

## GENERAL CLASS OF WEIGHTED FUZZY CLUSTER LOADING MODELS

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**ABSTRACT.** *Fuzzy clustering is known to have the power to reveal the complex structure of real data. However, it suffers from having results that are difficult to interpret. A weighted fuzzy cluster loading model can solve this problem by finding the estimates of fuzzy cluster loadings which can show the relationship between the obtained fuzzy clusters and given variables. In this model, the spatial heteroscedastic structure of the data is captured by using the classification structure obtained by the fuzzy clustering. The structure is unknown and complicated, so it is necessary to offer a truly comprehensive and highly flexible model to identify the structure. Therefore, we define the general class of the weighted fuzzy cluster loading models so as to accommodate the variety of different structures of the data. Numerical examples show a high capability to capture the latent structure by extending an ordinal weighted fuzzy cluster loading model to the general class of the models.*

**Keywords:** Fuzzy cluster, Additive linear models, Aggregation operator, Weighted least squares method

**1. Introduction.** Conventional clustering means classifying the given observation into exclusive clusters. So, we can clearly discriminate whether an object belongs to a cluster or not. However, such a partition is not sufficient to represent many real situations. Therefore, fuzzy clustering methods [1-5] are offered to create clusters with uncertainty boundaries. This method allows one object to belong to some overlapping clusters with some grades. The grade is represented as the degree of belongingness of objects to clusters.

However, replaced by the representativeness of fuzzy clustering to real complex data, the interpretation of the obtained fuzzy clusters causes us some confusion, because objects which have a similar degree of belongingness over the obtained fuzzy clusters can together form one more cluster.

In order to solve this problem, we have proposed a linear fuzzy cluster loading model [5,6]. The methodology of this model is essentially the same as an ordinal linear regression model. That is, we assume a linear relationship between the obtained degree of belongingness of an object to a cluster and given variables of data. The purpose of this model is to estimate the regression coefficients as the fuzzy cluster loadings.

However, if there is a clear classification structure in the data, obviously the data has heteroscedastic structure associated with heteroscedastic residuals. Thus in the linear fuzzy cluster loading model, the estimate of the least squares of regression coefficients of the model do not satisfy conditions necessary to be a maximum likelihood estimator or the best linear unbiased estimator.

In conventional statistical methodology, a weighted regression model [7] is well known for solving the problem of heteroscedastic residuals. In this model, local spatial variance