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RENAMABLE INTERVAL BOOLEAN FUNCTIONS

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Abstract. *Interval functions constitute quite a special class of Boolean functions for which it is very easy and fast to determine their functional value on a specified input vector. The value of an n-variable interval function specified by interval $[a, b]$ (where a and b are n-bit binary numbers) is true if and only if the input vector viewed as an n-bit number belongs to the interval $[a, b]$. In this paper we study the problem of deciding whether a given DNF represents an interval function and if so then we also want to output the corresponding interval. For general Boolean functions this problem is co-NP-hard. In our article we present a polynomial time algorithm which works for some restricted classes of functions such as monotone functions. This result is then extended to a "renamable" variant of interval functions, i.e. to their variable complementation closure.*

Keywords: Boolean function, DNF representation, Recognition problem

1. **Introduction.** The class of interval functions was introduced in [7], where the following problem was presented: Given two n-bit numbers a, b, find a shortest DNF representing a Boolean function f on n variables, which is true exactly on numbers from the interval $[a, b]$. This problem originated from the field of automatic generation of test patterns for hardware verification, see e.g. [5, 4].

The representation of an interval Boolean function using only the two n-bit numbers a, b can be rather useful. In particular, it is very short and the value of function on a given vector can be effectively computed. Therefore, we consider in this paper the reverse problem which can be used to construct this compact representation: Given a DNF $\mathcal{F}$, can we recognize if it represents an interval function? This problem is co-NP hard in general, as we shall show in Section 2. Moreover, this problem remains co-NP hard even if we fix in advance the order of variables determining their significancy (when we view a Boolean vector as an n-bit binary number), and test whether the input DNF represents an interval function with respect to this fixed order. On the other hand, even the general problem (without a fixed order of variables) is polynomialy solvable when we restrict our attention only to those classes of DNFs, for which we are able to solve the falsifiability problem in polynomial time. The most trivial example is the class of monotone functions.

As we shall see in Section 5, the class of interval functions is not closed under variable complementation, hence we also introduce a complementation closure of interval functions and describe, how to recognize, whether a given DNF represents a function which belongs