2\prod_{i=1}^{n}\left(1-\frac{a_i^2}{h^2}\right)^{\phi_i}}}, \\ s_{\sqrt{h^2-h^2\prod_{i=1}^{n}\left(1-\frac{b_i^2}{h^2}\right)^{\phi_i}}} \end{bmatrix}, \begin{bmatrix} s_{h\prod_{i=1}^{n}\left(\frac{c_i}{h}\right)^{\phi_i}}, \\ s_{h\prod_{i=1}^{n}\left(\frac{d_i}{h}\right)^{\phi_i}} \end{bmatrix}, \begin{bmatrix} s_{h\prod_{i=1}^{n}\left(\frac{e_i}{h}\right)^{\phi_i}}, \\ s_{h\prod_{i=1}^{n}\left(\frac{f_i}{h}\right)^{\phi_i}} \end{bmatrix} \right).
\end{aligned} \tag{11}$$

Moreover, the aggregated result $LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n)$ remains an LIVSFN.

Proof: (1) According to Definition 3.1, $LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \bigoplus_{i=1}^{n}(\phi_i\alpha_i)$ and by Definition 2.3,

$$\phi_i\alpha_i = \left(\begin{bmatrix} s_{\sqrt{h^2-h^2\left(1-\frac{a_i^2}{h^2}\right)^{\phi_i}}}, \\ s_{\sqrt{h^2-h^2\left(1-\frac{b_i^2}{h^2}\right)^{\phi_i}}} \end{bmatrix}, \begin{bmatrix} s_{h\left(\frac{c_i}{h}\right)^{\phi_i}}, \\ s_{h\left(\frac{d_i}{h}\right)^{\phi_i}} \end{bmatrix}, \begin{bmatrix} s_{h\left(\frac{e_i}{h}\right)^{\phi_i}}, \\ s_{h\left(\frac{f_i}{h}\right)^{\phi_i}} \end{bmatrix} \right).$$

We establish this result by mathematical induction on n, where n denotes the number of linguistic interval-valued spherical fuzzy numbers.

Base case $n = 2$: $LIVSFPWA(\alpha_1, \alpha_2) = \phi_1\alpha_1 \oplus \phi_2\alpha_2$

$$= \left(\begin{bmatrix} s_{\sqrt{\left(h^2-h^2\left(1-\frac{a_1^2}{h^2}\right)^{\phi_1}\right)+\left(h^2-h^2\left(1-\frac{a_2^2}{h^2}\right)^{\phi_2}\right)-\frac{\left(h^2-h^2\left(1-\frac{a_1^2}{h^2}\right)^{\phi_1}\right)\left(h^2-h^2\left(1-\frac{a_2^2}{h^2}\right)^{\phi_2}\right)}{h^2}}}, \\ s_{\sqrt{\left(h^2-h^2\left(1-\frac{b_1^2}{h^2}\right)^{\phi_1}\right)+\left(h^2-h^2\left(1-\frac{b_2^2}{h^2}\right)^{\phi_2}\right)-\frac{\left(h^2-h^2\left(1-\frac{b_1^2}{h^2}\right)^{\phi_1}\right)\left(h^2-h^2\left(1-\frac{b_2^2}{h^2}\right)^{\phi_2}\right)}{h^2}}} \end{bmatrix}, \begin{matrix} \begin{bmatrix} s_{h\left(\frac{c_1}{h}\right)^{\phi_1}\left(\frac{c_2}{h}\right)^{\phi_2}}, \\ s_{h\left(\frac{d_1}{h}\right)^{\phi_1}\left(\frac{d_2}{h}\right)^{\phi_2}} \end{bmatrix}, \begin{bmatrix} s_{h\left(\frac{e_1}{h}\right)^{\phi_1}\left(\frac{e_2}{h}\right)^{\phi_2}}, \\ s_{h\left(\frac{f_1}{h}\right)^{\phi_1}\left(\frac{f_2}{h}\right)^{\phi_2}} \end{bmatrix} \end{matrix} \right)$$

$$= \left(\begin{bmatrix} s_{\sqrt{h^2-h^2\left(1-\frac{a_1^2}{h^2}\right)^{\phi_1}\left(1-\frac{a_2^2}{h^2}\right)^{\phi_2}}}, \\ s_{\sqrt{h^2-h^2\left(1-\frac{b_1^2}{h^2}\right)^{\phi_1}\left(1-\frac{b_2^2}{h^2}\right)^{\phi_2}}} \end{bmatrix}, \begin{bmatrix} s_{h\left(\frac{c_1}{h}\right)^{\phi_1}\left(\frac{c_2}{h}\right)^{\phi_2}}, \\ s_{h\left(\frac{d_1}{h}\right)^{\phi_1}\left(\frac{d_2}{h}\right)^{\phi_2}} \end{bmatrix}, \begin{bmatrix} s_{h\left(\frac{e_1}{h}\right)^{\phi_1}\left(\frac{e_2}{h}\right)^{\phi_2}}, \\ s_{h\left(\frac{f_1}{h}\right)^{\phi_1}\left(\frac{f_2}{h}\right)^{\phi_2}} \end{bmatrix} \right).$$

Thus, for $n = 2$, Equation (11) holds.

Assume Equation (11) holds for $n = k$. We have to show that Equation (11) holds for $n = k + 1$. By induction hypothesis and Definition 2.3 we have

$$\begin{aligned}
&LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_{k+1})\\
&= LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_k) \oplus \phi_{k+1}\alpha_{k+1}
\end{aligned}$$

$$= \left(\begin{bmatrix} s_{\sqrt{h^2-h^2\prod_{i=1}^{k}\left(1-\frac{a_i^2}{h^2}\right)^{\phi_i}}}, \\ s_{\sqrt{h^2-h^2\prod_{i=1}^{k}\left(1-\frac{b_i^2}{h^2}\right)^{\phi_i}}} \end{bmatrix}, \begin{bmatrix} s_{h\left(\prod_{i=1}^{k}\left(\frac{c_i}{h}\right)^{\phi_i}\right)}, \\ s_{h\left(\prod_{i=1}^{k}\left(\frac{d_i}{h}\right)^{\phi_i}\right)} \end{bmatrix}, \begin{bmatrix} s_{h\left(\prod_{i=1}^{k}\left(\frac{e_i}{h}\right)^{\phi_i}\right)}, \\ s_{h\left(\prod_{i=1}^{k}\left(\frac{f_i}{h}\right)^{\phi_i}\right)} \end{bmatrix} \right)$$

$$\oplus \left(\begin{bmatrix} s_{\sqrt{h^2-h^2\left(1-\frac{a_{k+1}^2}{h^2}\right)^{\phi_{k+1}}}}, \\ s_{\sqrt{h^2-h^2\left(1-\frac{b_{k+1}^2}{h^2}\right)^{\phi_{k+1}}}} \end{bmatrix}, \begin{bmatrix} s_{h\left(\frac{c_{k+1}}{h}\right)^{\phi_{k+1}}}, \\ s_{h\left(\frac{d_{k+1}}{h}\right)^{\phi_{k+1}}} \end{bmatrix}, \begin{bmatrix} s_{h\left(\frac{e_{k+1}}{h}\right)^{\phi_{k+1}}}, \\ s_{h\left(\frac{f_{k+1}}{h}\right)^{\phi_{k+1}}} \end{bmatrix} \right)$$

$$= \left(\begin{bmatrix} s_{\sqrt{h^2-h^2\prod_{i=1}^{k+1}\left(1-\frac{a_i^2}{h^2}\right)^{\phi_i}}}, \\ s_{\sqrt{h^2-h^2\prod_{i=1}^{k+1}\left(1-\frac{b_i^2}{h^2}\right)^{\phi_i}}} \end{bmatrix}, \begin{bmatrix} s_{h\left(\prod_{i=1}^{k+1}\left(\frac{c_i}{h}\right)^{\phi_i}\right)}, \\ s_{h\left(\prod_{i=1}^{k+1}\left(\frac{d_i}{h}\right)^{\phi_i}\right)} \end{bmatrix}, \begin{bmatrix} s_{h\left(\prod_{i=1}^{k+1}\left(\frac{e_i}{h}\right)^{\phi_i}\right)}, \\ s_{h\left(\prod_{i=1}^{k+1}\left(\frac{f_i}{h}\right)^{\phi_i}\right)} \end{bmatrix} \right).$$

Therefore, by mathematical induction, Equation (11) holds.

(2) For a set of LIVSFNs $\alpha_i = ([s_{a_i}, s_{b_i}], [s_{c_i}, s_{d_i}], [s_{e_i}, s_{f_i}])$ $(i = 1, \ldots, n)$, it follows from Definition 2.2 that $d_i^2 + f_i^2 \le h^2 - b_i^2$ $(i = 1, 2, \ldots, n)$. Let $\phi_i = \frac{\omega_i(1+T(\alpha_i))}{\sum_{j=1}^{n}\omega_j(1+T(\alpha_j))}$, and then we have

$$\begin{aligned} & h^2 - h^2\prod_{i=1}^{n}\left(1-\frac{b_i^2}{h^2}\right)^{\phi_i} + h\prod_{i=1}^{n}\left(\frac{d_i}{h}\right)^{\phi_i} + h\prod_{i=1}^{n}\left(\frac{f_i}{h}\right)^{\phi_i} \\ =& h^2\left(1-\prod_{i=1}^{n}\left(\frac{h^2-b_i^2}{h^2}\right)^{\phi_i}\right) + h^2\prod_{i=1}^{n}\left(\frac{d_i^2}{h^2}\right)^{\phi_i} + h^2\prod_{i=1}^{n}\left(\frac{f_i^2}{h^2}\right)^{\phi_i} \\ \le& h^2\left(1-\prod_{i=1}^{n}\left(\frac{d_i^2+f_i^2}{h^2}\right)^{\phi_i}\right) + h^2\prod_{i=1}^{n}\left(\frac{d_i^2}{h^2}\right)^{\phi_i} + h^2\prod_{i=1}^{n}\left(\frac{f_i^2}{h^2}\right)^{\phi_i} = h^2. \end{aligned}$$

Based on the foregoing analysis, it follows that $LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n)$ remains an LIVSFN.

If the weight vector $\boldsymbol{\omega} = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^\top$, let $\theta = \frac{1+T(\alpha_i)}{\sum_{j=1}^{n}(1+T(\alpha_j))}$, the explicit formulation of the *LIVSFPA* operator is readily obtained as

$$\begin{aligned} & LIVSFPA(\alpha_1, \alpha_2, \ldots, \alpha_n) \\ =& \left(\begin{bmatrix} s_{\sqrt{h^2-h^2\prod_{i=1}^{n}\left(1-\frac{a_i^2}{h^2}\right)^{\theta}}}, \\ s_{\sqrt{h^2-h^2\prod_{i=1}^{n}\left(1-\frac{b_i^2}{h^2}\right)^{\theta}}} \end{bmatrix}, \begin{bmatrix} s_{h\prod_{i=1}^{n}\left(\frac{c_i}{h}\right)^{\theta}}, \\ s_{h\prod_{i=1}^{n}\left(\frac{d_i}{h}\right)^{\theta}} \end{bmatrix}, \begin{bmatrix} s_{h\prod_{i=1}^{n}\left(\frac{e_i}{h}\right)^{\theta}}, \\ s_{h\prod_{i=1}^{n}\left(\frac{f_i}{h}\right)^{\theta}} \end{bmatrix} \right). \end{aligned}$$

Theorem 3.2. *Let $\alpha_i = ([s_{a_i}, s_{b_i}], [s_{c_i}, s_{d_i}], [s_{e_i}, s_{f_i}])$ $(i = 1, 2, \ldots, n)$ be a collection of LIVSFNs, and let $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_n)^\top$ denote their weight vector satisfying $\omega_i > 0$ and $\sum_{i=1}^{n}\omega_i = 1$ $(i = 1, 2, \ldots, n)$. Then the aggregated result satisfies the following properties:*

1. *Idempotency: If $\alpha_i = \alpha = ([s_a, s_b], [s_c, s_d], [s_e, s_f])$ $(i = 1, 2, \ldots, n)$, then*
$$LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha = ([s_a, s_b], [s_c, s_d], [s_e, s_f]);$$
2. *Monotonicity: Let $\beta_i = ([s_{g_i}, s_{j_i}], [s_{k_i}, s_{l_i}], [s_{s_i}, s_{t_i}])$ $(i = 1, 2, \ldots, n)$ be another collection of LIVSFNs satisfying $a_i \le g_i$, $b_i \le j_i$, $c_i \le k_i$, $d_i \le l_i$, $e_i \ge s_i$, $f_i \ge t_i$ and $T(\alpha_i) = T(\beta_i)$, and then $LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n) \le LIVSFPWA(\beta_1, \beta_2, \ldots, \beta_n)$;*
3. *Boundness: Define the lower and upper bounds:*
$$a^- = \left([s_{\min\{a_i\}}, s_{\min\{b_i\}}], [s_{\min\{c_i\}}, s_{\min\{d_i\}}], [s_{\min\{e_i\}}, s_{\min\{f_i\}}]\right),$$
$$a^+ = \left([s_{\max\{a_i\}}, s_{\max\{b_i\}}], [s_{\max\{c_i\}}, s_{\max\{d_i\}}], [s_{\max\{e_i\}}, s_{\max\{f_i\}}]\right).$$

Then the aggregated result satisfies $\alpha^- \le LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n) \le \alpha^+$.

Proof: (1) Since $\alpha_i = \alpha = ([s_a, s_b], [s_c, s_d], [s_e, s_f])$ $(i = 1, 2, \ldots, n)$, it follows that

$$\sum_{i=1}^{n} \frac{\omega_i(1+T(\alpha_i))}{\sum_{j=1}^{n}\omega_j(1+T(\alpha_j))} = 1.$$

By Theorem 3.1, we obtain

$$\begin{aligned}
&LIVSFPWA\left(\alpha_1, \alpha_2, \ldots, \alpha_n\right)\\
&=\left(\begin{bmatrix} s_{\sqrt{h^2-h^2\prod_{i=1}^{n}\left(1-\frac{a_i^2}{h^2}\right)^{\phi_i}}}, \\ s_{\sqrt{h^2-h^2\prod_{i=1}^{n}\left(1-\frac{b_i^2}{h^2}\right)^{\phi_i}}} \end{bmatrix}, \begin{bmatrix} s_{h\prod_{i=1}^{n}\left(\frac{c_i}{h}\right)^{\phi_i}}, \\ s_{h\prod_{i=1}^{n}\left(\frac{d_i}{h}\right)^{\phi_i}} \end{bmatrix}, \begin{bmatrix} s_{h\prod_{i=1}^{n}\left(\frac{e_i}{h}\right)^{\phi_i}}, \\ s_{h\prod_{i=1}^{n}\left(\frac{f_i}{h}\right)^{\phi_i}} \end{bmatrix}\right)\\
&=\left(\begin{bmatrix} s_{\sqrt{h^2-h^2\left(1-\frac{a^2}{h^2}\right)^{\sum_{i=1}^{n}\phi_i}}}, \\ s_{\sqrt{h^2-h^2\left(1-\frac{b^2}{h^2}\right)^{\sum_{i=1}^{n}\phi_i}}} \end{bmatrix}, \begin{bmatrix} s_{h\left(\frac{c}{h}\right)^{\sum_{i=1}^{n}\phi_i}}, \\ s_{h\left(\frac{d}{h}\right)^{\sum_{i=1}^{n}\phi_i}} \end{bmatrix}, \begin{bmatrix} s_{h\left(\frac{e}{h}\right)^{\sum_{i=1}^{n}\phi_i}}, \\ s_{h\left(\frac{f}{h}\right)^{\sum_{i=1}^{n}\phi_i}} \end{bmatrix}\right)\\
&=\alpha = \left(\left[s_a, s_b\right], \left[s_c, s_d\right], \left[s_e, s_f\right]\right);
\end{aligned}$$

(2) Since $T(\alpha_i) = T(\beta_i)$ $(i = 1, 2, \ldots, n)$, we have

$$\frac{\omega_i(1+T(\alpha_i))}{\sum_{j=1}^{n}\omega_j(1+T(\alpha_j))} = \frac{\omega_i(1+T(\beta_i))}{\sum_{j=1}^{n}\omega_j(1+T(\beta_j))}.$$

Let $\phi_i = \frac{\omega_i(1+T(\alpha_i))}{\sum_{j=1}^{n}\omega_j(1+T(\alpha_j))}$ $(i = 1, 2, \ldots, n)$, and then it follows that

$$\begin{aligned}
&LIVSFPWA\left(\alpha_1, \alpha_2, \ldots, \alpha_n\right)\\
&=\left(\begin{gathered}\left[s_{\sqrt{h^2-h^2\prod_{i=1}^{n}\left(1-\frac{a_i^2}{h^2}\right)^{\phi_i}}}, s_{\sqrt{h^2-h^2\prod_{i=1}^{n}\left(1-\frac{b_i^2}{h^2}\right)^{\phi_i}}}\right], \left[s_{h\prod_{i=1}^{n}\left(\frac{c_i}{h}\right)^{\phi_i}}, s_{h\prod_{i=1}^{n}\left(\frac{d_i}{h}\right)^{\phi_i}}\right], \\ \left[s_{h\prod_{i=1}^{n}\left(\frac{e_i}{h}\right)^{\phi_i}}, s_{h\prod_{i=1}^{n}\left(\frac{f_i}{h}\right)^{\phi_i}}\right]\end{gathered}\right);\\
&LIVSFPWA\left(\beta_1, \beta_2, \ldots, \beta_n\right)\\
&=\left(\begin{gathered}\left[s_{\sqrt{h^2-h^2\prod_{i=1}^{n}\left(1-\frac{g_i^2}{h^2}\right)^{\phi_i}}}, s_{\sqrt{h^2-h^2\prod_{i=1}^{n}\left(1-\frac{j_i^2}{h^2}\right)^{\phi_i}}}\right], \left[s_{h\prod_{i=1}^{n}\left(\frac{k_i}{h}\right)^{\phi_i}}, s_{h\prod_{i=1}^{n}\left(\frac{l_i}{h}\right)^{\phi_i}}\right], \\ \left[s_{h\prod_{i=1}^{n}\left(\frac{s_i}{h}\right)^{\phi_i}}, s_{h\prod_{i=1}^{n}\left(\frac{t_i}{h}\right)^{\phi_i}}\right]\end{gathered}\right).
\end{aligned}$$

According to $a_i \le g_i$, $b_i \le j_i$, $c_i \le k_i$, $d_i \le l_i$, $e_i \ge s_i$, $f_i \ge t_i$, we can get

$$h^2 - h^2\prod_{i=1}^{n}\left(1-\frac{a_i^2}{h^2}\right)^{\phi_i} \le h^2 - h^2\prod_{i=1}^{n}\left(1-\frac{g_i^2}{h^2}\right)^{\phi_i};$$

$$h^2 - h^2\prod_{i=1}^{n}\left(1-\frac{b_i^2}{h^2}\right)^{\phi_i} \le h^2 - h^2\prod_{i=1}^{n}\left(1-\frac{j_i^2}{h^2}\right)^{\phi_i};$$

$$h\prod_{i=1}^{n}\left(\frac{c_i}{h}\right)^{\phi_i} \le h\prod_{i=1}^{n}\left(\frac{k_i}{h}\right)^{\phi_i}; \quad h\prod_{i=1}^{n}\left(\frac{d_i}{h}\right)^{\phi_i} \le h\prod_{i=1}^{n}\left(\frac{l_i}{h}\right)^{\phi_i};$$

$$h\prod_{i=1}^{n}\left(\frac{e_i}{h}\right)^{\phi_i} \ge h\prod_{i=1}^{n}\left(\frac{s_i}{h}\right)^{\phi_i}; \quad h\prod_{i=1}^{n}\left(\frac{f_i}{h}\right)^{\phi_i} \ge h\prod_{i=1}^{n}\left(\frac{t_i}{h}\right)^{\phi_i}.$$

1. If $a_i = g_i$, $b_i = j_i$, $c_i = k_i$, $d_i = l_i$, $e_i = s_i$, and $f_i = t_i$ $(i = 1, 2, \ldots, n)$, then $\alpha_i = \beta_i = ([s_{a_i}, s_{b_i}], [s_{c_i}, s_{d_i}], [s_{e_i}, s_{f_i}])$, implying
$$LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = LIVSFPWA(\beta_1, \beta_2, \ldots, \beta_n).$$
2. If there exists $i_* \in \{1, 2, \ldots, n\}$ that does not satisfy $a_{i_*} = g_{i_*}$, $b_{i_*} = j_{i_*}$, $c_{i_*} = k_{i_*}$, $d_{i_*} = l_{i_*}$, $e_{i_*} = s_{i_*}$, $f_{i_*} = t_{i_*}$, assuming
$$h^2 - h^2 \prod_{i=1}^{n} \left(1 - \frac{a_{i*}^2}{h^2}\right)^{\phi_i} \leq h^2 - h^2 \prod_{i=1}^{n} \left(1 - \frac{g_{i*}^2}{h^2}\right)^{\phi_i},$$
the score function can be obtained as follows:
$$S(LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n)) \leq S(LIVSFPWA(\beta_1, \beta_2, \ldots, \beta_n)),$$
then
$$LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n) < LIVSFPWA(\beta_1, \beta_2, \ldots, \beta_n).$$
In summary
$$LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq LIVSFPWA(\beta_1, \beta_2, \ldots, \beta_n).$$

(3) This property is a direct consequence of the idempotency property and the definition of the lower and upper bounds α^- and α^+.

Since $\alpha^- \leq \alpha_i \leq \alpha^+$ holds for all $i = 1, 2, \ldots, n$, and the *LIVSFPWA* operator is idempotent, it follows that
$$\alpha^- = LIVSFPWA(\alpha^-, \alpha^-, \ldots, \alpha^-) \leq LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n)$$
and
$$LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq LIVSFPWA(\alpha^+, \alpha^+, \ldots, \alpha^+) = \alpha^+.$$
Combining these two inequalities yields the desired result:
$$\alpha^- \leq LIVSFPWA(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \alpha^+.$$

3.2. **Similarity-based objective attribute weight determination method.** In practical applications, particularly when dealing with MAGDM problems, attribute weights are frequently unspecified. Within the framework of linguistic interval-valued spherical fuzzy sets, this section proposes a novel computational approach for determining unknown attribute weights through inter-attribute similarity comparison.

For MAGDM problems in linguistic interval-valued spherical fuzzy settings, let $P = \{P^1, P^2, \ldots, P^p\}$ be the set of experts, with the expert weight vector $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_p)^\top$, where $\sum_{k=1}^{p} \omega_k = 1$ and $0 \leq \omega_k \leq 1$ for $k = 1, \ldots, p$, $X = \{x_1, x_2, \ldots, x_n\}$ be the set of n objects, and $T = \{t_1, t_2, \ldots, t_m\}$ be the set of m attributes. The attribute weight vector is denoted by $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$, where λ_j $(j = 1, \ldots, m)$ represents the weight of attribute t_j $(j = 1, \ldots, m)$, with $\sum_{j=1}^{m} \lambda_j = 1$ and $0 \leq \lambda_j \leq 1$. The evaluation value given by expert P^k for the i-th object with respect to the j-th attribute is represented as
$$x_{ij}^k = \left(\left[s_{a_{ij}^k}, s_{b_{ij}^k}\right], \left[s_{c_{ij}^k}, s_{d_{ij}^k}\right], \left[s_{e_{ij}^k}, s_{f_{ij}^k}\right]\right),$$
where $i = 1, \ldots, n$; $j = 1, \ldots, m$; $k = 1, \ldots, p$. All evaluation values $P^k = \left(x_{ij}^k\right)_{n \times m}$ form an $n \times m$ multi-attribute decision matrix.

Definition 3.2. *Let $P^k = (x_{ij}^k)_{n \times m}$ be the p multi-attribute decision matrices provided by p experts, where each $x_{ij}^k = \left(\left[s_{a_{ij}^k}, s_{b_{ij}^k}\right], \left[s_{c_{ij}^k}, s_{d_{ij}^k}\right], \left[s_{e_{ij}^k}, s_{f_{ij}^k}\right]\right)$ $(i = 1, \ldots, n$; $j = 1, \ldots, m$; $k = 1, \ldots, p)$ represents the linguistic interval-valued spherical fuzzy evaluation given by* *expert P^k for the i-th object with respect to the j-th attribute. The average similarity degree of evaluations given by expert P^k for all objects under attribute t_j is defined as*

$$\overline{S}\left(t_j^k\right) = \frac{1}{n(n-1)} \sum_{i}^{n} \sum_{\substack{o=1 \\ o \neq i}}^{n} Sup\left(x_{ij}^k, x_{oj}^k\right) = \frac{1}{n(n-1)} \sum_{i}^{n} \sum_{\substack{o=1 \\ o \neq i}}^{n} \left(1 - d\left(x_{ij}^k, x_{oj}^k\right)\right). \tag{12}$$

Larger distance between LIVSFNs indicates greater dissimilarity and lower similarity, implying the attribute is more significant; Smaller distance suggests higher similarity and lower importance of the attribute.

Given the weight vector of experts $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_p)^\top$, where $\sum_{k=1}^{p} \omega_k = 1$ and $0 \leq \omega_k \leq 1$ for $k = 1, \ldots, p$, the total similarity degree for each attribute t_j $(j = 1, 2, \ldots, m)$ is defined as

$$\overline{S}(t_j) = \sum_{k=1}^{p} \omega_k \overline{S}\left(t_j^k\right). \tag{13}$$

The final weight λ_j for each attribute is calculated by

$$\lambda_j = \frac{1 - \overline{S}(t_j)}{\sum_{j=1}^{m} \left(1 - \overline{S}(t_j)\right)}. \tag{14}$$

3.3. **The proposed MAGDM framework based on *LIVSFPWA* operator.** Building upon the theoretical foundation of the *LIVSFPWA* operator, we now propose a decision-making method with the following procedural steps:

Step 1: Calculate the support degree between LIVSFNs x_{ij}^l and x_{ij}^k:

$$Sup\left(x_{ij}^k, x_{ij}^l\right) = 1 - d\left(x_{ij}^k, x_{ij}^l\right); \tag{15}$$

Step 2: Compute the total support T^k for each LIVSFN x_{ij}^k using expert weights:

$$T^k = T\left(x_{ij}^k\right) = \sum_{\substack{l=1 \\ l \neq k}}^{p} \omega_l Sup\left(x_{ij}^k, x_{ij}^l\right); \tag{16}$$

Step 3: Calculate attribute weights according to Definition 3.2. If weights are predetermined, proceed to Step 4;

Step 4: With expert weight vector $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_p)^\top$, where $\sum_{k=1}^{p} \omega_k = 1$ and $0 \leq \omega_k \leq 1$, let $\phi_k = \frac{\omega_k\left(1+T\left(x_{ij}^k\right)\right)}{\sum_{k=1}^{p} \omega_k\left(1+T\left(x_{ij}^k\right)\right)}$, aggregate all decision matrices $P^k = \left(x_{ij}^k\right)_{n \times m}$ using the *LIVSFPWA* operator to obtain the comprehensive matrix $\overline{P} = (x_{ij})_{n \times m}$:

$$\begin{aligned} x_{ij} &= LIVSFPWA\left(x_{ij}^1, x_{ij}^2, \ldots, x_{ij}^k\right) \\ &= \left(\begin{bmatrix} s_{\sqrt{h^2 - h^2 \prod_{k=1}^{p}\left(1 - \frac{{a_{ij}^k}^2}{h^2}\right)^{\phi_k}}}, \\ s_{\sqrt{h^2 - h^2 \prod_{k=1}^{p}\left(1 - \frac{{b_{ij}^k}^2}{h^2}\right)^{\phi_k}}} \end{bmatrix}, \begin{bmatrix} s_{h \prod_{k=1}^{p}\left(\frac{c_{ij}^k}{h}\right)^{\phi_k}}, \\ s_{h \prod_{k=1}^{p}\left(\frac{d_{ij}^k}{h}\right)^{\phi_k}} \end{bmatrix}, \begin{bmatrix} s_{h \prod_{k=1}^{p}\left(\frac{e_{ij}^k}{h}\right)^{\phi_k}}, \\ s_{h \prod_{k=1}^{p}\left(\frac{f_{ij}^k}{h}\right)^{\phi_k}} \end{bmatrix} \right); \end{aligned} \tag{17}$$

Step 5: Aggregate the comprehensive attribute values $z_i = ([s_{a_i}, s_{b_i}], [s_{c_i}, s_{d_i}], [s_{e_i}, s_{f_i}])$ for each alternative x_i $(i = 1, 2, \ldots, n)$ using the *LIVSFPWA* operator:

$$z_i = LIVSFPWA(z_{i1}, z_{i2}, ..., z_{im})$$

$$= \left(\begin{bmatrix} s_{\sqrt{h^2 - h^2 \prod_{i=1}^{n} \left(1 - \frac{a_{ij}^2}{h^2}\right)^{\lambda_j}}}, \\ s_{\sqrt{h^2 - h^2 \prod_{i=1}^{n} \left(1 - \frac{b_{ij}^2}{h^2}\right)^{\lambda_j}}} \end{bmatrix}, \begin{bmatrix} s_{h \prod_{i=1}^{n} \left(\frac{c_{ij}}{h}\right)^{\lambda_j}}, \\ s_{h \prod_{i=1}^{n} \left(\frac{d_{ij}}{h}\right)^{\lambda_j}} \end{bmatrix}, \begin{bmatrix} s_{h \prod_{i=1}^{n} \left(\frac{e_{ij}}{h}\right)^{\lambda_j}}, \\ s_{h \prod_{i=1}^{n} \left(\frac{f_{ij}}{h}\right)^{\lambda_j}} \end{bmatrix} \right); \tag{18}$$

Step 6: Compute the score functions $S(z_i)$ for all alternatives based on Definition 2.4, and then determine the final ranking of alternatives according to Theorem 2.1.

Step 7: Select the optimal alternative based on the ranking of comprehensive evaluation values.

4. **Illustrative Example.** To demonstrate the practicality and effectiveness of the proposed MAGDM framework based on the *LIVSFPWA* operator, an illustrative example on faculty evaluation is conducted in this section. The properties of the *LIVSFPWA* operator play a critical role in application. The boundedness property ensures that all intermediate aggregated values in the comprehensive matrix $\overline{P}$ and the final alternative values z_i are valid LIVSFNs, fairly representing the spectrum of expert opinions without producing spurious results. The idempotency and monotonicity properties work implicitly throughout the aggregation steps, ensuring the stability and logical consistency of the aggregation process.

Let $X = \{x_1, x_2, x_3, x_4\}$ be the four teachers under evaluation, and $P = \{P^1, P^2, P^3\}$ be the three experts with weight vector $\boldsymbol{\omega}$. We consider weight vector as $\boldsymbol{\omega} = (0.350, 0.450, 0.200)^\top$ for decision makers based on normal distribution method. The experts evaluate each of these teachers based on four attributes: teaching quality (t_1), instructional materials (t_2), research capability (t_3), communication clarity (t_4). The attribute weight vector is $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)$. Using the linguistic term set $\overline{S} = \{s_z | 0 \le z \le 8\}$, each expert provides their evaluation matrices in linguistic interval-valued spherical fuzzy format: $P^k = \left(x_{ij}^k\right)_{4\times4} = \left(\left[s_{a_{ij}^k}, s_{b_{ij}^k}\right], \left[s_{c_{ij}^k}, s_{d_{ij}^k}\right], \left[s_{e_{ij}^k}, s_{f_{ij}^k}\right]\right)_{4\times4}$ $(k = 1, 2, 3)$:

$$P^1 = \begin{pmatrix} ([s_3, s_5], [s_0, s_3], [s_2, s_3]) & ([s_5, s_6], [s_0, s_2], [s_1, s_2]) & ([s_5, s_6], [s_0, s_2], [s_1, s_2]) & ([s_4, s_5], [s_1, s_3], [s_1, s_2]) \\ ([s_3, s_5], [s_0, s_3], [s_2, s_3]) & ([s_4, s_5], [s_0, s_2], [s_2, s_3]) & ([s_2, s_4], [s_0, s_3], [s_3, s_4]) & ([s_3, s_5], [s_1, s_4], [s_1, s_2]) \\ ([s_5, s_6], [s_0, s_2], [s_1, s_2]) & ([s_3, s_4], [s_1, s_3], [s_2, s_3]) & ([s_3, s_4], [s_1, s_3], [s_2, s_3]) & ([s_4, s_5], [s_1, s_3], [s_1, s_2]) \\ ([s_4, s_5], [s_0, s_2], [s_2, s_3]) & ([s_5, s_6], [s_0, s_2], [s_1, s_2]) & ([s_3, s_5], [s_0, s_4], [s_1, s_3]) & ([s_3, s_6], [s_0, s_4], [s_1, s_2]) \end{pmatrix};$$

$$P^2 = \begin{pmatrix} ([s_4, s_5], [s_0, s_2], [s_2, s_3]) & ([s_4, s_5], [s_1, s_3], [s_1, s_2]) & ([s_2, s_3], [s_1, s_3], [s_3, s_4]) & ([s_3, s_3], [s_0, s_2], [s_3, s_5]) \\ ([s_3, s_5], [s_0, s_4], [s_1, s_3]) & ([s_1, s_2], [s_2, s_6], [s_1, s_4]) & ([s_3, s_4], [s_1, s_3], [s_2, s_3]) & ([s_3, s_6], [s_0, s_4], [s_1, s_2]) \\ ([s_3, s_4], [s_0, s_3], [s_2, s_4]) & ([s_3, s_6], [s_0, s_4], [s_1, s_2]) & ([s_2, s_5], [s_0, s_4], [s_2, s_3]) & ([s_5, s_7], [s_0, s_2], [s_1, s_1]) \\ ([s_4, s_5], [s_1, s_3], [s_1, s_2]) & ([s_1, s_2], [s_2, s_6], [s_1, s_4]) & ([s_3, s_3], [s_2, s_3], [s_2, s_3]) & ([s_4, s_5], [s_1, s_3], [s_1, s_2]) \end{pmatrix};$$

$$P^3 = \begin{pmatrix} ([s_2, s_4], [s_2, s_5], [s_1, s_2]) & ([s_3, s_4], [s_2, s_4], [s_1, s_2]) & ([s_5, s_6], [s_1, s_2], [s_1, s_1]) & ([s_5, s_6], [s_0, s_2], [s_1, s_2]) \\ ([s_4, s_5], [s_1, s_3], [s_1, s_2]) & ([s_4, s_5], [s_1, s_3], [s_1, s_2]) & ([s_2, s_4], [s_1, s_5], [s_1, s_3]) & ([s_5, s_6], [s_0, s_2], [s_1, s_2]) \\ ([s_3, s_5], [s_1, s_4], [s_1, s_2]) & ([s_2, s_3], [s_1, s_5], [s_1, s_4]) & ([s_3, s_4], [s_0, s_4], [s_1, s_3]) & ([s_3, s_5], [s_0, s_3], [s_2, s_3]) \\ ([s_5, s_7], [s_0, s_2], [s_1, s_1]) & ([s_2, s_4], [s_1, s_5], [s_1, s_3]) & ([s_3, s_4], [s_0, s_3], [s_2, s_4]) & ([s_4, s_6], [s_0, s_3], [s_1, s_2]) \end{pmatrix}.$$

Step 1: Calculate the support degree matrices $Sup\left(x_{ij}^k, x_{ij}^l\right)_{4\times4}$, where $k, l = 1, 2, 3$ and $i, j = 1, 2, 3, 4$:

$$Sup_{12(21)} = \begin{pmatrix} 0.9063 & 0.8438 & 0.6250 & 0.7734 \\ 0.9219 & 0.5547 & 0.8984 & 0.9063 \\ 0.6797 & 0.7578 & 0.8281 & 0.7422 \\ 0.9375 & 0.4688 & 0.7656 & 0.7969 \end{pmatrix},$$

$$Sup_{23(32)} = \begin{pmatrix} 0.7188 & 0.8750 & 0.6250 & 0.6641 \\ 0.8438 & 0.5781 & 0.8125 & 0.7813 \\ 0.8516 & 0.7500 & 0.9609 & 0.6875 \\ 0.7422 & 0.8359 & 0.8594 & 0.9063 \end{pmatrix},$$

$$Sup_{13(31)} = \begin{pmatrix} 0.8125 & 0.7188 & 0.9688 & 0.8438 \\ 0.8984 & 0.9375 & 0.8125 & 0.7813 \\ 0.7813 & 0.7578 & 0.8438 & 0.8984 \\ 0.7422 & 0.6328 & 0.8828 & 0.8906 \end{pmatrix}.$$

Step 2: Compute the total support degree matrix $T^k = \left(T\left(x_{ij}^k\right)\right)_{4\times 4}$ for each expert, where $k = 1, 2, 3$ and $i, j = 1, 2, 3, 4$:

$$T^1 = \begin{pmatrix} 0.5703 & 0.5235 & 0.4750 & 0.5168 \\ 0.5945 & 0.4371 & 0.5668 & 0.5641 \\ 0.4621 & 0.4926 & 0.5414 & 0.5137 \\ 0.5703 & 0.3375 & 0.5211 & 0.5367 \end{pmatrix},$$

$$T^2 = \begin{pmatrix} 0.4610 & 0.4703 & 0.3438 & 0.4035 \\ 0.4914 & 0.3098 & 0.4769 & 0.4735 \\ 0.4082 & 0.4152 & 0.4820 & 0.3973 \\ 0.4766 & 0.3313 & 0.4398 & 0.4602 \end{pmatrix},$$

$$T^3 = \begin{pmatrix} 0.6078 & 0.6453 & 0.6203 & 0.5942 \\ 0.6942 & 0.5883 & 0.6500 & 0.6250 \\ 0.6567 & 0.6027 & 0.7277 & 0.6238 \\ 0.5938 & 0.5976 & 0.6957 & 0.7195 \end{pmatrix}.$$

Step 3: According to Definition 3.2, we calculate the average similarity degrees $\overline{S}\left(t_j^k\right)$ for each expert's evaluations across all attributes. The results are presented in Table 1, indicating how consistently each expert evaluated the different attributes.

TABLE 1. Average similarity degrees of expert evaluations

	t_1	t_2	t_3	t_4
P^1	0.9141	0.8841	0.8477	0.9401
P^2	0.9076	0.7825	0.9066	0.7878
P^3	0.8307	0.8724	0.8307	0.9128

The total similarity degree for each attribute t_j is computed by aggregating the experts' evaluations with their respective weights: $\overline{S}(t_1) = 0.8945$, $\overline{S}(t_2) = 0.8360$, $\overline{S}(t_3) = 0.8708$, $\overline{S}(t_4) = 0.8661$.

The final attribute weight vector is obtained as $W = (0.2580, 0.2411, 0.2511, 0.2498)$.

Step 4: Aggregate all decision matrices $P^k = \left(x_{ij}^k\right)_{4\times 4}$ using the *LIVSFPWA* operator to obtain the comprehensive evaluation matrix $\overline{P} = (x_{ij})_{4\times 4}$ $(i, j = 1, 2, 3, 4)$:

$$\overline{P} = \begin{pmatrix} \begin{pmatrix} [s_{3.4627}, s_{4.9138}], \\ [s_{0.0000}, s_{2.6075}], \\ [s_{1.8604}, s_{2.8757}] \end{pmatrix} & \begin{pmatrix} [s_{4.2608}, s_{5.2741}], \\ [s_{0.0000}, s_{2.7530}], \\ [s_{1.0000}, s_{2.0000}] \end{pmatrix} & \begin{pmatrix} [s_{4.3721}, s_{5.3945}], \\ [s_{0.0000}, s_{2.2828}], \\ [s_{1.4310}, s_{2.1174}] \end{pmatrix} & \begin{pmatrix} [s_{3.9326}, s_{4.7220}], \\ [s_{0.0000}, s_{2.3596}], \\ [s_{1.5347}, s_{2.8589}] \end{pmatrix} \\ \begin{pmatrix} [s_{3.2283}, s_{5.0000}], \\ [s_{0.0000}, s_{3.4120}], \\ [s_{1.2837}, s_{2.7748}] \end{pmatrix} & \begin{pmatrix} [s_{3.4840}, s_{4.4626}], \\ [s_{0.0000}, s_{3.1199}], \\ [s_{1.3102}, s_{2.8531}] \end{pmatrix} & \begin{pmatrix} [s_{2.5315}, s_{4.0000}], \\ [s_{0.0000}, s_{3.2832}], \\ [s_{2.0414}, s_{3.3202}] \end{pmatrix} & \begin{pmatrix} [s_{3.4447}, s_{5.6855}], \\ [s_{0.0000}, s_{3.5909}], \\ [s_{1.0000}, s_{2.0000}] \end{pmatrix} \\ \begin{pmatrix} [s_{4.0713}, s_{5.0892}], \\ [s_{0.0000}, s_{2.6175}], \\ [s_{1.3420}, s_{2.6841}] \end{pmatrix} & \begin{pmatrix} [s_{2.9489}, s_{5.2215}], \\ [s_{0.0000}, s_{3.5763}], \\ [s_{1.3626}, s_{2.4652}] \end{pmatrix} & \begin{pmatrix} [s_{2.6319}, s_{4.3473}], \\ [s_{0.0000}, s_{3.3904}], \\ [s_{2.0000}, s_{3.0000}] \end{pmatrix} & \begin{pmatrix} [s_{4.4744}, s_{6.1581}], \\ [s_{0.0000}, s_{2.3064}], \\ [s_{1.0000}, s_{1.4936}] \end{pmatrix} \\ \begin{pmatrix} [s_{4.0000}, s_{5.0000}], \\ [s_{0.0000}, s_{2.6456}], \\ [s_{1.2398}, s_{2.2679}] \end{pmatrix} & \begin{pmatrix} [s_{3.6656}, s_{4.6830}], \\ [s_{0.0000}, s_{2.7167}], \\ [s_{1.6880}, s_{3.3760}] \end{pmatrix} & \begin{pmatrix} [s_{3.0000}, s_{5.0000}], \\ [s_{0.0000}, s_{4.0000}], \\ [s_{1.0000}, s_{3.0000}] \end{pmatrix} & \begin{pmatrix} [s_{3.0000}, s_{6.0000}], \\ [s_{0.0000}, s_{4.0000}], \\ [s_{1.0000}, s_{2.0000}] \end{pmatrix} \end{pmatrix}.$$

Step 5: Aggregate the comprehensive attribute values for each alternative x_i ($i = 1, 2, 3, 4$) using the *LIVSFPWA* operator with attribute weights:

$$z_1 = ([s_{4.0290}, s_{5.0896}], [s_{0.0000}, s_{2.4899}], [s_{1.4288}, s_{2.4363}]);$$
$$z_2 = ([s_{3.2128}, s_{4.8768}], [s_{0.0000}, s_{3.3446}], [s_{1.3544}, s_{2.6981}]);$$
$$z_3 = ([s_{3.6370}, s_{5.2753}], [s_{0.0000}, s_{2.9398}], [s_{1.3906}, s_{2.3715}]);$$
$$z_4 = ([s_{3.4622}, s_{5.2254}], [s_{0.0000}, s_{3.2713}], [s_{1.1995}, s_{2.6036}]).$$

Step 6: Calculate the score functions $S(z_i)$ ($i = 1, 2, 3, 4$) for each alternative according to Definition 2.4: $S(z_1) = 6.3366$, $S(z_2) = 6.1277$, $S(z_3) = 6.3115$, $S(z_4) = 6.2530$. Using Theorem 2.1, the ranking of alternatives is obtained: $z_1 \succ z_3 \succ z_4 \succ z_2$.

Step 7: Rank the alternatives based on comprehensive evaluation values: $x_1 \succ x_3 \succ x_4 \succ x_2$. Therefore, the optimal teacher selection is x_1.

In the process of evaluating four teachers, since spherical fuzzy sets serve as an extension of intuitionistic fuzzy sets, we selected the MAGDM of the LIVAIF Ordered Weighted Averaging (*LIVAIFOWA*) operator [26] based on linguistic interval-valued atanassov intuitionistic fuzzy sets for comparison. Additionally, we also selected MAGDM methods related to spherical fuzzy sets for comparison, including MAGDM based on the Linguistic Spherical Fuzzy Weighted Aggregation (*LSFWA*) operator [17] and MAGDM based on IVT-SF Aczel Alsina Power Weighted Average (*IVT-SFAAPWA*) operator [25]. Since both methods require specifying attribute weights, we used the attribute weights obtained in this paper: $(0.2580, 0.2411, 0.2511, 0.2498)^\top$. The expert weight vector, based on the normal distribution, is set as $\boldsymbol{\omega} = (0.350, 0.450, 0.200)^\top$. Additionally, since the method based on the *LIVAIFOWA* operator cannot handle hesitancy evaluations, we ignored such evaluations during processing. For the method based on the *LSFWA* operator, which cannot directly process interval values, we used averaging to handle interval inputs. The ranking results from different methods are shown in Table 2.

Table 2. Ranking of different methods

	$S(z_1)$	$S(z_2)$	$S(z_3)$	$S(z_4)$	Ranking
LIVAIFOWA operator	5.4431	5.0930	5.2256	5.3266	$x_1 \succ x_4 \succ x_3 \succ x_2$
LSFWA operator	5.0331	4.8316	4.9102	4.9071	$x_1 \succ x_3 \succ x_4 \succ x_2$
IVT-SFAAPWA operator	0.1887	0.1475	0.1730	0.1726	$x_1 \succ x_3 \succ x_4 \succ x_2$
LIVSFPWA operator	6.3366	6.1277	6.3115	6.2530	$x_1 \succ x_3 \succ x_4 \succ x_2$

Based on the ranking results of the above comparative experiments, it can be observed that

The ordering obtained using the *LIVAIFOWA* operator is $x_1 \succ x_4 \succ x_3 \succ x_2$, while the methods based on the *LSFWA* operator, the *IVT-SFAAPWA* operator, and the *LIVSFPWA* operator proposed in this paper all yield the ordering $x_1 \succ x_3 \succ x_4 \succ x_2$. Although the rankings of x_1 and x_2 are consistent across different methods, the relative order of the intermediate alternatives x_3 and x_4 varies.

This discrepancy primarily stems from the differences in the methods' capabilities to handle uncertain information: The *LIVAIFOWA* operator cannot handle hesitant evaluations, thus ignoring relevant hesitation information in the experiment; In contrast, the *LIVSFPWA* operator adopted in this study not only fully considers all endpoint information of linguistic interval-valued spherical fuzzy numbers but also incorporates the fuzziness of interval membership, non-membership, and hesitancy, enabling a more comprehensive and detailed characterization of uncertainty in the evaluation process.

Furthermore, based on the ranking results of the *LIVSFPWA* operator and the *LIVAI-FOWA* operator, it is evident that the fuzziness of interval hesitancy has a significant impact on the ordering outcome, indicating that this factor cannot be overlooked. Therefore, the ranking results obtained by the method proposed in this study are more reliable.

5. **Conclusions.** Information fusion is crucial for addressing uncertainty in decision-making environments. Firstly, this paper has proposed the *LIVSFPWA* operator based on linguistic interval-valued spherical fuzzy set theory to consolidate expert preferences. Given that assigning attribute weights presents a significant challenge in group decision-making, then we have introduced a method to determine unknown attribute weights by utilizing similarity measures among correlated information. Finally, we have proposed a solution approach for MAGDM problems based on the proposed operators. An illustrative example has demonstrated that this method has effectively overcome limitations of conventional interval-valued spherical fuzzy sets, improved the validity the authenticity of evaluation information in decision-making to some extent, and provided better solutions for practical problems.

In subsequent research, we will integrate linguistic interval-valued spherical fuzzy set theory with other aggregation operators and develop its innovative applications in domains such as artificial intelligence, image processing, and pattern recognition.

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Author Biography

Liwei Sha received the B.E. degree in Computer Science and Technology from Shandong Jianzhu University, China, in 2023. He is now pursuing his M.E. degree in Computer Technology at Shandong Jianzhu University, China.

Mr. Sha's current research interests include multiple criteria decision-making, fuzzy logic.

Huaizhe Zhao received the B.E. degree in Computer Science and Technology from Shandong Jianzhu University, China, in 2023. He is now pursuing his M.E. degree in Computer Technology at Shandong Jianzhu University, China.

Mr. Zhao's current research interests include multiple criteria decision-making, fuzzy logic.

Jianzhong Huang received the B.E. degree in Communication Command from Air Force Telecommunication Engineering College, China, in 2003. He is now employed at Shandong Medical Device and Pharmaceutical Packaging Inspection Institute.

Mr. Huang's current research interests include artificial intelligence applications in laboratory settings.

Tie Hou received the B.Sc. degree from Northeast Normal University, China, in 2001; the M.Sc. and Ph.D. degrees from Swansea University, UK, in 2006 and 2014.

Dr. Hou is currently a lecturer in the School of Computer and Artificial Intelligence, Shandong Jianzhu University, China. Her current research interests include logic, formal specification and verification, ontology reasoning and information visualization.

Li Zou received the B.Sc. degree in Basic Mathematics from Liaoning Normal University, China, in 1994; the M.Sc. degree in Basic Mathematics from Liaoning Normal University, China, in 1997; the Ph.D. degree in Traffic Information Engineering and Control from Southwest Jiaotong University, China, in 2010.

Prof. Zou is currently Head of the International Collaborative Joint Laboratory for Advanced Artificial Intelligence Technologies and Vice Director of the Computational Intelligence Center, Shandong Jianzhu University, China. Her research interests include intelligent decision-making systems, fundamental theory of artificial intelligence, and intelligent information processing. She has led four research projects funded by the National Natural Science Foundation of China and has published over 40 papers in renowned academic journals in these fields.