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COMPACT SETS OF $L-R$ FUZZY NUMBERS

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Abstract. *In this paper we discuss the compactness of fuzzy metric spaces and the Schauder-Tychonoff fixed point theorem to a compact set which is introduced from the complete metric fuzzy space. We prove that a subset of $L-R$ fuzzy numbers becomes a compact set in the normed space under uniform boundedness and the uniform equicontinuity. Moreover we show that an into continuous mapping on a uniformly bounded and uniformly equicontinuous subset in the complete metric fuzzy space of with $L-R$ shape functions has at least one fixed point by applying the Schauder-Tychonoff fixed point theorem in main results.*
Keywords: Fuzzy number, $L-R$ fuzzy number, Compactness, Equivalence relation, Complete metric space, Schauder-Tychonoff fixed point theorem

1. **Introduction.** Fuzzy numbers are characterized by membership functions which have three properties: normality , compact convex support and upper semi-continuity. Membership functions are described by $\alpha-$cut sets, *i.e.,* level sets for $0 \leq \alpha \leq 1$, which are compact convex subsets in $\mathbf{R}^n$ under the above assumptions of membership functions hold. In [7] the author discussed an embedding theorem where metric spaces of compact convex sets are complete. There are so many results on various kinds of complete metric fuzzy spaces in [2, 5]. For more results on this topic, we refer readers to the references therein.

In analyzing qualitative properties of differential equations, the Schauder-Tychonoff fixed point theorem in normed spaces is very useful, because it guarantees the existence of solutions for integral equations corresponding to the differential equations etc. The compactness property is playing an important property in proving the existence and stability property of solutions for many types of equations (*e.g.*, [1]). Another type of the Schauder-Tychonoff fixed point is shown in [3]. It can be easily seen that Various sets of fuzzy numbers are complete metric spaces with suitable metrics, but it is not possible to discuss the qualitative properties of solutions in the complete metric spaces by applying the Schauder-Tychonoff fixed point theorem rather than the contraction principle and the comparison method (*e.g.,* [2, 4, 5, 8, 9]).

In this paper we introduce a parametric representation of fuzzy numbers, which are strictly fuzzy convex, then the fuzzy numbers can be identified by bounded closed curves in the two-dimensional metric space. We consider fuzzy numbers with $L-R$ shape functions. By using the parametric representation of $L-R$ fuzzy numbers we give better sufficient conditions for the compactness to subsets of $L-R$ fuzzy numbers rather than the representation of membership functions of fuzzy numbers. Moreover we show that the set of all the fuzzy numbers becomes a normed space and establish sufficient conditions