

## BUILDING THE THREE-GROUP CAUSAL PATH IN THE VAR MODEL

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**ABSTRACT.** *To exhibit the leading/lagging relationship among groups in a VAR process, we construct a three-group causal path (an extended Granger causality) as well as an identification procedure for the pathway which includes the independent, the intermediate and the dependent groups. In addition, we impose the unidirectional restriction on the pathway. Consequently, our method can organize more detailed and practical causal structure in a dynamic system than the conventional methods. The property concerning the impulse response function is derived when the three-group causality occurs in the VAR model. Finally, we show that these techniques can be easily implemented in the U.S. economic model consisting of the stock return, the inflation rate and the industrial production growth rate.*

**Keywords:** Vector autoregressive process (VAR), Granger causality, Three-group causal path, Impulse response analysis

**1. Introduction.** For many years, the following question has been discussed in the fields of both academy and business: What kind of causalities can be used to make meaningful predictions concerning the economic system? Answers to this question have been provided with various economic theories and statistical methods. Several studies dealt with this question, relating it to the measure of the causality via spectral decomposition in a VECM model [1], to the causal connectivity analysis in neural mechanisms [2] or to frequency-modified causality of the network structure in a non-linear system [3]. Recently, the identification of the causality has applications in bidirectional function learning method [4], the dynamic economic models [5], the knowledge acquisition [6] and many others.

However, Granger's [7] causality has become a fairly popular technique to measure dynamic relationship between groups of variables in the time series process. It has been widely used in economics [8,9] since the 1960s, and its applications in neuroscience [10-12] have been in favor in the last years.

The concept of Granger causality is based on two-group framework in which all variables are partitioned into two groups  $x$  and  $z$ . We say that  $x$  is Granger-causal for  $z$  if  $x$  is useful in forecasting  $z$ . In this setting, the information set for forecasting  $z$  contains two groups  $x$  and  $z$ . However, in order to extend the idea of the two-group framework, Lütkepohl [13] and Dufour and Renault [14] considered a higher dimensional system in which all variables are partitioned into three groups, say  $x$ ,  $z$  and  $w$ . In the three-group setting,