

## DERIVATIVE STATE CONSTRAINED OPTIMAL $H_2$ CONTROL FOR UNSTABLE SYSTEMS

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**ABSTRACT.** *This paper presents the controller design method called the Derivative State Constrained optimal  $H_2$  control for an unstable system. The control scheme is obtained via the state-space approach with the quadratic derivatives of state variables in the standard performance index for the Linear Quadratic Gaussian as an extra weighting function. The proposed method can be reduced to the standard  $H_2$  optimal control problem by modifying the controlled output equations. The aim is to stabilize an unstable system and damp the closed-loop system dynamics. A two-wheeled balancing mobile robot, which is an inverted pendulum system, has been realized in continuous-time and discrete-time designs to verify the effectiveness of the proposed control scheme. Simulation and experimental results revealed that the swing of the pendulum's angle could be decreased extensively with excellent performance. In addition, the suitable weight selection for the control system is also suggested.*

**Keywords:** Inverted pendulum system (IPS), Derivative state constraints (DSC), Two-wheeled balancing mobile robot, Unstable systems

1. **Introduction.** It is known that the conventional controller design approaches for an unstable system require a good knowledge of system and accurate tuning in order to obtain good performance. Most research has shown the potential of designed control for this system by using modern control theory. The IPS is often used to evaluate various kinds of control theories since it is a complex multivariable, nonlinear and unstable system [1,7,10]. Recently, there have been some research focusing on IPS such as Two-Wheeled Balancing Robot or Two-Wheeled Mobile Robot and Two-Wheeled Inverted Pendulum Mobile Robot [5,6,11]. Their work aimed to design the controller for stabilizing the inverted pendulum that pivoted to the robot's pedestal while the movement occurred. A Two-Wheeled Balancing Mobile Robot has been built to use as a research and education tool at our university [8,9,12]. In [8], the teaching of analog controller design by LQR, LQE and LQG control structure for the unstable system was presented. However, the desired design [3,4] of the state feedback in the sense of modern control stabilizes the system and damps the closed-loop dynamics responses. Thus, in [9], an analog controller design by using the method in [4] was proposed and the controller designs using discrete state-space approaches for our IPS were reported in [12].

This paper describes the system hardware and controller design of a prototyped Two-Wheeled Balancing Mobile Robot as the IPS. The goal is to stabilize the inverted pendulum and controls the rate of changes with respect to time of the pendulum's angle output for smoother movement. Hence, the pendulum rod is approached to the vertical axis and its angle is deviated around the axis and also decreased at the same time. To