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MODELS OF FUZZY LOGIC IN A CATEGORY OF SETS WITH SIMILARITY RELATIONS

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Abstract. *Let Ω be a complete residuated lattice. By $\mathbf{SetF}(\Omega)$ we denote a category of sets with similarity relations (A, δ) with values in Ω. We investigate an interpretation of a first order predicate fuzzy logic in a model based on objects of this category $\mathbf{SetF}(\Omega)$. Some properties of interpretation of formulas are proved. The paper demonstrates that an interpretation of first order fuzzy logic in sets with similarities is a natural generalization of a fuzzy logic model theory based on classical fuzzy sets.*
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1. **Introduction.** Let us recall very shortly what does an interpretation (or model) of many sorted first order predicate fuzzy logic mean ([9]). Let J be a language of fuzzy logic which consists (classically) of a set $\mathcal{S}$ of sorts, a set of predicate symbols $P \in \mathcal{P}$, each of a sort $\iota_1 \times \cdots \times \iota_n$ for some $\iota_k \in \mathcal{S}$ and a set of functional symbols $f \in \mathcal{F}$, each of a sort $\iota_1 \times \cdots \times \iota_n \to \iota$. Moreover J contains also a set Ω of logical constants. Then a model of a language J is

$$\mathcal{D} = (\{A_\iota : \iota \in \mathcal{S}\}, \{P_\mathcal{D} : P \in \mathcal{P}\}, \{f_\mathcal{D} : f \in \mathcal{F}\}),$$

where

(a) A_ι is a set,
(b) $P_\mathcal{D} \underset{\sim}{\subset} A_{\iota_1} \times \cdots \times A_{\iota_n}$ is a fuzzy set with values in Ω,
(c) $f_\mathcal{D} : A_{\iota_1} \times \cdots \times A_{\iota_n} \to A_\iota$ is a map.

Let us also shortly recall how formulas and terms are interpreted in a model $\mathcal{D}$ of J. For a variable x let ι_x be a sort of x and let us set $\mathcal{D}(X) = \prod_{x \in X} A_{\iota_x}$. This set is then a domain for interpretation of formulas and terms. In fact, by well known procedures which is based on inductive principle we can obtain a simple proposition.

Proposition 1.1. *Let t be a term in J of a sort ι and let ψ be a formula in J with free variables in X. Then*

(a) An interpretation of t in $\mathcal{D}$ is a map $\|t\|_\mathcal{D} : \mathcal{D}(X) \to A_\iota$.
(b) An interpretation of ψ in $\mathcal{D}$ is a fuzzy set $\|\psi\|_\mathcal{D} \underset{\sim}{\subset} \mathcal{D}(X)$ with values in Ω, i.e. $\|\psi\|_\mathcal{D} : \mathcal{D}(X) \to \Omega$.

It is clear that the interpretation is based significantly on a set Ω. In case that $\Omega = (L, \wedge, \vee, \otimes, \to)$ is a complete residuated lattice (which is the most important case for fuzzy set theory) then this interpretation reflects natural relationships between logical and lattice connectives. For example, if $\psi \equiv \phi \Rightarrow \sigma$ then for any $\mathbf{a} \in \mathcal{D}(X)$ we have

$$\|\psi\|_\mathcal{D}(\mathbf{a}) = \|\phi\|_\mathcal{D}(\mathbf{a}) \to \|\sigma\|_\mathcal{D}(\mathbf{a})$$